Matrice

December 13, 2021

1. Date su matrice $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ i $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Izračunati $3A - 2B + 5E$.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \neq E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$3A - 2B + 5E = 3\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ -6 & -8 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 2 \\ 0 & 0 \end{bmatrix} \qquad \neq \begin{bmatrix} 6 & 2 \end{bmatrix}$$

2. Ako je moguće izračunati:

$A \cdot B \neq B \cdot A$

- $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}_{2\times 3} \cdot \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 2\cdot 3+1\cdot 2+1\cdot 1 & 2\cdot 1+1\cdot 1+1\cdot 0 \\ 3\cdot 3+0\cdot 2+1\cdot 1 & 3\cdot 1+0\cdot 1+1\cdot 0 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 9 & 3 \\ 10 & 3 \end{bmatrix}_{2\times 2}$

$\neq$

- $\begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix}_{3\times 2} \cdot \begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 6+3 & 3+0 & 3+1 \\ 4+3 & 2+0 & 2+1 \\ 2+0 & 1+0 & 1+0 \end{bmatrix}_{3\times 3} = \begin{bmatrix} 9 & 3 & 4 \\ 7 & 2 & 3 \\ 2 & 1 & 1 \end{bmatrix}_{3\times 3}$
- $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 3+4+3 \\ 0+2+6 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 10 \\ 8 \end{bmatrix} = 2\begin{bmatrix} 5 \\ 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1} \cdot \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3} =$ NIJE MOGUĆE
- $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}_{3\times 1} \cdot [2]_{1\times 1} = \begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}_{3\times 1} = 2\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$
- $[2]_{1\times 1} \cdot \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}_{3\times 1} =$ NIJE MOGUĆE

3. Date su matrice $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & -3 \end{bmatrix}$ i $C = \begin{bmatrix} 2 & 3 \\ 2 & 6 \\ 5 & 15 \end{bmatrix}$. Izračunati $A^2 + BC - 3E$.

$$A^2 + BC - 3E = \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}} + \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 1 \\ 3 & 6 & -3 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 2 & 3 \\ 2 & 6 \\ 5 & 15 \end{bmatrix}} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} 1+2 & 2+6 \\ 1+3 & 2+9 \end{bmatrix} + \begin{bmatrix} 2+4+5 & 3+12+15 \\ 6+12-15 & 9+36-45 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 8 \\ 4 & 11 \end{bmatrix} + \begin{bmatrix} 11 & 30 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 38 \\ 7 & 8 \end{bmatrix}$$

4. Odrediti A^{-1}, ako postoji:

Kvadratna matrica je regularna (ima inverznu) akko $\det(A) \neq 0$.

4.1 $A = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix} = 5 - 6 = -1 \neq 0 \Rightarrow \exists A^{-1}$$

I $\quad \operatorname{adj}(A) = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \frac{1}{-1} \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}$$

II $\quad [A \mid E] = \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ -3 & 5 & 0 & 1 \end{array}\right] \cdot 3 \sim \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & -1 & 3 & -1 \end{array}\right] /(-1) \sim \left[\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -1 \end{array}\right] \cdot 2$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & -5 & -2 \\ 0 & 1 & -3 & -1 \end{array}\right] = [E \mid A^{-1}] \qquad \Rightarrow A^{-1} = \begin{bmatrix} -5 & -2 \\ -3 & -1 \end{bmatrix}$$

4.2 $A = \begin{array}{c} \begin{array}{ccc} a_{11} & a_{12} & \end{array} \\ \begin{bmatrix} -1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$

$$\det(A) = \left|\begin{matrix} -1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{matrix}\right| \begin{matrix} -1 & 2 \\ 1 & 1 \\ 0 & 1 \end{matrix} = -1 + 2 - 2 = -1 \neq 0 \Rightarrow \exists A^{-1}$$

I

$$\operatorname{adj}(A) = \begin{bmatrix} +\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & 1 \\ -2 & -1 & 1 \\ 4 & 2 & -3 \end{bmatrix}^T = \begin{bmatrix} -1 & -2 & 4 \\ -1 & -1 & 2 \\ 1 & 1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = -\begin{bmatrix} -1 & -2 & 4 \\ -1 & -1 & 2 \\ 1 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -4 \\ 1 & 1 & -2 \\ -1 & -1 & 3 \end{bmatrix}$$

$$a_{11} \longrightarrow A_{11} = (-1)^{1+1} M_{11} = +\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$a_{12} \longrightarrow A_{12} = (-1)^{1+2} M_{12} = -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$$

$$A_{ij} = (-1)^{i+j} M_{ij}$$

$+$ ako je $i+j$ parno
$-$ ako je $i+j$ neparno

II

$$[A|E] = \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] + \sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 3 & 2 & 1 & 1 & 0 \end{array}\right] -3 \sim \left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 1 & -3 \end{array}\right] \begin{array}{l} /(-1) \\ \\ /(-1) \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right] -1 \sim \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right] 2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & -4 \\ 0 & 1 & 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 & -1 & 3 \end{array}\right] = [E|A^{-1}] \Rightarrow A^{-1} = \begin{bmatrix} 1 & 2 & -4 \\ 1 & 1 & -2 \\ -1 & -1 & 3 \end{bmatrix}$$

4.3 $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & 0 \end{vmatrix} \begin{matrix} 1 & -2 \\ 0 & 0 \\ 1 & 3 \end{matrix} = -2 - 3 = -5 \neq 0 \Rightarrow \exists A^{-1}$$

$$\operatorname{adj}(A) = \begin{bmatrix} +\begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 1 & 3 \end{vmatrix} \\ -\begin{vmatrix} -2 & 0 \\ 3 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & -2 \\ 1 & 3 \end{vmatrix} \\ +\begin{vmatrix} -2 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} -3 & 1 & 0 \\ 0 & 0 & -5 \\ -2 & -1 & 0 \end{bmatrix}^T = \begin{bmatrix} -3 & 0 & -2 \\ 1 & 0 & -1 \\ 0 & -5 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = -\frac{1}{5} \begin{bmatrix} -3 & 0 & -2 \\ 1 & 0 & -1 \\ 0 & -5 & 0 \end{bmatrix}$$

5. Rešiti matrične jednačine:

5.1 $AX = B$ ako je $A = \begin{bmatrix} 1 & 2 \\ 6 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

$\boxed{AX=B}$ $\boxed{A^{-1} \text{ sa leve strane}}$

$\cancel{AXA^{-1}=BA^{-1}}$

$\cancel{AA^{-1}X=BA^{-1}}$

$\cancel{A^{-1}AX=BA^{-1}}$

$\underbrace{A^{-1}A}_{E}X=A^{-1}B$

$EX=A^{-1}B$

$\boxed{X=A^{-1}B}$

$\det(A)=\begin{vmatrix} 1 & 2 \\ 6 & 3 \end{vmatrix}=3-12=-9\neq 0$

$\rightarrow \exists A^{-1}$

$adj(A)=\begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}$

$A^{-1}=\frac{1}{\det(A)}adj(A)=-\frac{1}{9}\begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}$

$ax=b$, $a,b,x\in\mathbb{R}$

$x=\frac{b}{a}$

$x=b\cdot a^{-1}=a^{-1}b$

$AA^{-1}=A^{-1}A=E$

$XE=EX=X$

$$X=A^{-1}B=-\frac{1}{9}\underset{2\times 2}{\begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}}\underset{2\times 1}{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}=-\frac{1}{9}\begin{bmatrix} 6 \\ -12 \end{bmatrix}=-\frac{6}{9}\begin{bmatrix} 1 \\ -2 \end{bmatrix}=-\frac{2}{3}\begin{bmatrix} 1 \\ -2 \end{bmatrix}=$$

$$=\frac{2}{3}\begin{bmatrix} -1 \\ 2 \end{bmatrix}=\begin{bmatrix} -\frac{2}{3} \\ -\frac{4}{3} \end{bmatrix}$$

5.2 $AX - 2X = B$ ako je $A = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 5 \\ -8 & -3 \end{bmatrix}$.

$$AX - 2X = B \qquad \left[AX - 2\underbrace{EX}_{X} = B \right]$$

$$\underbrace{(A - 2E)}_{M} X = B$$

desno jer je s desne strane matrice A

$$MX = B \quad / M^{-1} \text{ s leve strane}$$

$$\underbrace{M^{-1}M}_{E} X = M^{-1}B$$

$$\boxed{X = M^{-1}B}$$

$$(A-2E)X = B \quad / \ (A-2E)^{-1} \text{ s leve}$$

$$\underbrace{(A-2E)^{-1}(A-2E)}_{E} X = (A-2E)^{-1}B$$

$$\boxed{X = (A-2E)^{-1}B}$$

$$M = A - 2E$$

$$= \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}$$

$$\det(M) = \begin{vmatrix} 1 & -5 \\ 1 & -3 \end{vmatrix} = -3 + 5 = 2 \neq 0 \Rightarrow \exists M^{-1}$$

$$\operatorname{adj}(M) = \begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$X = M^{-1}B = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -8 & -3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -52 & -30 \\ -12 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} -26 & -15 \\ -6 & -4 \end{bmatrix}$$

5.3 $X - 2XA = B$ ako je $A = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$ i $B = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$.

$$x - 2xa = b$$
$$x(1-2a) = b$$
$$x = \frac{b}{1-2a}$$

$$2XA = X \cdot 2 \cdot A$$
$$= X \cdot A \cdot 2$$

$$X - 2XA = B$$
$$X(\underbrace{E - 2A}_{M}) = B$$
$$XM = B \quad / M^{-1} \text{ so desne strane}$$
$$X\underbrace{MM^{-1}}_{E} = BM^{-1}$$
$$\boxed{X = BM^{-1}}$$

$$M = E - 2A$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -2 \\ -6 & 5 \end{bmatrix}$$

$$\det(M) = \begin{vmatrix} 5 & -2 \\ -6 & 5 \end{vmatrix} = 25 - 12 = 13 \neq 0 \Rightarrow \exists M^{-1}$$

$$\operatorname{adj}(M) = \begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix}$$

$$\Rightarrow M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{13}\begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix}$$

$$X = BM^{-1} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}}_{2\times 2} \cdot \frac{1}{13} \underbrace{\begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix}}_{2\times 2}$$
$$= \frac{1}{13}\begin{bmatrix} 6 & 5 \\ -17 & -12 \end{bmatrix}$$

6. Rešiti matričnu jednačinu $AX - B = X$ za $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ i

$$B = \begin{bmatrix} 0 & 2 & -6 \\ 4 & -2 & -2 \\ -3 & 7 & -3 \end{bmatrix}.$$

$$AX - B = X$$

$$AX - X = B$$

$$\underbrace{(A - E)}_{M}X = B$$

$$MX = B \;\Big|\; M^{-1} \text{ sa leve strane}$$

$$\underbrace{M^{-1}M}_{E}X = M^{-1}B$$

$$\boxed{X = M^{-1}B}$$

$$M = A - E = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$$

$$\det(M) = \left|\begin{array}{ccc|cc} 0 & 2 & 0 & 0 & 2 \\ -1 & 0 & 1 & -1 & 0 \\ 3 & 1 & 0 & 3 & 1 \end{array}\right| = 6 \neq 0 \Rightarrow \exists M^{-1}$$

$$\operatorname{adj}(M) = \begin{bmatrix} +\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix} & +\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} -1 & 3 & -1 \\ 0 & 0 & 6 \\ 2 & 0 & 2 \end{bmatrix}^T =$$

$$= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 6 & 2 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{6} \begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 6 & 2 \end{bmatrix}$$

$$X = M^{-1} B = \frac{1}{6} \underset{3\times 3}{\begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 6 & 2 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 0 & 2 & -6 \\ 4 & -2 & -2 \\ -3 & 7 & -3 \end{bmatrix}}$$

$$= \frac{1}{6} \begin{bmatrix} -6 & 12 & 0 \\ 0 & 6 & -18 \\ 18 & 0 & -12 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & -3 \\ 3 & 0 & -2 \end{bmatrix}$$

7. Rešiti matričnu jednačinu $ABX = 4X + C$ za $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$,

$B = A^T$ i $C = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$.

$$ABX = 4X + C$$

$$ABX - 4X = C$$

$$\underbrace{(AB - 4E)}_{M}X = C$$

$$MX = C \quad / M^{-1} \text{ sa leve strane}$$

$$\underbrace{M^{-1}M}_{E}X = M^{-1}C$$

$$\boxed{X = M^{-1}C}$$

$$M = AB - 4E = \underset{3\times 2}{\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \end{bmatrix}} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 4 \\ 2 & 4 & 2 \\ 4 & 2 & 10 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -2 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 6 \end{bmatrix}$$

$$\det(M) = \begin{vmatrix} -2 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 6 \end{vmatrix} \begin{matrix} -2 & 2 \\ 2 & 0 \\ 4 & 2 \end{matrix} = 16 + \cancel{16} + \cancel{8} - \cancel{24} = 16 \neq 0 \Rightarrow \exists M^{-1}$$

$$\operatorname{adj}(M) = \begin{bmatrix} -4 & -4 & +4 \\ -4 & -28 & +12 \\ +4 & +12 & -4 \end{bmatrix}^T = \begin{bmatrix} -4 & -4 & 4 \\ -4 & -28 & 12 \\ 4 & 12 & -4 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \cdot \operatorname{adj}(M) = \frac{1}{16}\begin{bmatrix} -4 & -4 & 4 \\ -4 & -28 & 12 \\ 4 & 12 & -4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -1 & -1 & 1 \\ -1 & -7 & 3 \\ 1 & 3 & -1 \end{bmatrix}$$

$\mathbb{R}^2 \begin{bmatrix} -1 \\ 0 \\ 7 \end{bmatrix}$

$$X = M^{-1}C = \frac{1}{4}\begin{bmatrix} -1 & -1 & 1 \\ -1 & -7 & 3 \\ 1 & 3 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{2}\underset{3\times 3}{\begin{bmatrix} -1 & -1 & 1 \\ -1 & -7 & 3 \\ 1 & 3 & -1 \end{bmatrix}}\underset{3\times 1}{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}$$

$$= \frac{1}{2}\begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

ZA VEŽBU

8. Rešiti matričnu jednačinu $AXB = C$ za $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & -4 \\ 3 & 1 & 2 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ i $C = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.

$AXB = C \ / A^{-1}$ sa leve strane

$\underbrace{A^{-1}A}_{E}XB = A^{-1}C$

$XB = A^{-1}C \ / B^{-1}$ sa desne strane

$X\underbrace{BB^{-1}}_{E} = A^{-1}CB^{-1}$

$$\boxed{X = A^{-1}CB^{-1}}$$

$A^{-1} = ?$

$B^{-1} = ?$

$X = A^{-1}CB^{-1}$

9. Matričnim računom rešiti sistem linearnih jednačina

$$\begin{aligned} -x &- 2y + z = 1 \\ x &- y - z = -1 \\ & y - z = 0 \end{aligned}.$$

$$A=\begin{bmatrix} -1 & -2 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix};\quad \boxed{X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}};\quad B=\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$AX=B \,/\, A^{-1}$ sa leve strane

$$\underbrace{A^{-1}A}_{E}X=A^{-1}B$$

$$\boxed{X=A^{-1}B}$$

$$\det(A)=\begin{vmatrix} -1 & -2 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}\begin{matrix} -1 & -2 \\ 1 & -1 \\ 0 & 1 \end{matrix}$$

$$= \cancel{-1}+\cancel{1}-1-2=-3\neq 0$$

$$\Rightarrow \exists A^{-1}$$

$$\operatorname{adj}(A)=\begin{bmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}^T=\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$A^{-1}=\frac{1}{\det(A)}\operatorname{adj}(A)=\frac{1}{-3}\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$X=\frac{1}{-3}\underbrace{\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}}_{3\times 3}\underbrace{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}_{3\times 1}=-\frac{1}{3}\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_S=\{(-1,0,0)\}$$

ZA VEŽBU

10. Matričnim računom rešiti sistem linearnih jednačina

$$\begin{array}{rcrcrcr} 5x & - & 3y & + & 2z & = & 17 \\ -x & & & + & 7z & = & 9 \\ x & + & 3y & & & = & 7 \end{array}.$$

$$A = \begin{bmatrix} 5 & -3 & 2 \\ -1 & 0 & 7 \\ 1 & 3 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 17 \\ 9 \\ 7 \end{bmatrix}$$

$AX = B \;/ A^{-1}$ sa leve strane

$$\underbrace{A^{-1}A}_{E}X = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

$A^{-1} = \dots$

$X = A^{-1}B$

$R_S = ?$

ZA VEŽBU

11. Matričnim računom rešiti sistem linearnih jednačina

$$\begin{array}{rcrcrcl} -x & + & y & + & 2z & = & 2 \\ 2x & + & 3y & - & z & = & 7 \\ 2x & - & y & - & z & = & 3 \end{array}.$$

12. Odrediti rang matrica:

12.1 $A=\begin{bmatrix}1&2&3\\0&3&1\\0&0&2\end{bmatrix} \Rightarrow \text{rang}(A)=3$

12.2 $B=\begin{bmatrix}1&0&1\\0&0&1\end{bmatrix} \sim \begin{bmatrix}0&1&1\\0&0&1\end{bmatrix} \Rightarrow \text{rang}(B)=2$

12.3 $C=\begin{bmatrix}1&1&0\\1&0&1\\0&1&1\end{bmatrix}\cdot(-1) \sim \begin{bmatrix}1&1&0\\0&-1&1\\0&1&1\end{bmatrix} + \sim \begin{bmatrix}1&1&0\\0&-1&1\\0&0&2\end{bmatrix} \Rightarrow \text{rang}(C)=3$

12.4 $D=\begin{bmatrix}1\\2\\0\end{bmatrix}\cdot(-2) \sim \begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow \text{rang}(D)=1$

12.5 $F=\begin{bmatrix}1&0&0\\0&0&0\\-1&0&0\end{bmatrix} + \sim \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix} \Rightarrow \text{rang}(\ )=1$

12.6 $G=\begin{bmatrix}0&1&0&0\end{bmatrix} \Rightarrow \text{rang}(G)=1$

$\begin{bmatrix}0&0&0&0\end{bmatrix} \Rightarrow \text{rang}=0$

13. Odrediti rang matrica:

13.1 $A = \begin{array}{c} \begin{bmatrix} -2 & 1 & -1 \\ -4 & 2 & -2 \\ 2 & -1 & -1 \end{bmatrix} \end{array} \begin{array}{l} -2 \\ \\ + \end{array} \sim \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$$\Rightarrow \text{rang}(A) = 2$$

13.2 $B = \begin{bmatrix} 2 & 3 & 7 \\ 8 & 4 & 12 \\ 6 & 5 & 13 \end{bmatrix} \begin{array}{l} \\ -4 \\ -3 \end{array} \sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & -8 & -16 \\ 0 & -4 & -8 \end{bmatrix} \begin{array}{l} \\ \\ -\frac{1}{2} \end{array} \sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow \operatorname{rang}(B) = 2$$

$$13.3 \quad C = \begin{bmatrix} 5 & 3 & 1 & 2 & 8 \\ 10 & 13 & 5 & 21 & 16 \\ 2 & 4 & 0 & 7 & 1 \\ 1 & 2 & 4 & 5 & 6 \end{bmatrix} \sim \begin{array}{c} \begin{bmatrix} 1 & 5 & 3 & 2 & 8 \\ 5 & 10 & 13 & 21 & 16 \\ 0 & 2 & 4 & 7 & 1 \\ 4 & 1 & 2 & 5 & 6 \end{bmatrix} \end{array} \begin{array}{l} -5 \\ \\ \\ -4 \end{array}$$

$$\sim \begin{bmatrix} 1 & 5 & 3 & 2 & 8 \\ 0 & -15 & -2 & 11 & -24 \\ 0 & 2 & 4 & 7 & 1 \\ 0 & -19 & -10 & -3 & -26 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 5 & 2 & 8 \\ 0 & -2 & -15 & 11 & -24 \\ 0 & 4 & 2 & 7 & 1 \\ 0 & -10 & -19 & -3 & -26 \end{bmatrix} \begin{array}{l} \\ \\ 2 \\ 5 \end{array}$$

$$\sim \begin{bmatrix} 1 & 3 & 5 & 2 & 8 \\ 0 & -2 & -15 & 11 & -24 \\ 0 & 0 & -28 & 29 & -47 \\ 0 & 0 & 56 & -58 & 94 \end{bmatrix} \begin{array}{l} \\ \\ \\ 2 \end{array} \sim \begin{bmatrix} 1 & 3 & 5 & 2 & 8 \\ 0 & -2 & -15 & 11 & -24 \\ 0 & 0 & -28 & 29 & -47 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \text{rang}(C) = 3$$

ZA VEŽBU:IZ SKRIPTE
Zadatak 9.21, 9.23, 9.24
teži: 9.1, 9.2, 9.3