Bulova algebra- vežbe

November 8, 2021

OVO IDE NA PREDISPITNE OBAVEZE

1. Koja od sledećih tvrđenja su tačna u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$?

$(B, +, \cdot, ', 0, 1)$

$(\mathcal{P}(A), \cup, \cap, \bar{\ }, \emptyset, A)$

A - нам је опште универзални скуп

$X \cup \overline{X} = A$

- $a + ab = aa'$;
  $\quad a \neq 0$
- $a + 1 = 0'$;
  $\quad 1 = 1$
- $a \cdot 1 = 0'$;
  $\quad a \neq 1$
- $ab = (ab)'$;
- $a + a'b = a + b$;
- $1 \cdot 0 = 1'$;
  $\quad 0 = 0$
- $a + b = (ab)'$;
- $ab = (a' + b')'$;
- $a(a + b) = aa'$;
  $\quad a = 0$
- $a + 1 = a$;
  $\quad 1 = a$

$a \cdot a' = 0$

$X \cap \overline{X} = \emptyset$

$a + 1 = 1$

$X \cup A = A$

$X \quad \overline{X}$

$1 \cdot 0 = 0$

$A \cap \emptyset = \emptyset$

$(ab)' = a' + b' \neq ab$

$a + a'b = a + b$

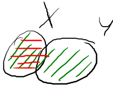
$X \cup (\overline{X} \cap Y) = (X \cup \overline{X}) \cap (X \cup Y)$

$= A \cap (X \cup Y)$

$= X \cup Y$

$(a' + b')' = (a')' \cdot (b')'$

$= a \cdot b$



$a(a+b)$

$X \cap (X \cup Y) = (X \cap X) \cup (X \cap Y)$

$= X \cup (X \cap Y)$

X

$x \leq y \Leftrightarrow x + y = y$

$$\underset{1}{1} + c \overset{}{=} \overset{=1}{1};$$

$1 \cdot 0 = 1;$

$0 \neq 1$

$a + a' = a;$

$1 \neq a$

$a' + a' = a';$

$a' = a'$

$a + bc = (a + b)(a + c);$

$1 + c = 0;$

$1 \neq 0$

$a \cdot 0 = 0;$

$0 = 0$

$a + a' = 0;$

$1 \neq 0$

$a' \cdot a' = a';$

$a' = a'$

$a \leq 1;$

$a \leq 0.$

$1 \cdot 0 = 0$

$A \cap \emptyset = \emptyset$

$a + a' = 1$

$X \cup \overline{X} = A$

$a' + a' = a'$

$\overline{X} \cup \overline{X} = \overline{X}$

$a + bc = (a + b) \cdot (a + c)$

$X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$

$a' \cdot a' = a'$

$\overline{X} \cap \overline{X} = \overline{X}$

$a \leq 1 \Longleftrightarrow a + 1 = 1$

$1 = 1$

$a \leq 0 \Longleftrightarrow a + 0 = 0$

$a \neq 0$

2. Dokazati da su u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$ sledeći iskazi ekvivalentni:

$$\text{(a) } xy = x; \quad \text{(b) } \underbrace{x + y = y}_{} \; (x \le y); \quad \text{(c) } x' + y = 1; \quad \text{(d) } xy' = 0.$$

$(a) \Rightarrow (b) \Rightarrow (c) \Rightarrow (d) \Rightarrow (a)$

$(a) \Rightarrow (b)$: $xy = x$ ← VAŽI

$x + y = y$ ← TREBA

$$x + y = xy + y = (x+1)y = 1 \cdot y = y$$

$y = 1 \cdot y$

DRUGI NAČIN: $x + y = xy + y = y + xy$

$= y + yx = y$

$(b) \Rightarrow (c)$: $x + y = y$ ← VAŽI

$x' + y = 1$ ← TREBA

$$x' + y = x' + (x + y) = (x' + x) + y = 1 + y = 1$$

$(c) \Rightarrow (d)$: $x' + y = 1$ ← VAŽI

$xy' = 0$ ← TREBA

$$xy' = (x')' \cdot y' = (x' + y)' = 1' = 0$$

DRUGI NAČIN:

$x' + y = 1 \quad /\,'$

$(x' + y)' = 1'$

$xy' = 0$

$(d) \Rightarrow (a)$: $xy' = 0$ ← VAŽI

$xy = x$ ← TREBA

$$xy + 0 = xy + xy' = x(y + y') = x \cdot 1 = x$$

3. Dokazati da su u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$ za sve $a, b, c \in B$ važi:

3.1 $a(bc') = (ab)(ac)'$;

$$(ab)(ac)' = (ab)\cdot(a' + c') = (ab)a' + (ab)c' =$$

$$= a'(ab) + (ab)c' = (a'a)b + a(bc')$$

$$= 0\cdot b + a(bc') = 0 + a(bc')$$

$$= a(bc')$$

3.2 $ab = 0 \iff ab' = a$;

$(\Rightarrow)$: $ab = 0 \Rightarrow ab' = a$

$ab = 0 \leftarrow$ VAŽI

$ab' = a \leftarrow$ TREBA

$$ab' = ab' + 0 = ab' + ab$$
$$= a(b' + b)$$
$$= a \cdot 1$$
$$= a$$

$(\Leftarrow)$: $ab' = a \Rightarrow ab = 0$

$ab' = a \leftarrow$ VAŽI

$ab = 0 \leftarrow$ TREBA

$$ab = (ab')b = a(b' \cdot b)$$
$$= a \cdot 0 = 0$$

$$a + (bc) = (a+b)\cdot(a+c)$$

3.3 $(ab) + (a' + b') = 1;$

$$(ab) + (a'+b') = (ab) + (ab)' = 1$$

DRUGI NAČIN:

$$(ab) + (a'+b') = (a'+b') + (ab) = ((a'+b')+a)\cdot((a'+b')+b)$$

$$= (a+(a'+b'))\cdot(a'+(b'+b))$$

$$= ((a+a')+b')\cdot(a'+(b'+b))$$

$$= (1+b')(a'+1)$$

$$= 1\cdot 1$$

$$= 1$$

3.4 $(c \leq a \wedge c \leq b) \iff c \leq ab.$

$(c+a=a \wedge c+b=b) \iff c+ab=ab$

$(\Rightarrow)$: $\left.\begin{array}{l} c+a=a \\ c+b=b \end{array}\right\}$ VAŽI

$c+ab=ab \leftarrow$ TREBA

$(\Leftarrow)$: $\boxed{c+ab=ab} \leftarrow$ VAŽI

$\left.\begin{array}{l} c+a=a \\ c+b=b \end{array}\right\} \leftarrow$ TREBA

$$c+ab=(c+a)\cdot(c+b) = a\cdot b$$

$$\begin{aligned} \underline{c+a} &= c+a\cdot 1 \\ &= c+a(b+b') \\ &= c+(ab+ab') \\ &= (c+ab)+ab' \\ &= ab+ab' \\ &= a(b+b') \\ &= a\cdot 1 \\ &= \underline{a} \end{aligned}$$

$(ab)'+ab=1$

$c'+c=1$

ZA VEŽBU $c+b=b$

$c+b=c+1\cdot b=c+(a+a')b=\ldots$

4. Svesti na DNF i SDNF sledeće Bulove izraze:

4.1 $I_1 = x(y'z)'$;

$a + a = a$

$$I_1 = x(y'z)' = x\left((y')' + z'\right) = x \cdot (y + z')$$

$$= xy + xz' \leftarrow \text{DNF}$$

$$= xy \cdot 1 + x \cdot 1 \cdot z'$$

$$= xy(z + z') + x(y + y')z'$$

$$= xyz + xyz' + xyz' + xy'z'$$

$$= xyz + xyz' + xy'z' \leftarrow \text{SDNF}$$

4.2 $I_2 = z\,(x' + y) + y'$;

$$I_2 = z(x'+y)+y' = zx' + zy + y' \leftarrow \text{DNF}$$

$$= zx'\cdot 1 + 1\cdot zy + y'\cdot 1\cdot 1$$

$$= zx'(y+y') + (x+x')zy + y'\overbrace{(x+x')(z+z')}^{xz+xz'+x'z+x'z'}$$

$$= x'yz + x'y'z + xyz + x'yz + xy'z + xy'z' + x'y'z + x'y'z'$$

$$= x'yz + x'y'z + xyz + xy'z + xy'z' + x'y'z' \leftarrow \text{SDNF}$$

4.3 $I_3 = (x + y'z)(y + z')$;

$$I_3 = (x+y'z)(y+z') = xy + xz' + \underbrace{y'zy}_{0} + y'\underbrace{zz'}_{0}$$

$$= xy + xz' + \underbrace{0\cdot z}_{0} + \underbrace{y'\cdot 0}_{0} \qquad = xy + xz' + 0 + 0$$

$$= \underline{xy} + \underline{xz'} \leftarrow \text{DNF}$$

$$= xy\cdot 1 + x\cdot 1\, z'$$

$$= xy(z+z') + x(y+y')z'$$

$$= xyz + x\underline{yz'} + x\underline{yz'} + xy'z'$$

$$= xyz + xyz' + xy'z' \leftarrow \text{SDNF}$$

ZA VE Ztau

4.4 $I_4 = (x' + y)' + y'z$;

4.5 $I_5 = (x+y)'(xy')'$; NAD $\{x, y, z\}$

$$I_5 = (x+y)'(xy')' = x'y' \cdot (x' + (y')') = x'y' \cdot (x'+y)$$

$$= \underbrace{x'y'x'}_{x'} + \cancel{x'y'y}_{0} = x'y \leftarrow \text{DNF}$$

$\to$ SDNF NAD $\{x, y\}$

$$= x'y \cdot 1$$

$$= x'y \cdot 1 = x'y \cdot (z+z')$$

$$= x'yz + x'yz' \leftarrow \text{SDNF}$$

ZA VEŽBU

4.6 $I_6 = y\,(x + yz)'$;

ZA VEŽBU

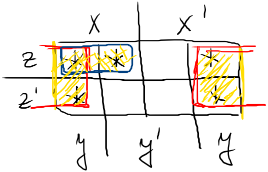
4.7 $I_7 = (x + y)(x' + y)\,z.$

5. Naći sve proste implikante i minimalne DNF Bulovih funkcija datih svojom tablicom vrednosti ili odgovarajućim Bulovim izrazom:

   5.1 $f(x, y, z) = xyz + xy'z + xyz' + x'y'z'$;

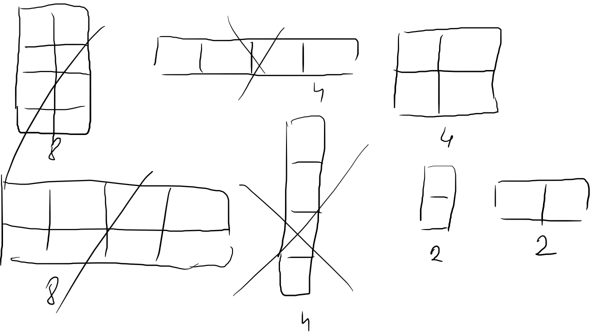
ZA VEŽBU

5.2 $f(x, y, z) = xyz + x'yz + xy'z + xyz' + x'yz'$;



PI : y, xz

$MNDF(f(x,y,z)) = y + xz$

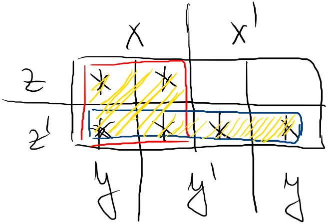


5.3

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
$f(x,y,z)$	1	0	1	0	1	1	1	1

;

$$SDNF(f(x,y,z)) = x'y'z' + x'yz' + xy'z' + xy'z + xyz' + xyz$$



PI: x, z'

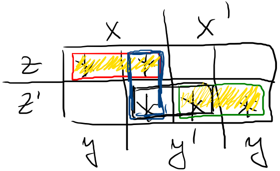
$$MDNF(f(x,y,z)) = x + z'$$

5.4

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
$f(x,y,z)$	1	0	1	0	1	1	0	1

;

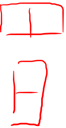
$$SDNF(f(x,y,z)) = x'y'z' + x'yz' + xy'z' + xy'z + xyz$$



$PI: xz,\ xy',\ x'z',\ y'z'$

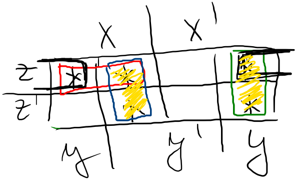
$$MDNF_1(f(x,y,z)) = xz + x'z' + y'z'$$

$$MDNF_2(f(x,y,z)) = xz + x'z' + xy'$$



5.5 $f(x,y,z) = xyz + xy' + x'y$;

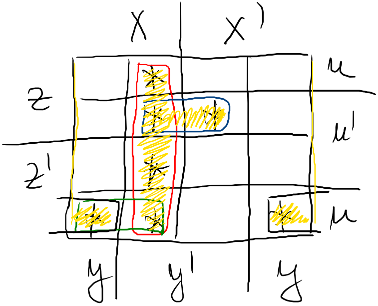
$$SDNF(f(x,y,z)) = xyz + xy'\cdot 1 + x'y\cdot 1$$
$$= xyz + xy'(z+z') + x'y(z+z')$$
$$= \underline{xyz} + \underline{xy'z} + \underline{xy'z'} + \underline{x'yz} + \underline{x'yz'}$$



PI: xz, xy', $x'y$, yz

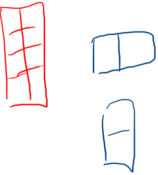
$$MDNF_1(f(x,y,z)) = xy' + x'y + xz$$
$$MDNF_2(f(x,y,z)) = xy' + x'y + yz$$

5.6 $f(x,y,z,u) = xy'zu + xy'zu' + xy'z'u' + xy'z'u + xyz'u + x'y'zu' + x'yz'u$;



PI: xy', $y'zu'$, $xz'u$, $yz'u$

MDNF$(f(x,y,z,u)) = xy' + y'zu' + yz'u$

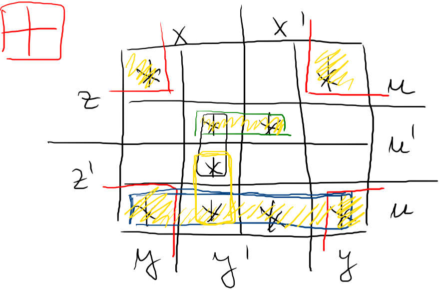


5.7

x	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
z	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
u	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
f	0	1	1	0	0	1	0	1	1	1	1	0	0	1	0	1

;

$$SDNF(f(x,y,z,u)) = x'y'z'u + x'y'zu' + x'yz'u + x'yzu + xy'z'u' + xy'z'u + xy'zu' + xyz'u + xyzu$$



PI: $yu,\ z'u,\ y'zu',\ xy'u',\ xy'z'$

$$MDNF_1(f(x,y,z,u)) = yu + z'u + y'zu' + xy'u'$$

$$MDNF_2(f(x,y,z,u)) = yu + z'u + y'zu' + xy'z'$$

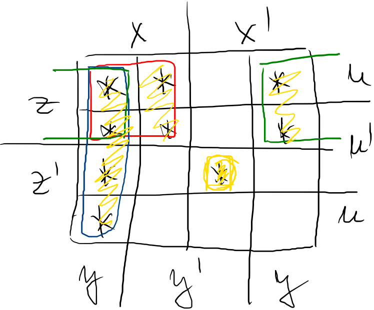
ZA VE ZDrm

5.8

x	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
z	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
u	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
f	1	1	1	0	0	0	1	0	1	0	1	1	1	0	0	0

;

5.9 $f(x,y,z,u) = \underline{xyzu} + \underline{xy'zu} + \underline{x'yzu} + \underline{xyzu'} + \underline{xy'zu'} + \underline{x'yzu'} + \underline{xyz'u'} + \underline{x'y'z'u'} + \underline{xyz'u}$;



PI: xz, xy, yz, $x'y'z'u'$

$MDNF(f(x,y,z,u)) = x'y'z'u' + xy + xz + yz$

ZA VEZBU

5.10 $f(x, y, z, u) = \underline{xy'} + xyz + x'y'z' + x'yzu'$.

$SDNF(f(x,y,z,u)) = xy \cdot 1 \cdot 1 + xyz \cdot 1 + x'y'z' \cdot 1 + x'yzu'$

$= xy(z+z')(u+u') + xyz(u+u') + x'y'z'(u+u') + x'yzu'$

ZA VEŽBU:IZ SKRIPTE
Zadatak 5.1, 5.3, 5.4, 5.6, 5.12 a,c, 5.14;
Primer 5.14;