8.50.

$p(x)$ – je 5 stepena, normiran

$\alpha+\beta i \Rightarrow \alpha-\beta i$

$q(x) = x^2+4$

$q(x) \mid p(x)$

$p(x) = q(x)\cdot t(x)$

i je jedan koren $p(x) \Rightarrow -i$ je jedan koren $p(x)$

$$\frac{p(x)}{x-1} \quad -20$$

$(x-i) \mid p(x)$

$(x+i) \mid p(x)$

$(x-i)(x+i) \mid p(x)$

$x^2+1 \mid p(x)$

$q(x) \mid p(x)$

$x^2+1 \mid p(x)$

$q(x)\cdot(x^2+1) \mid p(x)$

$\underbrace{(x^2+4)(x^2+1)}_{4.\ \text{stepena}} \mid \underbrace{p(x)}_{5\ \text{stepena}}$

$p(x) = (x^2+4)(x^2+1)(x-a)$

$$\frac{p(x)}{x-1}$$

$p(1) = -20$

$p(1) = 5\cdot 2\cdot(1-a)$

$-20 = 10\,(1-a)$

$-2 = 1-a$

$\boxed{a=3}$

$p(x) = (x^2+4)(x^2+1)(x-3)$ nad $\mathbb{R}$

$p(x) = (x+2i)(x-2i)(x+i)(x-i)(x-3)$ nad $\mathbb{C}$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & -5 \\ 2 & -1 & 4 & 0 & 3 \\ -3 & -1 & -4 & -3 & 2 \end{bmatrix} \begin{array}{l} \\ -2 \\ 3 \end{array} \sim \begin{bmatrix} 1 & 2 & 0 & 3 & -5 \\ 0 & -5 & 4 & -6 & 13 \\ 0 & 5 & -4 & 6 & -13 \end{bmatrix} \begin{array}{l} \\ \\ + \end{array} \sim$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 3 & -5 \\ 0 & -5 & 4 & -6 & 13 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ rang $= 2$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 5 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 2 & 3 & 5 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ -1 \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 0 & 1 & 2 & 3 & 5 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rang $= 3$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rang $= 2$

$$\begin{array}{cc} \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} & \begin{array}{c} \\ -2 \\ -1 \end{array} \end{array} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{rang} = 2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \; -1 \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \; -1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{rang} = 2$$

$$\sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$