Vektori

December 21, 2021

1. Neka su vektori $\vec{a}$ i $\vec{b}$ takvi da je $|\vec{a}| = 2$, $\left|\vec{b}\right| = 1$ i $\measuredangle\left(\vec{a}, \vec{b}\right) = \dfrac{\pi}{3}$.

   Izračunati:

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\measuredangle(\vec{a},\vec{b}) = 2\cdot 1\cdot\cos\frac{\pi}{3} = \cancel{2}\cdot\frac{1}{\cancel{2}} = 1$$

$$\begin{aligned}\left(\vec{a} + 2\vec{b}\right)\cdot\left(\vec{a} - \vec{b}\right) &= \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{a} - 2\vec{b}\,\vec{b}\\ &= |\vec{a}|^2 - \vec{a}\,\vec{b} + 2\vec{a}\cdot\vec{b} - 2|\vec{b}|^2\\ &= 4 + 1 - 2\\ &= 3\end{aligned}$$

$$\begin{aligned}\left(\vec{a} - \vec{b}\right)\cdot\left(\vec{a} + \vec{b}\right) &= \vec{a}\cdot\vec{a} + \cancel{\vec{a}\,\vec{b}} - \cancel{\vec{b}\,\vec{a}} - \vec{b}\cdot\vec{b}\\ &= |\vec{a}|^2 - |\vec{b}|^2\\ &= 4 - 1\\ &= 3\end{aligned}$$

$$\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\underbrace{\cos\measuredangle(\vec{a},\vec{a})}_{1}$$

$$\vec{a}\cdot\vec{a} = |\vec{a}|^2$$

$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$

$$(A-B)(A+B) = A^2 - B^2$$

$$\cancel{\vec{a}^2 - \vec{b}^2}$$

$$\alpha(\vec{a}\cdot\vec{b}) = (\alpha\vec{a})\,\vec{b} = \vec{a}\cdot(\alpha\vec{b})$$

2. Naći intenzitet vektora $\vec{a} = \vec{p} - 2\vec{q}$, ako je $|\vec{p}| = 2$, $|\vec{q}| = \sqrt{3}$ i $\measuredangle(\vec{p}, \vec{q}) = \frac{\pi}{6}$.

$$|\vec{a}|^2 = \vec{a} \cdot \vec{a} \Rightarrow |\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$$

$$\underline{\vec{a} \cdot \vec{a}} = (\vec{p} - 2\vec{q}) \cdot (\vec{p} - 2\vec{q})$$

$$= \vec{p} \cdot \vec{p} - 2\vec{p} \cdot \vec{q} - 2\vec{q} \cdot \vec{p} + 4\vec{q} \cdot \vec{q}$$

$$= |\vec{p}|^2 - 4\underline{\vec{p} \cdot \vec{q}} + 4|\vec{q}|^2$$

$$= 4 - 4 \cdot |\vec{p}| \cdot |\vec{q}| \cdot \cos\measuredangle(\vec{p}, \vec{q}) + 4 \cdot 3$$

$$= 16 - 4 \cdot 2 \cdot \sqrt{3} \cos\frac{\pi}{6}$$

$$= 16 - \cancel{8}^{4}\sqrt{3}\,\frac{\sqrt{3}}{\cancel{2}}$$

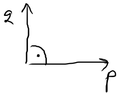
$$= \underline{4}$$

$$|\vec{a}| = \sqrt{4} = 2$$

3. Odrediti realan parametar α tako da vektori $\vec{p} = \alpha\vec{a} + 17\vec{b}$ i $\vec{q} = 3\vec{a} - \vec{b}$ budu uzajamno normalni, ako je $|\vec{a}| = 2$, $\left|\vec{b}\right| = 5$ i $\measuredangle\left(\vec{a}, \vec{b}\right) = \frac{2\pi}{3}$.

$\perp$ = NORMALNI = ORTOGONALNI

$\vec{p} \perp \vec{q}$



$\cos\measuredangle(\vec{p}, \vec{q}) = \cos\frac{\pi}{2} = 0$

$\vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\measuredangle(\vec{p}, \vec{q})$

$\Rightarrow \vec{p}\cdot\vec{q} = 0$

$\vec{p} \perp \vec{q} \iff \vec{p}\cdot\vec{q} = 0$

$$
\begin{aligned}
\vec{p}\cdot\vec{q} &= (\alpha\vec{a} + 17\vec{b})\cdot(3\vec{a} - \vec{b}) \\
&= 3\alpha\,\vec{a}\cdot\vec{a} - \alpha\,\vec{a}\cdot\vec{b} + 51\,\vec{b}\cdot\vec{a} - 17\,\vec{b}\cdot\vec{b} \\
&= 3\alpha|\vec{a}|^2 + (51-\alpha)\,\vec{a}\cdot\vec{b} - 17\cdot|\vec{b}|^2 \\
&= 3\alpha\cdot 4 + (51-\alpha)|\vec{a}||\vec{b}|\cdot\cos\measuredangle(\vec{a}, \vec{b}) - 17\cdot 25 \\
&= 12\alpha + (51-\alpha)\cdot 2\cdot 5\cdot\cos\frac{2\pi}{3} - 425 \\
&= 12\alpha + (51-\alpha)\,\overset{5}{\cancel{10}}\cdot\left(-\frac{1}{\cancel{2}}\right) - 425 \\
&= 12\alpha - 255 + 5\alpha - 425 \\
&= 17\alpha - 680
\end{aligned}
$$

$$
\begin{array}{r}
17\cdot 25 \\
\hline
85 \\
34 \\
\hline
425
\end{array}
$$

$[0, \pi]$

$\vec{p}\cdot\vec{q} = 17\alpha - 680$

$0 = 17\alpha - 680$

$17\alpha = 680$

$\boxed{\alpha = 40}$

4. Koji ugao obrazuju jedinični vektori $\vec{a}$ i $\vec{b}$ ako su vektori $\vec{p} = \vec{a} + 2\vec{b}$ i $\vec{q} = 5\vec{a} - 4\vec{b}$ uzajamno normalni?

$$|\vec{a}| = |\vec{b}| = 1$$

$$\vec{p} \perp \vec{q} \iff \vec{p} \cdot \vec{q} = 0$$

$$\vec{p} \cdot \vec{q} = 0$$

$$(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$$

$$5\vec{a}\cdot\vec{a} - 4\vec{a}\cdot\vec{b} + 10\,\vec{b}\cdot\vec{a} - 8\vec{b}\cdot\vec{b} = 0$$

$$5|\vec{a}|^2 + 6\,\underline{\vec{a}\cdot\vec{b}} - 8\,|\vec{b}|^2 = 0$$

$$5 + 6|\vec{a}||\vec{b}|\cos\measuredangle(\vec{a},\vec{b}) - 8 = 0$$

$$-3 + 6\cdot 1\cdot 1\cdot\cos\measuredangle(\vec{a},\vec{b}) = 0$$

$$6\cos\measuredangle(\vec{a},\vec{b}) = 3$$

$$\cos\measuredangle(\vec{a},\vec{b}) = \frac{1}{2}$$

$$\underline{\underline{\measuredangle(\vec{a},\vec{b}) = \frac{\pi}{3}}}$$

Напомена:

$$\underline{\underline{\vec{a}\cdot\vec{b}}} = |\vec{a}||\vec{b}|\cos\measuredangle(\vec{a},\vec{b}) = 1\cdot 1\cdot\frac{1}{2} = \underline{\frac{1}{2}}$$

5. Izračunati površinu paralelograma konstruisanog nad vektorima $\vec{p} = 2\vec{b} - \vec{a}$ i $\vec{q} = 3\vec{a} + 2\vec{b}$ ako je $|\vec{a}| = |\vec{b}| = 5$ i $\sphericalangle(\vec{a}, \vec{b}) = \frac{\pi}{4}$.

$P_\square = |\vec p \times \vec q|$

$\vec p \times \vec q = (2\vec b - \vec a) \times (3\vec a + 2\vec b)$

$= 6\vec b \times \vec a + 4\underbrace{\vec b \times \vec b}_{0} - 3\underbrace{\vec a \times \vec a}_{0} - 2\underbrace{\vec a \times \vec b}_{-\vec b \times \vec a}$

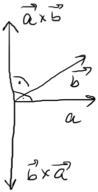
$= 6\vec b \times \vec a + 2\vec b \times \vec a$

$= 8\vec b \times \vec a$

$P_\square = |\vec p \times \vec q| = |8\vec b \times \vec a| = |8|\,|\vec b \times \vec a|$

$= 8|\vec b|\,|\vec a| \sin\sphericalangle(\vec a, \vec b) = 8 \cdot 5 \cdot 5 \sin\sphericalangle\frac{\pi}{4}$

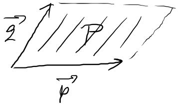
$= 200 \cdot \frac{\sqrt{2}}{2} = 100\sqrt{2}$



$\vec a \times \vec b = -\vec b \times \vec a$

$\alpha \vec a$

$|\alpha \vec a| = |\alpha|\,|\vec a|$



$P = |\vec p \times \vec q|$

$\alpha(\vec a \times \vec b) = (\alpha \vec a) \times \vec b$
$= \vec a \times (\alpha \vec b)$

$\vec a \times \vec a$

$\Rightarrow \vec a \times \vec b$:

1) $\vec a \times \vec b \perp \vec a \wedge \vec a \times \vec b \perp \vec b$

2) $|\vec a \times \vec b| = |\vec a|\,|\vec b| \cdot \sin\sphericalangle(\vec a, \vec b)$

3) pravilo desnog triedra

$\Rightarrow \vec a \times \vec a = 0$

6. Za koje vrednosti realnog parametra k će vektori $\vec{p} = k\vec{a} + 5\vec{b}$ i $\vec{q} = 3\vec{a} - \vec{b}$ biti kolinearni ako vektori $\boxed{\vec{a} \text{ i } \vec{b} \text{ nisu kolinearni?}}$

$$\vec{p} \parallel \vec{q} \iff \vec{p} \times \vec{q} = 0$$

$$\vec{p} \times \vec{q} = 0$$

$$(k\vec{a} + 5\vec{b}) \times (3\vec{a} - \vec{b}) = 0$$

$$3k\,\cancel{\vec{a}\times\vec{a}}^{\,0} - k\vec{a}\times\vec{b} + 15\vec{b}\times\vec{a} - 5\,\cancel{\vec{b}\times\vec{b}}^{\,0} = 0$$

$$-k\vec{a}\times\vec{b} - 15\vec{a}\times\vec{b} = 0$$

$$(-k-15)\,\vec{a}\times\vec{b} = 0$$

$$-k-15 = 0 \quad \vee \quad \underbrace{\vec{a}\times\vec{b} = 0}$$

$$\boxed{k = -15}$$

што не може никад да се деси јер $\vec{a}$ и $\vec{b}$ нису колинеарни па је $\vec{a}\times\vec{b} \neq 0$ увек.

$\vec{p}, \vec{q}$ kolinearni $\iff$

$\vec{p} \parallel \vec{q}$ (ili $\vec{p} \equiv \vec{q}$)

$\iff \vec{q} = \alpha \vec{p}, \ \alpha \in \mathbb{R}$

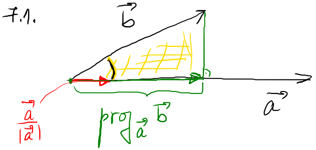
$\iff \vec{p} \times \vec{q} = 0$

7. Dati su vektori $\vec{a} = \vec{m} - 2\vec{n}$ i $\vec{b} = 2\vec{m} + \vec{n}$, gde je $|\vec{m}| = 2$, $|\vec{n}| = 3$ i $\measuredangle(\vec{m}, \vec{n}) = \frac{\pi}{3}$.

7.1 Odrediti projekciju vektora $\vec{b}$ na vektor $\vec{a}$.

7.2 Izračunati površinu trougla određenog vektorima $\vec{a}$ i $\vec{b}$.

7.1.



$$proj_{\vec{a}}\vec{b} = |proj_{\vec{a}}\vec{b}| \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$\cos\measuredangle(\vec{a}, \vec{b}) = \frac{|proj_{\vec{a}}\vec{b}|}{|\vec{b}|}$$

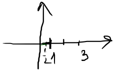
$$|proj_{\vec{a}}\vec{b}| = |\vec{b}| \cdot \cos\measuredangle(\vec{a}, \vec{b}) = \cancel{|\vec{b}|} \cdot \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cancel{|\vec{b}|}} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

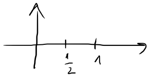
$$proj_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$$

$$\boxed{proj_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}}$$

$\vec{a}$ – неки вектор

$\frac{\vec{a}}{|\vec{a}|}$ – јединични вектор који одговара вектору $\vec{a}$





$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\measuredangle(\vec{a}, \vec{b})$$

$$\Rightarrow \cos\measuredangle(\vec{a}, \vec{b}) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$\vec{a}\cdot\vec{b} = (\vec{m}-2\vec{n})\cdot(2\vec{m}+\vec{n})$$
$$= 2\vec{m}\cdot\vec{m} + \vec{m}\vec{n} - 4\vec{n}\cdot\vec{m} - 2\vec{n}\cdot\vec{n}$$
$$= 2|\vec{m}|^2 - 3\vec{m}\cdot\vec{n} - 2|\vec{n}|^2$$
$$= 2\cdot 4 - 3|\vec{m}||\vec{n}|\cos\measuredangle(\vec{m},\vec{n}) - 2\cdot 9$$
$$= 8 - 3\cdot 2\cdot 3\cdot\cos\frac{\pi}{3} - 18$$
$$= -10 - \cancel{18}^{9}\cdot\frac{1}{\cancel{2}}$$
$$= -19$$

$$|\vec{a}|^2 = \vec{a}\cdot\vec{a}$$
$$= (\vec{m}-2\vec{n})\cdot(\vec{m}-2\vec{n})$$
$$= \vec{m}\cdot\vec{m} - 2\vec{m}\cdot\vec{n} - 2\vec{n}\cdot\vec{m} + 4\vec{n}\vec{n}$$
$$= |\vec{m}|^2 - 4\vec{m}\cdot\vec{n} + 4|\vec{n}|^2$$
$$= 4 - 4|\vec{m}||\vec{n}|\cos\measuredangle(\vec{m},\vec{n}) + 4\cdot 9$$
$$= 4 - 4\cdot 2\cdot 3\cdot\cos\frac{\pi}{3} + 36$$
$$= 40 - \cancel{24}^{12}\cdot\frac{1}{\cancel{2}}$$
$$= 28$$

$$\mathrm{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\,\vec{a} = -\frac{19}{28}(\vec{m}-2\vec{n}) = -\frac{19}{28}\vec{m} + \frac{19}{14}\vec{n}$$

b)

$$P_\Delta = \frac{1}{2}|\vec{a}\times\vec{b}|$$

$$P = |\vec{a}\times\vec{b}|$$

$$\vec{a}\times\vec{b} = (\vec{m}-2\vec{n})\times(2\vec{m}\times\vec{n})$$

$$= \cancel{2\vec{m}\times\vec{m}}^{\,0} + \vec{m}\times\vec{n} - 4\vec{n}\times\vec{m} - \cancel{2\vec{n}\times\vec{n}}^{\,0}$$

$$= \vec{m}\times\vec{n} + 4\vec{m}\times\vec{n}$$

$$= 5\,\underline{\vec{m}\times\vec{n}}$$

$$P_\Delta = \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|5\vec{m}\times\vec{n}| = \frac{5}{2}|\vec{m}\times\vec{n}|$$

$$= \frac{5}{2}|\vec{m}|\,|\vec{n}|\sin\angle(\vec{m},\vec{n})$$

$$= \frac{5}{\cancel{2}}\cancel{2}\cdot 3\sin\frac{\pi}{3}$$

$$= 15\frac{\sqrt{3}}{2}$$

$$= \frac{15}{2}\sqrt{3}$$

DOVDE SU VEKTORI
PO DEF I OSOBINAMA
BEZ KOORDINATA!

8. Za vektore $\vec{a} = 8\vec{i} + 2\vec{j} - 2\vec{k}$ i $\vec{b} = 4\vec{i} - 4\vec{j}$ izračunati:

$$\vec{a} = (8, 2, -2) \qquad \vec{b} = (4, -4, 0)$$

$$|\vec{a}| = \sqrt{8^2 + 2^2 + (-2)^2} = \sqrt{64+4+4} = \sqrt{72} = 6\sqrt{2}$$

$$|\vec{b}| = \sqrt{4^2 + (-4)^2 + 0^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$3\vec{a} + \vec{b} = 3(8,2,-2) + (4,-4,0) = (24,6,-6) + (4,-4,0) = (28, 2, -6)$$

$$2\vec{a} - \vec{b} = 2(8,2,-2) - (4,-4,0) = (16,4,-4) - (4,-4,0) = (12, 8, -4)$$

$$\vec{a} \cdot \vec{b} = (8,2,-2) \cdot (4,-4,0) = 8 \cdot 4 + 2 \cdot (-4) + (-2) \cdot 0 = 32 - 8 = 24$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & 2 & -2 \\ 4 & -4 & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} 2 & -2 \\ -4 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 8 & -2 \\ 4 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 8 & 2 \\ 4 & -4 \end{vmatrix}$$

$$= \vec{i}(0-8) - \vec{j}(0+8) + \vec{k}(-32-8)$$

$$= -8\vec{i} - 8\vec{j} - 40\vec{k} = (-8, -8, -40)$$

$$= -8(1, 1, 5)$$

$$\sphericalangle(\vec{a}, \vec{b}) = \frac{\pi}{3}$$

$$\cos \sphericalangle(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{24}{6\sqrt{2} \cdot 4\sqrt{2}} = \frac{1}{2}$$

$$\vec{a} = (a_1, a_2, a_3)$$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \boxed{\vec{i} \quad \vec{j} \quad \vec{k}} \\ a_1 \quad a_2 \quad a_3 \\ b_1 \quad b_2 \quad b_3 \end{vmatrix}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \sphericalangle(\vec{a}, \vec{b})$$

9. Dati su vektori $\vec{a} = (1,1,1)$, $\vec{b} = (1,1,0)$ i $\vec{c} = (1,-1,0)$. Naći vektor $\vec{x}$ tako da važi $\vec{x} \cdot \vec{a} = 3$ i $\vec{x} \times \vec{b} = \vec{c}$.

$\vec{x} = (x, y, z)$

$\vec{x} \cdot \vec{a} = 3$

$(x, y, z) \cdot (1,1,1) = 3$

$\boxed{x + y + z = 3}$

$\vec{x} \times \vec{b} = \vec{c}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 1 & 0 \end{vmatrix} = (1,-1,0)$$

$$\vec{i} \begin{vmatrix} y & z \\ 1 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} x & z \\ 1 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} x & y \\ 1 & 1 \end{vmatrix} = (1,-1,0)$$

$$\vec{i}\,(0 - z) - \vec{j}\,(0 - z) + \vec{k}\,(x - y) = (1,-1,0)$$

$$-z\vec{i} + z\vec{j} + (x-y)\vec{k} = (1,-1,0)$$

$$(-z, z, x-y) = (1,-1,0)$$

$$\underline{-z = 1 \ \wedge \ z = -1} \ \wedge \ x - y = 0$$

$\boxed{z = -1}$ $\qquad$ $\boxed{x = y}$

$x + x - 1 = 3$

$2x = 4$

$\boxed{x = 2}$

$\boxed{y = 2}$

$$\boxed{\vec{x} = (2, 2, -1)}$$

10. Izračunati površinu paralelograma konstruisanog nad vektorima $\vec{p} = 2\vec{i} + 3\vec{j}$ i $\vec{q} = \vec{i} - 4\vec{j}$.

$$\vec{p} = (2, 3, 0)$$
$$\vec{q} = (1, -4, 0)$$

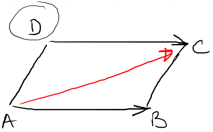
$$P_{\square} = |\vec{p} \times \vec{q}|$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ 1 & -4 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 3 & 0 \\ -4 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix}$$

$$= 0\vec{i} - 0\vec{j} - 11\vec{k} = (0, 0, -11)$$

$$P_{\square} = |(0, 0, -11)| = \sqrt{0^2 + 0^2 + (-11)^2} = \sqrt{121} = \underline{\underline{11}}$$

11. Odrediti koordinate temena D paralelograma $ABCD$ i dužinu dijagonale AC ako su data tri uzastopna temena $A(1,-2,0)$, $B(2,1,3)$ i $C(2,0,5)$.



$\vec{AB} = \vec{DC}$, $D(x,y,z)$

$\vec{AB} = \vec{OB} - \vec{OA} = (2,1,3) - (1,-2,0) = (1,3,3)$

$\vec{DC} = \vec{OC} - \vec{OD} = (2,0,5) - (x,y,z) = (2-x, -y, 5-z)$

$(1,3,3) = (2-x, -y, 5-z)$

$2-x=1 \qquad -y=3 \qquad 5-z=3$

$x=1 \qquad y=-3 \qquad z=2$

$\boxed{D(1,-3,2)}$

$\vec{AC} = \vec{OC} - \vec{OA} = (1,-2,0) - (2,0,5) = (-1,-2,-5)$

$|\vec{AC}| = \sqrt{(-1)^2+(-2)^2+(-5)^2} = \sqrt{1+4+25} = \sqrt{30}$

to dužina dijagonale

$\vec{AC} = \vec{AB} + \vec{BC}$

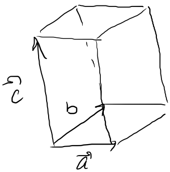
$\vec{BC} = \ldots$

12. Izračunati zapreminu paralelopipeda konstruisanog nad vektorima $\vec{a} = -\vec{i} + 2\vec{j} - 3\vec{k}$ i $\vec{b} = 4\vec{i} + \vec{k}$ i $\vec{c} = -2\vec{i} + 5\vec{j} - \vec{k}$.

$\vec{a} = (-1, 2, -3)$

$\vec{b} = (4, 0, 1)$

$\vec{c} = (-2, 5, -1)$



$$V = \left| \vec{a} \cdot (\vec{b} \times \vec{c}) \right|$$

↳ apsolutno vrednost

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \left| \begin{array}{ccc} -1 & 2 & -3 \\ 4 & 0 & 1 \\ -2 & 5 & -1 \end{array} \right| \begin{array}{cc} -1 & 2 \\ 4 & 0 \\ -2 & 5 \end{array} = -4 - 60 + 5 + 8 = -51$$

$$V = |-51| = 51$$

13. Date su tačke $A(1,1,1)$, $B(2,2,1)$ i $C(2,1,2)$. Izračunati ugao između vektora $\overrightarrow{AB}$ i $\overrightarrow{AC}$.

$$\sphericalangle(\overrightarrow{AB}, \overrightarrow{AC}) = ?$$

$$\cos \sphericalangle(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{1}{\sqrt{2}\,\sqrt{2}} = \frac{1}{2} \implies \sphericalangle(\overrightarrow{AB}, \overrightarrow{AC}) = \frac{\pi}{3}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (2,2,1) - (1,1,1) = (1,1,0)$$

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (2,1,2) - (1,1,1) = (1,0,1)$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = (1,1,0) \cdot (1,0,1) = 1 + 0 + 0 = 1$$

$$|\overrightarrow{AB}| = \sqrt{1+1+0} = \sqrt{2}$$

$$|\overrightarrow{AC}| = \sqrt{1+0+1} = \sqrt{2}$$

14. Odrediti realan parametar α tako da vektori $\vec{a} = 2\vec{i} - 3\vec{j}$ i $\vec{b} = \alpha\vec{i} + 4\vec{j}$ budu ortogonalni.

$$\vec{a} \perp \vec{b} \iff \underline{\vec{a} \cdot \vec{b} = 0}$$

$$\vec{a} = (2, -3, 0)$$
$$\vec{b} = (\alpha, 4, 0)$$

$$\vec{a} \cdot \vec{b} = (2, -3, 0) \cdot (\alpha, 4, 0) = 2\alpha - 12$$

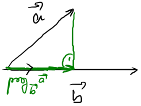
$$2\alpha - 12 = 0$$

$$\boxed{\alpha = 6}$$

NEMA POTREBA

$$\vec{a} \cdot \vec{b} = (2\vec{i} - 3\vec{j}) \cdot (\alpha\vec{i} + 4\vec{j})$$
$$= 2\alpha\, \vec{i} \cdot \vec{i} + 8\, \vec{i} \cdot \vec{j} - 3\alpha\, \vec{j} \cdot \vec{i} - 12\, \vec{j} \cdot \vec{j}$$
$$= 2\alpha |\vec{i}|^2 + 5\, \vec{i} \cdot \vec{j} - 12 |\vec{j}|^2$$
$$= 2\alpha \cdot 1 + 5 |\vec{i}| |\vec{j}| \cos\measuredangle(\vec{i}, \vec{j}) - 12 \cdot 1$$
$$= 2\alpha + 5 \cdot 1 \cdot 1 \cos\frac{\pi}{2} - 12$$
$$= 2\alpha - 12$$

15. Odrediti projekciju vektora $\vec{a} = \vec{i} + \vec{j} + 2\vec{k}$ na vektor $\vec{b} = \vec{i} - \vec{j} + 4\vec{k}$.



$\frac{\vec{b}}{|\vec{b}|}$ – јединични вектор који одговара вектору $\vec{b}$

$$proj_{\vec{b}}\vec{a} = |proj_{\vec{b}}\vec{a}| \frac{\vec{b}}{|\vec{b}|}$$

$$\cos\sphericalangle(\vec{b},\vec{a}) = \frac{|proj_{\vec{b}}\vec{a}|}{|\vec{a}|}$$

$$\Rightarrow |proj_{\vec{b}}\vec{a}| = |\vec{a}| \cos\sphericalangle(\vec{b},\vec{a}) = \cancel{|\vec{a}|} \frac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}\,|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$\Longrightarrow proj_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \cdot \frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b}$$

$$\vec{a} = (1, 1, 2)$$
$$\vec{b} = (1, -1, 4)$$
$$\vec{a}\cdot\vec{b} = (1,1,2)(1,-1,4) = 1 - 1 + 8 = 8$$
$$|\vec{b}|^2 = 1 + (-1)^2 + 4^2 = 1+1+16 = 18$$

$$proj_{\vec{b}}\vec{a} = \frac{\cancel{8}^{\,4}}{\cancel{18}_{\,9}} \cdot (1, -1, 4) = \left(\frac{4}{9}, -\frac{4}{9}, \frac{16}{9}\right)$$

16. Dati su vektori $\vec{a} = (2k-1, 2, k+2)$, $\vec{b} = (3, k-1, -1)$ i $\vec{c} = (p, 1, 3)$, gde $k \in \mathbb{R}$, $p \in \mathbb{R}^-$.

16.1 Odrediti vrednost parametara k i p tako da važi $\vec{a} \perp \vec{b}$ i $|\vec{c}| = \sqrt{26}$.

16.2 Za tako određene k i p pokazati da su vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ komplanarni.

16.1. $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$

$(2k-1, 2, k+2) \cdot (3, k-1, -1) = 0$

$6k - 3 + 2k - 2 - k - 2 = 0$

$7k - 7 = 0$

$\boxed{k = 1}$

$|\vec{c}| = \sqrt{26}$

$\sqrt{p^2 + 1^2 + 3^2} = \sqrt{26}$

$p^2 + 1 + 9 = 26$

$p^2 = 16$

$\cancel{p = 4} \quad \vee \quad \boxed{p = -4}$

$p \in \mathbb{R}^-$

16.2. $\vec{a} = (1, 2, 3)$

$\vec{b} = (3, 0, -1)$

$\vec{c} = (-4, 1, 3)$

$\vec{a}, \vec{b}, \vec{c}$ su komplanarni $\iff$ paralelni sa jednom ravni $\iff \vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & -1 \\ -4 & 1 & 3 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 0 \\ -4 & 1 \end{matrix} = 8 + 9 + 1 - 18 = 0 \checkmark$$

ZA VEŽBU:IZ SKRIPTE
Zadatak 10.12, 10.20, 10.21
Primer: 10.1