Sistemi linearnih jednačina- vežbe

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1. Rešiti sledeće sisteme linearnih jednačina:

1.1
$$\begin{aligned} x + 3y - 5z &= 6 \quad \cdot(-2),\ \cdot(-3)\\ 2x + z &= 3\ ;\\ 3x + 3y - 4z &= 9 \end{aligned}$$

$$\begin{aligned} 3y + x - 5z &= 6\\ 2x + z &= 3 \quad \cdot(-1)\\ \boxed{3y} + 3x - 4z &= 9 \end{aligned}$$

$$\begin{aligned} 3y + x - 5z &= 6\\ 2x + z &= 3\\ 2x + z &= 3 \quad \cdot(-1) \end{aligned}$$

$$\begin{aligned} 3y + x - 5z &= 6\\ 2x + z &= 3\\ \hline 0 &= 0 \ \checkmark \end{aligned}$$

$$\begin{aligned} x + 3y - 5z &= 6\\ 2x + z &= 3\\ 2x + z &= 3 \end{aligned}$$

$$\begin{aligned} 3y - 5z &= 6 - x\\ z &= 3 - 2x \end{aligned}$$

1 x neodređen

(jer sa desne strane jednakosti imamo samo promenljivu x)

$x = t,\ t \in \mathbb{R}$

$z = 3 - 2t$

$3y = 6 - t + 5(3 - 2t)$

$3y = 21 - 11t$

$y = 7 - \frac{11}{3}t$

$R_S = \{(t, 7 - \frac{11}{3}t, 3 - 2t) \mid t \in \mathbb{R}\}$

$$\begin{aligned} x + 3y - 5z &= 6\\ -6y + 11z &= -9\\ -6y + 11z &= -9 \quad \cdot(-1) \end{aligned}$$

$$\begin{aligned} x + 3y - 5z &= 6\\ -6y + 11z &= -9\\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} x + 3y &= 6 + 5z\\ -6y &= -9 - 11z \end{aligned}$$

1 x neodređen

$z = m,\ m \in \mathbb{R}$

$y = \frac{3}{2} + \frac{11}{6}m$

$x = 6 + 5m - \frac{9}{2} - \frac{11}{2}m$

$= \frac{3}{2} - \frac{1}{2}m$

$R_S = \{(\frac{3}{2} - \frac{1}{2}m, \frac{3}{2} + \frac{11}{6}m, m) \mid m \in \mathbb{R}\}$

$$1.2\quad \begin{array}{rcrcrcr} 3x & + & 3y & - & 5z & = & 6 \\ 2x & + & 2y & + & z & = & 3 \\ 3x & + & 3y & - & 4z & = & 9 \end{array}\ ;$$

$$\begin{aligned} -5z+3x+3y&=6 \\ z+2x+2y&=3 \\ -4z+3x+3y&=9 \end{aligned}$$

$$\begin{aligned} z+2x+2y&=3 \quad 5 \\ -5z+3x+3y&=6 \quad 4 \\ -4z+3x+3y&=9 \end{aligned}$$

$$\begin{aligned} z+2x+2y&=3 \\ 13x+13y&=21 \quad /:13 \\ 11x+11y&=21 \quad /:11 \end{aligned}$$

$$\begin{aligned} z+2x+2y&=3 \\ x+y&=\frac{21}{13} \\ x+y&=\frac{21}{11} \quad -1 \end{aligned}$$

$$\begin{aligned} z+2x+2y&=3 \\ x+y&=\frac{21}{11} \\ 0&=\underbrace{\frac{21}{11}-\frac{21}{13}}_{\frac{21\cdot 2}{11\cdot 13}} \end{aligned}$$

NEMOGUĆ SISTEM $\quad R_S=\emptyset$

$$1.3 \quad \begin{array}{rcrcr} x & + & y & = & 6 \\ 2x & + & y & = & 9 \\ 4x & + & 2y & = & 18 \end{array} \quad -2,\ -4$$

$$x + y = 6$$
$$-y = -3$$
$$-2y = -6 \quad -2$$

$$\cancel{x = 3}$$
$$\cancel{y = 3}$$

$$\boxed{x + y} = 6$$
$$-y = -3$$
$$0 = 0 \quad ✓$$

$$y = 3$$
$$x = 3$$
$$R_S = \{(3,3)\}$$

$$x + y = 6$$
$$2x + y = 9 \quad -2$$
$$4x + 2y = 20 \quad -4$$

$$x + y = 6$$
$$-y = -3 \quad \Rightarrow \quad \cancel{x = 3},\ \cancel{y = 3}$$
$$-2y = -4 \quad -2$$

$$x + y = 6$$
$$-y = -3$$
$$0 = 2$$

$$R_S = \emptyset$$

1.4 $\begin{array}{rcrcrcl} x & + & y & + & z & = & 5 \\ 3x & - & y & + & 2z & = & 12 \end{array}$ -3

$$\boxed{\begin{array}{l} x+y+z \\ \quad -4y-z \end{array}} \begin{array}{l} =5 \\ =-14 \end{array}$$

$$\boxed{\begin{array}{l} x+y \\ \quad -4y \end{array}} \begin{array}{l} =5-z \\ =-14+z \end{array}$$

Innsodreden

$z=t,\ t\in\mathbb{R}$

$y=\frac{7}{2}-\frac{1}{4}t$

$x=5-t-\frac{7}{2}+\frac{1}{4}t$

$=\frac{3}{2}-\frac{3}{4}t$

$$R_S=\left\{\left(\frac{3}{2}-\frac{3}{4}t,\ \frac{7}{2}-\frac{1}{4}t,\ t\right)\,\middle|\,t\in\mathbb{R}\right\}$$

1.5

$$\begin{aligned} x + y + 2z + 2u &= 5 \\ 2x + y - z - u &= 0 \\ 3x + 2y + z + u &= 5 \\ -x - y + 2z + u &= 3 \end{aligned}$$

-2, -3, +

$$x + y + 2z + 2u = 5$$
$$-y - 5z - 5u = -10$$
$$\cancel{-y - 5z - 5u = -10} \quad -1$$
$$4z + 3u = 8$$

$$x + y + 2z + 2u = 5$$
$$-y - 5z - 5u = -10$$
$$4z + 3u = 8$$

$$x + y + 2z = 5 - 2u$$
$$-y - 5z = -10 + 5u$$
$$4z = 8 - 3u$$

1 x neodreden

$u = t, \quad t \in \mathbb{R}$

$z = 2 - \frac{3}{4}t$

$-y = -10 + 5t + 5\left(2 - \frac{3}{4}t\right)$

$-y = \frac{5}{4}t$

$y = -\frac{5}{4}t$

$x = 5 - 2t - 2\left(2 - \frac{3}{4}t\right) + \frac{5}{4}t$

$= 1 + \frac{3}{4}t$

$R_S = \left\{\left(1 + \frac{3}{4}t, -\frac{5}{4}t, 2 - \frac{3}{4}t, t\right) \mid t \in \mathbb{R}\right\}$

1.6
$$\begin{array}{rcrcrcrcrcr}
2x & - & y & + & 3z & - & 2u & + & 4v & = & -1 \\
4x & - & 2y & + & 5z & + & u & + & 7v & = & 2 \\
2x & - & y & + & z & + & 8u & + & 2v & = & 1
\end{array}$$

-2 $\quad$ -1

$$\begin{array}{l}
2x - y + 3z - 2u + 4v = -1 \\
\qquad -z + 5u - v = 4 \\
\qquad -2z + 10u - 2v = 2
\end{array}$$

-2

$$\begin{array}{l}
2x - y + 3z - 2u + 4v = -1 \\
\qquad -z + 5u - v = 4 \\
\qquad\qquad 0 = 6
\end{array}$$

$\lightning$ $\qquad R_S = \emptyset$

1.7
$$\begin{array}{rcrcrcrcrcr} 2x & - & y & + & 3z & - & 2u & + & 4v & = & -1 \\ 4x & - & 2y & + & 5z & + & u & + & 7v & = & 2 \\ 2x & - & y & + & z & + & 8u & + & 2v & = & 7 \end{array}$$

-2

-1

$$2x - y + 3z - 2u + 4v = -1$$
$$-z + 5u - v = 4$$
$$-2z + 10u - 2v = 8 \quad -2$$

$$2x - y + 3z - 2u + 4v = -1$$
$$-z + 5u - v = 4$$
$$0 = 0 \quad \checkmark$$

$$-y + 3z = 1 - 2x + 2u - 4v$$
$$-z = 4 - 5u + v$$

3x neodrejen

$x = t,\ t \in \mathbb{R}$

$u = m,\ m \in \mathbb{R}$

$v = p,\ p \in \mathbb{R}$

$z = -4 + 5m - p$

$y = -1 + 2t - 2m + 4p + 3(-4 + 5m - p)$

$= -13 + 2t + 13m + p$

$$R_S = \{(t, -13 + 2t + 13m + p, -4 + 5m - p, m, p) \mid t, m, p \in \mathbb{R}\}$$

2. Odrediti realne parametre c i d tako da sistem bude neodređen i rešiti ga u slučaju neodređenosti.

$$\begin{aligned} 5x &+ 3y + z = -5 \\ x &- 2y + z = 2 \\ cx &+ 2y - z = d \end{aligned}$$

$D_s \neq 0 \Rightarrow$ sistem određen

$D_s = 0 \Rightarrow$ sistem nemoguć ili neodređen

Da bi sistem bio neodređen $D_s = 0$.

$$D_s = \begin{vmatrix} 5 & 3 & 1 \\ 1 & -2 & 1 \\ c & 2 & -1 \end{vmatrix} \begin{matrix} 5 & 3 \\ 1 & -2 \\ c & 2 \end{matrix} = \cancel{10} + 3c + 2 + 2c - \cancel{10} + 3 = 5c + 5$$

$5c + 5 = 0$

$\boxed{c = -1}$

Kada je $D_s = 0$ onda je sistem ili nemoguć ili neodređen i moramo se vratiti na Gausov postupak.

$$\begin{aligned} 5x + 3y + z &= -5 \\ x - 2y + z &= 2 \\ -x + 2y - z &= d \end{aligned}$$

$$\begin{aligned} x - 2y + z &= 2 \\ -x + 2y - z &= d \\ 5x - 2y + z &= 2 \end{aligned}$$

$$\begin{aligned} x - 2y + z &= 2 \\ \boxed{0 = d + 2} \\ 8y - 4z &= -8 \quad /:4 \end{aligned}$$

za $d \neq -2$

za $\boxed{d = -2}$ NEODREĐEN

$$\begin{aligned} x - 2y + z &= 2 \\ 2y - z &= -2 \end{aligned}$$

$$\begin{aligned} x + z &= 2 + 2y \\ -z &= -2 - 2y \end{aligned}$$

1x neodređen

$y = t,\ t \in \mathbb{R}$

$z = 2 + 2t$

$x = \cancel{2} + \cancel{2t} - \cancel{2} - \cancel{2t}$

$x = 0$

$R_s = \{(0, t, 2 + 2t) \mid t \in \mathbb{R}\}$

3. Odrediti realan parametar b tako da sistem bude nemoguć.

$$\begin{array}{rcrcrcl} x & + & y & - & 3z & = & 1 \\ 2x & + & y & - & 2z & = & 1 \\ x & + & y & + & z & = & b \\ x & + & 2y & - & 3z & = & 1 \end{array} \quad \begin{array}{l} \\ \cdot(-2) \\ \cdot(-1) \\ \cdot(-1) \end{array}$$

$$\begin{array}{rcl} x+y-3z & = & 1 \\ -y+4z & = & -1 \\ 4z & = & b-1 \\ y & = & 0 \end{array}$$

$$\boxed{y=0}$$

$$\begin{array}{rcl} x-3z & = & 1 \\ 4z & = & -1 \\ 4z & = & b-1 \end{array} \quad \cdot(-1)$$

$$\begin{array}{rcl} y & = & 0 \\ x-3z & = & 1 \\ 4z & = & -1 \\ 0 & = & b \end{array}$$

$$b \neq 0$$

4. U zavisnosti od realnog parametra a, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$\begin{aligned} ax + (a-1)y + z &= 1 \\ x - y + az &= a \\ - ay + az &= 2 \end{aligned}$$

$$D_S = \begin{vmatrix} a & a-1 & 1 \\ 1 & -1 & a \\ 0 & -a & a \end{vmatrix} \begin{matrix} a & a-1 \\ 1 & -1 \\ 0 & -a \end{matrix} = -a^2 - a + a^3 - a(a-1)$$

$$= -a^2 - \cancel{a} + a^3 - a^2 + \cancel{a}$$

$$= a^3 - 2a^2$$

$$= a^2(a-2)$$

I $D_S \neq 0$ za $a \neq 0 \wedge a \neq 2 \Rightarrow$ sistem određen

II $D_S = 0$ za $a = 0 \vee a = 2 \Rightarrow$ sistem nemoguć ili neodređen

i moramo dalje preći na Gausov postupak!

$\boxed{a=0}$

$$\begin{aligned} -y + z &= 1 \\ x - y \quad &= 0 \\ 0 &= 2 \end{aligned}$$

$\lightning \quad R_S = \emptyset$

$\boxed{a=2}$

$$\begin{aligned} 2x + y + z &= 1 \\ x - y + 2z &= 2 \\ -2y + 2z &= 2 \,/ :2 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \quad / \cdot (-2) \\ 2x + y + z &= 1 \\ -y + z &= 1 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ 3y - 3z &= -3 \,/ :3 \\ -y + z &= 1 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ y - z &= -1 \\ -y + z &= 1 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ y - z &= -1 \end{aligned}$$

$0 = 0$ w

$$\begin{aligned} x - y &= 2 - 2z \\ y &= -1 + z \end{aligned}$$

1x neodređen

$z = t, t \in \mathbb{R}$

$y = -1 + t$

$x = 2 - 2t + t - 1$

$x = 1 - t$

$R_S = \{(1-t, t-1, t) \mid t \in \mathbb{R}\}$

ZA VEŽBU

5. U zavisnosti od realnog parametra a, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & a \\ x & + & (a+1)\,y & + & z & = & 2a \\ x & + & y & + & az & = & -a \end{array}$$

6. U zavisnosti od realnog parametra a, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$\begin{aligned} x - 2y + (a+1)z &= 3 \\ 5x + 2y &= 1 \\ ax + 2z &= 2 \end{aligned}$$

$$D_S = \begin{vmatrix} 1 & -2 & a+1 \\ 5 & 2 & 0 \\ a & 0 & 2 \end{vmatrix} \begin{matrix} 1 & -2 \\ 5 & 2 \\ a & 0 \end{matrix} = 4 - 2a(a+1) + 20$$

$$= 4 - 2a^2 - 2a + 20 = -2a^2 - 2a + 24$$

$$= -2(a^2 + a - 12)$$

$$a_{1,2} = \frac{-1 \pm \sqrt{1+48}}{2} = \frac{-1 \pm 7}{2} = \begin{cases} 3 \\ -4 \end{cases}$$

I $D_S \neq 0$ za $a \neq 3 \wedge a \neq -4$

$\Rightarrow$ sistem određen

II $D_S = 0$ za $\boxed{a=3}$ $\vee$ $\boxed{a=-4}$

$\Rightarrow$ sistem nemoguć ili neodređen

$\boxed{a=3}$

$$\begin{aligned} x - 2y + 4z &= 3 \\ 5x + 2y &= 1 \\ 3x + 2z &= 2 \end{aligned}$$

$$\begin{aligned} x - 2y + 4z &= 3 \\ 12y - 20z &= -14 \quad /:2 \\ 6y - 10z &= -7 \end{aligned}$$

$$\begin{aligned} x - 2y + 4z &= 3 \\ 6y - 10z &= -7 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} x - 2y &= 3 - 4z \\ 6y &= -7 + 10z \end{aligned}$$

1 x neodređen

$z = t, t \in \mathbb{R}$

$y = -\frac{7}{6} + \frac{5}{3}t$

$x = 3 - 4t - \frac{7}{3} + \frac{10}{3}t$

$= \frac{2}{3} - \frac{2}{3}t$

$R_S = \{(\frac{2}{3} - \frac{2}{3}t, -\frac{7}{6} + \frac{5}{3}t, t) \mid t \in \mathbb{R}\}$

$\boxed{a=-4}$

$$\begin{aligned} x - 2y - 3z &= 3 \\ 5x + 2y &= 1 \\ -4x + 2z &= 2 \end{aligned}$$

$$\begin{aligned} x - 2y - 3z &= 3 \\ 12y + 15z &= -14 \\ -8y - 10z &= 14 \quad /:2 \end{aligned}$$

$$\begin{aligned} x - 2y - 3z &= 3 \\ -4y - 5z &= 7 \\ 12y + 15z &= -14 \end{aligned}$$

$$\begin{aligned} x - 2y - 3z &= 3 \\ -4y - 5z &= 7 \\ 0 &= 7 \end{aligned}$$

$R_S = \emptyset$

7. U zavisnosti od realnog parametra a, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga.

$$\begin{aligned} (a-1)x &= 0 \\ x + (a-1)y &= 0 \\ y + (a-1)z &= 0 \end{aligned}$$

$$D_S = \left|\begin{matrix} a-1 & 0 & 0 \\ 1 & a-1 & 0 \\ 0 & 1 & a-1 \end{matrix}\right| \begin{matrix} a-1 & 0 \\ 1 & a-1 \\ 0 & 1 \end{matrix} = (a-1)^3$$

I $D_S \neq 0$ za $a \neq 1 \Rightarrow$ sistem je određen i rešenje mu je $(0,0,0)$

II $D_S = 0$ za $a = 1 \Rightarrow$ sistem je neodređen (homogen sistem nikada ne može biti nemoguć)

$\boxed{a=1}$

$$\begin{aligned} 0 &= 0 \\ x &= 0 \\ y &= 0 \end{aligned} \qquad z = t, \; t \in \mathbb{R}$$

$R_S = \{(0,0,t) \mid t \in \mathbb{R}\}$

8. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$
\begin{array}{rcrcrcr}
x & + & 2y & + & z & = & 3 \\
2x & + & 3y & - & 3z & = & b \\
x & + & ay & + & 6z & = & -2
\end{array}
$$

$$D_s = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & -3 \\ 1 & a & 6 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 3 \\ 1 & a \end{matrix} = 18 - 6 + 2a - 3 + 3a - 24$$

$$= 5a - 15 = 5(a-3)$$

I $D_s \neq 0$ za $a \neq 3$ $(b \in \mathbb{R})$ $\Rightarrow$ sistem određen

II $D_s = 0$ za $a = 3$ $\Rightarrow$ sistem nemoguć ili neodređen

$\boxed{a=3}$

$$
\begin{array}{l}
x+2y+z=3 \quad \cdot(-2) \\
2x+3y-3z=b \\
x+3y+6z=-2 \quad \cdot(-1)
\end{array}
$$

$$
\begin{array}{l}
x+2y+z=3 \\
-y-5z=b-6 \\
y+5z=-5 \quad +
\end{array}
$$

$$
\begin{array}{l}
x+2y+z=3 \\
-y-5z=b-6 \\
0=b-11
\end{array}
$$

za $b \neq 11 \Rightarrow R_s = \emptyset$

za $b = 11$

$$
\begin{array}{l}
x+2y+z=3 \\
-y-5z=5
\end{array}
$$

$$
\begin{array}{l}
x+2y=3-z \\
-y=5+5z
\end{array}
$$

1x neodređen

$z=t,\ t \in \mathbb{R}$

$y=-5-5t$

$x=3-t+10+10t$

$x=13+9t$

$R_s=\{(13+9t, -5-5t, t) \mid t \in \mathbb{R}\}$

ZA VEŽBU

9. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$\begin{array}{rcrcrcr} -ax & & & - & 3z & = & 3b \\ x & + & (a+1)\,y & & & = & 1 \\ ax & - & 2y & + & 4z & = & -2 \end{array}$$

10. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$
\begin{array}{rcrcrcr}
(a-3)x & + & y & - & z & = & ab \\
-2x & + & 2(a-2)y & - & 2z & = & 3b \\
(a-2)x & + & 2y & & & = & b
\end{array}
$$

$$D_S = \begin{vmatrix} a-3 & 1 & -1 \\ -2 & 2(a-2) & -2 \\ a-2 & 2 & 0 \end{vmatrix} \begin{matrix} a-3 & 1 \\ -2 & 2(a-2) \\ a-2 & 2 \end{matrix}$$

$$= -2(a-2)+4+2(a-2)^2+4(a-3)$$

$$= -2a+4+\cancel{4}+2a^2-8a+\cancel{8}+4a-12$$

$$= 2a^2-6a+4 = 2(a^2-3a+2)$$

$$a_{1,2} = \frac{3\pm\sqrt{9-8}}{2} = \frac{3\pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

I $D_S \neq 0$ za $a \neq 1 \wedge a \neq 2$

$\rightarrow$ sistem određen

II $D_S = 0$ za $(a=1) \vee \boxed{a=2}$

$\rightarrow$ sistem nemoguć ili neodređen

$\boxed{a=1}$

$$
\begin{aligned}
-2x+y-z &= b \\
-2x-2y-2z &= 3b \\
-x+2y &= b
\end{aligned}
$$

$$
\begin{aligned}
-z-2x+y &= b \quad \cdot(-2) \\
-2z-2x-2y &= 3b \\
-x+2y &= b
\end{aligned}
$$

$$
\begin{aligned}
-z-2x+y &= b \\
2x-4y &= b \\
-x+2y &= b
\end{aligned}
$$

$$
\begin{aligned}
-z-2x+y &= b \\
-x+2y &= b \\
2x-4y &= b \quad \cdot 2
\end{aligned}
$$

$$
\begin{aligned}
-z-2x+y &= b \\
-x+2y &= b \\
0 &= 3b
\end{aligned}
$$

za $b \neq 0$ $\rightarrow$ $R_S = \emptyset$

za $b = 0$

$$
\begin{aligned}
-z-2x+y &= 0 \\
-x+2y &= 0
\end{aligned}
$$

$$
\begin{aligned}
-z-2x &= -y \\
-x &= -2y
\end{aligned}
$$

1x neodređen

$y = t, \quad t \in \mathbb{R}$

$x = 2t$

$z = -3t$

$R_S = \{(2t, t, -3t) \mid t \in \mathbb{R}\}$

$\boxed{a=2}$

$$-x+y-z=2b$$
$$-2x\quad -2z=3b \quad /\cdot(-2)$$
$$2y=b$$

$$-x+y-z=2b$$
$$-2y=-b$$
$$2y=b \quad /+$$

$$-x+y-z=2b$$
$$-2y=-b$$
$$0=0 \checkmark$$

$$-x+y=2b+z$$
$$2y=b$$

1x neodreden

$$z=m,\ m\in\mathbb{R}$$
$$y=\frac{b}{2}$$
$$x=\frac{b}{2}-2b-m=-\frac{3}{2}b-m$$

$$R_S=\left\{\left(-\frac{3}{2}b-m,\ \frac{b}{2},\ m\right)\ \middle|\ m\in\mathbb{R}\right\}$$

ZA VEŽBU

11. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja sistem linearnih jednačina i rešiti ga u slučaju neodređenosti.

$$\begin{array}{rcrcl} ax & & + \; bz & = & -a \\ & (a+b)\,y & & = & b \\ bx & & + \; az & = & 2a \end{array}$$

12. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja i rešiti sistem linearnih jednačina

$$\begin{array}{rcrcrcl} ax & + & y & + & z & = & 1 \\ -2ax & + & (a-3)\,y & + & az & = & b-2 \end{array}$$

$$\begin{aligned} ax + y + z &= 1 \\ (a-1)y + (a+2)z &= b \end{aligned}$$

I $a \neq 0 \wedge a \neq 1$

$$\begin{aligned} ax + y &= 1 - z \\ (a-1)y &= b - (a+2)z \end{aligned}$$

1x neodređen

$z = t,\ t \in \mathbb{R}$

$y = \dfrac{b-(a+2)t}{a-1}$

$x = \dfrac{1}{a}\left(1 - t - \dfrac{b-(a+2)t}{a-1}\right)$

$R_S = \left\{\left(\dfrac{1}{a}\left(1-t-\dfrac{b-(a+2)t}{a-1}\right), \dfrac{b-(a+2)t}{a-1}, t\right) \middle| t \in \mathbb{R}\right\}$

II $\boxed{a=0}$

$$\begin{aligned} y + z &= 1 \\ 3z &= b \end{aligned}$$

$z = \dfrac{b}{3}$

$y = 1 - \dfrac{b}{3}$

$x = m,\ m \in \mathbb{R}$

$R_S = \left\{\left(m, 1-\dfrac{b}{3}, \dfrac{b}{3}\right) \middle| m \in \mathbb{R}\right\}$

III $\boxed{a=1}$

$$\begin{aligned} x + y + z &= 1 \\ 3z &= b \end{aligned}$$

$$\begin{aligned} x + z &= 1 - y \\ 3z &= b \end{aligned}$$

$y = p,\ p \in \mathbb{R}$

$z = \dfrac{b}{3}$

$x = 1 - p - \dfrac{b}{3}$

$R_S = \left\{\left(1-p-\dfrac{b}{3}, p, \dfrac{b}{3}\right) \middle| p \in \mathbb{R}\right\}$

13. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja i rešiti sistem linearnih jednačina

$$\begin{array}{rcrcrcrcl} x & + & 2y & - & 3az & + & 4u & = & 1 \\ ax & - & 2y & + & z & - & 2u & = & b \end{array}.$$

14. U zavisnosti od realnih parametara a i b, diskutovati prirodu rešenja i rešiti sistem linearnih jednačina

$$\begin{array}{rcrcrcrcl} -x & + & (a-2)\,y & + & az & + & (a-1)\,u & = & 1 \\ ax & + & (a-2)\,y & + & az & - & u & = & b \\ ax & + & (a-2)\,y & - & z & + & au & = & b \end{array}.$$

ZA VEŽBU:IZ SKRIPTE
Zadatak 9.5, 9.6, 9.7, 9.8, 9.9,
teži: 9.10, 9.17