$z=|z|e^{\varphi i}$

$z=|z|(\cos\varphi+i\sin\varphi)$

Primer: Predstaviti u eksponencijalnom i trigonometrijskom obliku kompleksne brojeve

a) $z_1 = 2\sqrt{3}+2i$, b) $z_2 = -5+5i$, c) $z_3 = 4-4\sqrt{3}i$,

d) $z_4 = -3-3i$, e) $z_5 = 5$, f) $z_6 = -6$,

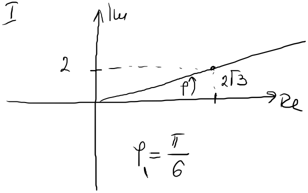
g) $z_7 = 5i$, h) $z_8 = -6i$.

a) $z_1 = 2\sqrt{3}+2i$

$$z = 4e^{\frac{\pi}{6}i} = 4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

$$|z_1| = \sqrt{(2\sqrt{3})^2+2^2} = \sqrt{12+4} = 4$$

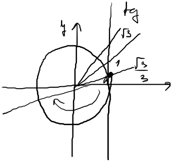
I



$\varphi_1 = \frac{\pi}{6}$

II

$$tg\,\varphi_1 = \frac{y}{x} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$\varphi_1 \in \left\{\frac{\pi}{6}, \cancel{-\frac{5\pi}{6}}\right\}$

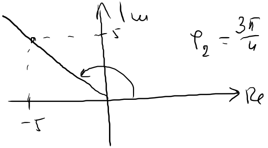
$\varphi_1 = \frac{\pi}{6}$

III

$$\left.\begin{array}{l}\sin\varphi_1 = \frac{2}{4} = \frac{1}{2}\\ \cos\varphi_1 = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}\end{array}\right\}\ \varphi_1 = \frac{\pi}{6}$$

b) $z_2=-5+5i$

$|z_2|=\sqrt{(-5)^2+5^2}=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$



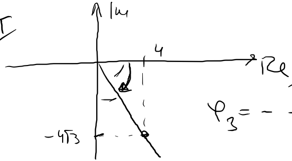
$\varphi_2=\frac{3\pi}{4}$

$z_2=5\sqrt{2}\,e^{\frac{3\pi}{4}i}=5\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

c) $z_3=4-4\sqrt{3}i=8\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$

$|z_3|=\sqrt{16+48}=8$

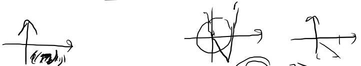
I



$\varphi_3=-\frac{\pi}{3}$

II $\sin\varphi_3=\frac{-4\sqrt{3}}{8}=-\frac{\sqrt{3}}{2}$

$\cos\varphi_3=\frac{4}{8}=\frac{1}{2}$ $\Bigg\}$ $\varphi_3=-\frac{\pi}{3}$



III $tg\,\varphi_3=\frac{-4\sqrt{3}}{4}=-\sqrt{3}\Rightarrow\varphi_3=\left\{-\frac{\pi}{3},\ \cancel{\frac{2\pi}{3}}\right\}$

$\Rightarrow z_3=8e^{-\frac{\pi}{3}i}=8\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)$

d) $z_4=-3-3i$

$|z_4|=\sqrt{9+9}=3\sqrt{2}$

I $\sin\varphi_4=\frac{-3}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$

$\cos\varphi_4=\frac{-3}{3\sqrt{2}}=-\frac{\sqrt{2}}{2}$ $\Bigg\}$ $\varphi_4=-\frac{3\pi}{4}$

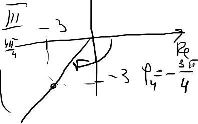
$\Rightarrow z_4=3\sqrt{2}\,e^{-\frac{3\pi}{4}i}$

$=3\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$

II $tg\,\varphi_4=\frac{-3}{-3}=1$

$\varphi_4\in\left\{\cancel{\frac{\pi}{4}},\ -\frac{3\pi}{4}\right\}$

III

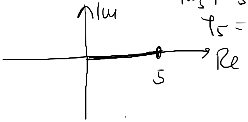


$\varphi_4=-\frac{3\pi}{4}$

e) $z_5 = 5$

$|z_5| = 5$

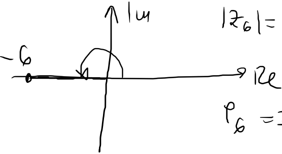
$\varphi_5 = 0$



$z_5 = 5\cdot e^{0i} = 5(\cos 0 + i\sin 0)$

f) $z_6 = -6$

$|z_6| = 6$



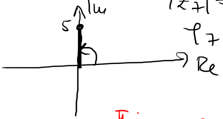
$\varphi_6 = \pi$

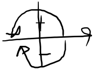
$z_6 = 6e^{\pi i} = 6(\cos\pi + i\sin\pi)$

g) $z_7 = 5i$

$|z_7| = 5$

$\varphi_7 = \frac{\pi}{2}$



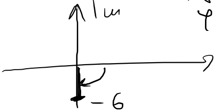


$z_7 = 5\cdot e^{\frac{\pi}{2}i} = 5(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

h) $z_8 = -6i$

$|z_8| = 6$

$\varphi_8 = -\frac{\pi}{2}$



$\frac{3\pi}{2} \notin [-\pi, \pi]$

$z_8 = 6e^{-\frac{\pi}{2}i}$

$= 6(\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2}))$

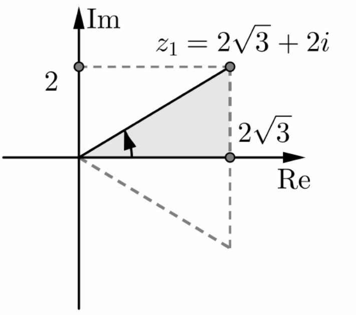
Rešenje: Kako je svaki kompleksan broj u eksponencijalnom (trigonometrijskom) obliku određen svojim modulom i argumentom, to za svaki od ovih brojeva treba odrediti moduo i argument.

a) Kako je za kompleksan broj

$$z_1 = 2\sqrt{3} + 2i$$

moduo $|z_1| = \sqrt{12+4} = 4$, a
argument $\arg(z_1) = \frac{\pi}{6}$,
to je

$$z_1 = 4e^{\frac{\pi}{6}i} = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right).$$

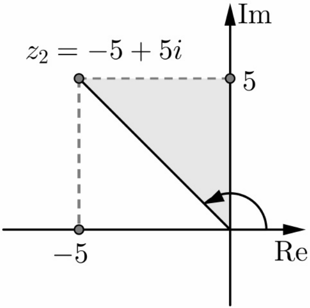


b) Kako je za kompleksan broj

$$z_2 = -5 + 5i$$

moduo $|z_2| = \sqrt{25+25} = 5\sqrt{2}$,
a argument $\arg(z_2) = \dfrac{3\pi}{4}$,
to je

$$z_2 = 5\sqrt{2} e^{\frac{3\pi}{4} i} = 5\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right).$$

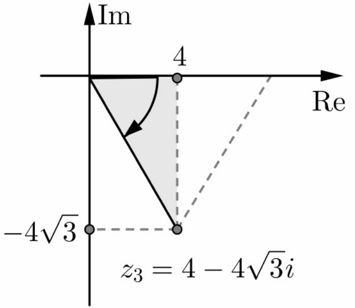


c) Kako je za kompleksan broj

$$z_3 = 4 - 4\sqrt{3}i$$

moduo $|z_3| = \sqrt{16+48} = 8$, a argument $\arg(z_3) = -\frac{\pi}{3}$,
to je

$$z_3 = 8e^{-\frac{\pi}{3}i}$$
$$= 8\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right).$$



d) Kako je za kompleksan broj

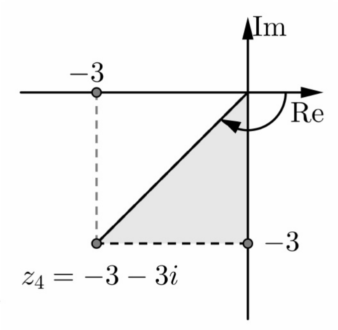
$$z_4 = -3 - 3i$$

moduo $|z_4| = \sqrt{9+9} = 3\sqrt{2}$, a

argument $\arg(z_4) = -\dfrac{3\pi}{4}$,

to je

$$\begin{aligned} z_4 &= 3\sqrt{2}e^{-\frac{3\pi}{4}i} \\ &= 3\sqrt{2}\left(\cos\left(-\tfrac{3\pi}{4}\right) + i\sin\left(-\tfrac{3\pi}{4}\right)\right). \end{aligned}$$

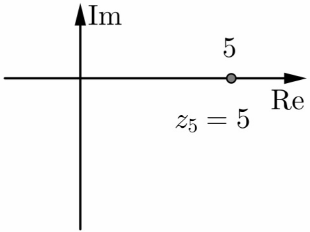


e) Sa slike se vidi da je za kompleksan broj

$$z_5 = 5$$

moduo $|z_5| = 5$, a argument $\arg(z_5) = 0$, pa je

$$\begin{aligned} z_5 &= 5e^{0i} \\ &= 5(\cos 0 + i \sin 0). \end{aligned}$$

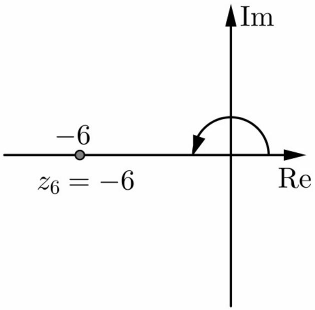


f) Sa slike se vidi da je za kompleksan broj

$$z_6 = -6$$

moduo $|z_6| = 6$, a argument $\arg(z_6) = \pi$, pa je

$$\begin{aligned} z_6 &= 6e^{\pi i} \\ &= 6(\cos\pi + i\sin\pi). \end{aligned}$$

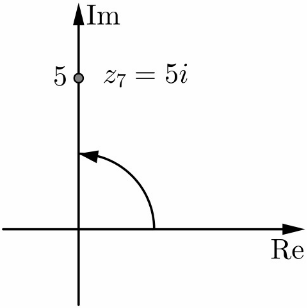


g) Sa slike se vidi da je za kompleksan broj

$$z_7 = 5i$$

moduo $|z_7| = 5$, a
argument $\arg(z_7) = \frac{\pi}{2}$, pa je

$$\begin{aligned} z_7 &= 5e^{\frac{\pi}{2}i} \\ &= 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right). \end{aligned}$$

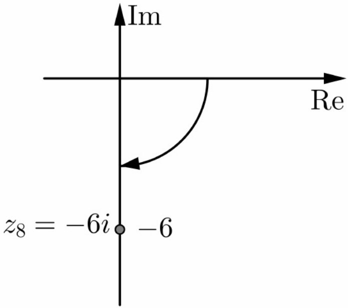


h) Sa slike se vidi da je za kompleksan broj

$$z_8 = -6i$$

moduo $|z_8| = 6$, a argument $\arg(z_8) = -\frac{\pi}{2}$, pa je

$$z_8 = 6e^{-\frac{\pi}{2}i}$$
$$= 6\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right).$$



$\boxed{e^{\varphi i} = \cos\varphi + i\sin\varphi}$

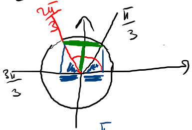
$\cos(-\alpha) = \cos\alpha$
$\sin(-\alpha) = -\sin\alpha$

Primer: Predstaviti u algebarskom obliku kompleksne brojeve

a) $z_1 = 2e^{\frac{\pi}{4}i}$, b) $z_2 = e^{-\frac{2\pi}{3}i}$, c) $z_3 = 3e^{\pi i}$, d) $z_4 = 5e^{-\frac{\pi}{2}i}$

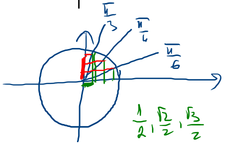
a) $z_1 = 2e^{\frac{\pi}{4}i} = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \sqrt{2} + \sqrt{2}i$

b) $z_2 = e^{-\frac{2\pi}{3}i} = \cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right) = \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

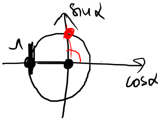


$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$



c) $z_3 = 3e^{\pi i} = 3(\cos\pi + i\sin\pi)$
$= 3(-1 + i\cdot 0) = -3$



d) $z_4 = 5\cdot e^{-\frac{\pi}{2}i} = 5\cdot\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$
$= 5\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$
$= 5(0 - i\cdot 1)$
$= -5i$

Rešenje:

a) $z_1 = 2e^{\frac{\pi}{4}i} = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \sqrt{2} + \sqrt{2}i,$

b) $z_2 = e^{-\frac{2\pi}{3}i} = \cos\frac{2\pi}{3} - i\sin\frac{3\pi}{3} = \frac{\sqrt{3}}{2} - \frac{1}{2}i,$

c) $z_3 = 3e^{\pi i} = 3(\cos\pi + i\sin\pi) = 3(-1+0) = -3,$

d) $z_4 = 5e^{-\frac{\pi}{2}i} = 5\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right) = 5(0-i) = -5i.$

Kompleksni brojevi

$$z_1 = |z_1|\left(\cos\varphi_1 + i\sin\varphi_1\right) = |z_1|e^{\varphi_1 i} \quad \text{i}$$

$$z_2 = |z_2|\left(\cos\varphi_2 + i\sin\varphi_2\right) = |z_2|e^{\varphi_2 i}$$

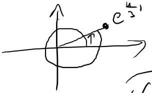
su **jednaki** akko je

$$|z_1| = |z_2| \quad \wedge \quad \varphi_1 = \varphi_2 + 2k\pi, \quad k \in \mathbb{Z}.$$

$\frac{7\pi}{3} \notin (-\pi, \pi]$

$e^{\frac{7\pi}{3} i} = e^{\left(\frac{\pi}{3}+2\pi\right) i}$

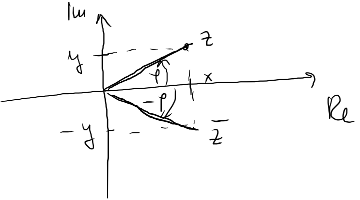
$= e^{\frac{\pi}{3} i}$



$e^{\frac{\pi}{3} i} = e^{\frac{7\pi}{3} i} = e^{\frac{13\pi}{3} i}$

$\boxed{(-\pi, \pi]}$

Konjugovano kompleksan broj broja $z = |z|(\cos\varphi + i\sin\varphi) = |z|e^{\varphi i}$ je $\overline{z} = |z|(\cos\varphi - i\sin\varphi) = |z|e^{-\varphi i}$.



$z = x + yi$

$\overline{z} = x - yi$

$|\overline{z}| = |z|$

$z = 5e^{\frac{3\pi}{4} i}$

$\overline{z} = 5e^{-\frac{3\pi}{4} i}$

Operacije sa kompleksnim brojevima u eksponencijalnom (trigonometrijskom) obliku

$$2^x \cdot 2^y = 2^{x+y}$$

$$\frac{2^x}{2^y} = 2^{x-y}$$

Neka je

$$z_1 = |z_1| (\cos \varphi_1 + i \sin \varphi_1) = |z_1| e^{\varphi_1 i} \quad \text{i}$$

$$z_2 = |z_2| (\cos \varphi_2 + i \sin \varphi_2) = |z_2| e^{\varphi_2 i}.$$

Tada je

$$z_1 \cdot z_2 = |z_1||z_2| (\cos (\varphi_1 + \varphi_2) + i \sin (\varphi_1 + \varphi_2)) = |z_1||z_2| e^{(\varphi_1 + \varphi_2) i},$$

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} (\cos (\varphi_1 - \varphi_2) + i \sin (\varphi_1 - \varphi_2)) = \underline{\frac{|z_1|}{|z_2|} e^{(\varphi_1 - \varphi_2) i}}.$$

Stepenovanje kompleksnog broja $z = |z|(\cos\varphi + i\sin\varphi) = |z|e^{\varphi i}$ se radi po formuli

$$z^n = |z|^n(\cos n\varphi + i\sin n\varphi) = |z|^n e^{n\varphi i}, \;\; n \in \mathbb{N}.$$

$(2^x)^y = 2^{xy}$

$(\cos\varphi + i\sin\varphi)^n = \cos n\varphi + i\sin n\varphi$

Korenovanje kompleksnog broja $z = |z|(\cos\varphi + i\sin\varphi) = |z|e^{\varphi i}$ se radi po formuli

$$z_k = \sqrt[n]{|z|}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right) = \sqrt[n]{|z|}e^{\frac{\varphi+2k\pi}{n}i},$$

$$k = 0, 1, \ldots, n-1, \;\; n \in \mathbb{N}.$$

$\cancel{z = |z|e^{\varphi_i}}$

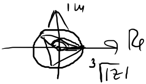
$\cancel{\sqrt[n]{z} = z^{\frac{1}{n}}}$

$\cancel{= |z|^{\frac{1}{n}} e^{\frac{\varphi_i}{n}}}$

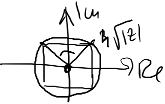
$\sqrt[n]{z} \neq z^{\frac{1}{n}}$

Za kompleksan broj z važi da $\sqrt[n]{z}$ $(z \neq 0)$ ima n različitih vrednosti, koje predstavljene u kompleksnoj ravni obrazuju temena pravilnog n-tougla upisanog u kružnicu sa centrom u koordinatnom početku, poluprečnika $\sqrt[n]{|z|}$. Svako od tih temena može se dobiti od prethodnog temena njegovom rotacijom oko koordinatnog početka za ugao $\dfrac{2\pi}{n}$.

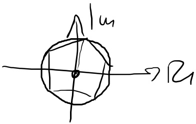
$\sqrt[3]{z}$



$\sqrt[4]{z}$



$\sqrt[5]{z}$

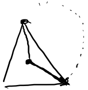


$$z = |z| e^{\varphi i}$$

$$\sqrt[n]{z} = \sqrt[n]{|z|}\, e^{\frac{\varphi + 2k\pi}{n} i}, \quad k = 0, 1, \dots, n-1$$

$k = 0 \qquad \frac{\varphi}{n} \qquad e^{\frac{\varphi}{n} i} = e^{\left(\frac{\varphi}{n} + 2\pi\right) i}$

$k = n \qquad \frac{\varphi + 2n\pi}{n} = \frac{\varphi}{n} + 2\pi$



$\frac{2\pi}{n}$

$\sqrt[n]{z}, \quad z \neq 0$

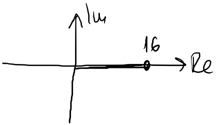
$\sqrt[n]{|z|}$

Primer: U skupu kompleksnih brojeva, izračunati $\sqrt[4]{16}$ u algebarskom obliku i rešenja predstaviti u kompleksnoj ravni.

$$\sqrt[4]{16} = \sqrt[4]{16e^{0i}} = \sqrt[4]{16}\cdot e^{\frac{0+2k\pi}{4}i} = 2e^{\frac{k\pi}{2}i}$$

$k=0,1,2,3$ $\quad k=0,1,2,3$

$$16 = 16\cdot e^{0i}$$



$k=0, \quad z_0 = 2\cdot e^{\frac{0\cdot\pi}{2}i} = \boxed{2e^{0i}} = 2$

$k=1, \quad z_1 = 2\cdot e^{\frac{1\cdot\pi}{2}i} = \boxed{2e^{\frac{\pi}{2}i}} = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2(0+i) = 2i$

$k=2, \quad z_2 = 2\cdot e^{\frac{2\pi}{2}i} = \boxed{2e^{\pi i}} = 2(\cos\pi + i\sin\pi) = 2(-1+i\cdot 0) = -2$

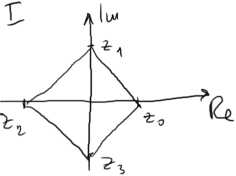
$k=3, \quad z_3 = 2\cdot e^{\frac{3\pi}{2}i} = \boxed{2e^{-\frac{\pi}{2}i}} = 2\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$

$= 2\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$

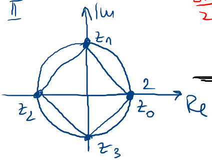
$= 2(0 - i\cdot 1) = -2i$

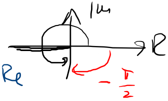
$\frac{3\pi}{2} \notin (-\pi, \pi)$

I



II





Rešenje: Kako je $16 = 16e^{0i}$, to je

$$\sqrt[4]{16} = \sqrt[4]{16e^{0i}} = 2e^{\frac{0+2k\pi}{4}i} = 2e^{\frac{k\pi}{2}i},\ k = 0, 1, 2, 3.$$

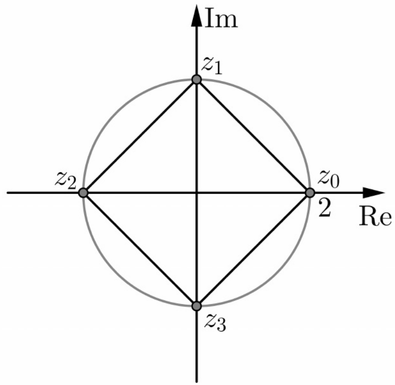
Za svako k dobija se po jedno rešenje:

$$k = 0 \implies z_0 = 2e^{0i} = 2,$$

$$k = 1 \implies z_1 = 2e^{\frac{\pi}{2}i} = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i,$$

$$k = 2 \implies z_2 = 2e^{\pi i} = 2(\cos\pi + i\sin\pi) = -2,$$

$$k = 3 \implies z_3 = 2e^{\frac{3\pi}{2}i} = 2e^{-\frac{\pi}{2}i} = 2\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right) = -2i.$$


Im
z_1
z_2
z_0
2
Re
z_3

Primer: U skupu kompleksnih brojeva rešiti jednačinu

$$z^3 + 27 = 0.$$

Rešenja napisati u algebarskom obliku i predstaviti u kompleksnoj ravni.

$$z^3 + 27 = 0$$

$$z^3 = -27$$

$$z = \sqrt[3]{-27}$$

$$-27 = 27e^{\pi i}$$

$$z = \sqrt[3]{-27} = \sqrt[3]{27 \cdot e^{\pi i}} = \underbrace{\sqrt[3]{27}}_{3} \cdot e^{\frac{\pi + 2k\pi}{3} i}, \quad k = 0, 1, 2$$

$$k=0, \quad z_0 = \boxed{3e^{\frac{\pi}{3}i}} = 3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i = \frac{3}{2}(1 + \sqrt{3}i)$$

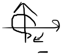
$$k=1, \quad z_1 = 3e^{\frac{3\pi}{3}i} = \boxed{3e^{\pi i}} = 3(\cos\pi + i\sin\pi) = 3(-1 + i\cdot 0) = -3$$

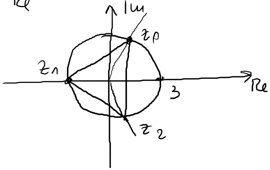
$$k=2 \quad z_2 = 3\cdot e^{\frac{5\pi}{3}i} = \boxed{3\cdot e^{-\frac{\pi}{3}i}} = 3\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = 3\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$\frac{5\pi}{3} \notin (-\pi, \pi]$$

$$\pi = \frac{3\pi}{3}$$

$$2\pi = \frac{6\pi}{3}$$





Rešenje: Jednačina $z^3 + 27 = 0$ ekvivalentna je sa $z = \sqrt[3]{-27}$. Kako je $-27 = 27e^{\pi i}$, to je

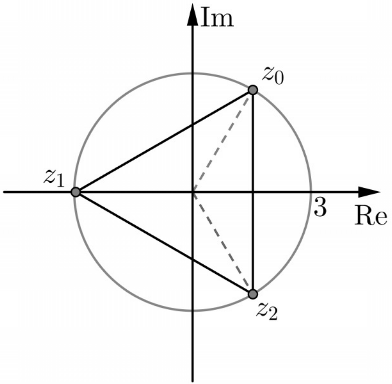
$$z = \sqrt[3]{27e^{\pi i}} = 3e^{\frac{\pi + 2k\pi}{3} i}, \; k = 0, 1, 2.$$

Za svako k dobija se po jedno rešenje:

$$k = 0 \implies z_0 = 3e^{\frac{\pi}{3} i} = 3\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2} i,$$

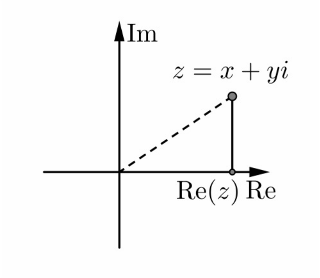
$$k = 1 \implies z_1 = 3e^{\pi i} = 3\left(\cos\pi + i \sin\pi\right) = -3,$$

$$k = 2 \implies z_2 = 3e^{\frac{5\pi}{3} i} = 3e^{-\frac{\pi}{3} i} = 3\left(\cos\frac{\pi}{3} - i \sin\frac{\pi}{3}\right) = \frac{3}{2} - \frac{3\sqrt{3}}{2} i.$$

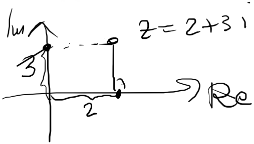


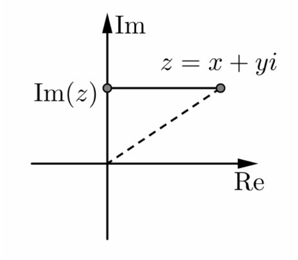
Im
z_0
z_1
3
Re
z_2

GEOMETRIJSKE INTERPRETACIJE U KOMPLEKSNOJ RAVNI

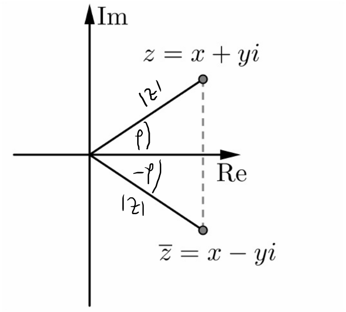
- 

  Projekcija tačke z na realnu osu je $\mathrm{Re}\,(z)$.

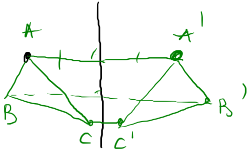
  

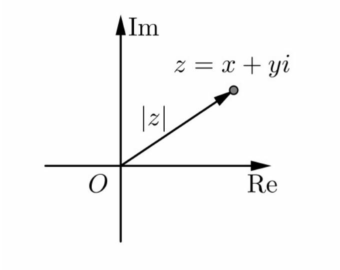
- 

  Projekcija tačke z na imaginarnu osu je $\mathrm{Im}\,(z)$.



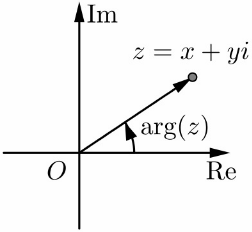
Konjugovano kompleksan broj $\overline{z}$ je tačka koja je osnosimetrična tački z u odnosu na realnu osu.



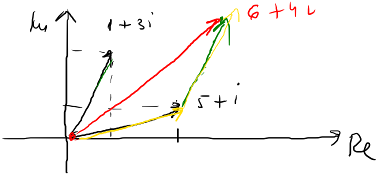


Moduo kompleksnog broja z, $|z|$, je intenzitet vektora $\overrightarrow{Oz}$.

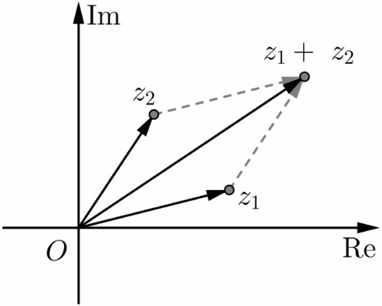
▶



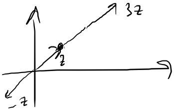
Argument kompleksnog broja z, $\arg(z)$, je mera orijentisanog ugla kojeg zaklapaju pozitivan deo realne ose i vektor $\overrightarrow{Oz}$.

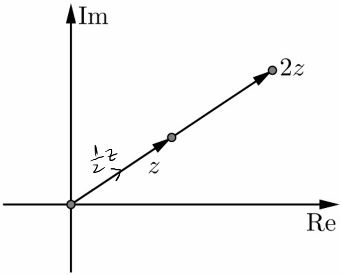


▶



Sabiranje kompleksnog broja z_1 sa kompleksnim brojem z_2 predstavlja translaciju vektora $\overrightarrow{Oz_1}$ za vektor $\overrightarrow{Oz_2}$.



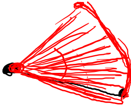


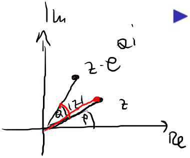
Množenje kompleksnog broja z sa realnim brojem $\alpha \neq 0$ predstavlja homotetiju sa centrom u koordinatnom početku i koeficijentom α. (slika je data za $\alpha = 2$)

$z_1 = -3$

$z_2 = -3 \cdot e^{\frac{2\pi}{3} i}$

$z_3 = -3\, e^{-\frac{2\pi}{3} i}$

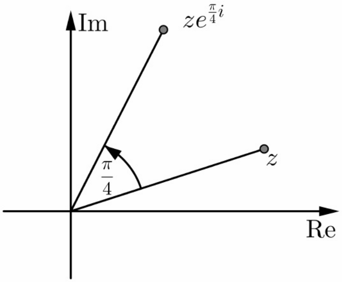




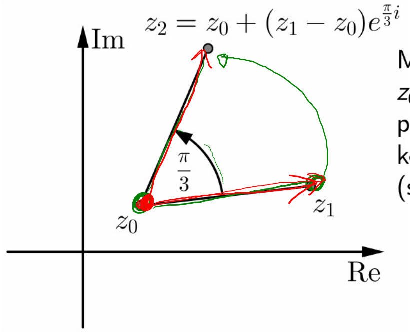
$z = |z| e^{\varphi i}$

$z \cdot e^{\alpha i} = |z| e^{\varphi i} \cdot e^{\alpha i}$

$= |z|\, e^{(\varphi + \alpha) i}$



Množenje kompleksnog broja z sa kompleksnim brojem $e^{\theta i}$ predstavlja rotaciju broja z oko koordinatnog početka za ugao θ. (slika je data za $\theta = \frac{\pi}{4}$)

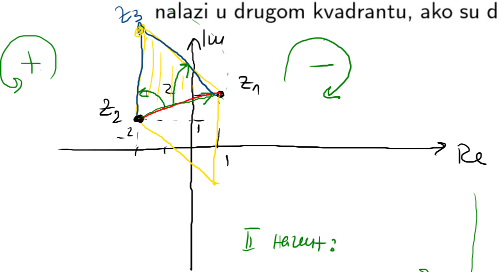


Množenje kompleksnog broja $z_1 - z_0$ sa kompleksnim brojem $e^{\theta i}$ predstavlja rotaciju broja z_1 oko kompleksnog broja z_0 za ugao θ (slika je data za $\theta = \frac{\pi}{3}$), tj.

$$\boxed{\rho_{z_0,\theta}\left(\overrightarrow{z_0z_1}\right) = \overrightarrow{z_0z_2}} \Longleftrightarrow \boxed{z_2 - z_0 = (z_1 - z_0)\,e^{\theta i}}.$$

$$\rho_{z_0,\frac{\pi}{3}}\left(\overrightarrow{z_0z_1}\right) = \overrightarrow{z_0z_2} \Longleftrightarrow z_2 - z_0 = (z_1 - z_0)\cdot e^{\frac{\pi}{3}i}$$

Primer: Odrediti treće teme jednakostraničnog trougla $z_1 z_2 z_3$ koje se nalazi u drugom kvadrantu, ako su data temena $z_1 = 1+2i$ i $z_2 = -2+i$.



$$\boxed{S_{z_2,\frac{\pi}{3}}\left(\overrightarrow{z_2 z_1}\right) = \overrightarrow{z_2 z_3}}$$

$$z_3 - z_2 = (z_1 - z_2)\cdot e^{\frac{\pi}{3}i}$$

$$\Rightarrow$$

$$z_3 = z_2 + (z_1 - z_2)\cdot e^{\frac{\pi}{3}i}$$

$$= -2+i+(1+2i-(-2+i))\,e^{\frac{\pi}{3}i}$$

$$= -2+i+(3+i)\cdot e^{\frac{\pi}{3}i}$$

$$= -2+i+(3+i)\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

$$= -2+i+(3+i)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

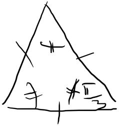
$$= -2+i+\frac{3}{2}+\frac{3\sqrt{3}}{2}i+\frac{1}{2}i-\frac{\sqrt{3}}{2}$$

$$\boxed{z_3 = \left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)+\left(\frac{3}{2}+\frac{3\sqrt{3}}{2}\right)i}$$

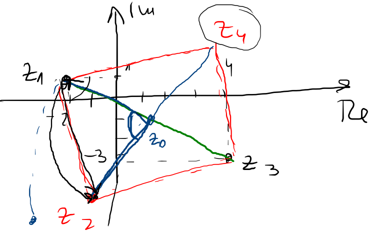
II način:

$$S_{z_1,-\frac{\pi}{3}}\left(\overrightarrow{z_1 z_2}\right) = \overrightarrow{z_1 z_3}$$

$$z_3 - z_1 = (z_2 - 1)\,e^{-\frac{\pi}{3}i}$$



Primer: Odrediti preostala temena kvadrata $z_1 z_2 z_3 z_4$, ako se zna da su temena $z_1 = -2 + i$ i $z_3 = 4 - 3i$ naspramna.



$$z_0 = \frac{z_1+z_2}{2} = \frac{-2+i+4-3i}{2} = 2-i$$

$$\rho_{z_0,\frac{\pi}{2}}(\overrightarrow{z_0 z_1}) = \overrightarrow{z_0 z_2}$$

$$z_2 - z_0 = (z_1 - z_0)e^{\frac{\pi}{2}i}$$

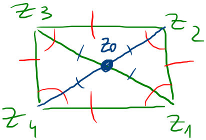
$$z_2 = z_0 + (z_1 - z_0)e^{\frac{\pi}{2}i}$$

$$= 2-i+(-2+i-2+i)\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$= 2-i+(-4+2i)(0+1\cdot i)$$

$$= \cancel{2}-i-4i-\cancel{2}$$

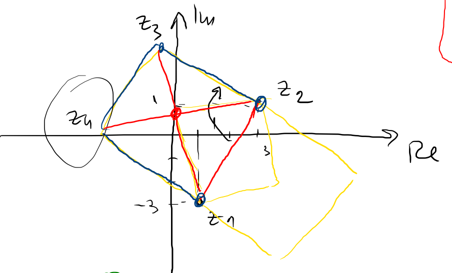
$$= -5i$$



$$z_0 = \frac{z_2+z_4}{2} \Rightarrow z_4 = 2z_0 - z_2 = 2(2-i)+5i = 4+3i$$

Primer: Odrediti preostala temena kvadrata $z_1z_2z_3z_4$, ako se zna da se jedno od njih nalazi u drugom kvadrantu, a temena $z_1 = 1 - 3i$ i $z_2 = 3 + i$ su susedna.

$\overrightarrow{z_2z_3} = z_3 - z_2$



$$\boxed{\rho_{z_2,-\frac{\pi}{2}}\left(\overrightarrow{z_2z_1}\right) = \overrightarrow{z_2z_3}}$$

$$z_3 - z_2 = (z_1 - z_2)\, e^{-\frac{\pi}{2} i}$$

$$z_3 = z_2 + (z_1 - z_2)\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$$

$$= 3 + i + (1 - 3i - 3 - i)(-i)$$

$$= 3 + i + (-2 - 4i)(-i)$$

$$= 3 + i + 2i - 4$$

$$= -1 + 3i$$

$$z_0 = \frac{z_1 + z_3}{\cancel{2}} = \frac{z_2 + z_4}{\cancel{2}}$$

$$z_4 = z_1 + z_3 - z_2$$

$$z_4 = \cancel{1} - \cancel{3i} - \cancel{1} + \cancel{3i} - 3 - i = -3 - i$$