Bulova algebra- vežbe

1. Koja od sledećih tvrđenja su tačna u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$?
   - $a + ab = aa'$;
   - $a + 1 = 0'$;
   - $a \cdot 1 = 0'$;
   - $ab = (ab)'$;
   - $a + a'b = a + b$;
   - $1 \cdot 0 = 1'$;
   - $a + b = (ab)'$;
   - $ab = (a' + b')'$;
   - $a\,(a + b) = aa'$;
   - $a + 1 = a$;
   - $1 + c = 1$;
   - $1 \cdot 0 = 1$;
   - $a + a' = a$;
   - $a' + a' = a'$;
   - $a + bc = (a + b)\,(a + c)$;
   - $1 + c = 0$;
   - $a \cdot 0 = 0$;
   - $a + a' = 0$;
   - $a' \cdot a' = a'$;
   - $a \leq 1$;
   - $a \leq 0$.
2. Dokazati da su u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$ sledeći iskazi ekvivalentni:

   $$\text{(a) } xy = x; \quad \text{(b) } x + y = y; \quad \text{(c) } x' + y = 1; \quad (d)\ xy' = 0.$$

3. Dokazati da su u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$ za sve $a, b, c \in B$ važi:
   (a) $a\,(bc') = (ab)\,(ac)'$;
   (b) $ab = 0 \iff ab' = a$;
   (c) $(ab) + (a' + b') = 1$;
   (d) $(c \leq a \ \wedge\ c \leq b) \iff c \leq ab$.
4. Svesti na DNF i SDNF sledeće Bulove izraze:
   (a) $I_1 = x\,(y'z)'$;

(b) $I_2 = z\,(x' + y) + y'$;

(c) $I_3 = (x + y'z)\,(y + z')$;

(d) $I_4 = (x' + y)' + y'z$;

(e) $I_5 = (x + y)'\,(xy')'$;

(f) $I_6 = y\,(x + yz)'$;

(g) $I_7 = (x + y)\,(x' + y)\,z$.

5. Naći sve proste implikante i minimalne DNF Bulovih funkcija datih svojom tablicom vrednosti ili odgovarajućim Bulovim izrazom:

(a) $f\,(x, y, z) = xyz + xy'z + xyz' + x'y'z'$;

(b) $f\,(x, y, z) = xyz + x'yz + xy'z + xyz' + x'yz'$;

(c)

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
$f\,(x, y, z)$	1	0	1	0	1	1	1	1

;

(d)

x	0	0	0	0	1	1	1	1
y	0	0	1	1	0	0	1	1
z	0	1	0	1	0	1	0	1
$f\,(x, y, z)$	1	0	1	0	1	1	0	1

;

(e) $f\,(x, y, z) = xyz + xy' + x'y$;

(f) $f\,(x, y, z, u) = xy'zu + xy'zu' + xy'z'u' + xy'z'u + xyz'u + x'y'zu' + x'yz'u$;

(g)

x	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
z	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
u	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$f\,(x, y, z)$	0	1	1	0	0	1	0	1	1	1	1	0	0	1	0	1

;

(h)

x	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
z	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
u	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$f\,(x, y, z)$	1	1	1	0	0	0	1	0	1	0	1	1	1	0	0	0

;

(i) $f\,(x, y, z, u) = xyzu + xy'zu + x'yzu + xyzu' + xy'zu' + x'yzu' + xyz'u' + x'y'z'u' + xyz'u$;

(j) $f\,(x, y, z, u) = xy' + xyz + x'y'z' + x'yzu'$.

ZA VEŽBU:IZ SKRIPTE
Zadatak 5.1, 5.3, 5.4, 5.6, 5.12 a,c, 5.14;
Primer 5.14;