$A = [0,1]$, $x*y = x+y-xy$, $(A, *)$

1) $A \neq \emptyset$

2) ZATVORENOST: $\forall a, b \in A, \ a*b \in A$? ✓

$a*b = a+b-ab$

$a+b-ab \in [0,1]$

$0 \leq a+b-ab \leq 1$?

$a+b-ab \geq 0$

$a(1-b)+b \geq 0$

$a \in [0,1]$ $0 \leq b \leq 1 \ / (-1)$

$b \in [0,1]$ $0 \geq -b \geq -1$

$1-b \in [0,1]$ $-1 \leq -b \leq 0 \ /+1$

$a, b, 1-b \geq 0$ $1-1 \leq 1-b \leq 1$

$\Rightarrow a(1-b)+b \geq 0$ $0 \leq 1-b \leq 1$

$1-b \in [0,1]$

$a+b-ab \leq 1$

$a+b-ab-1 \leq 0$

$a(1-b)-(1-b) \leq 0$

$(a-1)(1-b) \leq 0$ ✓

$a-1 \leq 0$ $a-1 \geq 0$

$1-b \geq 0$ $1-b \leq 0$

$a \leq 1$ $a \geq 1$

$b \leq 1$ ✓ $b \geq 1$

jer $a, b \in [0,1]$

$a \in [0,1] \Rightarrow 0 \leq a \leq 1$

$-1 \leq a-1 \leq 0$

$a-1 \leq 0$

$b \in [0,1] \Rightarrow 0 \leq b \leq 1 \ /^{(\cdot)}$

$1-b \geq 0$ $-1 \leq -b \leq 0 \ /+1$

$0 \leq 1-b \leq 1$

$$x * y = x + y - xy \qquad 0 \le x, y \le 1$$

1) pp. $2 \ge x + y \ge 0$

2) $1 \ge xy \ge 0$

$$\langle \; x + y \ge xy \; \rangle$$

$$[0, 2] \ge [0, 1]$$

$$x * y = x + y - xy$$

kom, asoc $_1$ ...

NEUTRALNI el. $\exists e \in [0,1], \forall a \in [\bar{a},1], e * a = a$?

$$e * a = a$$

$$e + a - ea = a$$

$$e(1-a) = 0$$

$\boxed{e = 0}$ \qquad $a = 1$

$\in [0,1]$

$$0 * 1 = 0 + 1 - 0 \cdot 1 = 1$$

INVERZNI el.

$\forall a \in [0,1], \exists a' \in [0,1], a' * a = 0$?

$$a' * a = 0$$

$$a' + a - a'a = 0$$

$$a'(1-a) = -a$$

$\boxed{a' = -\dfrac{a}{1-a}}$

$a \in [0,1]$

$\boxed{a = 1}$ NEMA INVERZNI el.

$-\dfrac{a}{1-a} \in [0,1]$

$$0 \leq -\dfrac{a}{1-a} \leq 1$$

$a \in [0,1]$

$0 \leq a \leq 1 \quad \big|^{(-1)}$

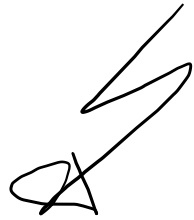$-1 \leq -a \leq 0 \quad ^{+1}$

$0 \leq 1-a \leq 1$

$1-a \in [0,1]$

$$0 \le -\frac{a}{1-a} \le 1 \quad / \cdot \quad \boxed{1-a \geqslant 0}$$

$$\underbrace{0 \le -a \le 1-a}$$

$$a \le 0$$

$$-a \le 1-a$$
$$0 \le 1$$

$$\Rightarrow \quad -\frac{a}{1-a} \notin [0,1]$$

$$0 * 0 = 0 + 0 - 0 \cdot 0 = 0$$
$$0 - \text{IHA} \quad \text{INVERZNI}$$

IDEMPOTENTNOST: $\forall a \in [0,1]$, $a*a = a$?

$a*a = a$

$\cancel{a} + a - a \cdot a = \cancel{a}$

$a - a^2 = 0$

$0 - 0 = 0$ ✓ $\frac{1}{2} - \frac{1}{4} \neq 0$

$1 - 1 = 0$ ✓

NE VAŽI

NILPOTENTNOST

$\exists 0 \in [0,1], \forall a \in [0,1]$

$0 * a = 0$?

$0 * a = 0$

$\cancel{0} + a - 0 a = \cancel{0}$

$0 - \cancel{a} = \cancel{a}$ $/ a \neq 0$

$\boxed{0 = 1}$

$1 * 0 = 1 + 0 - 1 \cdot 0$
$= 1$

ZA SVAKI OSIM 0   VAŽI i za
                    $a = 0$

1 ∄ NILPOTENTNI ZA
      [$\forall a \in [0,1]$]

KANCELACIJA

$a * b = a + b - ab$

$1 * \frac{1}{2} = 1 + \frac{1}{2} - \frac{1}{2} = 1$

$1 * \frac{1}{4} = 1 + \frac{1}{4} - \frac{1}{4} = 1$

$\frac{1}{2} \neq \frac{1}{4}$ $0 * \frac{1}{2} = \frac{1}{2}$

$0 * \frac{1}{4} = \frac{1}{4}$