

$$11.2 \quad p(x) = x^5 - 6x^3 + 6x^2 - 7x + 6.$$

Морено Нату 5 члены фп в момент 5-го сеанса

$$\begin{array}{l} 1|6 \Rightarrow 1 \in \{\pm 1, \pm 2, \pm 3, \pm 6\} \\ 2|1 \Rightarrow 2 \in \{1\} \end{array} \left\{ \begin{array}{l} 1 \in \{\pm 1, \pm 2, \pm 3, \pm 6\} \\ 2 \end{array} \right.$$

Нулю отличие $p(x)$
 $\Rightarrow (x-a) | p(x)$
 \Rightarrow при делении $p(x)$
 со $x-a$ остаток
 не 0

$$\begin{array}{r|rrrrrr} & 1 & 0 & -6 & 6 & -7 & 6 \\ \hline 1 & 1 & 1 & -5 & 1 & -6 & 0 \\ \hline 1 & 1 & 2 & -3 & -2 & -8 \end{array} \Rightarrow 1 \text{ je Hgwo } p(x) \\ \Rightarrow p(x) = (x-1)(x^3 + x^2 - 5x^2 + x - 6)$$

$$\begin{array}{r} \boxed{-1} & 1 & 0 & -5 & 6 & \boxed{-12} \\ 2 & 1 & 3 & 1 & 3 & \boxed{0} \\ \hline -1 & 1 & 11 & 25 \end{array} \Rightarrow \underline{2} \text{ je myne } p(x) \Rightarrow p(x) = (x-1)(x-2)(x^3+3x^2+x+3)$$

$$\begin{array}{r|rrr} & 1 & 1 & -1 & 5 \\ \hline -2 & 1 & 1 & -1 & 5 \\ \hline -3 & 1 & 0 & 1 & 0 \end{array} \Rightarrow -3 \text{ je lösung von } p(x) \Rightarrow p(x) = (x-1)(x-2)(x+3)(x^2+1) - \text{abakusprinzzip für } \mathbb{R}$$

$$\begin{aligned} \frac{x^2+1}{x^2+1=0} & x = \sqrt{-1} \Rightarrow \pm i \text{ ай хүснэгт } p(x) \\ x^2 = -1 & x = \pm i \Rightarrow p(x) = (x-1)(x-2)(x+3) \setminus (x-i)(x+i) - \text{дэлхийн хамгийн тохижуулсан } C \\ |i^2 = -1 \Rightarrow i \neq -1 & \text{ & ГЭРЭЛТЭН} \end{aligned}$$

$$p(x) = (x-1)(x+2)(x-3)^2 \quad -\text{ділиться на } x-3 \text{ з залишком } 0 \quad \text{на} \quad \mathbb{R} \text{ або } \mathbb{C}$$

$$\begin{aligned} p(x) &= (x-1)(x+2)(x-i)(x+i) \quad -\text{на} \quad \mathbb{C} \\ &= (x-1)(x+2)(x^2+1) \quad -\text{на} \quad \mathbb{R} \end{aligned}$$

$$\begin{aligned} p(x) &= (x^2+1)(x^2+4) \quad -\text{на} \quad \mathbb{R} \\ &= (x-i)(x+i)(x-2i)(x+2i) \quad -\text{на} \quad \mathbb{C} \end{aligned}$$

$$\begin{aligned} p(x) &= (x-3)(x+5)^2(x^2-1) \\ &= (x-3)(x+5)^2(x-1)(x+1) \quad -\text{на} \quad \mathbb{R} \\ &\quad \text{и на} \quad \mathbb{C} \end{aligned}$$

$$\begin{aligned} p(x) &= (x^2-4)(x^2+x+1) \\ &= (x-2)(x+2)(x^2+x+1) \quad -\text{на} \quad \mathbb{R} \\ &= (x-2)(x+2)\left(x-\frac{-1+\sqrt{3}i}{2}\right)\left(x-\frac{-1-\sqrt{3}i}{2}\right) \quad -\text{на} \quad \mathbb{C} \end{aligned}$$

$$\begin{aligned} x^2+1 &= 0 \\ x^2 &= -1 \\ x &= \pm i \end{aligned}$$

$$\begin{aligned} x^2+4 &= 0 \\ x^2 &= -4 \\ x &= \pm 2i \end{aligned}$$

$$\begin{aligned} x^2-1 &= 0 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

$$\begin{aligned} a^4 &= \underbrace{a \cdots a}_n \\ (x-3)^2 &= (x-3)(x-3) \end{aligned}$$

$$\begin{aligned} ax^2 + bx + c &= 0 \\ x_1, x_2 &- \text{розв'язки} \\ a(x-x_1)(x-x_2) &= 0 \end{aligned}$$

$$\begin{aligned} x^2+x+1 &= 0 \\ x_{1,2} &= \frac{-1 \pm \sqrt{-3}}{2} \notin \mathbb{R} \\ &= \frac{-1 \pm \sqrt{3}i}{2} = \underbrace{\frac{-1+\sqrt{3}i}{2}}_{-\frac{1-\sqrt{3}i}{2}} \end{aligned}$$

12. Naći sve nule polinoma $p(x)$, a zatim ga faktorisati nad poljima \mathbb{R} i

C:

12.1 $p(x) = 3x^5 + 8x^4 - 10x^2 - 3x + 2$;
-npreda go nađemo 5 nivoja sa p(x) Č-ovih cvećena

$$\begin{aligned} \text{N/2} &\Rightarrow N \in \{\pm 1, \pm 2\} \\ 2/3 &\Rightarrow 2 \in \{1, 3\} \end{aligned} \quad \left\{ \begin{array}{l} 1 \in \{\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}\} \\ 2 \in \{1, 3\} \end{array} \right.$$

$$\begin{array}{c|cccccc} & 3 & 8 & 0 & -10 & -3 & 2 \\ \textcircled{1} & 3 & 11 & 11 & 1 & -2 & \textcircled{0} \\ \textcircled{-1} & 3 & 4 & 25 & 26 & \cancel{24} & \\ \textcircled{-1} & 3 & 8 & 3 & -2 & \textcircled{0} & \\ \textcircled{-1} & 3 & 5 & -2 & \textcircled{0} & & \end{array}$$

$$3x^2 + 5x - 2 = 0$$
$$x_{1,2} = \frac{-5 \pm \sqrt{25+24}}{6} = \frac{-5 \pm 7}{6} = \left(\begin{array}{l} \frac{1}{3} \\ -2 \end{array} \right)$$

Nule učinkova p(x) su:

$1, \frac{1}{3}, -2$ - jedno učinko
 -1 - dvoje učinko tri nivoja

Haj C u Haj IR

$$p(x) = 3(x-1)\left(x-\frac{1}{3}\right)(x+2)(x+1)^2$$

$$ax^2 + bx + c$$

$$mx + u$$

$$(x+1)^2 = (x+1)(x+1)$$

$$12.2 \quad p(x) = x^5 - 2x^4 - x + 2.$$

$$\begin{array}{l} 1|2 \Rightarrow 1 \in \{\pm 1, \pm 2\} \\ 2|1 \Rightarrow 2 \in \{1\} \end{array} \quad \left. \begin{array}{l} 1 \in \{\pm 1, \pm 2\} \\ 2 \in \{1\} \end{array} \right\} \frac{1}{2} \in \{\pm 1, \pm 2\}$$

	1	-2	0	0	-1	2
1	1	-1	-1	-1	-2	0
1	1	0	-1	-2	4	
-1	1	-2	1	-2	0	
-1	1	-3	4	-6		
2	1	0	1	0		

$$\boxed{x^2+1} = 0$$

$$x^2 = -1$$

$$x = \pm i$$

Натуралният корен $p(x)$ е:

$1, -1, 2, \pm i$ - действителните

Натуралният корен $p(x)$ е:

$$p(x) = (x-1)(x+1)(x-2)(x-\underline{i})(x+\underline{i})$$

Натуралният корен $p(x)$ е:

$$p(x) = (x-1)(x+1)(x-2)(x^2+1)$$

$$12.3 \quad p(x) = 2x^6 + 9x^5 + 22x^4 + 45x^3 + 58x^2 + 36x + 8;$$

$$\left. \begin{array}{l} p \neq 0 \Rightarrow p \in \{\pm 1, \pm 2, \pm 4, \pm 8\} \\ q \mid 2 \Rightarrow q \in \{1, 2\} \end{array} \right\} \left. \begin{array}{l} p \in \{\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}\} \\ q \in \{1, 2\} \end{array} \right\}$$

	2	9	22	45	58	36	8	\leftarrow обр. коэффициентов с у равнением, то не можно ищем подразумеван броя кое кочт же ищется и собираем подразумев брояв не можно подразумев нулу
-1	2	7	15	30	28	8	0	
-1	2	5	10	20	8	0		
-1	2	3	7	13	-5			
-2	2	1	8	4	0			
-2	2	-3	14	-24				
-4	2	-7	36	-140				
$-\frac{1}{2}$	2	0	8	0				
	$2x^2 + 8 = 0$		$x^2 = -4$					
	$2(x^2 + 4) = 0$		$x = \pm 2i$					
	$\underline{\underline{x^2 + 4 = 0}}$							

Натурални: $-1 - \text{однодубликат}$
 $-2, -\frac{1}{2}, 2i, -2i - \text{јестно изјутре}$

$$\text{Нагл. C: } p(x) = 2(x+1)^2(x+2)\left(x+\frac{1}{2}\right)(x-2i)(x+2i)$$

$$\text{Нагл. R: } p(x) = 2(x+1)^2(x+2)\left(x+\frac{1}{2}\right)(x^2+4)$$

$$12.4 \quad p(x) = x^5 + x^4 - 6x^3 - 4x^2 + 8x = x(x^4 + x^3 - 6x^2 - 4x + 8)$$

$$\begin{array}{l} p|8 \Rightarrow p \in \{1, \pm 2, \pm 4, \pm 8\} \\ q|1 \Rightarrow q \in \{1\} \end{array} \quad \left\{ \begin{array}{l} p \in \{\pm 1, \pm 2, \pm 4, \pm 8\} \\ q \in \{\pm 1\} \end{array} \right.$$

$$\begin{array}{r} 1 & 1 & -6 & -4 & 8 \\ \hline 1 & 1 & 2 & -4 & -8 & 0 \\ 1 & 1 & 3 & -1 & -9 & \\ \hline -1 & 1 & 1 & -5 & -3 & \\ \hline 2 & 1 & 4 & 4 & 0 & \end{array}$$

$$x^2 + 4x + 4 = (x+2)^2$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16+16}}{2} = -2$$

Натуралният корен $p(x)$ е

$1, 2, 0$ - реални корени

-2 - двойен корен

$$\text{Нагледно за } \mathbb{C} \text{ и нагледно за } \mathbb{R} \\ p(x) = x(x-1)(x-2)(x+2)^2$$

14. Odrediti realne parametre a i b tako da brojevi -1 i 2 budu korenji polinoma $p(x) = x^4 + (a+1)x^3 - 9x^2 + bx + 12$, a zatim za te vrednosti parametara a i b faktorisati polinom $p(x)$ nad poljima \mathbb{R} i \mathbb{C} .

$$\begin{array}{l} \text{a je koren (koje je)} p(x) \\ \text{takav da } p(a) = 0 \\ \text{takav da } x-a \mid p(x) \end{array}$$

$$p(x) = x^4 + (a+1)x^3 - 9x^2 + bx + 12$$

$$\boxed{-1 \text{ i } 2 \text{ su koreni polinoma } p(x) \Rightarrow p(-1) = 0 \text{ i } p(2) = 0}$$

$$p(-1) = 1 - (a+1) - 9 - b + 12 \Rightarrow \boxed{\begin{aligned} & -a - 1 - 9 - b + 12 = 0 \\ & \hline -a - b = -3 \end{aligned}}$$

$$p(2) = 16 + 8(a+1) - 36 + 2b + 12 \Rightarrow \boxed{\begin{aligned} & 16 + 8a + 8 - 36 + 2b + 12 = 0 \\ & \hline 8a + 2b = 0 \end{aligned}} \quad | :2$$

$$\boxed{p(x) = x^4 - 9x^2 + 4x + 12}$$

$$p(x) = (x+1)(x-2)^2(x+3)$$

teg C v teg R

$$\left| \begin{array}{r|rrrr} & 1 & 0 & -9 & 4 & 12 \\ -1 & 1 & -1 & -8 & 12 & 0 \\ 2 & 1 & 1 & -6 & 0 & 0 \end{array} \right.$$

$$x^2 + x - 6 = 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

$$\left\{ \begin{array}{l} a+b=3 \\ 4a+b=0 \\ \hline 3a=-3 \\ \hline a=-1 \\ \hline b=4 \end{array} \right.$$

ZA VEŽBU IZ SKRIPTE

Zadatak 8.37, 8.40, 8.41, 8.42, 8.45, 8.49, 8.50, 8.51;

Primer 8.3, 8.5;