

P8106 HW 2

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```
library(caret)
library(splines)
library(mgcv)
library(pdp)
library(earth)
library(tidyverse)
library(ggplot2)

set.seed(2021)
```

Data preparation

```
college_df = read_csv('./data/College.csv') %>%
  janitor::clean_names()

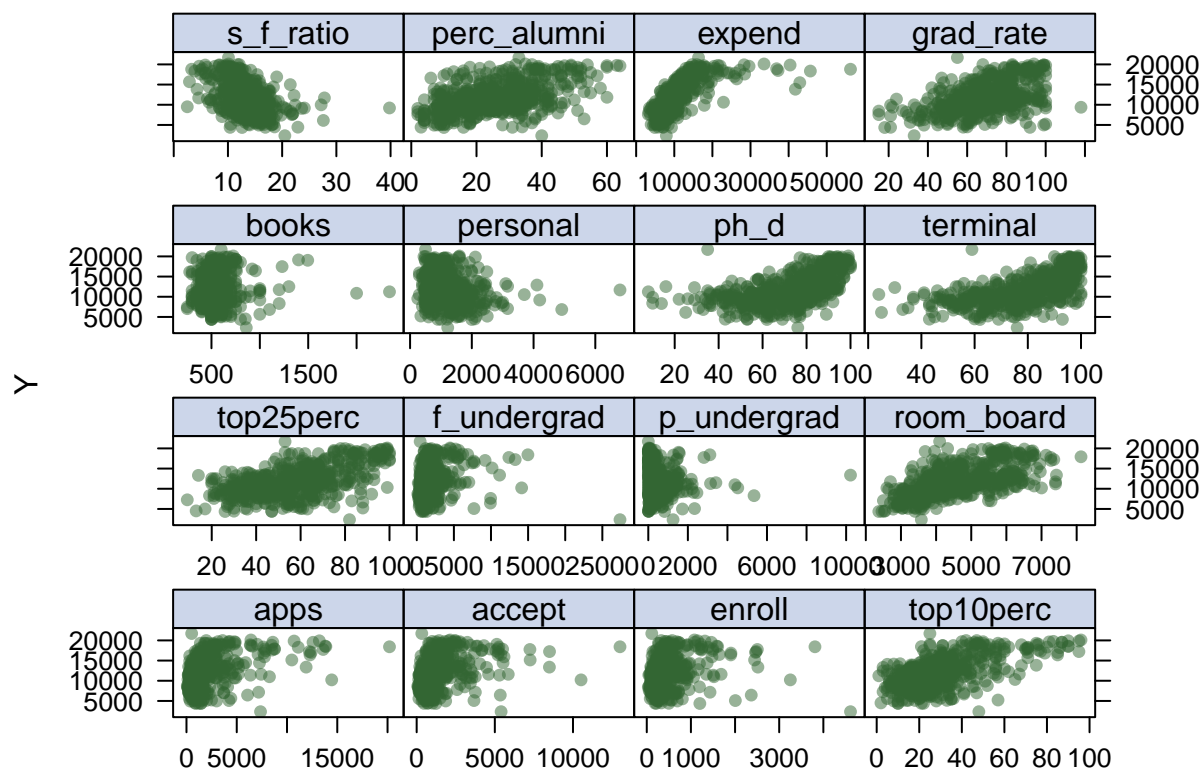
#skimr::skim(college_df)

college_train = college_df %>%
  filter(college != "Columbia University") %>%
  select(-college)
```

a) Perform exploratory data analysis

```
# matrix of predictors
x = model.matrix(outstate ~ ., college_train)[,-1]
# vector of response
y = college_train$outstate

theme1 <- trellis.par.get()
theme1$plot.symbol$col <- rgb(.2, .4, .2, .5)
theme1$plot.symbol$pch <- 16
theme1$plot.line$col <- rgb(.8, .1, .1, 1)
theme1$plot.line$lwd <- 2
theme1$strip.background$col <- rgb(.0, .2, .6, .2)
trellis.par.set(theme1)
featurePlot(x, y, plot = "scatter", labels = c("", "Y"),
            type = c("p"), layout = c(4, 4))
```



b) Fit smoothing spline models

```
#df
fit.ss <- smooth.spline(college_train$terminal, college_train$outstate)
fit.ss$df

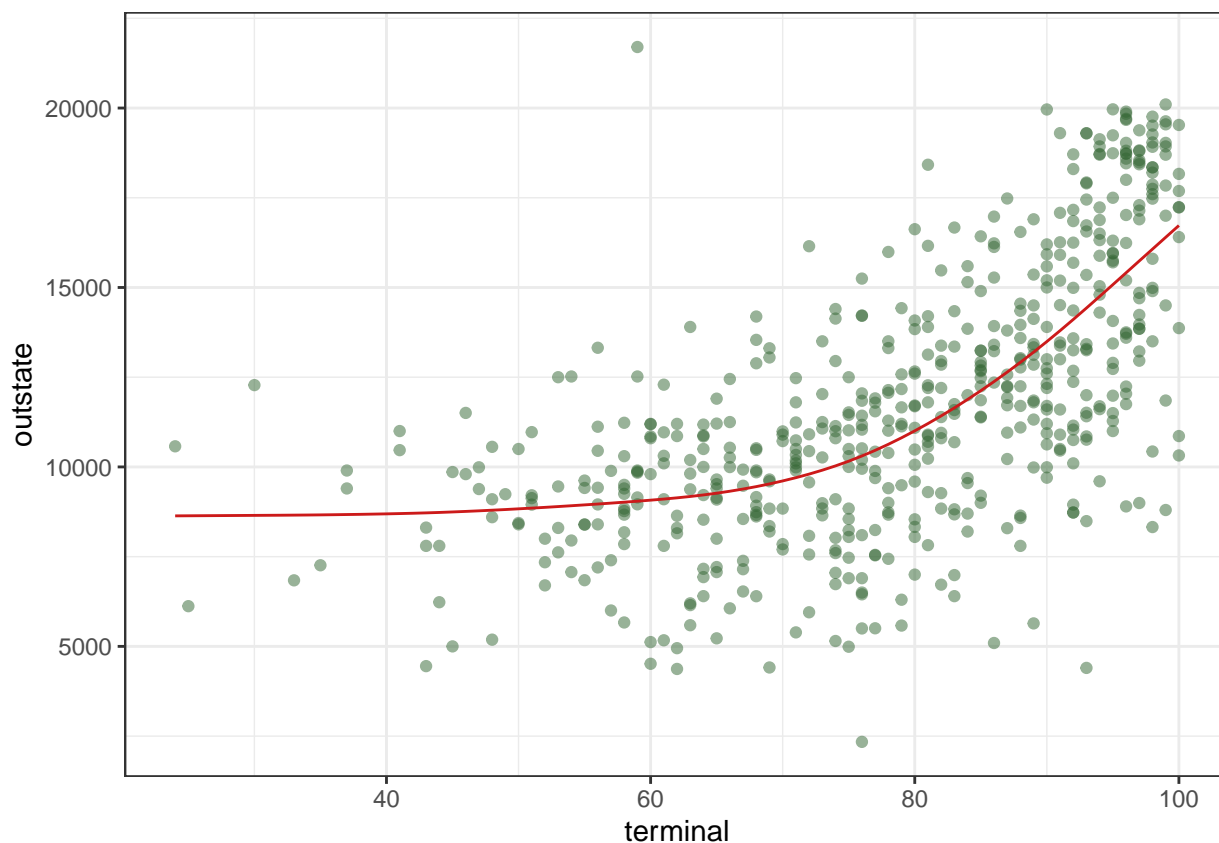
## [1] 4.468629

#plot the resulting fits
terminallims <- range(college_train$terminal)
terminal.grid <- seq(from = terminallims[1], to = terminallims[2])

pred.ss = predict(fit.ss, x = terminal.grid)
pred.ss.df = data.frame(pred = pred.ss$y, terminal = terminal.grid)

p = ggplot(data = college_train, aes(x = terminal, y = outstate)) +
  geom_point(color = rgb(.2, .4, .2, .5))

p +
  geom_line(aes(x = terminal, y = pred), data = pred.ss.df, color = rgb(.8, .1, .1, 1)) + theme_bw()
```



Choose my own degree of freedom:

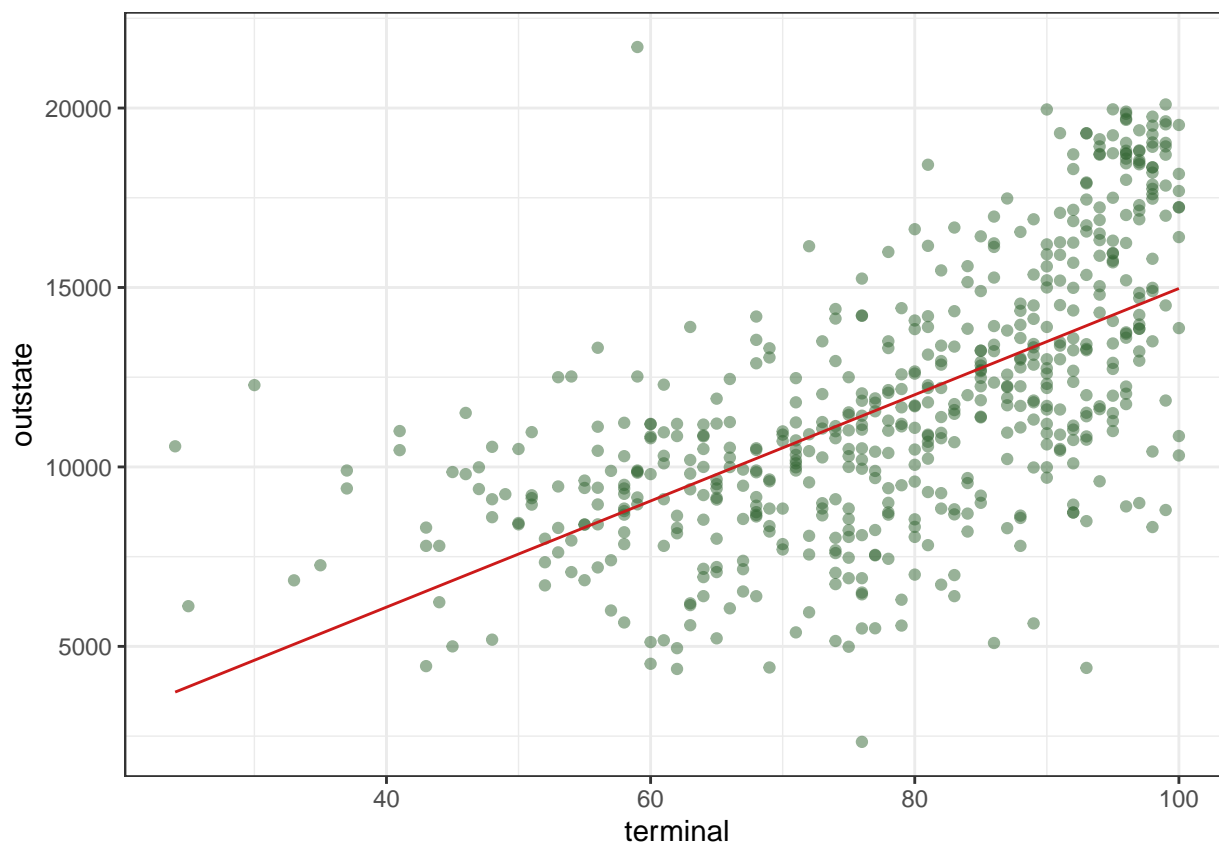
```
#2 degree of freedom
fit.ss <- smooth.spline(college_train$terminal, college_train$outstate, df=2)
df_2 = fit.ss$df

#plot the resulting fits
terminallims <- range(college_train$terminal)
terminal.grid <- seq(from = terminallims[1], to = terminallims[2])

pred.ss = predict(fit.ss, x = terminal.grid)
pred.ss.df = data.frame(pred = pred.ss$y, terminal = terminal.grid)

p = ggplot(data = college_train, aes(x = terminal, y = outstate)) +
  geom_point(color = rgb(.2, .4, .2, .5))

p +
  geom_line(aes(x = terminal, y = pred), data = pred.ss.df, color = rgb(.8, .1, .1, 1)) + theme_bw()
```



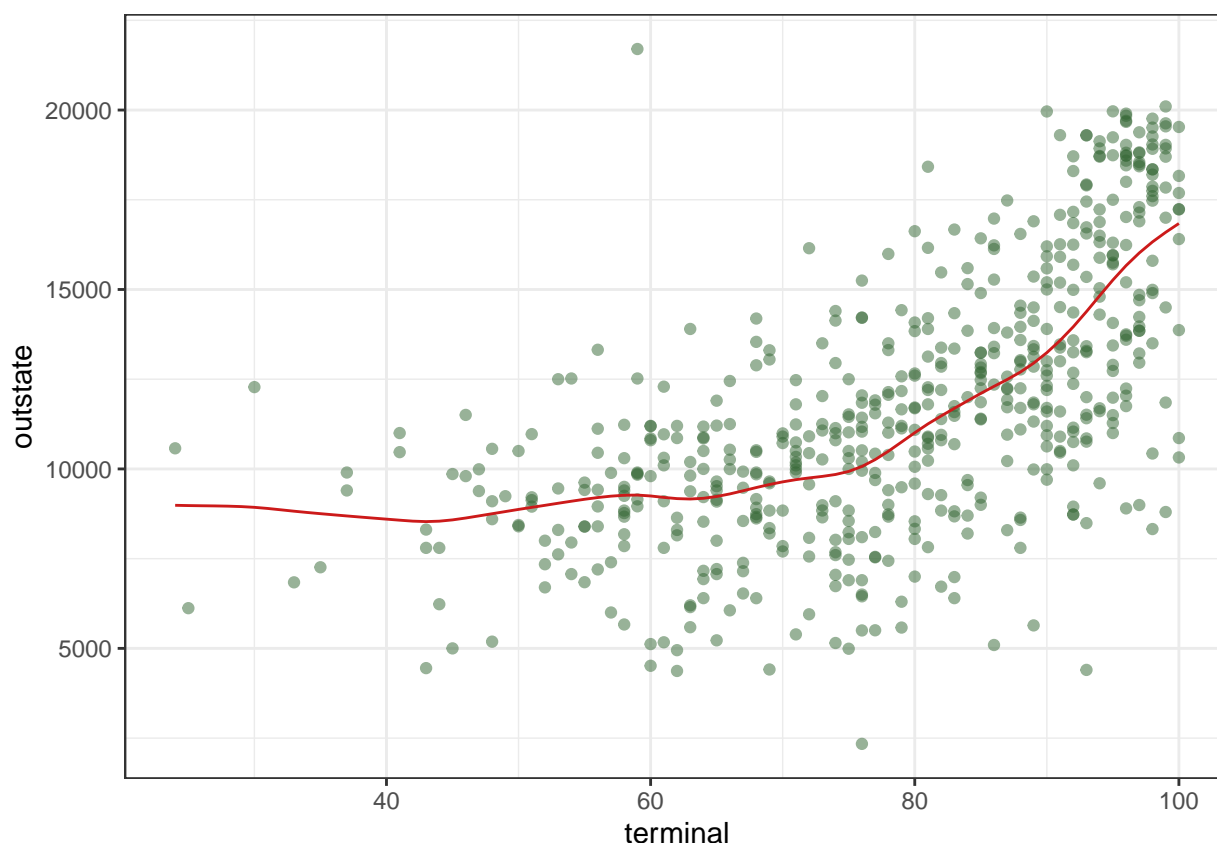
```
#10 degree of freedom
fit.ss <- smooth.spline(college_train$terminal, college_train$outstate, df=10)
df_10 = fit.ss$df

#plot the resulting fits
terminallims <- range(college_train$terminal)
terminal.grid <- seq(from = terminallims[1], to = terminallims[2])

pred.ss = predict(fit.ss, x = terminal.grid)
pred.ss.df = data.frame(pred = pred.ss$y, terminal = terminal.grid)

p = ggplot(data = college_train, aes(x = terminal, y = outstate)) +
  geom_point(color = rgb(.2, .4, .2, .5))

p +
  geom_line(aes(x = terminal, y = pred), data = pred.ss.df, color = rgb(.8, .1, .1, 1)) + theme_bw()
```



The degree of freedom obtained by generalized cross validation is 4.468629. From the first plot, we can see that there is a non linear relationship between terminal and outstate. When we only use terminal as a predictor and the degree of freedom obtained by generalized cv to fit the data, the fitted curve is very smooth. The smoothing spline, which is the red line, shows the prediction of the smoothing spline fits the data.

When picking $df = 2$ and $df = 10$, we can see that $df = 2$ shows a linear line, and $df = 10$ shows a wiggly line. We can conclude that larger values make the line much more wiggly, while smaller degrees of freedom are more linear.

c) Fit a generalized additive model (GAM) using all the predictors

```
ctrl1 <- trainControl(method = "cv", number = 5)
set.seed(7)
gam.fit <- train(x, y,
method = "gam",
tuneGrid = data.frame(method = "GCV.Cp", select = c(TRUE,FALSE)),
trControl = ctrl1)
gam.fit$bestTune
```

```
## select method
## 1 FALSE GCV.Cp
```

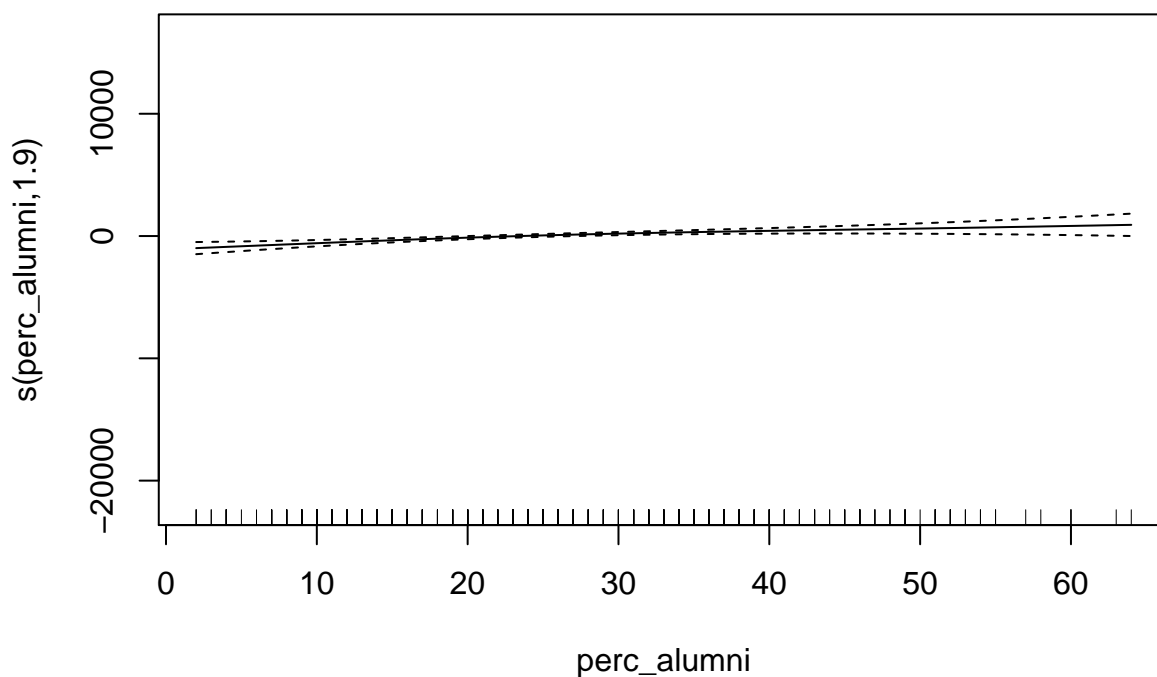
```
gam.fit$results
```

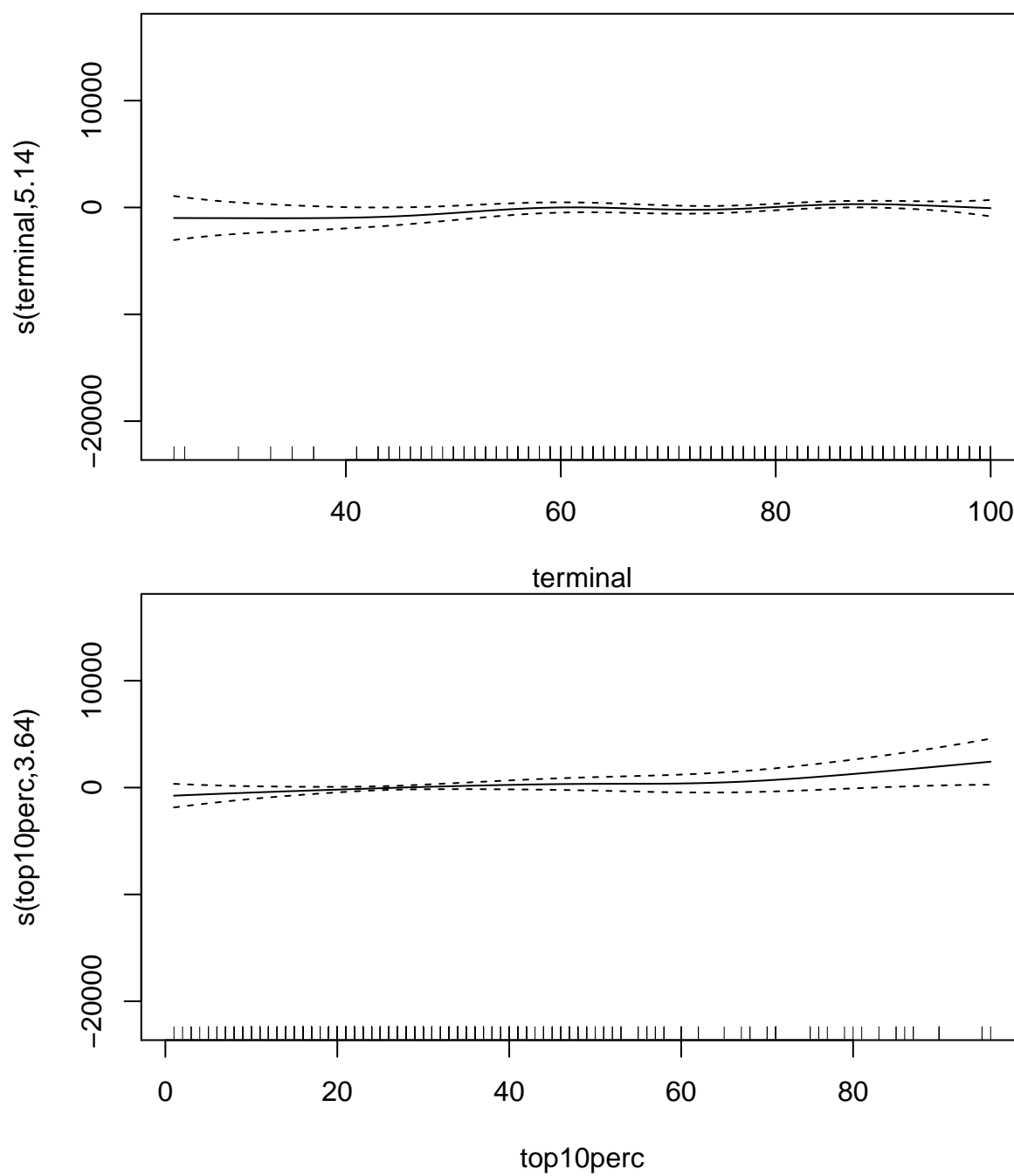
```
##   method select      RMSE Rsquared      MAE  RMSESD RsquaredSD  MAESD
## 1 GCV.Cp FALSE 1816.709 0.7654877 1384.619 204.1885 0.04437825 107.4501
## 2 GCV.Cp TRUE 1905.812 0.7476713 1415.683 301.6884 0.06002304 101.9745
```

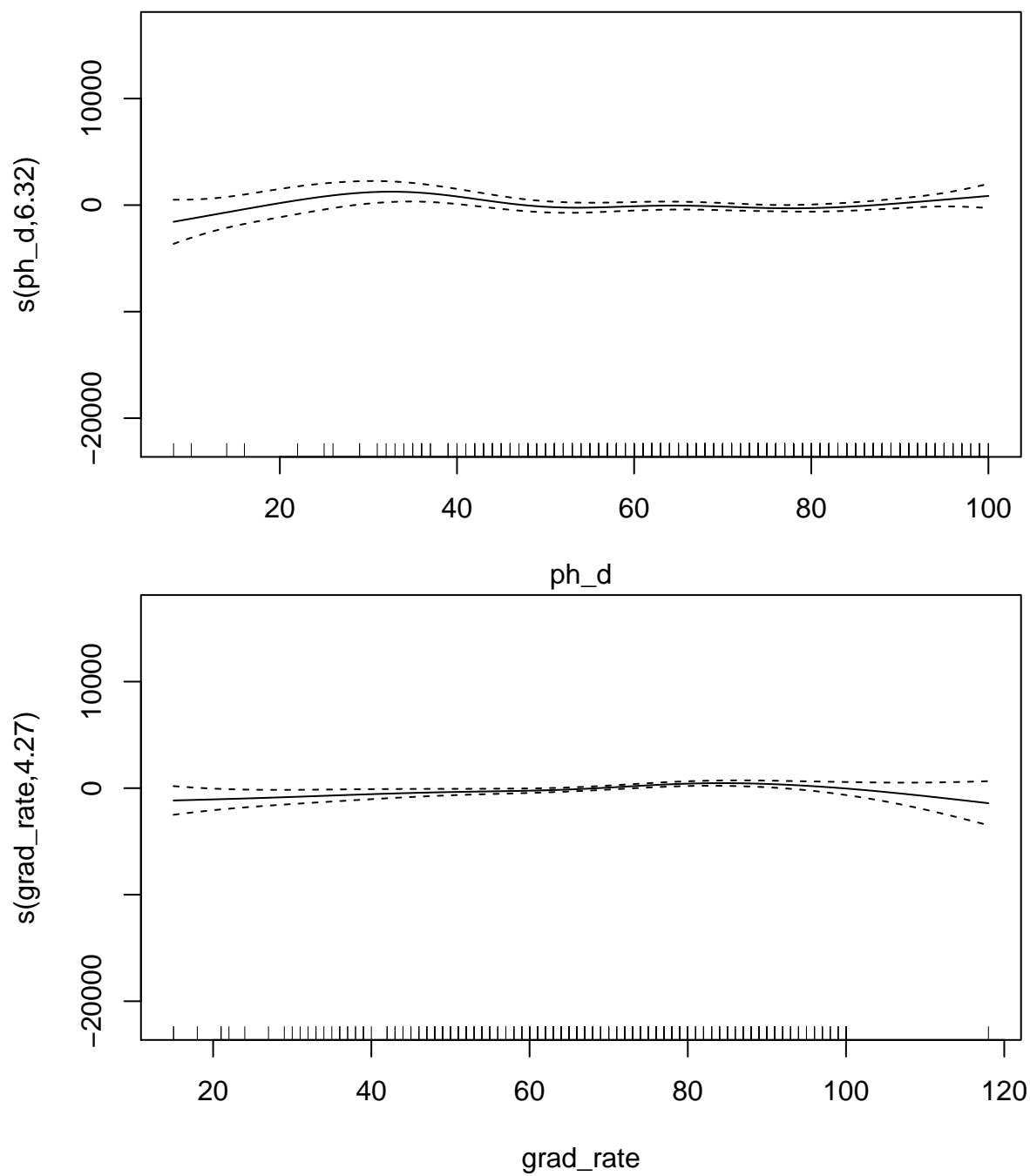
```
gam.fit$finalModel
```

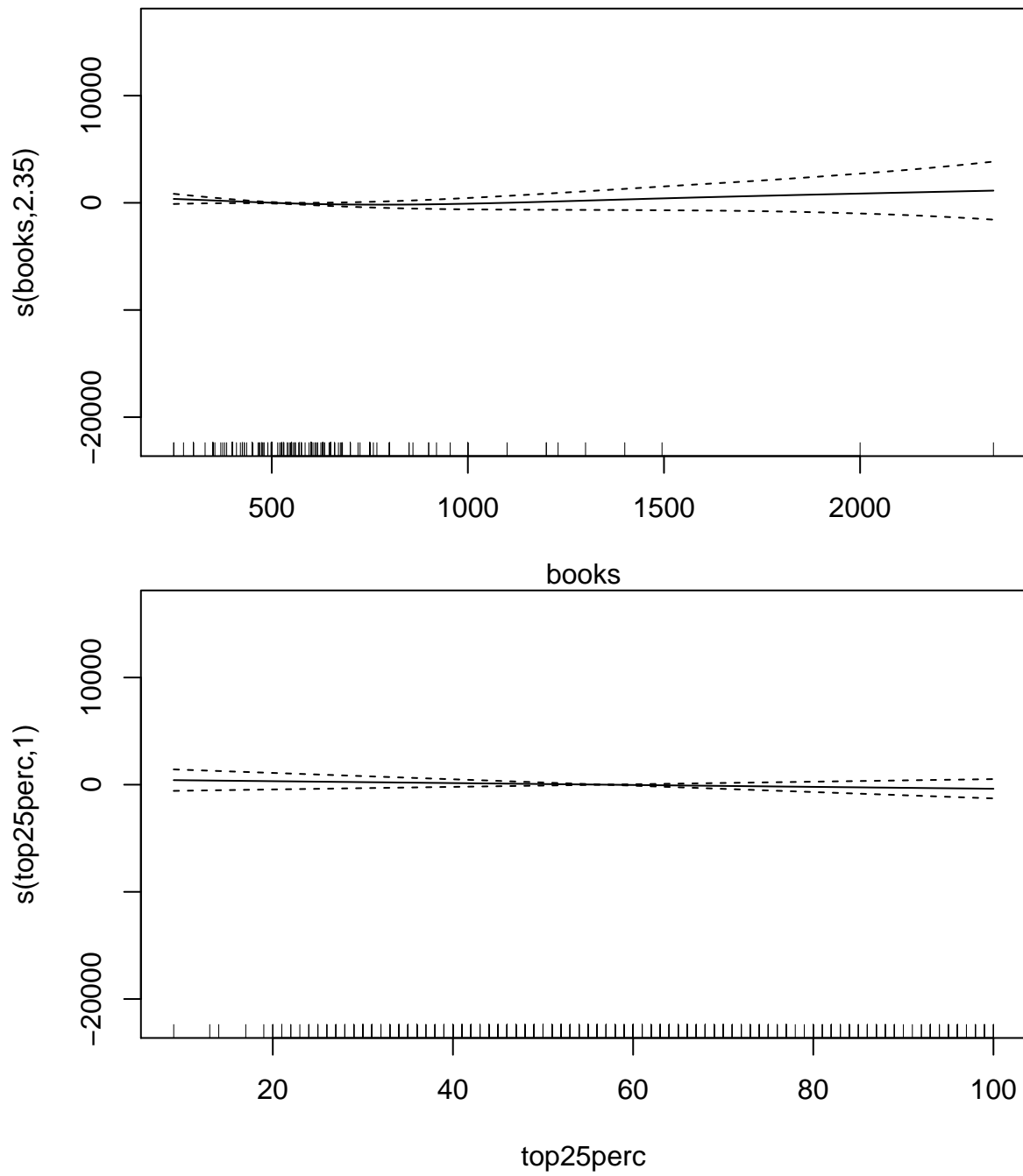
```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## .outcome ~ s(perc_alumni) + s(terminal) + s(top10perc) + s(ph_d) +
##           s(grad_rate) + s(books) + s(top25perc) + s(s_f_ratio) + s(personal) +
##           s(p_undergrad) + s(enroll) + s(room_board) + s(accept) +
##           s(f_undergrad) + s(apps) + s(expend)
##
## Estimated degrees of freedom:
## 1.90 5.14 3.64 6.32 4.27 2.35 1.00
## 4.33 1.00 1.00 1.00 2.13 3.58 6.28
## 4.59 6.45 total = 55.98
##
## GCV score: 2761951
```

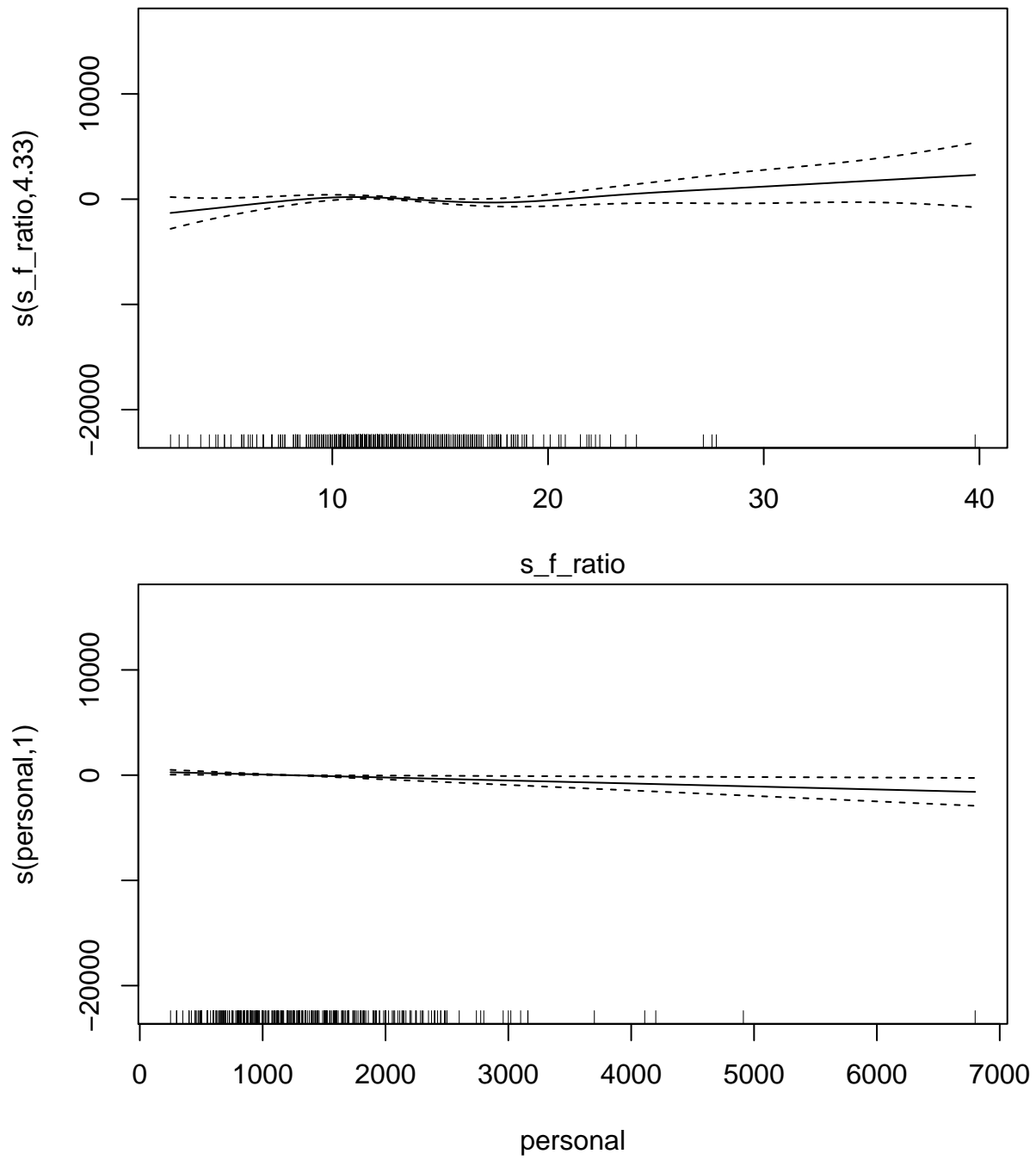
```
plot(gam.fit$finalModel)
```

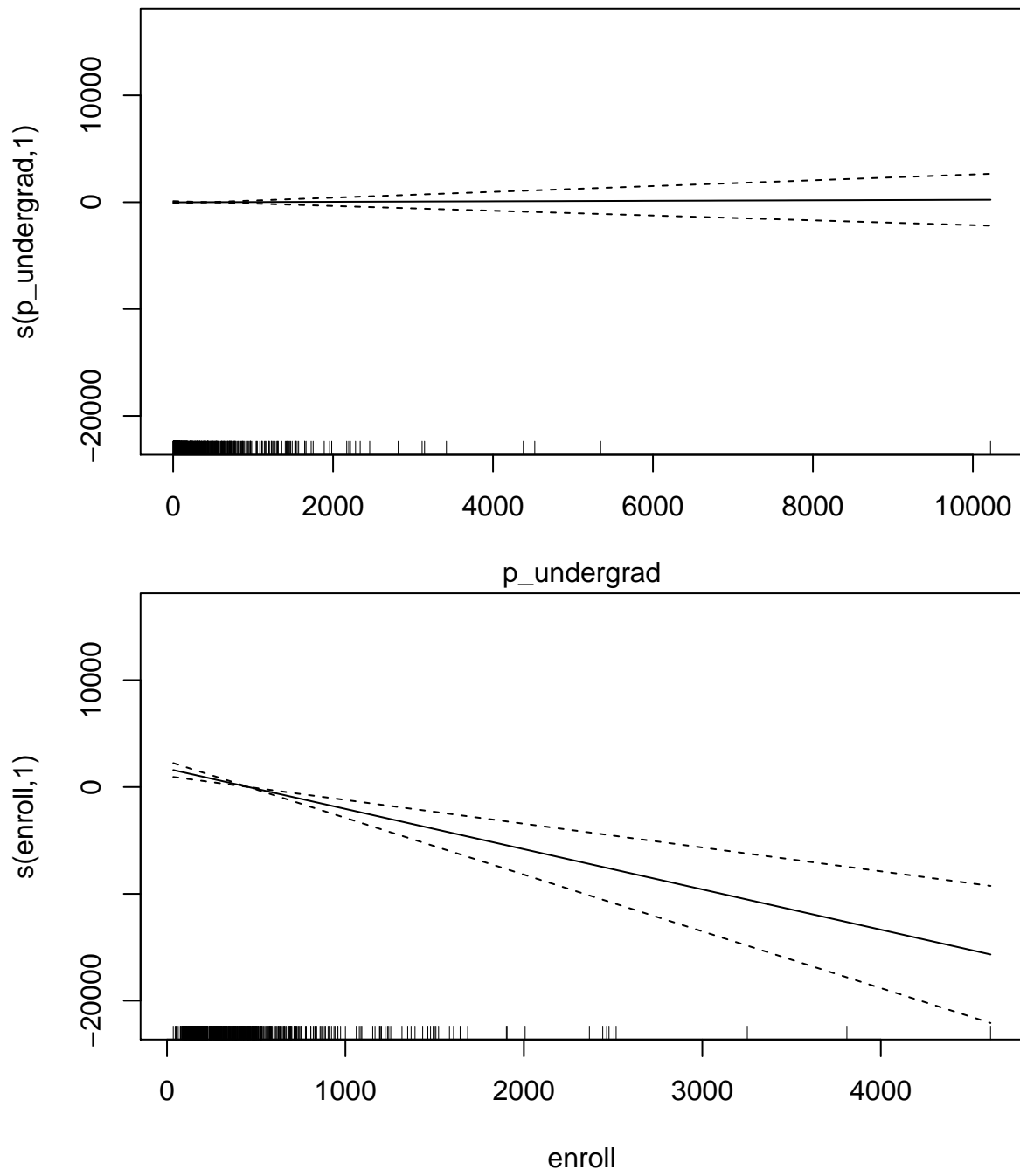


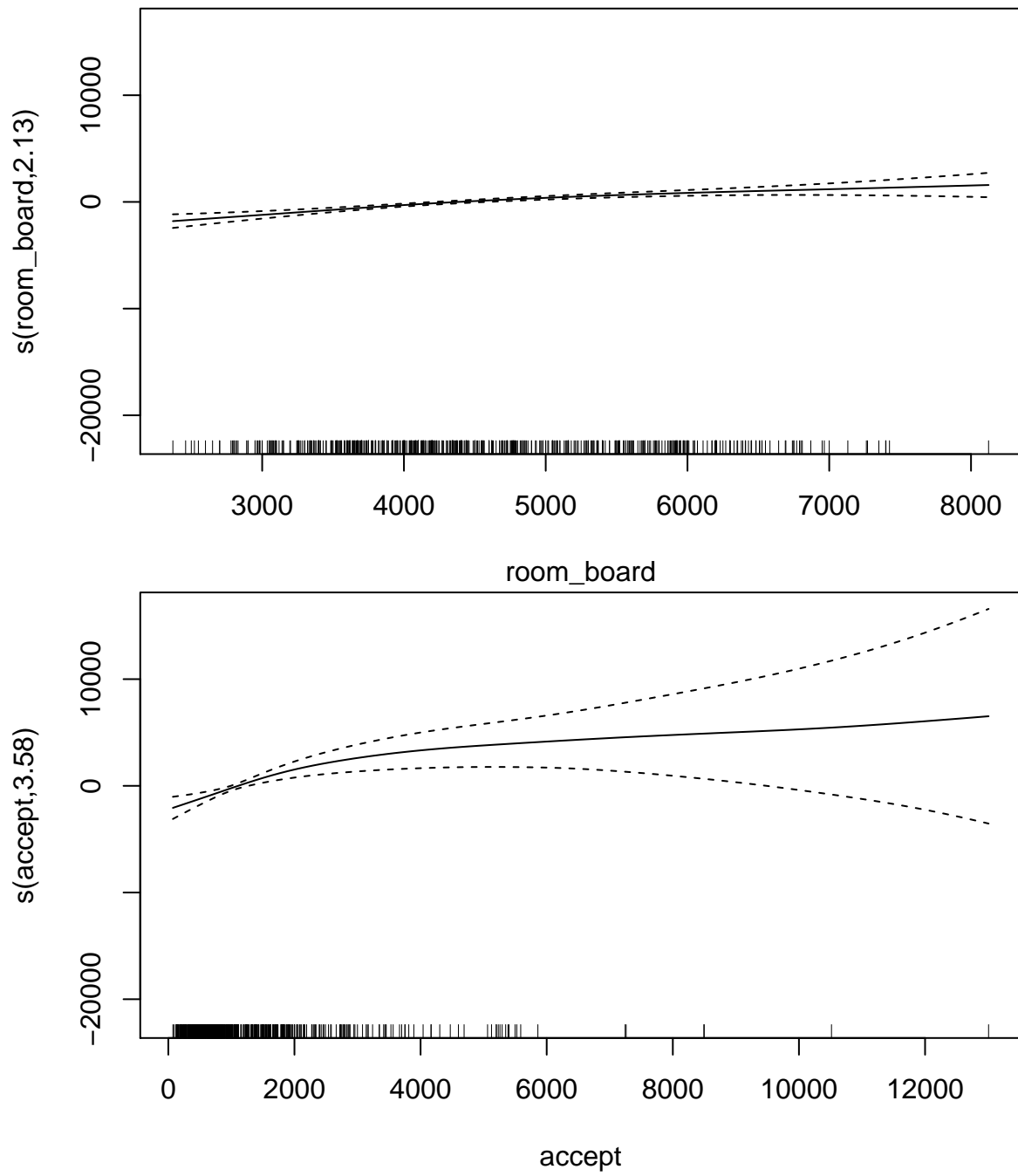


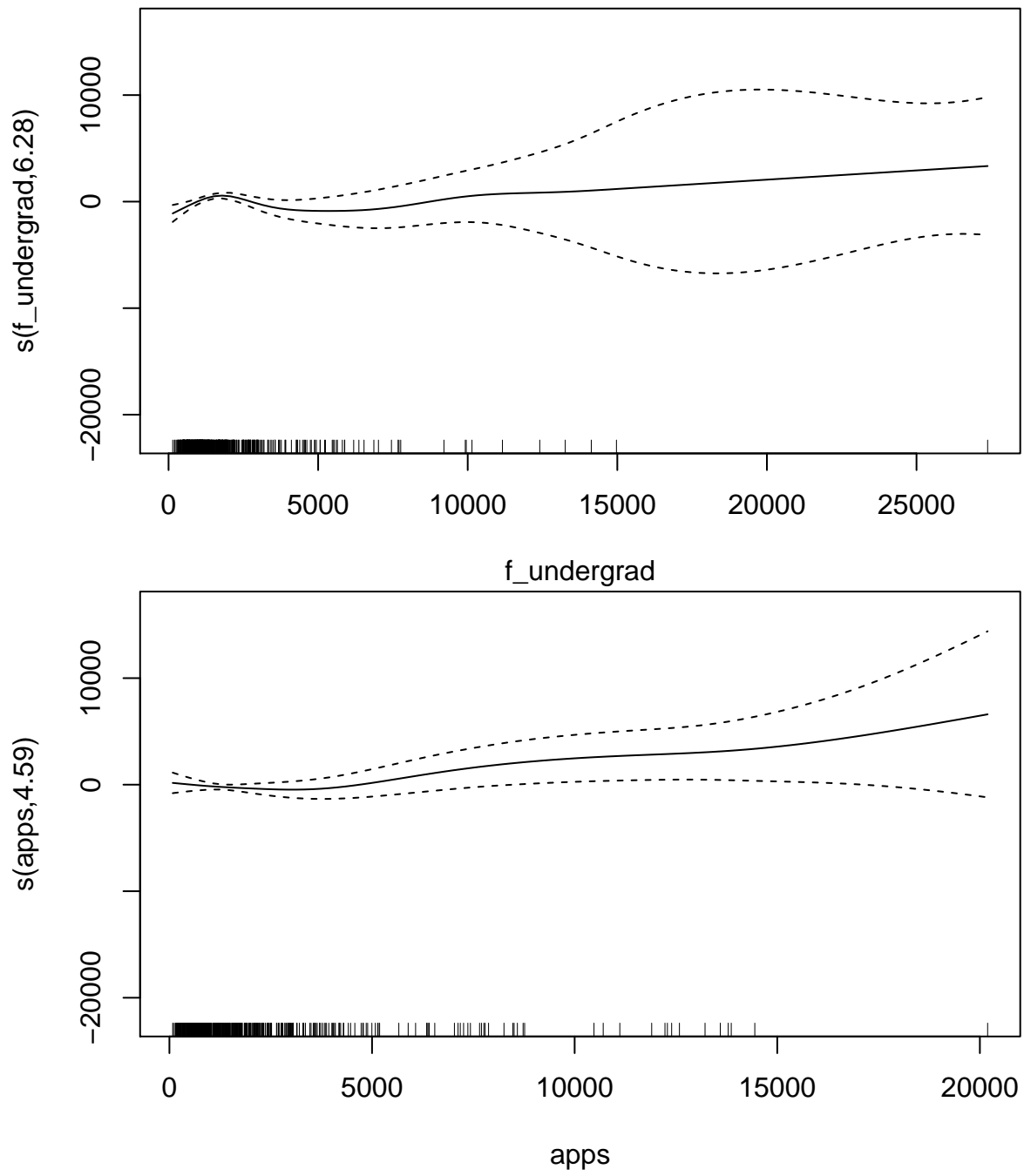


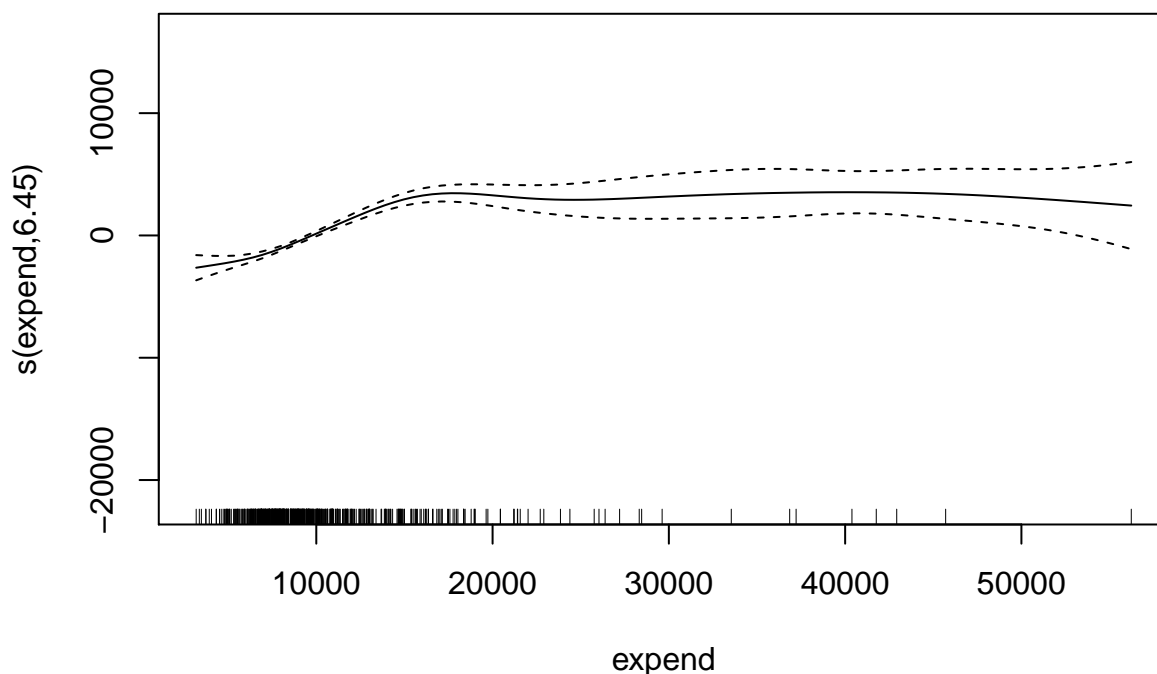












From the results by caret, we can see that the output of bestTune showed that selecting FALSE is better than selecting TRUE. Comparing the results in caret, RMSE of selecting FALSE is smaller than that of selecting TRUE. From the final model, we can see that it added smooth function to every variable. Both GCV and df score are large. The result of caret also indicates that there are some potential tensor interaction between predictors.

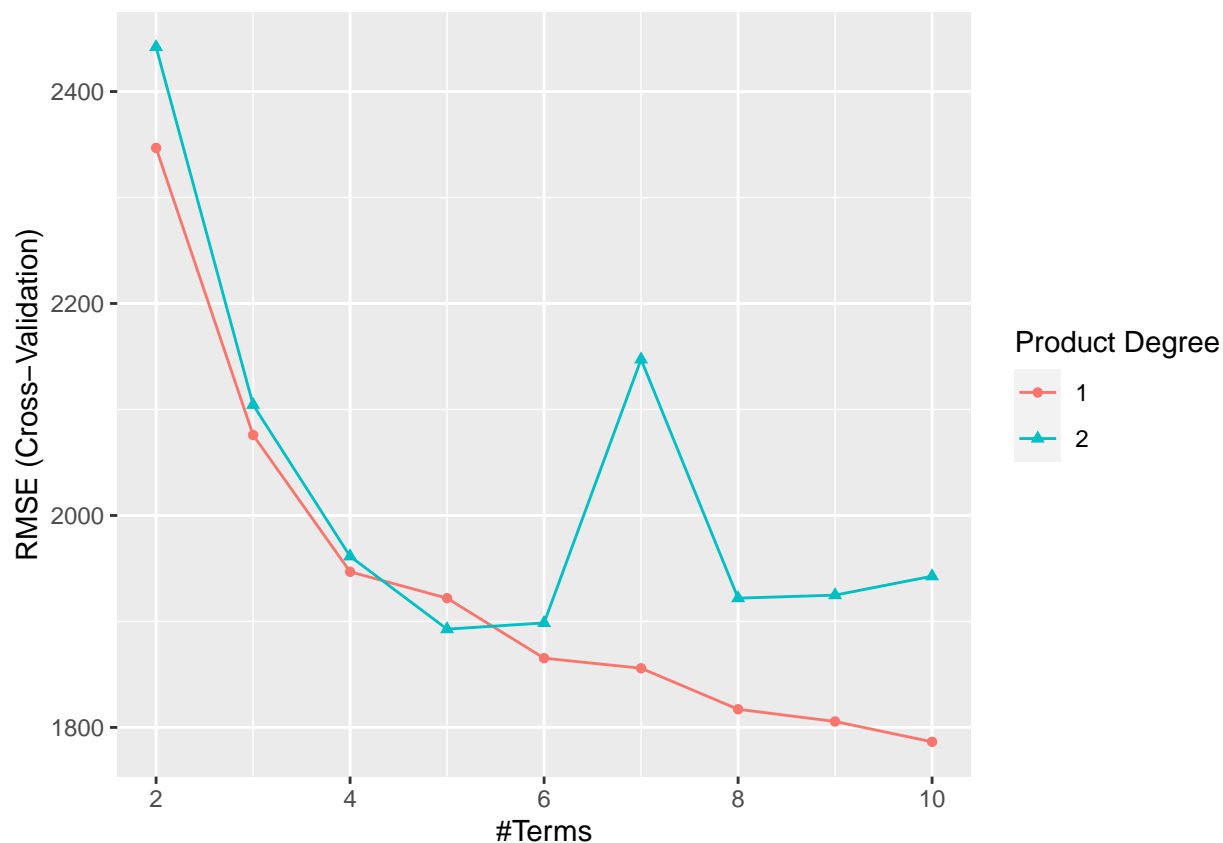
d) Train a multivariate adaptive regression spline (MARS) model using all the predictors

```
ctrl1 <- trainControl(method = "cv", number = 5)

mars_grid <- expand.grid(degree = 1:2,
                        nprune = 2:10)

set.seed(1)
mars.fit <- train(x, y,
                  method = "earth",
                  tuneGrid = mars_grid,
                  trControl = ctrl1)

ggplot(mars.fit)
```



```
mars.fit$bestTune
```

```
##  nprune degree
##  9      10      1
```

```
coef(mars.fit$finalModel)
```

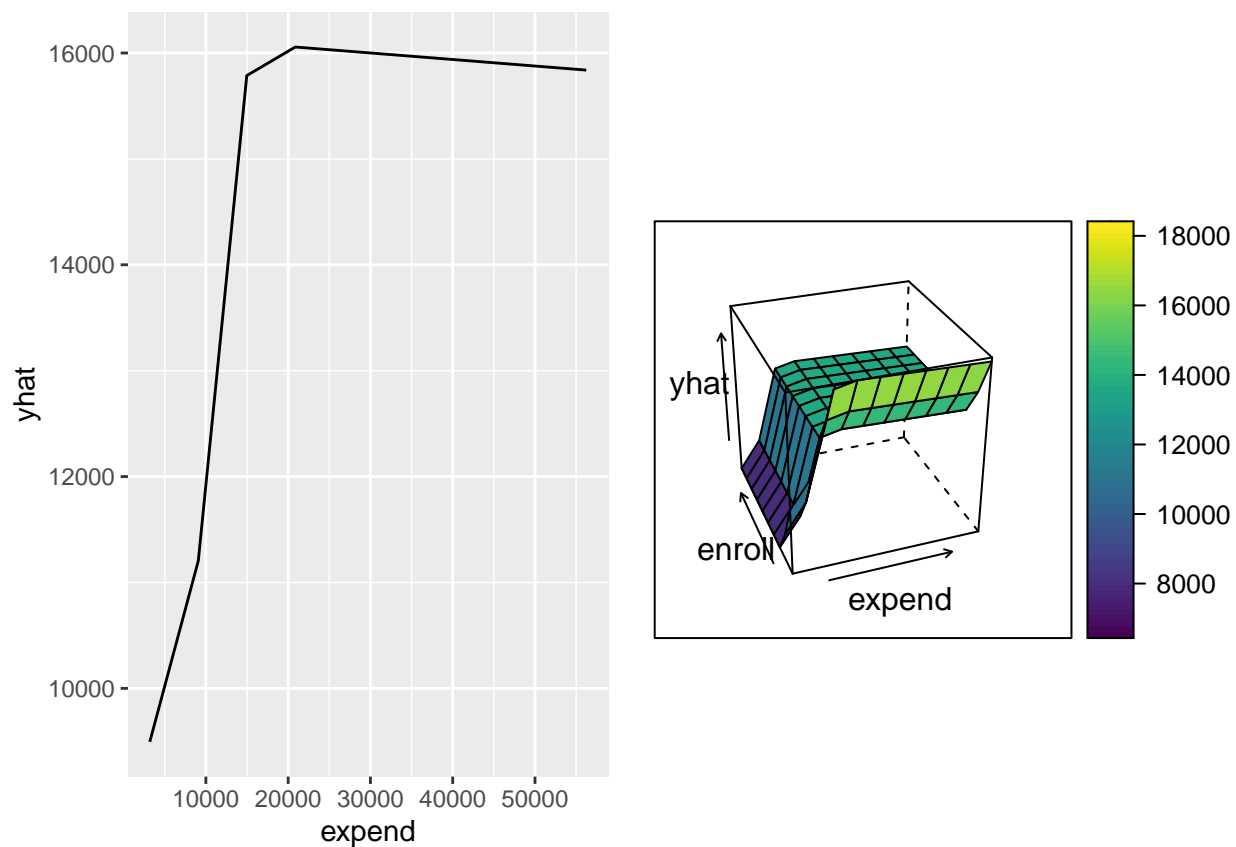
```
##      (Intercept)      h(expend-15365)  h(4450-room_board) h(f_undergrad-1355)
##      10856.8275542      -0.7836173      -1.4272043      -0.3818847
## h(1355-f_undergrad) h(22-perc_alumni)      h(apps-3712)      h(913-enroll)
##      -1.6799143      -105.5570689      0.4334737      4.5019587
##      h(2193-accept)      h(expend-6881)
##      -1.9769988      0.7774546
```

```
#partial dependence plot
```

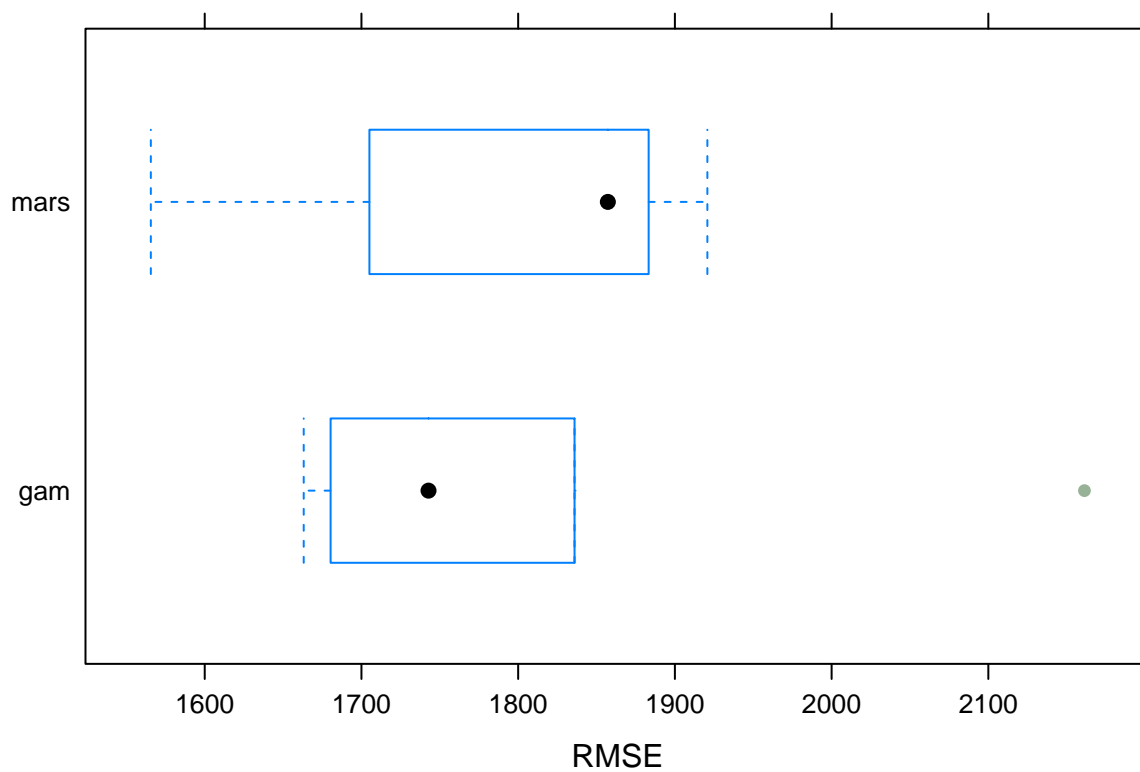
```
p1 = pdp::partial(mars.fit, pred.var = c("expend"), grid.resolution = 10) %>% autoplot()
```

```
p2 = pdp::partial(mars.fit, pred.var = c("expend","enroll"), grid.resolution = 10) %>% plotPartial(level,
```

```
grid.arrange(p1, p2, ncol = 2)
```

```
bwplot(resamples(list(mars = mars.fit,
gam = gam.fit)), metric = "RMSE")
```



The final model using MARS is:

$$f(x) = 10856.83 - 0.78h(\text{expend}-15365) - 1.43h(4450-\text{room_board}) - 0.38h(\text{f_undergrad}-1355) - 1.68h(1355-\text{f_undergrad}) - 105.56h(22-\text{perc_alumni}) + 0.43h(\text{apps}-3712) + 4.50h(913-\text{enroll}) - 1.97h(2193-\text{accept}) + 0.78h(\text{expend}-6881)$$

From the boxplot, we can see that GAM has smaller RMSE than MARS.