# Equivalence Teaching Tool

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# Contents

1	Intr	oduction	4
	1.1	Motivation	4
	1.2	Approach	4
	1.3	Objectives	5
	1.4	Achievements	6
2	$\mathbf{Log}$	ic	7
	2.1	Propositional Logic	7
	2.2	First Order Logic	8
	2.3	Truth tables	8
3	Rela	ated Work	9
	3.1	Logic Daemon	9
	3.2	Pandora	10
	3.3	Logic Solver	11
	3.4	Truth Tables	12
	3.5	LogicCalc	12
	3.6	Propositional Logic Calculator	13
	3.7	Previous Year Projects	14
4	Par	ser	16
	4.1	Parser Generation	16
		4.1.1 ANTLR4 and alternatives	16
	4.2	Grammar	16
		4.2.1 Tokens	16
		4.2.2 Order of Precedence	16
		4.2.3 Error Handling - Incorrect grammar syntax to handle er-	
		rors semantically	16
		4.2.4 Extension to First Order Logic	16
	4.3	Parser	16
		4.3.1 Parser	16
		4.3.2 Walker	16
5	Inte	ernal Tree Representation	17

	5.1	1	17
		0	17
	5.2		17
		5.2.1 Node indexing - proof no two nodes can have the same	
		depth and key	17
		5.2.2 Tree operations, eg. adding a new node	17
	5.3	Nodes	17
		5.3.1 Binary Operators	17
			17
			17
	_		
6		1	18
	6.1	8	18
	6.2	11	18
	6.3	11	18
	6.4	11	18
		6.4.1 Generating equivalent formulae	18
7	And	Iroid VS Web Applications	19
•	7.1	1 1	19
	,		19
			19
		0 1	19 19
	7.2	· ·	19 19
	1.4	11	19 19
			19 19
		1.2.2 Tolliation Tree would not retailed on phone app	
8	And	lroid 2	20
	8.1	Technical Background	20
	8.2	Features	20
		8.2.1 Multi-Touch Gestures	20
	8.3	Market Share	20
9	The	App	21
J	9.1		21
	9.1		21
	0.2		21 21
	9.2	1	
		0 0 1	21
		9.2.2 Generate equivalence formula - repeated application of	~ 4
			21
	9.3	v	21
		8 1	21
	9.4		21
			21
			21
	9.5	Formation Tree Representation	21

	9.6 9.7	9.5.1 The Formation Tree 21   9.5.2 The NP-Complete Problem 22   9.5.3 What Makes An Attractive Tree? 22   9.5.4 Reingold and Tilford's Algorithm 23   9.5.5 Walker's Algorithm 23   9.5.6 Abego TreeLayout 23   9.5.7 Drawing the Formation Tree 23   9.5.8 Responding to Touch Events 24   Rule Application 25   9.6.1 How rules are displayed within the app 25   Difficulty 25   9.7.1 Length of generated equivalences 25
		9.7.2 Number of variables/operators
		9.7.3 Number of rules different
	9.8	Help
	0.0	9.8.1 Detecting cycles
10		uation 26
	10.1	Testing
		10.1.1 JUnit tests
		10.1.2 Survey - People who do/don't understand logic 26
	10.2	Performance
11	Con	clusions 27
		Completed objectives
		Comparisons to related work
		•
<b>12</b>		re Work 28
		Improvements to completed objectives
	12.2	Device support
		12.2.1 Reduced app without trees for phones
	12.3	More complex help
		12.3.1 Calculating the optimal route
		12.3.2 Suggested next move
	10:	12.3.3 Future dead end detection
	12.4	First Order Logic
		12.4.1 Extend grammar
		12.4.2 Add and handle new rules
13	Usei	Guide 30
		How to use the app
		<del></del>

# Introduction

### 1.1 Motivation

All logics are based on propositional logic in some form, so it is important that new students learn how to use it. Propositional logic consists of syntax, semantics and proof theory; syntax is the formal language which is used to express concepts, semantics provide meaning for the language and proof theory provides a way to convert one formula into another using a defined set of rules.

We know that new students learning propositional logic can struggle to understand the rules and how they should be applied to formulae. To help with this our idea is to create an equivalence teaching tool; this will be a tablet application which will allow a user to apply rules to a formula until they have reached the desired equivalence.

## 1.2 Approach

We decided early on in the project that the tool should be an Android tablet application as opposed to a web or mobile one; this allows for an intuitive, interactive design while still having a large screen space.

The tool can be divided up into its main component parts: the parser, tree constructor, tree processing and tablet interface. The parser will be generated by the ANTLR4 parser generator, the tree constructor, processing code and the tablet interface will be written in Java with the interface using the Android SDK.

The parser requires a grammar to generate the relevant parser components. This will be used to parse the initial equivalence and return the ANTLR4 tree representation of the string.

The tree constructor is required to take the ANTLR4 representation of a tree and convert it into a more useful data structure which can be modified and

displayed easily. The operators and atoms of the formula will be represented as nodes and leaves respectively.

Tree processing will be used to internally calculate which rules are applicable to each node and will allow us to subsequently manipulate the tree by applying these rules.

The interface will have an intuitive design displaying the current formula's formation tree (generated by the tree constructor) and allowing a user to click on the operators to apply a rule.

## 1.3 Objectives

The application should have some key features. These are outlined below:

#### 1. Graphical tree representation of formulae

Representing the formula as a tree structure allows a user to see exactly how the formula should be read and can help them understand the order of operations. It also allows the user to click on an operator to select a rule for it; this is much more intuitive than just clicking on the whole formula and not knowing which section the rule would be applied to.

#### 2. Undo/Redo functionality

Previous equivalent formulae will be displayed above the current formula. When an old formula is selected it will expand into tree form and the formulae below it are faded out (*Undo*). This will allow a user to perform rules on the old tree or select one of the later faded trees (*Redo*). When a rule is applied, the faded formulae will be removed from the history allowing the user to continue on from that point (Figure 1.1).

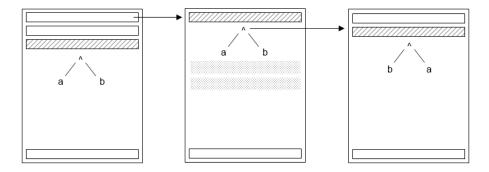


Figure 1.1: Undo functionality - Clicking a previous equivalence shows the formula tree for that equivalence and sets the undone history as translucent. Applying a rule then overwrites the undone history.

#### 3. Generated equivalences

We want to allow the user to have equivalences automatically generated for them to solve. This would be implemented by running the tool on a generated formula to give a significantly different equivalent one. The key advantage of having this feature as well as allowing manual entry of equivalences would be that the user will have a continuous supply of new equivalent logic formulae after completing all those set by their lecturer. This also requires generating the initial formula for the tool to be run upon or allowing the user to manually enter the first formula and have the second one generated for them.

#### 4. Difficulty setting

Extending the idea of generated equivalences, we could allow the user to select a difficulty. This would be calculated by the length of the generated formula and how many rules were applied to get its equivalent formula. We could also provide a recommended difficulty based on how many previous equivalences they have completed and how far from optimal their solutions were.

#### 5. Help

Providing the user with help is key to their improvement. Once an equivalence has been set up, the tool should calculate the optimal route from the starting formula to the desired end point. Upon finishing the equivalence the user will receive a message telling them how far from the optimal solution they were and give them the option to try again or to view the optimal solution. Throughout there will also be a help button available that will suggest the next step to the user on request; this includes the recommendation to undo certain steps if the user reaches a cycle. Pointing out mistakes could also be enabled, so if the user has completed a cycle or is heading towards a dead end they will be prompted to consider a different strategy.

#### 1.4 Achievements

We have been challenged with developing an android application to help new students learn propositional logic and to complete logical equivalences. In this section we will discuss our motivation, approach and objectives.

# Logic

We are assuming a basic understanding of propositional logic, including the operators and rules that are defined in the system. For more information on these please visit the Wikipedia article[1].

## 2.1 Propositional Logic

Propositional logic is a branch of logic that studies ways of combining and modifying whole sentences, statements or propositions to form more complex propositions. It is a formal system containing logical relations and properties which are derived from these methods of joining or altering statements.

A logical system contains three major parts:

- 1. Syntax the formal language that is used to express concepts.
- 2. Semantics provide meaning for the language.
- 3. Proof theory provides a way to convert one formula into another using a defined set of rules.

#### Logical definitions:

- Atomic A formula whose logical form is  $\top$ ,  $\bot$  or p for an atom p.
- Negated atomic A formula of the form  $\neg p$ .
- Negated formula A formula of the form  $\neg A$  for a formula A.
- Conjunction A formula of the form  $A \wedge B$ .
- Disjunction A formula of the form  $A \vee B$ .
- Implication A formula of the form  $A \rightarrow B$ , where A is the antecedent and B is the consequent.
- Literal A formula that is either atomic or negated atomic.
- Clause A disjunction of one or more literals.

A *statement* is defined as a meaningful declarative sentence that is either true or false. For example, a statement could be:

- 'Socrates is a man.'
- 'All men live on Earth.'

A statement can be constructed of multiple parts, for example, the above statements can be combined into:

• 'Socrates is a man and all men live on Earth.'

Each part of this statement can be considered a proposition. Propositional logic involves studying the connectives that join these such as 'and' and 'or' (to form conjunctions and disjunctions), the rules that determine the truth values of the propositions and what that means for the validity of the statement.

## 2.2 First Order Logic

### 2.3 Truth tables

It is necessary to understand the meanings of the symbols used in a language. Truth tables are mathematical tables used in logic to compute the functional values of logical expressions. They can be used to determine whether or not a propositional logic statement is logically valid.

A *situation* determines whether each propositional atom is true or false. A truth table shows all the situations the input variables can be in. We write 1 for true and 0 for false as shown below:

A	В	A∧B	A∨B
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Truth tables can be used to define any operators, including any new ones which might be desired.

# Related Work

## 3.1 Logic Daemon

Created in Texas A&M University, the Logic Daemon[5] is an online logic proof checker. It comprises a simple web page with two small text input boxes for the premises and conclusion and then one large text box for applying primitive rules (Figure: 3.1).

While this tool does allow us to apply rules to prove equivalences we do not find it very intuitive to use at all. The interface itself looks quite unclear and isn't user friendly so it would not be suited to new students. Applying the rules to the premises is also quite confusing as it shows all the rules and does not alert the user to the fact they cannot be applied until they have already clicked on it. We found this a frustrating way of attempting a proof; as such, we intend to only display the rules that can be applied in the current situation.

However, it is a useful tool for those more knowledgeable about logic for checking proofs once they have got to grips with the interface. It also has a 'Get Help' button to suggest which rule to use next. We hope to use the idea of a help button in our project but to build upon it in order to provide more extensive help such as showing when an action has been repeated (i.e. a cycle has been reached).

We hope that our tool will be more intuitive to use because we will display formulae as formation trees with interactive nodes to apply rules with. This should provide a more straight forward way of presenting the rules to new students.

The website also provides a simple equivalence checker, well-formed formula checker and countermodel checker. These are all similar tools useful in logic but again the interface is not particularly intuitive or user friendly.

The Logic Daemon tool also provides support for first order logic which we are not planning on covering in our main tool. However, this could be implemented as an extension at the end of our project.

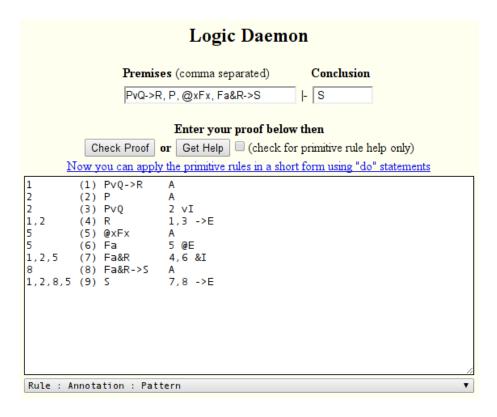


Figure 3.1: Logic Daemon

### 3.2 Pandora

Pandora[6] is a tool created by Imperial College London; it stands for 'Proof Assistant for Natural Deduction using Organised Rectangular Areas'. It can be used to prove that a goal formula follows from the given formulae. Pandora allows the user to repeatedly apply natural deduction rules (Figure: 3.2).

We do not intend to include natural deduction in our tool as we want to focus more on solving propositional logic equivalences. Pandora is a much nicer tool than the Logic Daemon and we find it much more intuitive and easy to use.

Pandora allows a user to apply rules forwards or backwards. We know from using Pandora that some proofs are easier to complete working backwards; as such, this is an idea we hope to incorporate into our tool by allowing the user to expand either the top or bottom formula to show its formation tree and to apply rules.

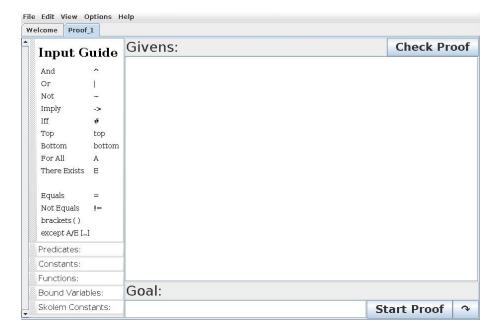


Figure 3.2: Pandora

### 3.3 Logic Solver

This is an Android application to show truth tables and possible logical equivalences. It looks like it has been made as a phone application as on a tablet it fills very little of the screen and it is difficult to click on some of the links (Figure 3.3).

The user can apply rules to the formula they enter into the application; this is done by selecting the rule they want out of a list which is then applied to the formula. However they cannot use this application to solve an equivalence as they can only enter one formula and then apply rules to that. We want to improve upon this in our own application by allowing the user to enter two formulae and work from either end to solve it.

The list of rules that is offered to the user only shows the rules that can be applied to the current formula. We want to also offer this behaviour as there are far too many rules to display them all to the user and expect them to sort through which could be applied.

Formulae are displayed simply in the application; we think this could be improved upon and want to display our current formula as its formation tree.



Figure 3.3: Logic Solver Android Application

### 3.4 Truth Tables

Truth Tables is also an Android application mainly for use on a phone. It simply generates and displays truth tables based on a logical formula (Figure 3.4).

We currently do not plan on adding truth tables to our application as we do not believe it is as useful for learning and understanding propositional logic as our other ideas. However, this could be added as an extension at the end of the project depending on how much time is left.

# 3.5 LogicCalc

LogicCalc is an Android application for solving problems with propositional logic. It allows the user to create workbooks to save and print their proofs (Figure 3.5). However we had trouble running it on our Nexus tablet.

We like the idea of saving proofs in workbooks for future reference. This is something we had not considered before and are curious to explore as an extension. This functionality would be useful for students completing exercises that needed a paper hand in as they would be able to save and print them off.

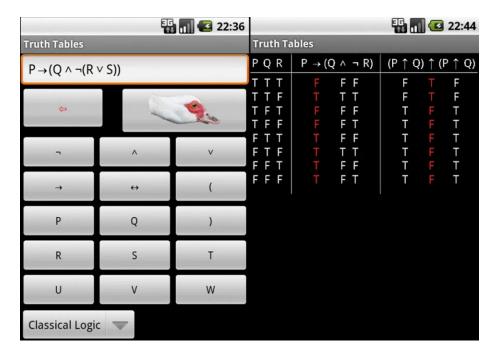


Figure 3.4: Truth Tables Android Application

## 3.6 Propositional Logic Calculator

The Propositional Logic Calculator [7] finds all of the models of a given propositional formula. The website tells us that the only limitation for this calculator is that we have only three atomic propositions to choose from: p,q and r (Figure: 3.6).

Propositions are entered using the keyboard they provide on the calculator and the reasoning process is initiated by clicking 'ENTER'. It then calculates the truth value assignments that will make the formula true in the 'MODELS' section and the truth value assignments making the formula false in the 'COUNTERMODELS' section.

This tool is used simply for calculating truth values of a formula. We will not be implementing this in our equivalences tool because we do not think that knowing the truth values for a formula is as useful for learning and understanding propositional logic as our other ideas.

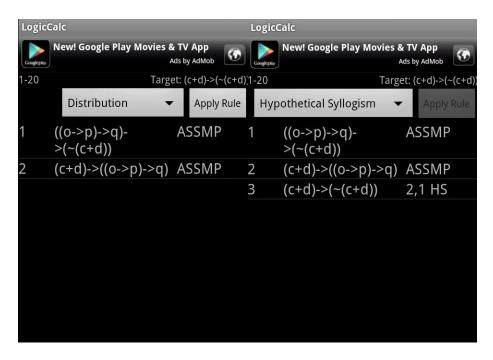


Figure 3.5: LogicCalc Android Application

# 3.7 Previous Year Projects

Once we have made more progress on our tool we will look into finding out what previous years did when they were assigned similar projects. We do not want to look into these this early in the project so we can generate our own implementation ideas as these projects will be very similar to ours.

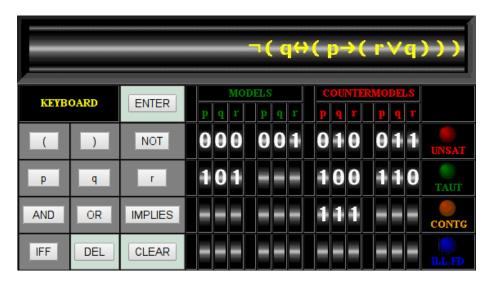


Figure 3.6: Propositional Logic Calculator

# Parser

- 4.1 Parser Generation
- 4.1.1 ANTLR4 and alternatives
- 4.2 Grammar
- **4.2.1** Tokens
- 4.2.2 Order of Precedence
- 4.2.3 Error Handling Incorrect grammar syntax to handle errors semantically
- 4.2.4 Extension to First Order Logic
- 4.3 Parser
- **4.3.1** Parser
- 4.3.2 Walker

# Internal Tree Representation

- 5.1 Compiler
- 5.1.1 Converts string into formation tree
- 5.2 Formation Tree
- 5.2.1 Node indexing proof no two nodes can have the same depth and key
- 5.2.2 Tree operations, eg. adding a new node
- 5.3 Nodes
- 5.3.1 Binary Operators
- 5.3.2 Unary Operators
- **5.3.3** Atoms

# Tree Manipulation

- 6.1 The Rule Engine
- 6.2 Rule Selection which rules can be applied to a node
- 6.3 Rule Application how rules are applied to a node
- 6.4 Random Rule Application
- 6.4.1 Generating equivalent formulae

# Android VS Web Applications

- 7.1 Android vs Web app
- 7.1.1 Intuitive to use
- 7.1.2 Learning experience
- 7.1.3 None currently available for Android
- 7.2 Tablet vs Phone app
- 7.2.1 Screensize
- 7.2.2 Formation Tree would not feasible on phone app

# Android

## 8.1 Technical Background

### 8.2 Features

#### 8.2.1 Multi-Touch Gestures

Android is an operating system based on the Linux kernel which was designed primarily for touchscreen mobile devices such as smartphones and tablet computers[3].

The user interface is based around direct manipulation. This means using touch inputs that loosely correspond to real-world actions, for example swiping, tapping and pinching to control items on screen. The response to user actions is designed to be immediate and provides a fluid touch interface. It often provides haptic feedback (forces, vibrations or motions) to the user using the vibration capabilities.

Android takes advantage of internal hardware such as accelerometers, gyroscopes and proximity sensors in some applications to respond to other user actions, such as re-orientating the screen from portrait to landscape.

## 8.3 Market Share

Android's main competitor in the mobile platform market is Apple's iOS. In research conducted in the fourth quarter of 2012, Kantar Worldpanel Comtech showed sales of all Android smartphones worldwide outpacing the iPhone by a huge margin: 70 percent to 21 percent of the smartphone market [4].

In the tablet market the iPad dominated Android's 7" tablets with 53.8 percent to 42.7, which is lower than Android's smartphone market but steadily increasing.

# The App

- 9.1 Truth Tables
- 9.1.1 Ensure equivalences are equal before beginning
- 9.2 Random Generation of Equivalences
- 9.2.1 Generating single equivalence
- 9.2.2 Generate equivalence formula repeated application of random rules
- 9.3 Custom Keyboard
- 9.3.1 Logic symbols and variables
- 9.4 Undo/Redo Functionality
- 9.4.1 Stack of trees maintained
- 9.4.2 OnClick listeners
- 9.5 Formation Tree Representation
- 9.5.1 The Formation Tree

Each logical connective in an formula can be written enclosed in parentheses. For example:

$$(a \rightarrow b) \land \neg a$$
 can be written as  $((a \rightarrow b) \land (\neg a))$ 

Due to this property we can create a binary tree unique to each proposition; this is called a formation tree. Each connective and propositional atom is represented by a node and a leaf respectively. This provides a clear and attractive way of displaying the formula but is too expensive for every day use. In our application we wish to display the formation tree of the current equivalences.

Displaying the formation trees allows a user to see an interactive representation of the equivalences where touching the nodes and leaves in the tree allows the user to apply rules to various parts of the tree.

### 9.5.2 The NP-Complete Problem

When planning to draw the formation trees to the app we assumed there would be a simple, classic algorithm for drawing neat, aesthetically pleasing trees. However, we discovered the problem was not that simple. Drawing an attractive Tree layout is an NP-complete problem[11] and there are many tree-drawing algorithms attempting to solve the problem of drawing attractive trees. We reviewed many of these algorithms to find one suitable to use.

#### 9.5.3 What Makes An Attractive Tree?

Although generating an attractive tree is a matter of taste, certain principles are widely agreed upon to be key to drawing an aesthetically pleasing tree. The first three are taken from Wetherell and Shannon's tree drawing algorithm for producing tidy drawings using the smallest width possible [8].

**Aesthetic 1**: Nodes at the same level of the tree should lie along a straight line, and the straight lines defining the levels should be parallel.

**Aesthetic 2**: A left son should be positioned to the left of its father and a right son to the right.

**Aesthetic 3**: A father should be centered over its sons.

Although not mentioned in the original article, Aesthetic 1 was also meant to guarantee the edges in the tree do not intersect except at the nodes by requiring that the relative order of nodes over any level be the same as the level order traversal of the tree. Wetherall and Shannon's algorithm is fairly basic and has a deficiency that compromises the overall attractiveness of the resulting tree. It produces drawings that could be made narrower within the constraints of the aesthetics and are not entirely pleasing to the eye - nodes in certain subtrees are drawn too far apart due to the fact that their shape is influenced by the positioning of nodes outside that subtree. This leads to the asymmetry of the resulting tree. Therefore when Reingold and Tilford[9] set out to create a better tree drawing algorithm they introduced a fourth aesthetic:

**Aesthetic 4**: A tree and its mirror image should produce drawings that are reflections of one another; moreover, a subtree should be drawn the same way regardless of where it occurs in the tree.

Fulfilling this aesthetic requires us breaking Wetherell and Shannon's aim of minimum width so that all all isomorphic subtrees are drawn the same. This is considered a more important principle than using the minimum width as our main aim is to create an attractive tree. This is an acceptable drawback as Supowit and Reingold proved that determining the minimum width under these aesthetics is NP-hard[10]. Therefore, we can use Reingold and Tilford's algorithm to draw our formation tree to produce an attractive result.

### 9.5.4 Reingold and Tilford's Algorithm

### 9.5.5 Walker's Algorithm

### 9.5.6 Abego TreeLayout

Abego TreeLayout is an external library which efficiently creates compact and highly cusomisable tree layouts. It's based on Walker's algorithm with enhancements suggested by Buchheim, Jnger, and Leipert[13]. The software builds tree layouts in linear time so that even trees with many nodes are built quickly. Tree-Layout separates the layout of the tree from the actual rendering, therefore it is suitable for drawing trees in an Android application because it is not limited to a specific output or format.

#### 9.5.7 Drawing the Formation Tree

We created a custom view DrawView to draw the trees to the screen in the application. This is created by simply extending the Android View class and overriding the onDraw() function. To add a custom view to our user interface we also need to define custom attributes in a resource element as shown in Listing 1.

Listing 1: DrawView resource element

In onDraw() we determine whether we are drawing the top or bottom formation tree and call setUpTreeLayout() (shown in Listing 2) which uses the Abego TreeLayout to manage the layout.

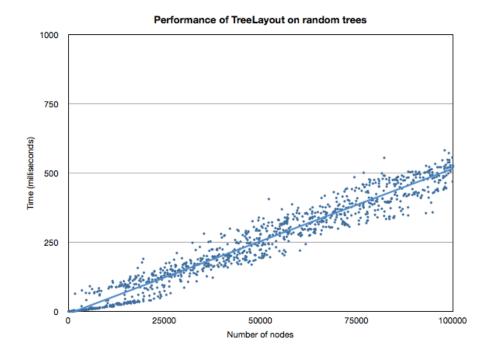


Figure 9.1: Illustrates the linear time behaviour of the Abego TreeLayout algorithm[14] and shows the applicability for large numbers of nodes.

To use TreeLayout we create a TreeLayout instance with our tree (accessible through the TreeForTreeLayout inferface), a FixedNodeExtentProvider (defines a fixed size for each node) and a DefaultConfiguration (configures the layout by defining parameters such as gapBetweenNodes and gapBetweenLevels).

TreeLayout calculates the position of each node so we can then paint the nodes and edges to the screen.

### 9.5.8 Responding to Touch Events

By overriding onTouchEvent() in DrawView we can get the event action performed on the view. Using this we can check if the touched position is within the bounds of any of the nodes drawn to the screen. If the position is within the bounds of a node we display applicable rules for that node by calling setRules() on the current activity. Selecting a node also forces a redraw so that we can highlight the selected node.

Listing 2: setUpTreeLayout() is called by onDraw() in DrawView

# 9.6 Rule Application

- 9.6.1 How rules are displayed within the app
- 9.7 Difficulty
- 9.7.1 Length of generated equivalences
- 9.7.2 Number of variables/operators
- 9.7.3 Number of rules different
- 9.8 Help
- 9.8.1 Detecting cycles

# **Evaluation**

- 10.1 Testing
- 10.1.1 JUnit tests
- 10.1.2 Survey People who do/don't understand logic

### 10.2 Performance

Once the application has been built we will need to find a way to test that it has met the objectives we set out to achieve. The main objective is to create an intuitive Android application that helps students improve in solving equivalences; as such, we will survey a wide range of test subjects. These tests will be split into two groups: people who understand logic and equivalences well and people who are new to propositional logic.

We need to ask people who understand logic (e.g. lecturers) so we can confirm that it works as a teaching tool. People who are new to logic (e.g. first year students) are necessary so we can evaluate how effective it is as a learning tool. This can be carried out through asking the target groups to use the application and provide feedback.

# Conclusions

- 11.1 Completed objectives
- 11.2 Comparisons to related work

# Future Work

- 12.1 Improvements to completed objectives
- 12.2 Device support
- 12.2.1 Reduced app without trees for phones
- 12.3 More complex help
- 12.3.1 Calculating the optimal route
- 12.3.2 Suggested next move
- 12.3.3 Future dead end detection
- 12.4 First Order Logic
- 12.4.1 Extend grammar
- 12.4.2 Add and handle new rules

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# User Guide

13.1 How to use the app