Derivatives 512 Project 3

Yixuan Huo, Quancheng Shao

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Our code: (click) https://colab.research.google.com/drive/11zOjVGNGGwgZWYcXY2OMBOpra7Eyx4Nd?usp=

This report implements Monte Carlo Simulation with LSMC engine to price Morgan Stanley's callable contingent income security. Our valuation with 500,000 simulated paths (USD 956.95) is within 3.35 USD of the issuer's estimate (USD 953.60), indicating their pricing likely relies on 70% moneyness (the coupon barrier) implied volatility at maturity and 2-year historical stocks' correlation.

1 Data Collection (raw data in the appendix)

The **underliers** are SP 500 (SPX), NASDAQ-100 (NDXT), Russell 2000 (RTY). We set the data retrieval date to the pricing date **16:00 on March 10, 2025**. The implied volatility (IV) data were obtained using the Bloomberg `"OVM"` function. We set the moneyness level at "55%, 60%, 70%, 85%, 100%, 110%", including the coupon barrier and downside protection. **Our Monte Carlo Model is sensitive to the volatility input. The model restricts to choose only one volatility as input, but the IVs are different for different moneyness and time to maturity. Thus, we choose a range of volatilities, aiming to capture the potential fluctuation ranges of the implied volatilities before maturity and see the valuation dynamics. We get the IVs at the observation dates. We choose to use the 70% IVs at maturity as default**, as shown in Table 1. The reason why we choose 70% maturity IVs as default is that 70% is the monthly coupon barrier, tends to reflect market expectations for future volatility, and implied volatilities often drift toward larger values over time (70% maturity IV is larger then ATM maturity IVs). It captures the market's view of the product's valuation, yielding a more accurate valuation than the ATM IVs. (IV table in the appendix. Notably, IVs for NDXT are the same in the same date among different moneyness, possibly since its low liquidity.)

Table 1: 70% Implied Volatility at Maturity

Index	**Ticker**	**70% Maturity IV (%)**
S&P 500	SPX	26.909
NASDAQ-100	NDXT	24.379
Russell 2000	RTY	29.079

The risk-free rates were obtained from Bloomberg using the `"42 - USD - OIS"` function. Due to Bloomberg's standard two-business-day reporting lag, the discount factors were retrieved as of **16:00 on March 6, 2025**. The relevant discount factors used for interpolation are shown below: To interpolate the discount factor at the maturity date $T = 1.7671$ years, we apply:

$$D(T) = D_1 + (D_2 - D_1) \cdot \frac{T - t_1}{t_2 - t_1}$$

Table 2: Discount Factors Used for Interpolation

Date	**Discount Factor**	**Role**
2026-09-10	0.943864	D_1 (first reference)
2026-12-15	0.935158	$D(T)$ (interpolated)
2027-03-10	0.927449	D_2 (second reference)

This results in:

$$D(T) = 0.935158$$

From the interpolated discount factor, the continuously compounded zero rate is:

$$r = -\frac{\ln D(T)}{T} = -\frac{\ln(0.935158)}{1.7671} \approx 3.7937\%$$

We assume a continuous dividend yield for simplicity without losing accuracy. The Bloomberg `"OVME"` function was used to approximate these values:

Table 3: Continuous Dividend Yields

Index	**Ticker**	**Dividend Yield (%)**
S&P 500	SPX	1.327
NASDAQ-100	NDXT	0.511
Russell 2000	RTY	1.522

For the dividend yield data, **we chose to use a continuous dividend yield, which makes the valuation simple without losing accuracy.** We approximated this value using data from the Bloomberg `"OVME"` function. It provides a reasonable proxy for the continuous dividend yield, as shown in Table 3.

For correlation data, we use "CORR" function in bloomberg, and set daily frequency. We choose the correlation of the past 2 years as for Mar 10th, 2025.

$$\text{Corr\_2y} = \begin{bmatrix} \rho_{\text{SPX,SPX}} & \rho_{\text{SPX,NDXT}} & \rho_{\text{SPX,RTY}} \\ \rho_{\text{NDXT,SPX}} & \rho_{\text{NDXT,NDXT}} & \rho_{\text{NDXT,RTY}} \\ \rho_{\text{RTY,SPX}} & \rho_{\text{RTY,NDXT}} & \rho_{\text{RTY,RTY}} \end{bmatrix} = \begin{bmatrix} 1.000 & 0.858 & 0.761 \\ 0.858 & 1.000 & 0.624 \\ 0.761 & 0.624 & 1.000 \end{bmatrix}$$

2 Algorithm

Model Assumptions and Inputs

Let us denote the following inputs and variables. The parameters used are (data in arrays are all in [SPX, NDXT, RTY] order):

- Number of simulations: $N = 500,000$
- Spot prices at Mar 10th, 2025: $S_0 = (S_0^{SPX}, S_0^{NDXT}, S_0^{RTY}) = [5614.56, 9548.55, 2019.067]$.
- Risk-free rate: $r = 3.7937\%$ (derived from interpolated discount factors between 2026-09-10 and 2027-03-10: $D(T) = D_1 + (D_2 - D_1) \cdot \frac{T-t_1}{t_2-t_1}$, and $r = -\frac{\ln D(T)}{T} = -\frac{\ln(0.935158)}{1.7671} \approx 0.037937$)
- Continuously compounded dividend yield for underliers $q = (q_{SPX}, q_{NDXT}, q_{RTY}) = [1.327\%, 0.511\%, 1.522\%]$

- 70% Implied Volatilities at Maturity for underliers: $\sigma = (\sigma_{RTY}, \sigma_{RTY}, \sigma_{RTY}) = [26.909\%, 24.379\%, 29.079\%]$.
- Correlation matrix for the past 2 years:

$$\text{Corr} = \begin{bmatrix} \rho_{\text{SPX,SPX}} & \rho_{\text{SPX,NDXT}} & \rho_{\text{SPX,RTY}} \\ \rho_{\text{NDXT,SPX}} & \rho_{\text{NDXT,NDXT}} & \rho_{\text{NDXT,RTY}} \\ \rho_{\text{RTY,SPX}} & \rho_{\text{RTY,NDXT}} & \rho_{\text{RTY,RTY}} \end{bmatrix} = \begin{bmatrix} 1.000 & 0.858 & 0.761 \\ 0.858 & 1.000 & 0.624 \\ 0.761 & 0.624 & 1.000 \end{bmatrix}$$

- Maturity: $T = 1.766$ years
- Principal: \$1,000
- Downside protection: Soft barrier at 60% of S0 on the worst asset at maturity.
- `coupon_amount`: Fixed coupon paid (\$5.833) at observation dates if barrier is met.
- `coupon_barrier` $= 0.7$: If all assets exceed 70% of their initial value at obs dates, coupon is paid.
- `obs_dates`: List of observation dates for coupon determination.
- `redemption_dates`: Dates when early redemption can occur.
- `pricing_date`, `maturity_date`: '2025-03-10' and '2026-12-15'.

Monte Carlo Simulation of Asset Paths

We simulate $N = 500,000$ paths for each of the 3 correlated indices using the risk-neutral dynamics. The superscript of S $i = 0, 1, 2$ represents three indices SPX, NDXT, RTY:

$$dS_t^i = S_t^i(r - q_i)dt + \sigma_i S_t^i dW_t^i,$$

where $\text{corr}(dW_t^i, dW_t^j) = \rho_{ij}$.

Let $t_0, t_1, \ldots, t_K$ be the sorted union of observation, redemption, and maturity dates, measured in year fractions from `pricing_date`. For each path, we use geometric Brownian motion (GBM) with Cholesky-decomposed correlated normals:

$$S_{t_k}^i = S_{t_{k-1}}^i \cdot \exp\left[\left(r - q_i - \frac{1}{2}\sigma_i^2\right)(t_k - t_{k-1}) + \sigma_i\sqrt{t_k - t_{k-1}}Z_i\right],$$

with $Z \sim \mathcal{N}(0, \rho)$.

Valuation Backward Induction

We define:

- $V_{n,k}$: Estimated value of path n at time step k (the value of k represent the length of sorted set of observation, redemption, and maturity dates).

Final Payoff at Maturity:

$$V_{n,K} = \begin{cases} \texttt{principal}, & \text{if } \min_i \frac{S_{n,K}^i}{S_{n,0}^i} \geq 0.6, \\ \texttt{principal} \cdot \min_i \left(\frac{S_{n,K}^i}{S_{n,0}^i}\right), & \text{otherwise.} \end{cases}$$

Backward Induction: For time steps $k = K - 1, \ldots, 0$, and assets [SPX, NDXT, RTY] $i = 1, 2, 3$:

1. *Discount future value:*

$$V_{n,k} = e^{-r\Delta t} \cdot V_{n,k+1}$$

2. *Coupon Only:* If k is an observation index and all $S^i_{n,k} \geq 0.7 \cdot S^i_{n,0}$,

$$V_{n,k} = V_{n,k} + \$5.833 \cdot e^{-r*(\text{days between obs date and coupon date}) / 365}$$

3. *Early redemption:* If k is a redemption index:

   - Construct Laguerre polynomial **basis** of degree 2: $[1, L_0, L_1, L_2, L_0 \cdot L_1, L_0 \cdot L_2, L_1 \cdot L_2]$. The basis is used in a least-squares regression to approximate the continuation value:

   $$\hat{C}_k(X) = \mathbb{E}[V_{k+1} \mid \text{state at } t_k] \approx \sum_j \beta_j \phi_j(X_t).$$

   where ϕ_j are the constructed basis functions and β_j the regression coefficients estimated from in-the-money paths.

   - Use `principal` as the Immediate redemption value to compare with the fitted continuation value:

   $$V_{n,k} = \begin{cases} \texttt{Immediate Pay}, & \text{if } \hat{C}_k > \texttt{Immediate Pay} \\ V_{n,k+1} \cdot e^{-r\Delta t}, & \text{otherwise} \end{cases}$$

   - Coupon is added to immediate redemption value if coupon barrier is met

4. After backward iteration, the present value is estimated as the average across all paths using $N =$ 500,000 paths:

$$\texttt{Estimated PV} = \frac{1}{N} \sum_{n=1}^{N} V_{n,0} = \$956.95$$

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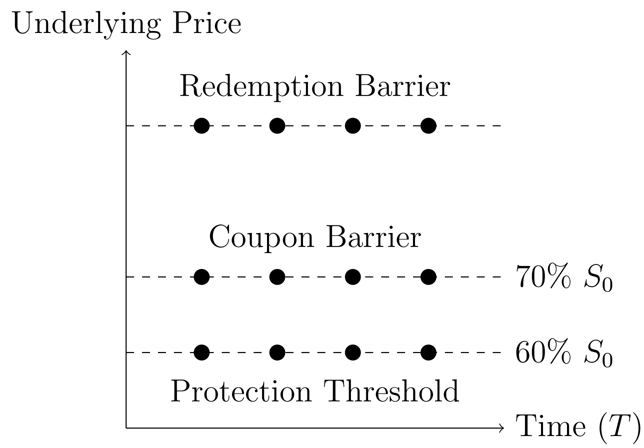
Laguerre Basis Construction:

Laguerre polynomials are orthogonal on the interval $[0, \infty)$ with respect to the weight function e^{-x}. They are used here to capture non-linear relationships in the continuation value estimation. For a scalar input x, the d-degree Laguerre polynomial basis is:

$$\texttt{laguerre\_basis}(x, d) = [L_0(x), L_1(x), \ldots, L_d(x)]$$

, where: $L_0(x) = 1, L_1(x) = 1 - x, L_2(x) = 1 - 2x + \frac{1}{2}x^2, \ldots$

In the multivariate case with 3 assets, at each time step, we normalize the spot prices by their initial values and evaluate the univariate Laguerre basis on each asset: $L_0 = [L_0(x_1), L_1(x_1), L_2(x_1)]$ on asset 1; $L_1 = [L_0(x_2), L_1(x_2), L_2(x_2)]$ on asset 2; $L_2 = [L_0(x_3), L_1(x_3), L_2(x_3)]$ on asset 3 , where $x_i = S^i_t / S^i_0$ for $i = 1, 2, 3$. We stack these and include cross terms. The cross-product terms like $L_0 \cdot L_1$ are element-wise multiplications of the degree-1 and higher terms (excluding the constant term) from different assets' Laguerre expansions.: Basis $= [1, L_0, L_1, L_2, L_0 \cdot L_1, L_0 \cdot L_2, L_1 \cdot L_2]$



3 Results and Discussion

Our valuation with 500,000 simulated paths (USD 956.95) is within 3.35 USD of the issuer's estimate (USD 953.60), indicating their pricing likely relies on 70% moneyness implied volatility at maturity and 2-year historical stocks' correlation, and for LSMC they might use 2-degree Laguerre polynomial basis with cross terms.

3.1 Randomness and Convergence Analysis

Monte Carlo simulation inherently involves randomness, as it estimates the expected present value of future payoffs by averaging across numerous simulated paths. To evaluate the **stability and reliability** of our pricing engine, we conduct a **randomness stress test** by running the full pricing simulation **500 times**, each with **500,000 independent paths** and a different random seed.

The resulting distribution of estimated prices is shown in Figure 1.

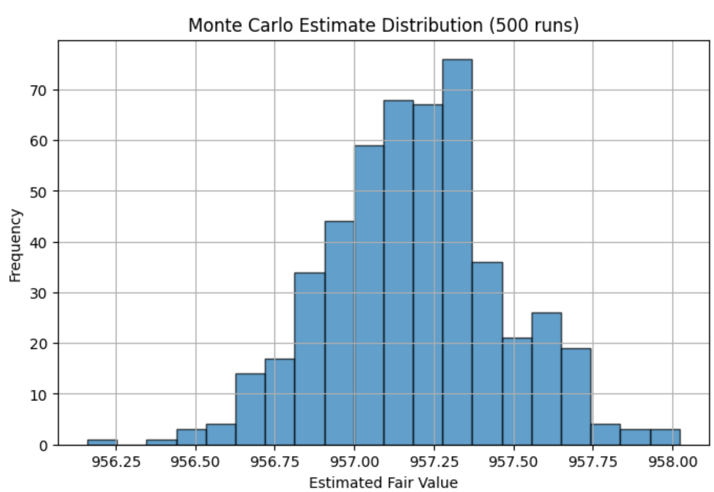


Figure 1: Distribution of Estimated Present Values over 500 Monte Carlo Runs

Over the 500 runs, the **mean estimated note value** is: $\hat{V}_{\text{mean}} = 957.1898$ USD with a **standard deviation** of: $\hat{\sigma}_{\text{MC}} = 0.276$ USD. This small standard deviation implies that the Monte Carlo engine exhibits **strong numerical convergence**, and the **pricing error due to randomness is negligible** in practice—less than ± 0.54 USD ($= 1.96 * std.dev.$) with 95% confidence, based on a normal approximation.

This experiment confirms that, under the given parameters (paths, volatility, correlation, interest rate), the LSMC Monte Carlo model produces **consistent, low-variance price estimates**.

3.2 Sensitivity Analysis– Overall

Each key parameter is bumped individually; re-pricing is run with 500 k Monte Carlo paths. Sensitivities are reported per unit change.

Table 4: Numerical Sensitivity Table

Metric	Sensitivity (USD)
$\Delta_{\text{SPX}}, \Delta_{\text{NDXT}}, \Delta_{\text{RTY}}$	0.00
ν_{SPX}	-267.64
ν_{NDXT}	-187.04
ν_{RTY}	-419.78
ρ	-488.90
Corr–Sensitivity	111.66

The following points expand on Table 4, explaining the economic intuition behind each of the key Greeks and correlation sensitivity:

Equity Delta (Δ) The note exhibits virtually zero first–order equity delta because both the coupon and principal payouts depend solely on whether the underlying indices breach fixed barrier levels (70% for coupons, 60% for principal) at predetermined observation dates. Small parallel shifts in SPX, NDXT or RTY spot prices ($\pm 1\%$) do not change the probability of barrier events when current levels are comfortably above the barriers, so the PV remains essentially unchanged.

Vega (ν) The note carries substantial negative vega: a one-unit increase in implied volatility of SPX reduces PV by approximately USD 267.6; for NDXT the sensitivity is USD 187.0; for RTY it is USD 419.8. Higher volatility implies fatter tails in the log-normal distribution of future index levels, increasing the chance that a given index will fall below the coupon or principal barrier on any observation date. Since barrier breaches reduce expected coupon and principal payments, higher volatilities erode the note's value.

Rho (ρ) The negative rho of –488.9 USD per unit change highlights the note's sensitivity to discount rates. A 25bp (0.0025) parallel upward shift in the OIS discount curve reduces PV by approximately

$$488.9 \times 0.0025 \approx 1.22 \text{ USD}.$$

This large impact arises because the note's cash flows extend over more than 1.75 years; higher rates steepen the discounting of all future coupon and principal payments.

Correlation Sensitivity (Corr–Sensitivity) In the "worst-of" payoff structure, higher correlation plays a positive role. Each one-percentage-point increase in the pairwise correlations makes the three indices move more in unison and reduces the dispersion of their worst-performer ratio, which in turn dramatically lowers the chance that any single index will lag far behind the others and trigger a principal haircut. As a result, expected coupon and principal payments rise, boosting the note's present value. Our numbers show a per-unit correlation change corresponds to a PV gain of about USD 111.7; with an actual 5-percentage-point bump, the PV increases by roughly USD 5.6.

3.3 Sensitivity Analysis on the Implied Volatility

Morgan Stanley's reported valuation of the product is 953.60$ per security, while our valuation using the 70% IV at maturity with N = 500, 000 paths yields a price of 956.95$ per security. This close agreement suggests that Morgan Stanley's model is likely using 70% IV at maturity as the primary input for their valuation. This assumption is reasonable because 70% IV, which is the monthlt coupon barrier, tends to reflect market expectations for future volatility, and implied volatilities often drift toward larger values over time (70% maturity IV is larger then ATM maturity IVs).

The valuation of the structured note is highly sensitive to the choice of implied volatility (IV). To account for the uncertainty in market-implied volatilities, we explore a range of maturity IVs across multiple moneyness levels—this allows us to stress-test the model under plausible volatility regimes and capture valuation sensitivity.

The product features have coupon, early call barriers, and a downside protection threshold at maturity. Our analysis adopts maturity IVs across multiple moneyness levels, capturing realistic market conditions at the pricing date. Table 5 summarizes the volatility vectors, their averages, and the resulting note valuations. A clear inverse relationship emerges between average σ and valuation: as IV decreases over the underliers, the estimated note value increases from \$921.45 to \$992.57. The monotonic decreasing property as the IVs decrease (the moneyness increase) is clearly shown in Figure 2.

Table 5: Maturity Volatility Vectors, and Estimated Note Values

Sigma Scenario	SPX	NDXT	RTY	Note Value (USD)
$\sigma_{55\%}$	0.31961	0.24379	0.33718	921.45
$\sigma_{60\%}$	0.30024	0.24379	0.32121	934.85
$\sigma_{70\%}$	0.26909	0.24379	0.29079	956.95
$\sigma_{85\%}$	0.22785	0.24379	0.25412	979.33
$\sigma_{100\%}$	0.19344	0.24379	0.23453	988.88
$\sigma_{110\%}$	0.17082	0.24379	0.22384	992.57

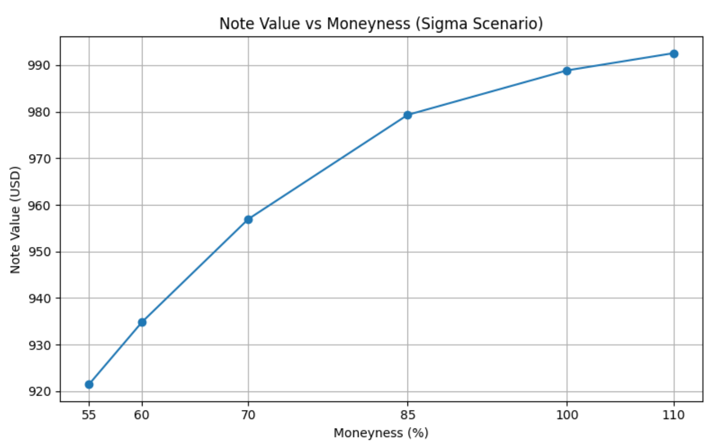


Figure 2: Valuation v.s. Implied volatility

This behavior is consistent with product structure. Higher volatility increases the likelihood of three adverse outcomes for investors: (1) triggering early redemption, (2) failing to meet the 70% coupon barrier on observation dates, and (3) breaching the 60% downside protection threshold at maturity. The third point is particularly important in our case. At maturity, if the worst-performing index falls below 60% of its initial level, the principal is proportionally reduced by the worst case.

Thus, higher volatility not only increases path dispersion (reducing coupon and call probability) but also magnifies the risk of principal loss. The result is a lower expected value under high-volatility scenarios. Conversely, lower volatility stabilizes index paths, reducing downside risk and improving expected cash flows—hence raising the valuation.

3.4 Sensitivity Analysis on the Correlation Matrix

The performance of a "worst-of" structured note is influenced not only by individual asset paths but also by the degree of co-movement among the underlying indices. In these structures, higher correlation reduces dispersion between the best- and worst-performing indices. This benefits the investor by lowering the chance that one index significantly underperforms the others—thereby increasing the likelihood of receiving contingent coupons, or preserving principal at maturity.

To evaluate this sensitivity, we examine three historical correlation matrices estimated over different lookback periods: 2 years, 1.5 years, and 1 year. Each matrix is summarized by its average pairwise correlation and the lowest off-diagonal entry, capturing both general co-movement and tail risk exposure. The resulting estimated valuations for the note are reported in Table 6. (the full correlation data can be found in the Appendix.)

Table 6: Correlation Matrices by Lookback Period and Their Valuation Impact

Lookback Period	Average Correlation	Minimum Correlation	Valuation (USD)
2-Year	0.746	0.621	958.72
1.5-Year	0.793	0.689	960.43
1-Year	0.775	0.665	956.95

Despite moderate variations in average correlation (from 0.746 to 0.793) and minimum correlation (from 0.621 to 0.689), we observe a consistent pattern: higher correlation slightly increases the note valuation. This aligns with intuition—greater co-movement among indices reduces the chance that one underperforms significantly, thus improving the probability of receiving coupons and preserving principal above the downside barrier. Overall, the correlation matrix has a very moderate effect on the valuation.

4 Conclusion

The valuation of the callable contingent income note was conducted using a Monte Carlo simulation with LSMC regression across 500,000 paths, yielding a fair value of $956.95—within $3.35 of Morgan Stanley's estimate ($953.60). This close match suggests their pricing likely relies on 70% implied volatility at maturity and recent 2-year historical correlation inputs.

Our analysis confirms that volatility assumptions drive valuation significantly: lower volatilities lead to higher valuations, while higher volatilities reduce expected payouts due to increased likelihood of early redemption, missed coupons, and principal loss. Correlation has a moderate but consistent influence—higher cross-asset correlation slightly raises valuation by reducing dispersion and downside risk.

Overall, the model robustly captures product dynamics under realistic market conditions, with strong numerical convergence and transparent parameter sensitivity.

5 Appendix

Morgan Stanley's reported valuation: \$953.60 per security.

Term sheet: https://www.sec.gov/Archives/edgar/data/895421/000183988225015267/ms6936$_4$24b2−07888.*htm*

Figure 3: Screen shot of risk free rate data selected from bloomberg

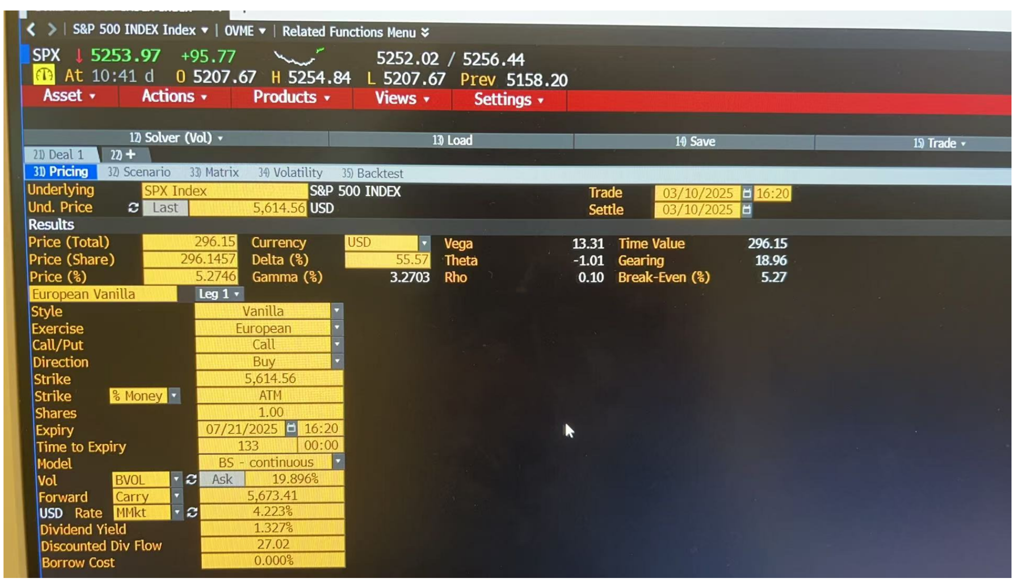


Figure 4: Screen shot of SPX dividend yield data selected from bloomberg

Figure 5: Screen shot of NDXT dividend yield data selected from bloomberg

Figure 6: Screen shot of RTY dividend yield data selected from bloomberg

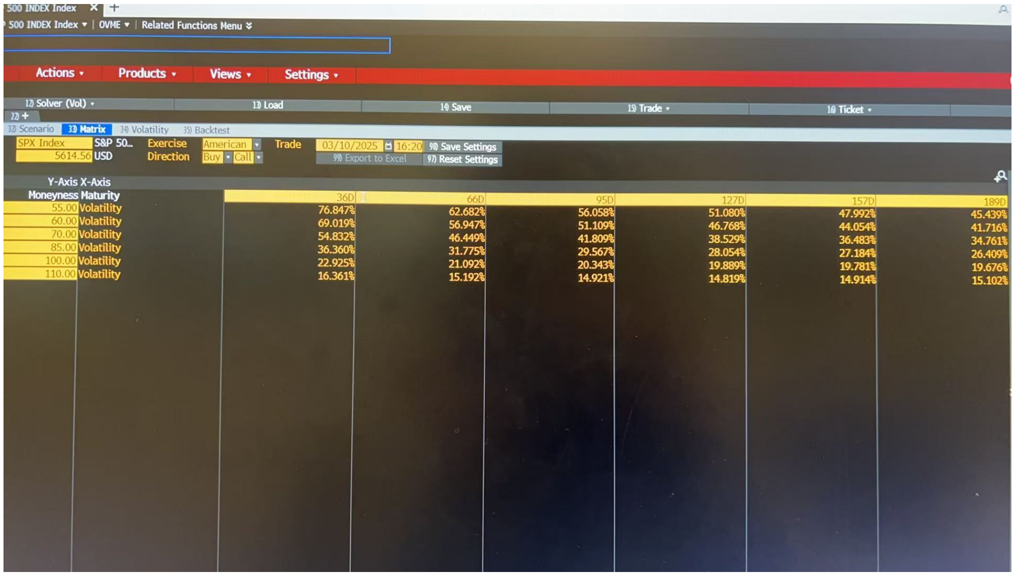


Figure 7: Screen shot of SPX implied volatility data selected from bloomberg

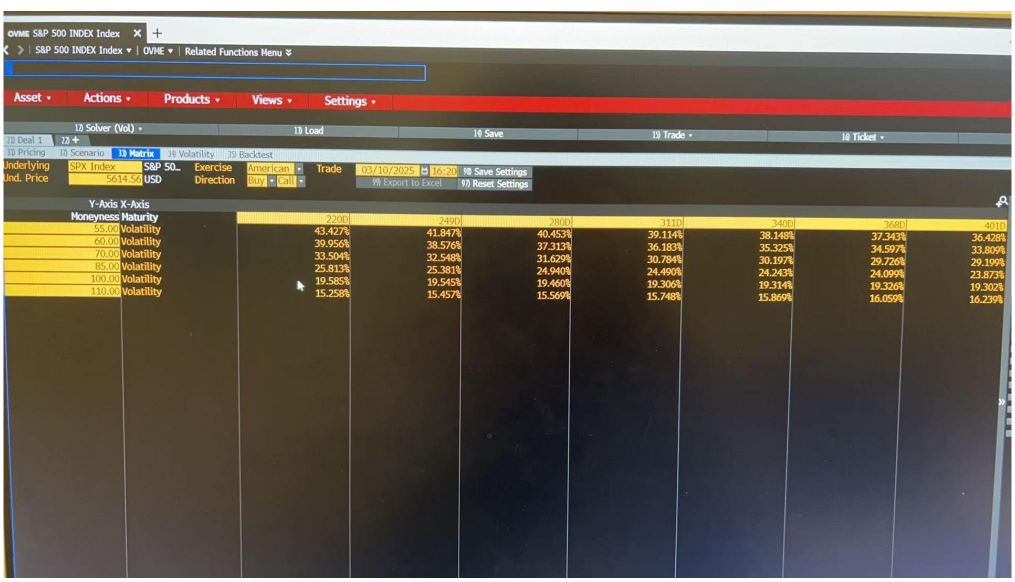


Figure 8: Screen shot of SPX implied volatility data selected from bloomberg

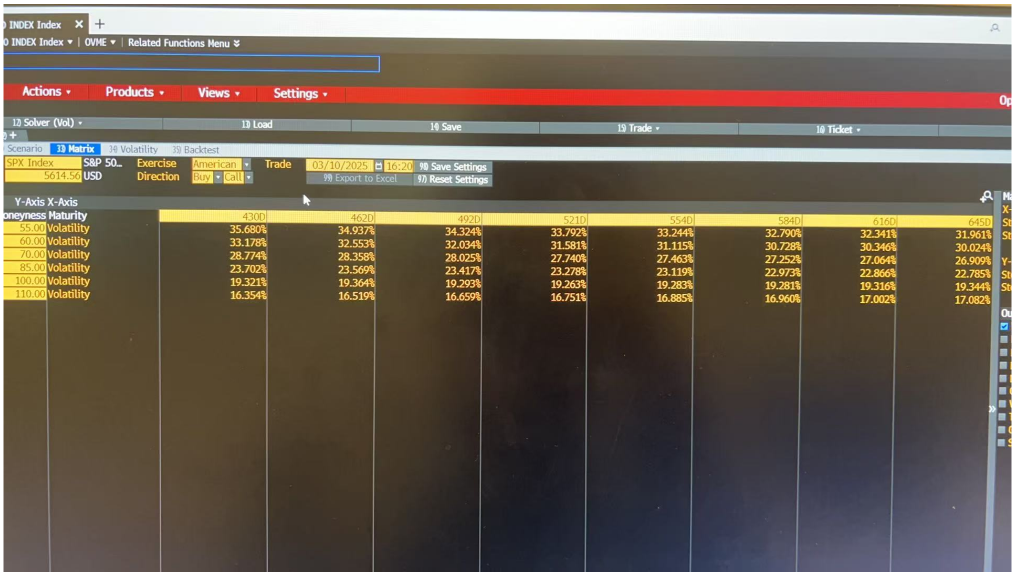


Figure 9: Screen shot of SPX implied volatility data selected from bloomberg

Figure 10: Screen shot of NDXT implied volatility data selected from bloomberg

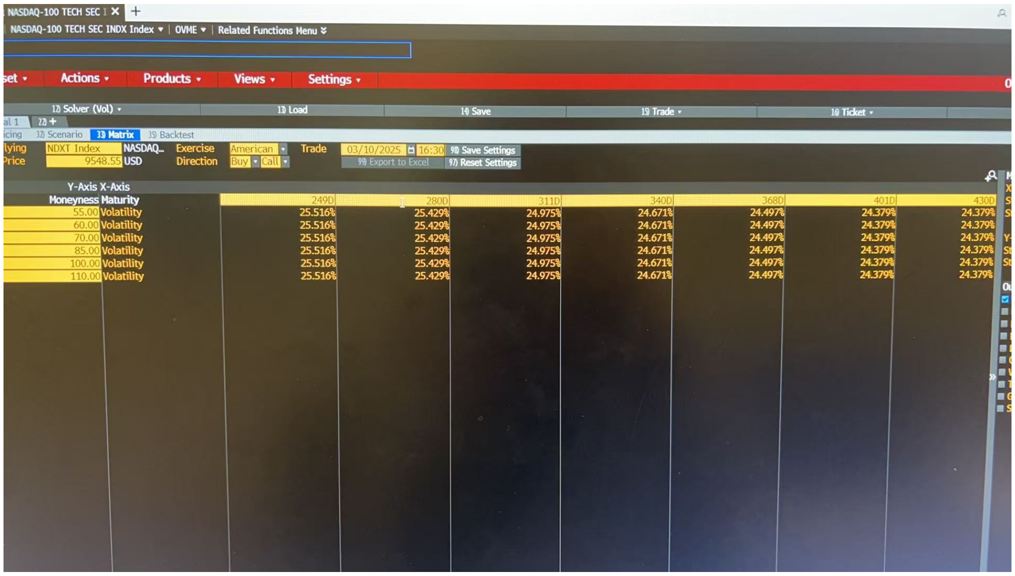


Figure 11: Screen shot of NDXT implied volatility data selected from bloomberg

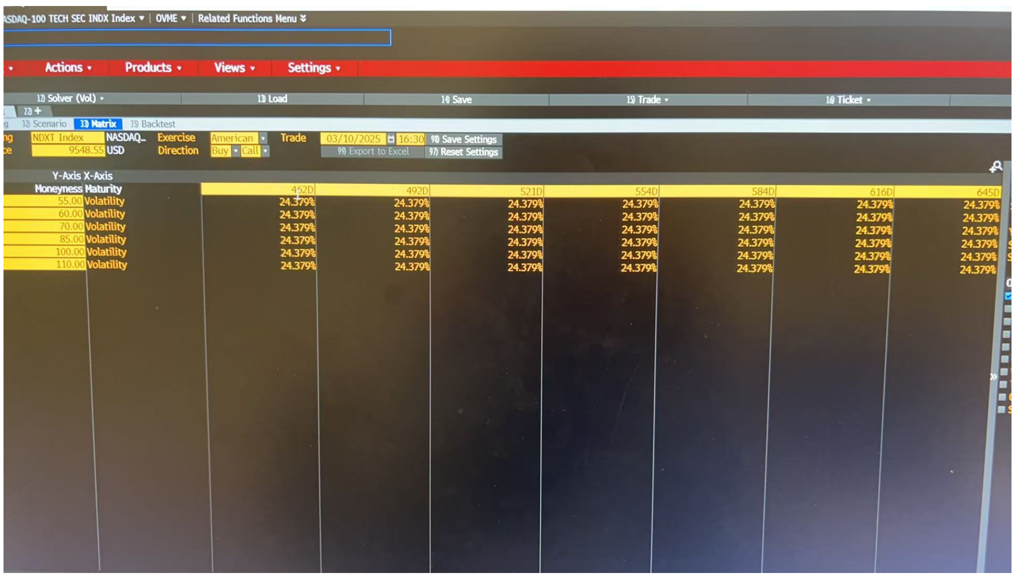


Figure 12: Screen shot of NDXT implied volatility data selected from bloomberg

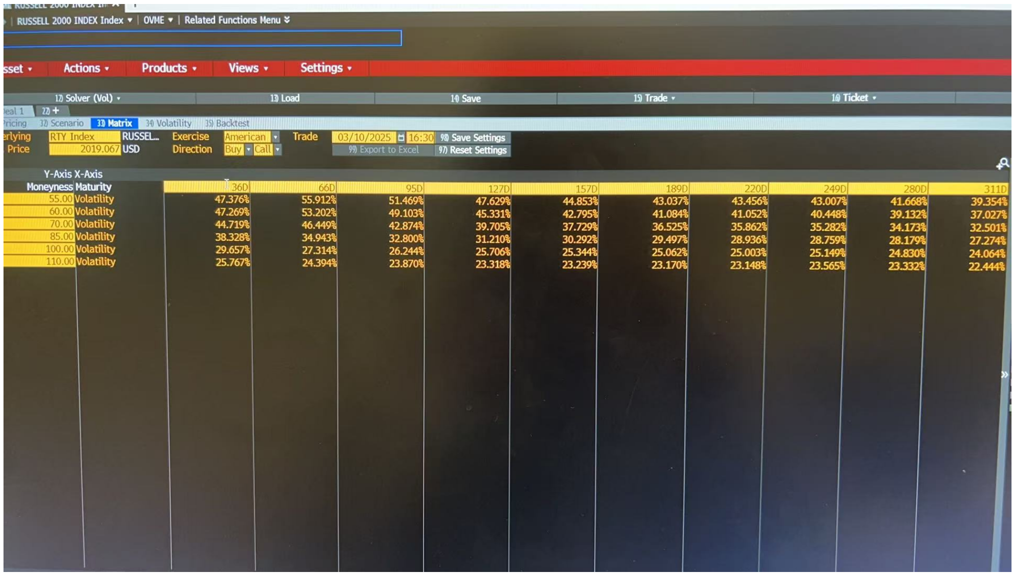


Figure 13: Screen shot of RTY implied volatility data selected from bloomberg

Figure 14: Screen shot of RTY implied volatility data selected from bloomberg

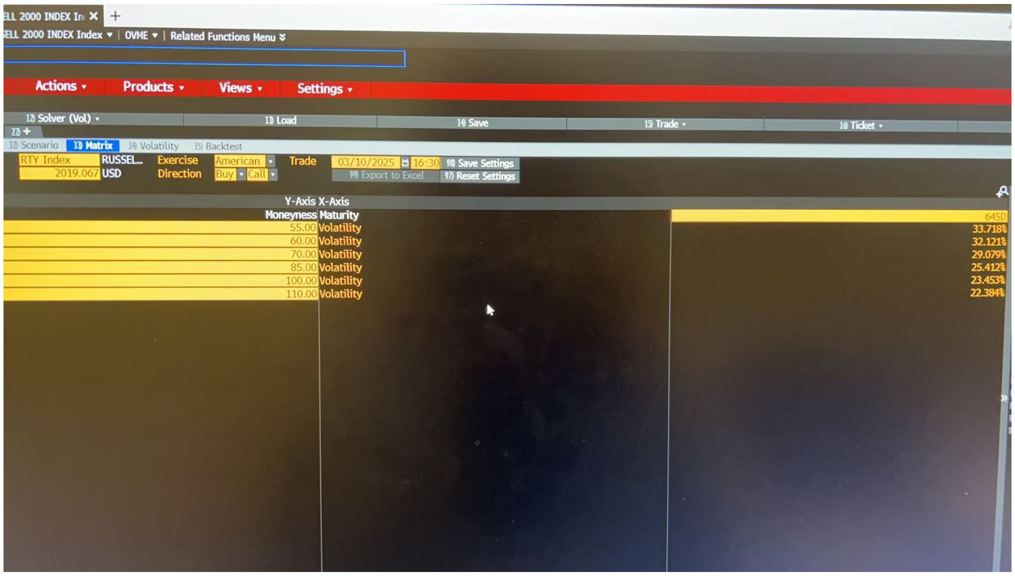


Figure 15: Screen shot of RTY implied volatility data selected from bloomberg

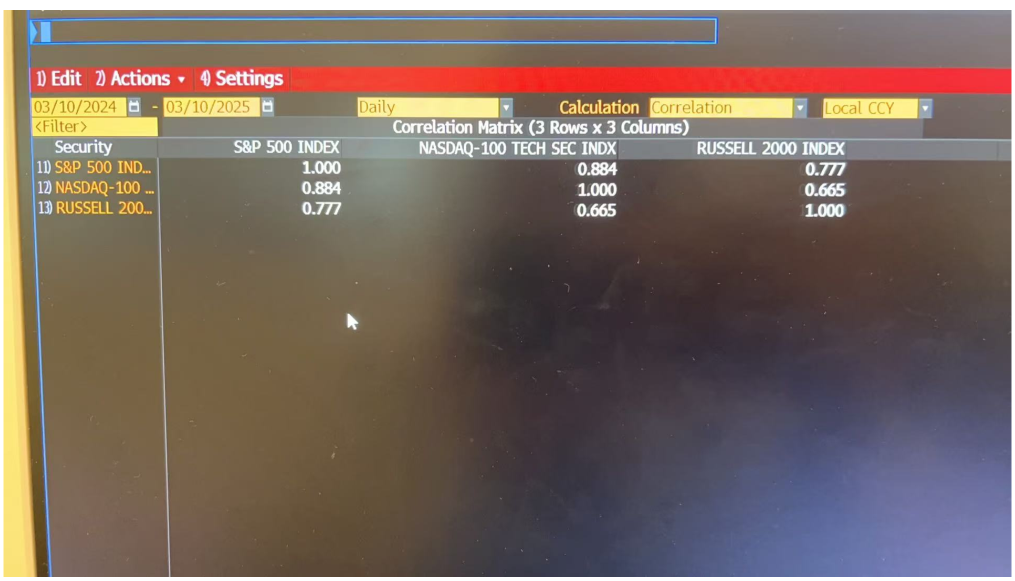


Figure 16: corr 1y from bloomberg

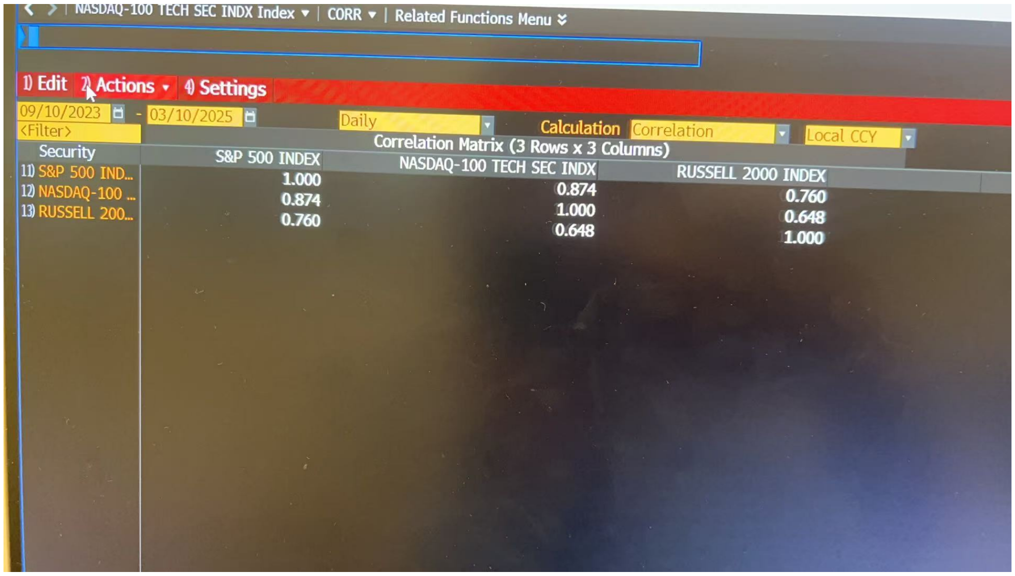


Figure 17: corr 1.5y from bloomberg

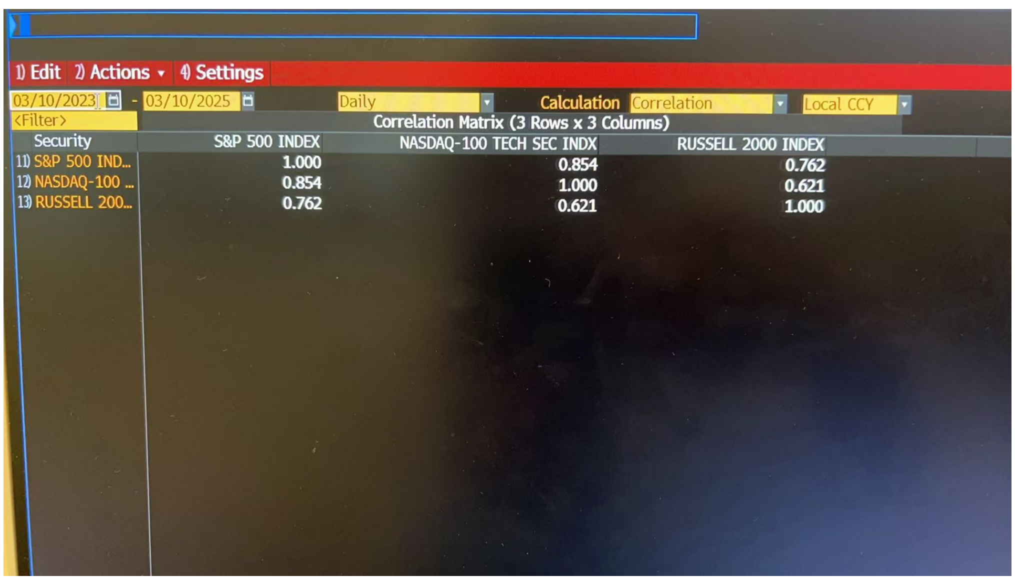


Figure 18: corr 2y from bloomberg