Models of Default Risk

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Introduction

- We consider two general approaches to modelling default risk, a risk characterizing almost all fixed-income securities.
- The "structural" approach was developed by Black and Scholes (1973) and Merton (1974) and values a firm's default-risky debt as an explicit function of the firm's capital structure and the value and risk of a firm's assets.
- The "reduced form" approach simply assumes that default is a Poisson process with a time-varying default intensity and default recovery rate without explicitly modeling a firm's assets and capital structure.
- Examples of the reduced form approach include Jarrow, Lando, and Turnbull (1997), Madan and Unal (1998), and Duffie and Singleton (1999).

Structural Approach Assumptions

- Consider a model similar to Merton (1974) where a firm owns risky assets with date t market value $A(t)$ and dynamics

$$dA/A = (\mu - \delta)\, dt + \sigma dz \tag{1}$$

  where μ and σ are the expectation and the standard deviation of the rate of return on assets and δ is the rate at which assets are paid out as dividends to the firm's shareholders.
- Along with shareholders' equity, the firm has issued a zero-coupon debt that promises to pay the amount B at date $T > t$, where $\tau \equiv T - t$.
- The date t market values of shareholders' equity and the debt are $E(t)$ and $D(t, T)$, respectively, so that $A(t) = E(t) + D(t, T)$.

Assumptions (continued)

- At date T, the firm pays B to the debtholders if there is sufficient firm asset value. Else, bankruptcy occurs and the debtholders take ownership of the firm's assets. Thus, the payoff to debtholders is

$$\begin{aligned} D(T,T) &= \min\left[B, A(T)\right] \\ &= B - \max\left[0, B - A(T)\right] \end{aligned} \tag{2}$$

- Let $P(t,T)$ be the current date t price of a default-free, zero-coupon bond that pays \$1 at date T and assume that the Vasicek (1977) model holds for the default-free term structure specified earlier in (9.41) to (9.43).

Market Value of Debt

- Recognizing that debt's payoff in (2) equals the default-free value B less the value of a put option written on the firm's assets with strike B, it is valued using option pricing results in Chapters 9 and 10:

$$\begin{aligned} D(t,T) &= P(t,T)B - P(t,T)BN(-h_2) + e^{-\delta\tau}AN(-h_1) \\ &= P(t,T)BN(h_2) + e^{-\delta\tau}AN(-h_1) \end{aligned} \tag{3}$$

where $h_1 = \left[\ln\left[e^{-\delta\tau}A/(P(t,T)B)\right] + \frac{1}{2}v^2\right]/v$, $h_2 = h_1 - v$, and $v(\tau)$ is given in (9.61).

- Note that the debt's promised yield-to-maturity is $R(t,T) \equiv \frac{1}{\tau}\ln[B/D(t,T)]$ and its "credit spread" is $R(t,T) - \frac{1}{\tau}\ln[1/P(t,T)]$.

Market Value of Shareholders' Equity

- Given (3), shareholders' equity equals

$$
\begin{aligned}
E(t) &= A(t) - D(t,T) \\
&= A - P(t,T)\,BN(h_2) - e^{-\delta\tau} AN(-h_1) \\
&= A\left[1 - e^{-\delta\tau} N(-h_1)\right] - P(t,T)\,BN(h_2)
\end{aligned}
\tag{4}
$$

- Equity is similar to a call option on the firm's assets since its payoff is $\max\left[A(T) - B, 0\right]$.
- However, it differs if the firm pays dividends to equityholders prior to the debt's maturity, as reflected in the first term in the last line of (4).

Discussion of Structural Models

- Merton (1974) analyzes the properties of debt and equity formulas similar to equations (3) and (4).
- Note that an equity formula such as (4) is useful when firms have publicly traded equity, since observation of the market value of equity and its volatility can be used to estimate $A(t)$ and σ, which can then be used to value $D(t, T)$.
- There is now a vast literature on structural models that modify and extend the original Merton (1974) framework.
- Examples include Black and Cox (1976), Leland (1994), and Collin-Dufresne and Goldstein (2001).

The Reduced-Form Approach

- As before, let $D(t, T)$ be the date t value of a default-risky, zero-coupon bond that promises to pay B at its maturity date of T.

- Let $\lambda(t)\,dt$ be the instantaneous probability of default occurring during the interval $(t, t + dt)$, so that $\lambda(t)$ is the physical default intensity, or "hazard rate."

- Then the bond's (physical) survival probability from dates t to τ is

$$E_t\left[e^{-\int_t^{\tau}\lambda(u)du}\right] \tag{5}$$

A Zero-Recovery Bond

- First, consider a bond that, if it defaults, has zero recovery value, so that $D(T, T) = B$ if there is no default or $D(T, T) = 0$ if default occurs over the interval from dates t to T.
- Applying risk-neutral pricing, this bond's value, $D_Z(t, T)$, is

$$D_Z(t, T) = \widehat{E}_t \left[e^{-\int_t^T r(u)du} D(T, T) \right] \tag{6}$$

where $r(t)$ is the date t instantaneous default-free interest rate, and $\widehat{E}_t[\cdot]$ is the date t risk-neutral expectations operator.

State Variables and Pricing Kernel

- Suppose the default-free term structure and $\lambda(t)$ depend on an $n \times 1$ vector of state variables, $\mathbf{x}$, that follows the process

$$d\mathbf{x} = \mathbf{a}(t, \mathbf{x})\, dt + \mathbf{b}(t, \mathbf{x})\, \mathbf{dz} \tag{7}$$

where $\mathbf{x} = (x_1 ... x_n)'$, $\mathbf{a}(t, \mathbf{x})$ is an $n \times 1$ vector, $\mathbf{b}(t, \mathbf{x})$ is an $n \times n$ matrix, and $\mathbf{dz} = (dz_1 ... dz_n)'$ is an $n \times 1$ vector of independent Brownian motions so that $dz_i dz_j = 0$ for $i \neq j$.

- Assuming complete markets, the stochastic discount factor for pricing the firm's default-risky bond is

$$dM/M = -r(t, \mathbf{x})\, dt - \Theta(t, \mathbf{x})' \mathbf{dz} - \psi(t, \mathbf{x}) [dq - \lambda(t, \mathbf{x})\, dt] \tag{8}$$

where $\Theta(t, \mathbf{x})$ is an $n \times 1$ vector of the market prices of risk associated with $\mathbf{dz}$ and $\psi(t, \mathbf{x})$ is the market price of risk associated with the actual default event.

Default as a Poisson Process

- The default event is recorded by dq, which if default occurs $q(t)$ jumps from 0 (the no-default state) to 1 (the absorbing default state) at which time $dq = 1$.
- The risk-neutral default intensity, $\widehat{\lambda}(t, \mathbf{x})$, is then given by $\widehat{\lambda}(t, \mathbf{x}) = [1 - \psi(t, \mathbf{x})]\, \lambda(t, \mathbf{x})$.
- Default is a "doubly stochastic" process, also referred to as a Cox process, because it depends on the Brownian motion vector $\mathbf{dz}$ that drives $\mathbf{x}$ and determines how the likelihood of default, $\widehat{\lambda}(t, \mathbf{x})$, changes over time, but it also depends on the Poisson process dq that determines the arrival of default.
- Hence, default risk reflects two types of risk premia, $\Theta(t, \mathbf{x})$ and $\psi(t, \mathbf{x})$.

Value of the Zero-Recovery Bond

- Based on (5), we can solve for $D_Z(t, T)$:

$$\begin{aligned} D_Z(t, T) &= \widehat{E}_t\left[e^{-\int_t^T r(u)du} e^{-\int_t^T \widehat{\lambda}(u)du} B\right] \qquad (9) \\ &= \widehat{E}_t\left[e^{-\int_t^T \left[r(u)+\widehat{\lambda}(u)\right]du}\right] B \end{aligned}$$

- Note that (9) is similar to valuing a default-free bond except that the discount rate $r(u) + \widehat{\lambda}(u)$, rather than just $r(u)$, is used.
- Given specific functional forms for $r(t, \mathbf{x})$, $\widehat{\lambda}(t, \mathbf{x})$, $\mathbf{x}$ and $\Theta(t, \mathbf{x})$, (9) can be computed.

Specifying Recovery Values

- Suppose that if the bond defaults at date τ, where $t < \tau \leq T$, bondholders recover an amount $w(\tau, \mathbf{x})$ at date τ.
- Then the risk-neutral probability density of defaulting at τ is

$$e^{-\int_t^\tau \widehat{\lambda}(u)du} \widehat{\lambda}(\tau) \tag{10}$$

- In (10), $\widehat{\lambda}(\tau)$ is discounted by $\exp\left[-\int_t^\tau \widehat{\lambda}(u)\, du\right]$ because default at date τ is conditioned on not having defaulted previously.

Valuing a Bond with Recovery Value

- Thus, the present value of recovery, $D_R(t, T)$, is:

$$\begin{aligned} D_R(t,T) &= \widehat{E}_t\left[\int_t^T e^{-\int_t^\tau r(u)du} w(\tau)\, e^{-\int_t^\tau \widehat{\lambda}(u)du}\widehat{\lambda}(\tau)\, d\tau\right] \\ &= \widehat{E}_t\left[\int_t^T e^{-\int_t^\tau \left[r(u)+\widehat{\lambda}(u)\right]du}\widehat{\lambda}(\tau)\, w(\tau)\, d\tau\right] \quad (11) \end{aligned}$$

- Putting this together with (9) gives the bond's total value:

$$\begin{aligned} D(t,T) &= D_Z(t,T) + D_R(t,T) \qquad (12) \\ &= \widehat{E}_t\left[e^{-\int_t^T \left[r(s)+\widehat{\lambda}(s)\right]ds} B \right. \\ &\quad \left. + \int_t^T e^{-\int_t^\tau \left[r(s)+\widehat{\lambda}(s)\right]ds}\widehat{\lambda}(\tau)\, w(\tau)\, d\tau\right] \end{aligned}$$

Recovery Proportional to Par Value

- Let us consider particular specifications for $w(\tau, \mathbf{x})$.
- Let the default date be τ and assume $w(\tau, \mathbf{x}) = \delta(\tau, \mathbf{x}) B$, where $\delta(\tau, \mathbf{x})$ can be a constant, say, $\bar{\delta}$. Then (11) is

$$D_R(t, T) = \bar{\delta} B \int_t^T k(t, \tau) \, d\tau \tag{13}$$

where

$$k(t, \tau) \equiv \widehat{E}_t \left[e^{-\int_t^\tau \left[r(u) + \widehat{\lambda}(u) \right] du} \widehat{\lambda}(\tau) \right] \tag{14}$$

has a closed-form solution when $r(u, \mathbf{x})$ and $\widehat{\lambda}(u, \mathbf{x})$ are affine functions of $\mathbf{x}$ and the vector $\mathbf{x}$ in (7) has a risk-neutral process that is also affine.
- (13) can be computed by numerical integration of $k(t, \tau)$.

Recovery Proportional to Par, Payable at Maturity

- Assume that if default occurs at date τ, bondholders recover $\delta(\tau, \mathbf{x}) B$ at date T, which is equivalent to $w(\tau, \mathbf{x}) = \delta(\tau, \mathbf{x}) P(\tau, T) B$. Then (11) is

$$
\begin{aligned}
& D_R(t, T) \\
= \; & \widehat{E}_t \left[\int_t^T e^{-\int_t^\tau \left[r(u) + \widehat{\lambda}(u) \right] du} \widehat{\lambda}(\tau) \, \delta(\tau, \mathbf{x}) \, e^{-\int_\tau^T r(u) du} B d\tau \right] \\
= \; & \widehat{E}_t \left[\int_t^T e^{-\int_t^\tau \widehat{\lambda}(u) du} \widehat{\lambda}(\tau) \, \delta(\tau, \mathbf{x}) \, e^{-\int_t^T r(u) du} B d\tau \right] \\
= \; & \widehat{E}_t \left[e^{-\int_t^T r(u) du} \int_t^T e^{-\int_t^\tau \widehat{\lambda}(u) du} \widehat{\lambda}(\tau) \, \delta(\tau, \mathbf{x}) \, d\tau \right] B \quad (15)
\end{aligned}
$$

Recovery Proportional to Par, Payable at Maturity

- If $\delta(\tau, x) = \overline{\delta}$, note that $\int_t^T \exp\left[-\int_t^\tau \widehat{\lambda}(u)\,du\right]\widehat{\lambda}(\tau)\,d\tau$ is the total risk-neutral probability of default from t to T and equals $1 - \exp\left[-\int_t^T \widehat{\lambda}(u)\,du\right]$. Thus,

$$\begin{aligned} D_R(t,T) &= \widehat{E}_t\left[e^{-\int_t^T r(u)du}\left(1 - e^{-\int_t^T \widehat{\lambda}(u)du}\right)\right]\overline{\delta}B \\ &= \widehat{E}_t\left[e^{-\int_t^T r(u)du} - e^{-\int_t^T \left[r(u)+\widehat{\lambda}(u)\right]du}\right]\overline{\delta}B \\ &= \overline{\delta}BP(t,T) - \overline{\delta}D_Z(t,T) \qquad (16) \end{aligned}$$

- Therefore, the total value of the bond is

$$D(t,T) = D_Z(t,T) + D_R(t,T) = \left(1 - \overline{\delta}\right)D_Z(t,T) + \overline{\delta}BP(t,T) \qquad (17)$$

so only a value for the zero-recovery bond is required.

Recovery Proportional to Market Value

- Assume that at default, bondholders lose a proportion $L(\tau, \mathbf{x})$ of the bond's value just prior to default:

$$D\left(\tau^{+}, T\right) = w(\tau, \mathbf{x}) = D\left(\tau^{-}, T\right)\left[1 - L(\tau, \mathbf{x})\right] \tag{18}$$

- Treating the defaultable bond as a contingent claim and applying Itô's lemma:

$$dD(t, T)/D(t, T) = (\alpha_D - \lambda k_D)\, dt + \boldsymbol{\sigma}_D' \mathbf{dz} - L(t, \mathbf{x})\, dq \tag{19}$$

  where α_D and the $n \times 1$ vector $\boldsymbol{\sigma}_D$ are given by the usual Itô's lemma expressions, the expected jump size $k_D(\tau^-) \equiv E_{\tau^-}\left[D(\tau^+, T) - D(\tau^-, T)\right]/D(\tau^-, T) = -L(\tau, \mathbf{x})$, so that the drift term in (19) is $\alpha_D + \lambda(t, \mathbf{x})\, L(t, \mathbf{x})$.

Recovery Proportional to Market Value (continued)

- The risk-neutral process for $D(t,T)$ replaces α_D with $r(t)$:

$$dD(t,T)/D(t,T) = \left(r(t,\mathbf{x}) + \widehat{\lambda}(t,\mathbf{x})\widehat{L}(t,\mathbf{x})\right)dt + \boldsymbol{\sigma}_D'\mathbf{d}\widehat{\mathbf{z}} - \widehat{L}(t,\mathbf{x})\,dq \tag{20}$$

  where $\widehat{L}(t,\mathbf{x})$ is the risk-neutral loss given default.

- Similar to (11.17), $D(t,T)$ satisfies the PDE:

$$\frac{1}{2}\text{Trace}\left[\mathbf{b}(t,\mathbf{x})\,\mathbf{b}(t,\mathbf{x})'\,\mathbf{D}_{xx}\right] + \widehat{\mathbf{a}}(t,\mathbf{x})'\,\mathbf{D}_x - R(t,\mathbf{x})\,D + D_t = 0 \tag{21}$$

  where $\mathbf{D}_x$ is the $n\times 1$ vector of first derivatives, $\mathbf{D}_{xx}$ is the $n\times n$ matrix of second derivatives, $\widehat{\mathbf{a}}(t,\mathbf{x}) = \mathbf{a}(t,\mathbf{x}) - \mathbf{b}(t,\mathbf{x})\,\Theta$, and $R(t,\mathbf{x}) \equiv r(t,\mathbf{x}) + \widehat{\lambda}(t,\mathbf{x})\widehat{L}(t,\mathbf{x})$.

Solution for the Defaultable Bond Value

- The PDE (21) is standard except that $R(t, \mathbf{x})$ replaces $r(t, \mathbf{x})$ in the standard PDE. Thus, the Feynman-Kac solution is

$$D(t, T) = \widehat{E}_t \left[e^{-\int_t^T R(u, \mathbf{x}) du} \right] B \tag{22}$$

where $R(t, \mathbf{x}) \equiv r(t, \mathbf{x}) + \widehat{\lambda}(t, \mathbf{x}) \widehat{L}(t, \mathbf{x})$ is the "default-adjusted" discount rate.

- The product $s(t, \mathbf{x}) \equiv \widehat{\lambda}(t, \mathbf{x}) \widehat{L}(t, \mathbf{x})$ is the "credit spread" on an instantaneous-maturity, defaultable bond, and since $\widehat{\lambda}(t, \mathbf{x})$ and $\widehat{L}(t, \mathbf{x})$ are not individually identified, a single functional form can be specified for $s(t, \mathbf{x})$.

Examples

- Let $\mathbf{x} = (x_1 \;\; x_2)'$ be a two-dimensional vector, $\widehat{\mathbf{a}}(t, \mathbf{x}) = (\kappa_1 (\overline{x}_1 - x_1) \quad \kappa_2 (\overline{x}_2 - x_2))'$, and $\mathbf{b}(t, \mathbf{x})$ is a diagonal matrix with elements of $\sigma_1 \sqrt{x_1}$ and $\sigma_2 \sqrt{x_2}$.
- Also assume $r(t, \mathbf{x}) = x_1(t)$ and $\widehat{\lambda}(t, \mathbf{x}) = x_2(t)$, so that the default-free term structure and $\widehat{\lambda}(t, \mathbf{x})$ are independent.
- Defining $\overline{r} \equiv \overline{x}_1$ and $\theta_1 \equiv \sqrt{\kappa_1^2 + 2\sigma_1^2}$, the CIR bond price is

$$P(t, T) = A_1(\tau)\, e^{-B_1(\tau) r(t)}, \text{ where} \tag{23}$$

$$A_1(\tau) \equiv \left[\frac{2\theta_1 e^{(\theta_1 + \kappa_1)\frac{\tau}{2}}}{(\theta_1 + \kappa_1)(e^{\theta_1 \tau} - 1) + 2\theta_1} \right]^{2\kappa_1 \overline{r} / \sigma_1^2} \tag{24}$$

$$B_1(\tau) \equiv \frac{2\left(e^{\theta_1 \tau} - 1\right)}{(\theta_1 + \kappa_1)(e^{\theta_1 \tau} - 1) + 2\theta_1} \tag{25}$$

Examples (continued)

- Also define $\overline{\lambda} \equiv \overline{x}_2$, then based on (9) we have

$$\begin{aligned} D_Z(t,T) &= \widehat{E}_t\left[e^{-\int_t^T \left[r(s)+\widehat{\lambda}(s)\right]ds}\right]B \\ &= \widehat{E}_t\left[e^{-\int_t^T r(s)ds}\right]\widehat{E}_t\left[e^{-\int_t^T \widehat{\lambda}(s)ds}\right]B \\ &= P(t,T)\,V(t,T)\,B \qquad (26) \end{aligned}$$

where

$$V(t,T) = A_2(\tau)\,e^{-B_2(\tau)\widehat{\lambda}(t)} \qquad (27)$$

and where $A_2(\tau)$ is the same as $A_1(\tau)$ in (24), and $B_2(\tau)$ is the same as $B_1(\tau)$ in (25) except that κ_2 replaces κ_1, σ_2 replaces σ_1, $\overline{\lambda}$ replaces $\overline{r}$, and $\theta_2 \equiv \sqrt{\kappa_2^2 + 2\sigma_2^2}$ replaces θ_1.

Example: Recovery Proportional to Par, Payable at Maturity

- If recovery is a fixed proportion, $\overline{\delta}$, of par, payable at maturity, then from (17):

$$\begin{aligned} D(t,T) &= \left(1-\overline{\delta}\right) D_Z(t,T) + \overline{\delta} B P(t,T) \\ &= \left[\overline{\delta} + \left(1-\overline{\delta}\right) V(t,T)\right] P(t,T) B \end{aligned} \qquad (28)$$

- In (27), $V(t,T)$ is analogous to a bond price in the standard Cox, Ingersoll, and Ross term structure model and is inversely related to $\widehat{\lambda}(t)$ and strictly less than 1 whenever $\widehat{\lambda}(t)$ is strictly positive, which can be ensured when $2\kappa_2\overline{\lambda} \geq \sigma_2^2$.

- Thus, (28) confirms that the defaultable bond's value declines as its risk-neutral default intensity rises.

Example: Recovery Proportional to Market Value

- Assume recovery is proportional to market value and $s(t,\mathbf{x}) \equiv \widehat{\lambda}(t,\mathbf{x})\widehat{L}(t,\mathbf{x}) = x_2$ and define $\overline{s} \equiv \overline{x}_2$. Then from (22):

$$\begin{aligned} D(t,T) &= \widehat{E}_t\left[e^{-\int_t^T [r(u)+s(u)]du}\right] B \\ &= \widehat{E}_t\left[e^{-\int_t^T r(u)du}\right] \widehat{E}_t\left[e^{-\int_t^T s(u)du}\right] B \\ &= P(t,T)\,S(t,T)\,B \qquad (29) \end{aligned}$$

where

$$S(t,T) = A_2(\tau)\, e^{-B_2(\tau)s(t)} \qquad (30)$$

and where $A_2(\tau)$ is the same as $A_1(\tau)$ in (24) and $B_2(\tau)$ is the same as $B_1(\tau)$ in (25) except that κ_2 replaces κ_1, σ_2 replaces σ_1, $\overline{s}$ replaces $\overline{r}$, and $\theta_2 \equiv \sqrt{\kappa_2^2 + 2\sigma_2^2}$ replaces θ_1.

- $D(t,T)$ is like $P(t,T)$ but $R(t) = r(t) + s(t)$ replaces $r(t)$.

Coupon Bonds

- Suppose a defaultable coupon bond promises n cashflows, with the i^{th} promised cashflow being equal to c_i and being paid at date $T_i > t$.
- Then the value of this coupon bond in terms of our zero-coupon bond formulas is

$$\sum_{i=1}^{n} D\left(t, T_i\right) \frac{c_i}{B} \tag{31}$$

Credit Default Swaps

- A credit default swap is a contract in which one party, the protection buyer, makes periodic payments until the contract's maturity as long as a particular issuer does not default.
- The other party, the protection seller, receives these payments in return for paying the difference between the issuer's debt's par value and its recovery value if default occurs prior to the swap's maturity.
- Let the contract specify equal periodic payments of c at future dates $t+\Delta$, $t+2\Delta$, ..., $t+n\Delta$. Since these payments are contingent on default not occurring, their value is

$$\frac{c}{B}\sum_{i=1}^{n} D_Z\left(t, t+i\Delta\right) \tag{32}$$

where $D_Z\left(t, T\right)$ is given in (9).

Credit Default Swaps (continued)

- Let $w(\tau, \mathbf{x})$ be the recovery value of the defaultable debt underlying the swap contract. Then similar to (11), the value of the swap protection is

$$\widehat{E}_t \left[\int_t^{t+n\Delta} e^{-\int_t^{\tau} \left[r(u) + \widehat{\lambda}(u) \right] du} \widehat{\lambda}(\tau) \left[B - w(\tau) \right] d\tau \right] \quad (33)$$

- Suppose the protection seller's payment in the event of default is $B - w(\tau) = B - \overline{\delta} B = B\left(1 - \overline{\delta}\right)$. Then (33) is

$$B\left(1 - \overline{\delta}\right) \int_t^{t+n\Delta} k(t, \tau)\, d\tau \quad (34)$$

where $k(t, \tau)$ is defined in (14).

- Given functional forms for $r(t, \mathbf{x})$, $\widehat{\lambda}(t, \mathbf{x})$, $w(t, \mathbf{x})$, and $\mathbf{x}$, the value of the swap payments, c, that equates (32) to (33) can be determined.

Implementing a Reduced-Form Approach

- A general issue when implementing the reduced-form approach is determining the proper current values $\widehat{\lambda}(t)$, $s(t)$, or $w(t)$ that may not be directly observable.
- One or more of these default variables might be inferred by setting the actual market prices of one or more of an issuer's bonds to their theoretical formulas.
- Then, based on the "implied" values of $\widehat{\lambda}(t)$, $s(t)$, or $w(t)$, one can determine whether a given bond of the same issuer is over- or underpriced relative to other bonds.
- Alternatively, these implied default variables can be used to set the price of a new bond of the same issuer or a credit derivative (such as a default swap) written on the issuer's debt.

Summary

- There are two main branches of modeling defaultable fixed-income securities.
- The *structural approach* models default based on the interaction between a firm's assets and its liabilities. Potentially, it can improve our understanding between capital structure and corporate bond and loan prices.
- In contrast, the *reduced-form approach* abstracts from specific characteristics of a firm's financial structure, but it permits a more flexible modeling of default probabilities and may better describe actual the prices of an issuer's debt.