

4.2 Suppose that you are testing the following hypotheses where the variance is known: $H_0: \mu = 100$, $H_1: \mu > 100$.

Find the P-value for the following values of the test statistic.

(a) $Z_0 = 2.50$

$P(z > 2.5) = P(z \leq -2.5) = 0.0062$

(b) $Z_0 = 1.95$

$P(z > 1.95) = P(z \leq -1.95) = 0.0256$

(c) $Z_0 = 2.05$

$P(z > 2.05) = P(z \leq -2.05) = 0.0202$

(d) $Z_0 = 2.36$

$P(z > 2.36) = P(z \leq -2.36) = 0.0091$

4.3 Suppose that you are testing the following hypotheses where the variance is unknown: $H_0: \mu = 100$, $H_1: \mu \neq 100$.

The sample size is $n = 20$. Find bounds on the P-value for the following values of the test statistic.

$N = 20$ 自由度 = $20 - 1 = 19$ 雙尾檢定

(a) $t_0 = 2.75$

$P\text{-value} = 2 * (1 - T.DIST(2.75, 19, TRUE)) = 0.013$

(b) $t_0 = 1.86$

$P\text{-value} = 2 * (1 - T.DIST(1.86, 19, TRUE)) = 0.078$

(c) $t_0 = -2.05$

$P\text{-value} = 2 * (1 - T.DIST(2.05, 19, TRUE)) = 0.054$

(d) $t_0 = -1.86$

$P\text{-value} = 2 * (1 - T.DIST(1.86, 19, TRUE)) = 0.078$

4.5 The inside diameters of bearings used in an aircraft landing gear assembly are known to have a standard deviation of $\sigma = 0.002$ cm. A random sample of 15 bearings has an average inside diameter of 8.2535 cm.

(a) Test the hypothesis that the mean inside bearing diameter is 8.25 cm. Use a two-sided alternative and $\alpha = 0.05$.

$Z_{0.0025} = 1.96$ $n = 15$

$|Z_0| = |(\bar{x} - \mu_0) / (\sigma / \sqrt{n})| = |(8.2535 - 8.25) / (0.002 / \sqrt{15})| = 6.77772 > 1.96$

Reject H_0 , we cannot say the mean inside bearing diameter is 8.25 cm.

(b) Find the P-value for this test.

The P-value is < 0.00001 (out of range)

(c) Construct a 95% two-sided confidence interval on the mean bearing diameter.

$[(\bar{x} - Z_{\alpha/2}) / (\sigma / \sqrt{n}), (\bar{x} + Z_{\alpha/2}) / (\sigma / \sqrt{n})]$

$= [8.2535 - 1.96(0.002 / \sqrt{15}), 8.2535 + 1.96(0.002 / \sqrt{15})]$

$= [8.25249, 8.25451]$

4.8 A new process has been developed for applying photoresist to 125-mm silicon wafers used in manufacturing integrated circuits. Ten wafers were tested, and the following photoresist thickness measurements (in angstroms $\times 1000$) were observed: 13.3987, 13.3957, 13.3902, 13.4015, 13.4001, 13.3918, 13.3965, 13.3925, 13.3946, and 13.4002.

$\bar{x} = 13.39618 = 13.3962$ $n = 10$ σ 未知 \rightarrow 使用 t 分配 $s = 0.0039$ 自由度 = 9

	sum	sum/10	$(x - \bar{x})^2$	s
13.3987	133.9618	13.39618	6.3504E-06	0.003908623
13.3957			2.304E-07	
13.3902			3.57604E-05	
13.4015			2.83024E-05	
13.4001			1.53664E-05	
13.3918			1.91844E-05	
13.3965			1.024E-07	
13.3925			1.35424E-05	
13.3946			2.4964E-06	
13.4002			1.61604E-05	

(a) Test the hypothesis that mean thickness is $13.4 \times 1000 \text{ \AA}$. Use and assume a two-sided alternative.

$$t_{0.025}(9) = 2.262$$

$$t = (\bar{x} - \mu) / (s / \sqrt{n}) = (13.3962 - 13.4) / (0.0039 / \sqrt{10}) = -3.08119 < 2.262$$

reject H_0 , we cannot say that mean thickness is $13.4 \times 1000 \text{ \AA}$.

(b) Find a 99% two-sided confidence interval on mean photoresist thickness. Assume that thickness is normally distributed.

$$t_{0.005}(9) = 3.25$$

$$[\bar{x} - 3.25(s / \sqrt{n}), \bar{x} + 3.25(s / \sqrt{n})]$$

$$= [13.3962 - 3.25(0.0039 / \sqrt{10}), 13.3962 + 3.25(0.0039 / \sqrt{10})]$$

$$= [13.39219, 13.40021]$$

(c) Does the normality assumption seem reasonable for these data?

Yes, because most of them are in the range of the 99% two-sided confidence interval.