8.2 Consider the piston ring data in Table 6.3. Estimate the process capability assuming that specifications are 74.00 ± 0.035 mm.

TABLE 6.3
Inside Diameter Measurements (mm) for Automobile Engine Piston Rings

Sample Number	Observations					$\bar{x}_i$	s_i
1	74.030	74.002	74.019	73.992	74.008	74.010	0.0148
2	73.995	73.992	74.001	74.011	74.004	74.001	0.0075
3	73.988	74.024	74.021	74.005	74.002	74.008	0.0147
4	74.002	73.996	73.993	74.015	74.009	74.003	0.0091
5	73.992	74.007	74.015	73.989	74.014	74.003	0.0122
6	74.009	73.994	73.997	73.985	73.993	73.996	0.0087
7	73.995	74.006	73.994	74.000	74.005	74.000	0.0055
8	73.985	74.003	73.993	74.015	73.988	73.997	0.0123
9	74.008	73.995	74.009	74.005	74.004	74.004	0.0055
10	73.998	74.000	73.990	74.007	73.995	73.998	0.0063
11	73.994	73.998	73.994	73.995	73.990	73.994	0.0029
12	74.004	74.000	74.007	74.000	73.996	74.001	0.0042
13	73.983	74.002	73.998	73.997	74.012	73.998	0.0105
14	74.006	73.967	73.994	74.000	73.984	73.990	0.0153
15	74.012	74.014	73.998	73.999	74.007	74.006	0.0073
16	74.000	73.984	74.005	73.998	73.996	73.997	0.0078
17	73.994	74.012	73.986	74.005	74.007	74.001	0.0106
18	74.006	74.010	74.018	74.003	74.000	74.007	0.0070
19	73.984	74.002	74.003	74.005	73.997	73.998	0.0085
20	74.000	74.010	74.013	74.020	74.003	74.009	0.0080
21	73.982	74.001	74.015	74.005	73.996	74.000	0.0122
22	74.004	73.999	73.990	74.006	74.009	74.002	0.0074
23	74.010	73.989	73.990	74.009	74.014	74.002	0.0119
24	74.015	74.008	73.993	74.000	74.010	74.005	0.0087
25	73.982	73.984	73.995	74.017	74.013	73.998	0.0162
						Σ = 1,850.028 $\bar{\bar{x}}$ = 74.001	0.2351 $\bar{s}$ = 0.0094

μ = x-bar-bar = 74.001 n = 5 d2 = 2.326

$\bar{R}$ = 0.023

σ = $\bar{R}$ /d2 = 0.023/2.326 = 0.01

[73.965, 74.035]

C_p = (USL-LSL) / 6σ = (74.035-73.965)/(6*0.01) = 1.17

C_{pl} = ($\hat{\mu}$ -LSL) / 3σ = (74.001-73.965)/(3*0.01) = 1.2

C_{pu} = (USL - $\hat{\mu}$) / 3σ = (74.035-74.001)/(3*0.01)=1.13

C_{pk} = min(C_{pl},C_{pu}) = 1.13 → <1.17 off-center

8.5. A process is in statistical control with x-bar-bar = 199 and $\overline{R}$ = 3.5. The control chart uses a sample size of n = 4. Specifications are at 200 ± 8. The quality characteristic is normally distributed.

(a) Estimate the potential capability of the process.

N = 4 $\rightarrow$ d2 = 2.059

$\sigma = \overline{R} / d2 = 3.5 / 2.059 = 1.7$

$C_p = (USL-LSL) / 6\sigma = (208 - 192) / (6 * 1.7) = 1.57$

P = 1/1.57 = 64%

The process uses approximately 64% of the specification band.

(b) Estimate the actual process capability.

$C_{pl} = (\hat{\mu} - LSL) / 3\sigma = (208 - 199)/(3*1.7) = 1.76$

$C_{pu} = (USL - \hat{\mu}) / 3\sigma = (199 - 192)/(3*1.7)=1.37$

$C_{pk} = \min(C_{pl},C_{pu}) = 1.37 \rightarrow <1.57$ off-center

(c) How much improvement could be made in process performance if the mean could be centered at the nominal value?

$P_{actual} = P\{x < LSL\} + P\{x > USL\}$

$= P\{x < LSL\} + [1 - P\{x \le USL\}]$

$= P\{z < ((LSL - \mu)/\sigma)\} + 1 - P\{z \le ((\mu - USL)/\sigma)\}$

$= P\{z < ((192-199)/1.7)\} + 1 - P\{z \le ((208 - 199)/1.7)\}$

$= P(z < -4.1176) + 1 - P(z \le 5.2941)$

$= 0.0000191$

$P_{move} = 2 * P\{z < (192 - 200)/1.7\} = 2 * 0.0000013 = 0.0000026$

Improve = 0.000191 – 0.0000026 = 0.000165

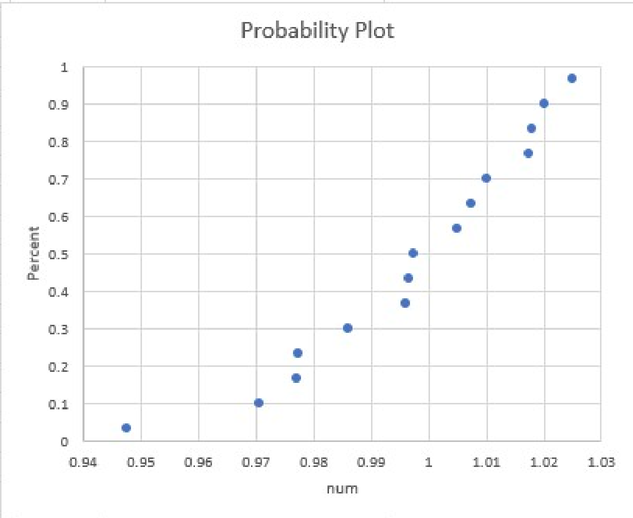
8.8 The weights of nominal 1-kg containers of a concentrated chemical ingredient are shown in Table 8E.2. Prepare a normal probability plot of the data and estimate process capability. Does this conclusion depend on process stability?

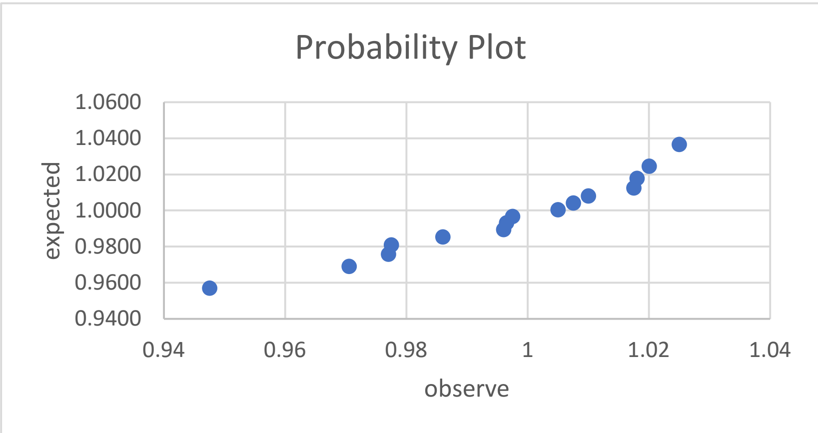
TABLE 8E.2

Weights of Containers

0.9475	0.9775	0.9965	1.0075	1.0180
0.9705	0.9860	0.9975	1.0100	1.0200
0.9770	0.9960	1.0050	1.0175	1.0250

ID	observed	Rank	Plotting Position(Rank-0.5)/n	expected from a normal distribution		
1	0.9475	1	0.0333	0.9570	Mean=	0.9968
6	0.9705	2	0.1000	0.9690	stdev=	0.0217
11	0.977	3	0.1667	0.9758		
2	0.9775	4	0.2333	0.9810		
7	0.986	5	0.3000	0.9854		
12	0.996	6	0.3667	0.9894		
3	0.9965	7	0.4333	0.9931		
8	0.9975	8	0.5000	0.9968		
13	1.005	9	0.5667	1.0004		
4	1.0075	10	0.6333	1.0041		
9	1.01	11	0.7000	1.0081		
14	1.0175	12	0.7667	1.0125		
5	1.018	13	0.8333	1.0177		
10	1.02	14	0.9000	1.0245		
15	1.025	15	0.9667	1.0365		





P50 = 0.9975 P84 = 1.02

σ = P84 – P50 = 1.02 – 0.9975 = 0.0225

6σ = 6 * 0.0225 = 0.135

Yes, this conclusion depends on process stability.

8.9. Consider the package weight data in Exercise 8.13. Suppose there is a lower specification at 0.985 kg. Calculate an appropriate process capability ratio for this material. What percentage of the packages produced by this process is estimated to be below the specification limit?

LSL = 0.985kg

$C_p = (\hat{\mu} - LSL)/(3 * \sigma) = (0.9975 - 0.985) / (3 * 0.0225) = 0.19$

$\hat{p} = P\{z < ((LSL - \mu)/\sigma)\} = P\{z < (0.985 - 0.9975)/0.0225\} = P\{z < -0.556\} = 0.2891$

8.15 The molecular weight of a particular polymer should fall between 2,100 and 2,350. Fifty samples of this material were analyzed with the results $\bar{x} = 2275$ and $s = 60$. Assume that molecular weight is normally distributed.

(a) Calculate a point estimate of C_{pk}.

$C_{pu} = (USL - \hat{\mu}) / 3\sigma = (2350 - 2275)/(3 * 60) = 0.42$

$C_{pl} = (\hat{\mu} - LSL)/ 3\sigma = (2275 - 2100) / (3 * 60) = 0.97$

$C_{pk} = \min(C_{pl}, C_{pu}) = 0.42$

(b) Find a 95% confidence interval on C_{pk}.

$\alpha = 0.05 \quad Z_{\alpha/2} = 1.96$

$C_{pk}[1 - Z_{\alpha/2}\sqrt{\ }(1/(9*n*C\text{^}2_{pk}) + 1/(2*(n-1)))] = 0.42[1-1.96\sqrt{\ }(1/(9*50*0.42\text{^}2) + 1/(2*(50-1)))] = 0.2957$

$C_{pk}[1 + Z_{\alpha/2}\sqrt{\ }(1/(9*n*C\text{^}2_{pk}) + 1/(2*(n-1)))] = 0.42[1+1.96\sqrt{\ }(1/(9*50*0.42\text{^}2) + 1/(2*(50-1)))] = 0.5443$

[0.2957, 0.5443]