11.1 The data shown in Table 11E.1 come from a production process with two observable quality characteristics: x_1 and x_2. The data are sample means of each quality characteristic, based on samples of size n = 25. Assume that mean values of the quality characteristics and the covariance matrix were computed from 50 preliminary samples: $\bar{\bar{x}} = \begin{bmatrix} 55 \\ 3o \end{bmatrix}$, $s = \begin{bmatrix} 200 & 130 \\ 130 & 120 \end{bmatrix}$

Construct a T^2 control chart using these data. Use the phase II limits.

M = 50; n = 25; p = 2; let α = 0.001

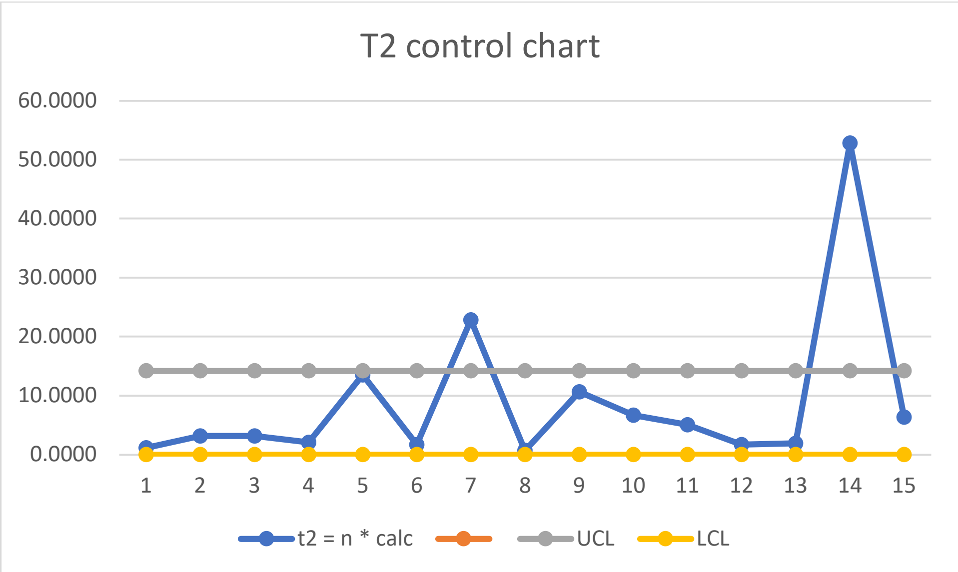
UCL ={ [p(m + 1)(n – 1)]/[mn – m – p + 1]} * $F_{\alpha,p,mn-m-p+1}$

={ [2(50 + 1)(25 – 1)]/[50*25 – 50 – 2 + 1]} * $F_{0.001,2,50*25-50-2+1}$

=(2448 / 1199) * 6.948 = 14.186

LCL = 0

sample no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
xbar1	58	60	50	54	63	53	42	55	46	50	49	57	58	75	55
xbar2	32	33	27	31	38	30	20	31	25	29	27	30	33	45	27
diff1	3	5	-5	-1	8	-2	-13	0	-9	-5	-6	2	3	20	0
diff2	2	3	-3	1	8	0	-10	1	-5	-1	-3	0	3	15	-3
s	200	130			xbarbar	55			s^-1=	0.0169	-0.0183				
	130	120				30				-0.0183	0.0282				
matrix calc	0.0451	0.1268	0.1268	0.0817	0.5408	0.0676	0.9127	0.0282	0.4254	0.2676	0.2028	0.0676	0.0761	2.1127	0.2535
t2 = n * cal	1.1268	3.1690	3.1690	2.0423	13.5211	1.6901	22.8169	0.7042	10.6338	6.6901	5.0704	1.6901	1.9014	52.8169	6.3380
UCL	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186	14.186
LCL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
OOC	x	x	x	x	x	x	o	x	x	x	x	x	x	o	x



Process 7, 14 out of control

11.3 Consider a T^2 control chart for monitoring p = 6 quality characteristics. Suppose that the subgroup size is n = 3 and there are 30 preliminary samples available to estimate the sample covariance matrix.

(a) Find the phase II control limits assuming that α = 0.005.

UCL = [p(m + 1)(n – 1)/(mn – m – p + 1)]*$F_{\alpha ,p,mn-m-p+1}$

=6(31)(2)/[30*3-30-6+1] *$F_{0.005 ,6,55}$

= 6.76364*3.531

=23.882

LCL = 0

(b) Compare the control limits from part (a) to the chi-square control limit. What is the magnitude of the difference in the two control limits?

Chi-square limit: UCL = $X^2_{\alpha ,p}$ = $X^2_{0.005,6}$ = 18.548

The Phase II UCL is almost 30% larger than the chi-square limit

(c) How many preliminary samples would have to be taken to ensure that the exact phase II control limit is within 1% of the chi-square control limit?

Ans.

18.548*1.01=18.733

UCL = 6(m+1)2/(m*3-m-6+1) $F_{0.005,6,mn-m-p+1}$= 18.733

M = 720

11.4 Rework Exercise 11.3, assuming that the subgroup size is n = 5.

(a)

UCL = [p(m + 1)(n – 1)/(mn – m – p + 1)]*$F_{\alpha ,p,mn-m-p+1}$

=6*31*4/(150-30-6+1)*$F_{0.005,6,115}$

=21.309

(b)

UCL = $X^2_{\alpha ,p}$ = $X^2_{0.005,6}$ = 18.548

The phase II UCL is almost 15% larger than the chi-square limit

(c)

18.548*1.01=18.733

UCL = 6(m+1)4/(m*5-m-6+1) $F_{0.005,6,mn-m-p+1}$= 18.733

M = 410