ECON526: Quantitative Economics with Data Science Applications

Introduction to Directed Acyclic Graphs

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Overview

Summary

- Previously in the course, we talked at a high level about some of the barriers to causal inference
- We used the potential outcomes framework to discuss the idea of a treatment effect
- Then we discussed the idea of a randomized experiment as a way to identify a treatment effect
  - → However, we mentioned that there are many situations where we cannot run a randomized experiment
- Today, we will discuss the idea of using a **graphical model** as a way to analyze whether you can truly identify a treatment effect

Directed Graphical Models

Conditional Independence

- Recall that two random variables X and Y are **conditionally independent** given a third random variable Z if and only if the following holds:
  - → $P(X|Z) = P(X|Z \cap Y)$
  - → Equivalently, $P(X \cap Y|Z) = P(X|Z)P(Y|Z)$
  - → We will denote this as $X \perp Y|Z$
- In the context of potential outcomes, we require that $(Y_0, Y_1) \perp T|X$
  - → This means that the potential outcomes are independent of the treatment assignment, given the covariates

Directed Graphical Models

- Complete independence is rare in complex systems. However, we can often find conditional independence relationships, that help inform our choice of statistical model.
- We can visualize conditional independence relationships using a **Bayesian** or **directed graphical model**
- A Bayesian graphical model is a directed graph where:
  - $\longrightarrow$ The nodes or vertices represent random variables
  - $\longrightarrow$ The links or edges represent conditional independence relationships

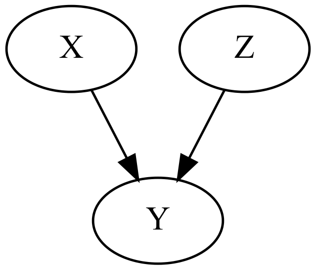
Directed Graphical Models

- Fundamentally, a graphical model is a way to represent how a joint probability distribution factorizes into a product of conditional distributions.
- For our purposes, we will usually interpret the edges as causal relationships.
- Furthermore, the graphs we draw will be **acyclic**, meaning that there are no loops in the graph.
  - → In other words, there is no way to start at a node and follow the arrows to get back to the same node
  - → This is because we are interested in **causal** relationships, and a cycle would imply that there is a feedback loop where a variable causes itself
- Since these graphs are both directed and acyclic, we call them **directed acyclic graphs** or **DAGs**

Directed Graphical Models

- A directed graphical model might look something like this:

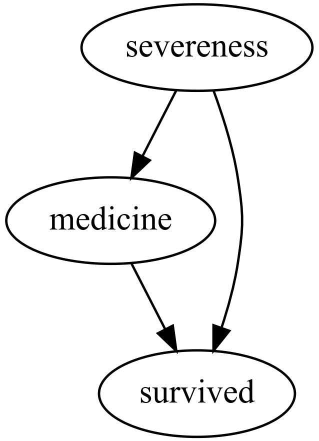
```python
import graphviz as gr

g = gr.Digraph()
g.node('X')
g.node('Y')
g.node('Z')
g.edge('X', 'Y')
g.edge('Z', 'Y')
g
```



- Here, we have three random variables: X, Y, and Z
  - $\longrightarrow$ X and Z are independent
  - $\longrightarrow$ Y depends on both X and Z

Directed Graphical Models

```
g = gr.Digraph()
g.edge("medicine", "survived")
g.edge("severeness", "survived")
g.edge("severeness", "medicine")
g
```



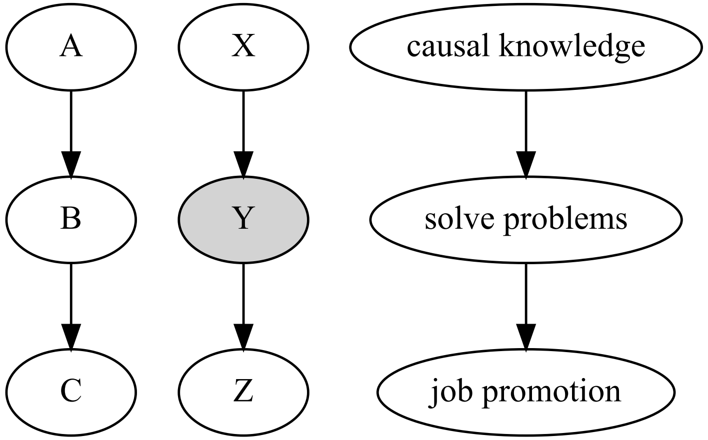
- We use arrows to indicate the direction of the conditional independence relationship
  - → For example, $medicine \rightarrow survived$ means that $survived$ depends on $medicine$, but not the other way around

Directed Graphical Models

- Directed graphical models are useful because they allow us to visualize conditional independence relationships, which can often be difficult to keep track of
- However, they are also useful because they allow us to determine whether or not we can identify a treatment effect.
- There are three very common sub-structures that appear in graphical models, that inform how dependence will flow through the model.

Directed Graphical Models

```
g = gr.Digraph()
g.edge("A", "B")
g.edge("B", "C")

g.edge("X", "Y")
g.edge("Y", "Z")
g.node("Y", "Y", style="filled")


g.edge("causal knowledge", "solve problems")
g.edge("solve problems", "job promotion")

g
```



- In this stylized example of a **directed path**, we are postulating that knowing causal inference is the only way to solve business problems

  → This is obviously not true, but it is a useful example for our purposes

Directed Graphical Models

- This model highlights the following statistical process:
  - → Knowing causal inference gives you the ability to solve business problems
  - → Solving business problems makes you more likely to get a job promotion
- Notice that this does not imply that causal knowledge is independent of job promotion
  - → That is, if we know the value of job promotion, we can still learn something about causal knowledge
  - → If we observe that a promotion happens in this model, this tells us that it is more likely that the person knows causal inference

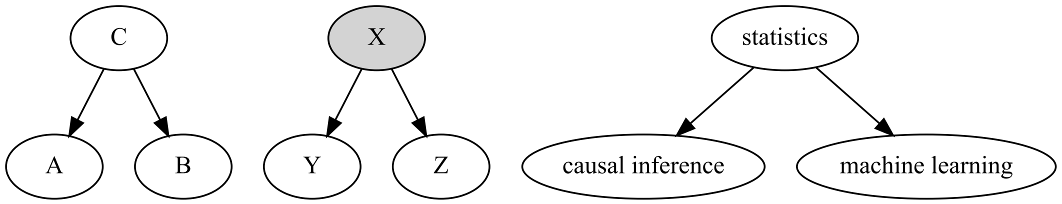
Directed Graphical Models

- Now let's condition on the intermediate variable Y
  - $\longrightarrow$ This is the variable that represents the ability to solve business problems
  - $\longrightarrow$ In the graph, we have colored this variable grey to indicate that we are conditioning on it
- Conditioning on Y means that we are assuming that we know whether or not the person solved a business problem
- In this case, conditioning on Y breaks the dependence relationship between X and Z
  - $\longrightarrow$ That is, X and Z are now independent, given Y
  - $\longrightarrow$ Mathematically, $A \perp C$, but $X \not\perp Z|Y$

Directed Graphical Models

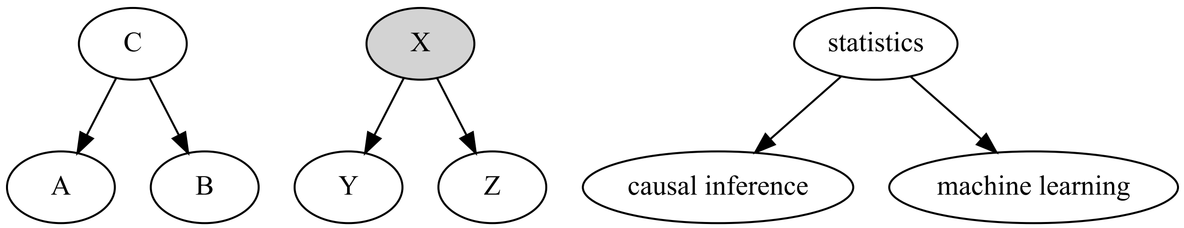
Now let’s look at another common structure:

```
g = gr.Digraph()
g.edge("C", "A")
g.edge("C", "B")

g.edge("X", "Y")
g.edge("X", "Z")
g.node("X", "X", style="filled")

g.edge("statistics", "causal inference")
g.edge("statistics", "machine learning")

g
```



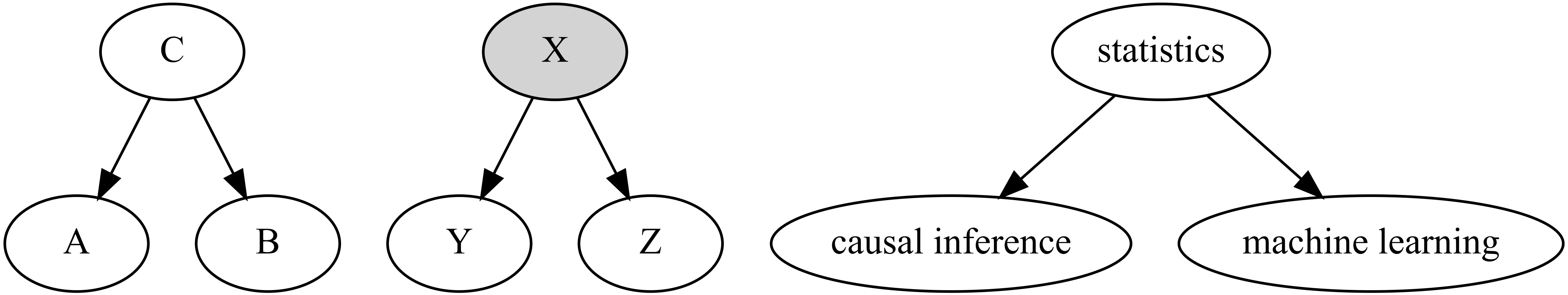
- In this model, we are postulating that statistics is a prerequisite for both causal inference and machine learning.

Directed Graphical Models



- This model highlights the following statistical process:
  - $\longrightarrow$ Knowing statistics gives you the ability to do causal inference
  - $\longrightarrow$ Knowing statistics gives you the ability to do machine learning
- When we don't condition for the root node, C, there is still a dependence relationship between A and B
  - $\longrightarrow$ That is, if we know that an individual has causal knowledge, this tells us that they are more likely to know statistics, and thus to also know machine learning

Directed Graphical Models

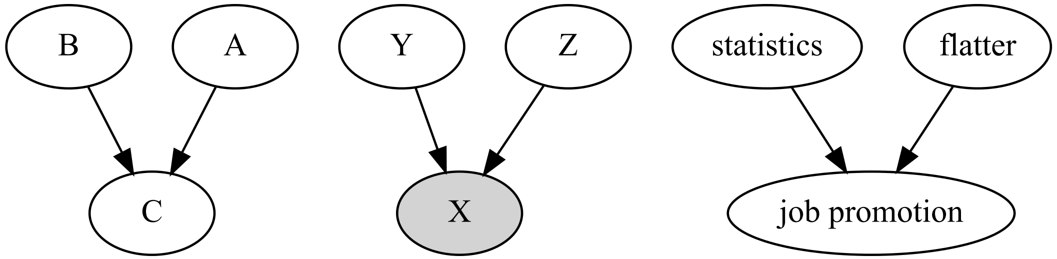


- By conditioning on X, we break the dependence relationship between Y and Z
  - $\rightarrow$ That is, $A \not\perp B$, but $Y \perp Z | X$
- We would call this a **fork** structure

Directed Graphical Models

Finally, let's look at a third common structure, called a **collider**:

```
g = gr.Digraph()
g.edge("B", "C")
g.edge("A", "C")

g.edge("Y", "X")
g.edge("Z", "X")
g.node("X", "X", style="filled")

g.edge("statistics", "job promotion")
g.edge("flatter", "job promotion")

g
```



- In this model, we are postulating that statistics and flattery are both determinants of getting a job promotion.

Building a Causal Graph

Building a Causal Graph

- Now that we have seen some examples of causal graphs, we should think about how to draw a causal graph for a problem in practice.
- Drawing a causal graph is a way of encoding the assumptions that you are making about the data generating process.
- The first step is to identify the variables that are relevant to the problem at hand.

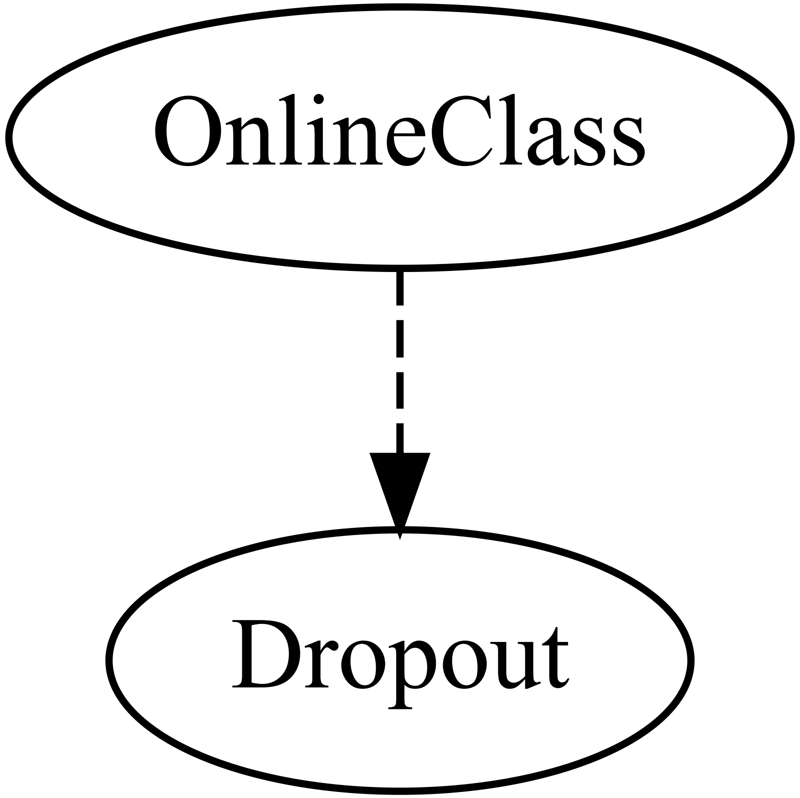
  → Do your research!

Building a Causal Graph

- In reality, there are many variables that are relevant to a problem.
- Real data-generating processes are therefore extremely large and complex.
- Think about what the data generating process might look like for something similar the online class example we have been using
  - → (if we didn't have a randomized experiment)

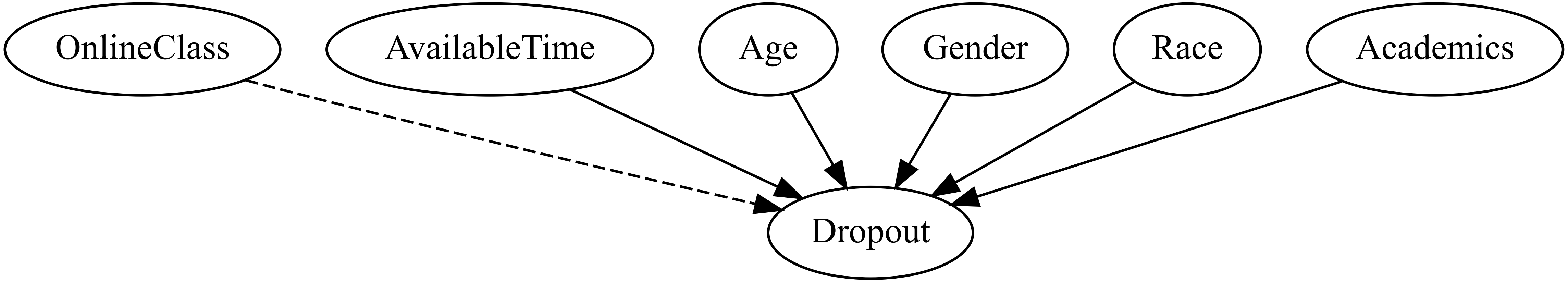
Building a Causal Graph

- Let's say we were interested in the effect of online classes on college dropout rates. The treatment effect we want to identify is:



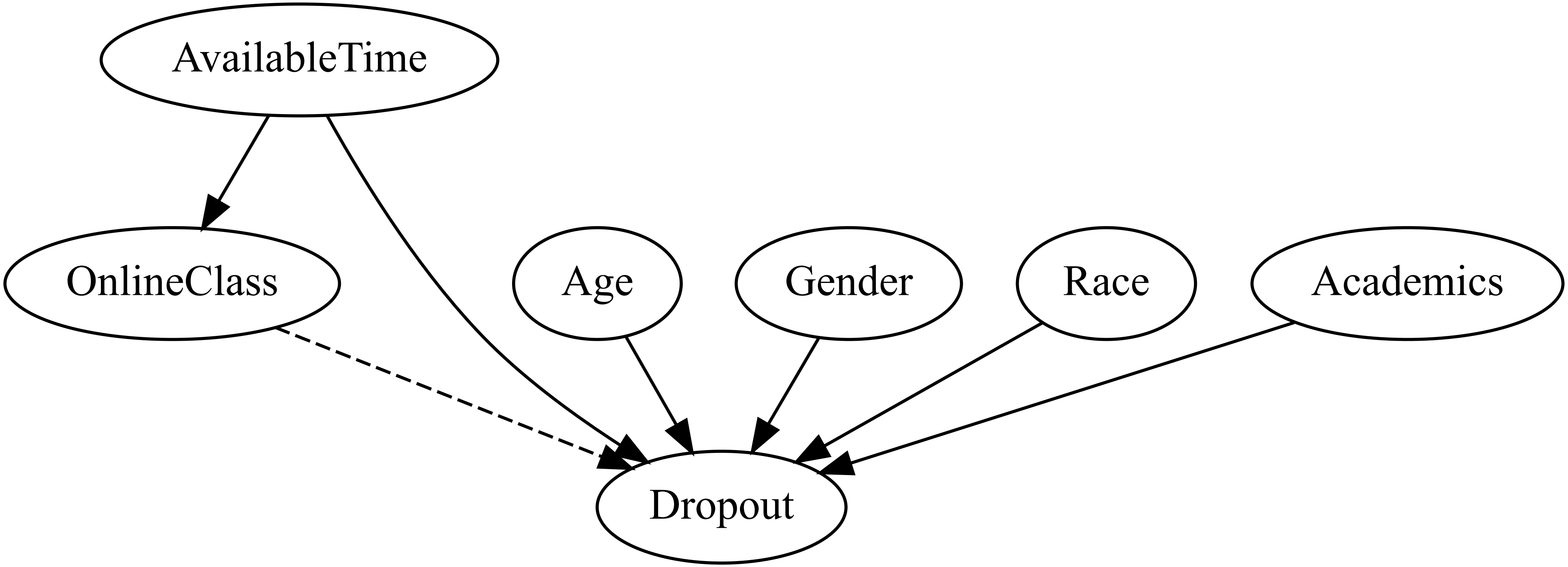
Building a Causal Graph

- In addition to `OnlineClass`, there are **many** variables that might cause a student to drop out of college.



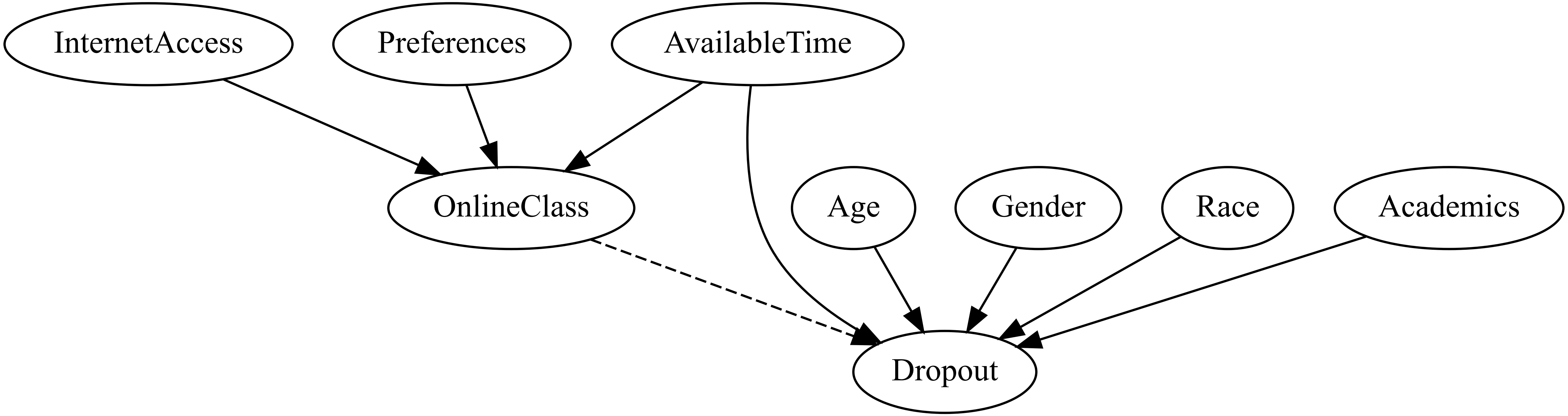
Building a Causal Graph

- Some of these variables might **also** determine whether or not a student takes an online class.



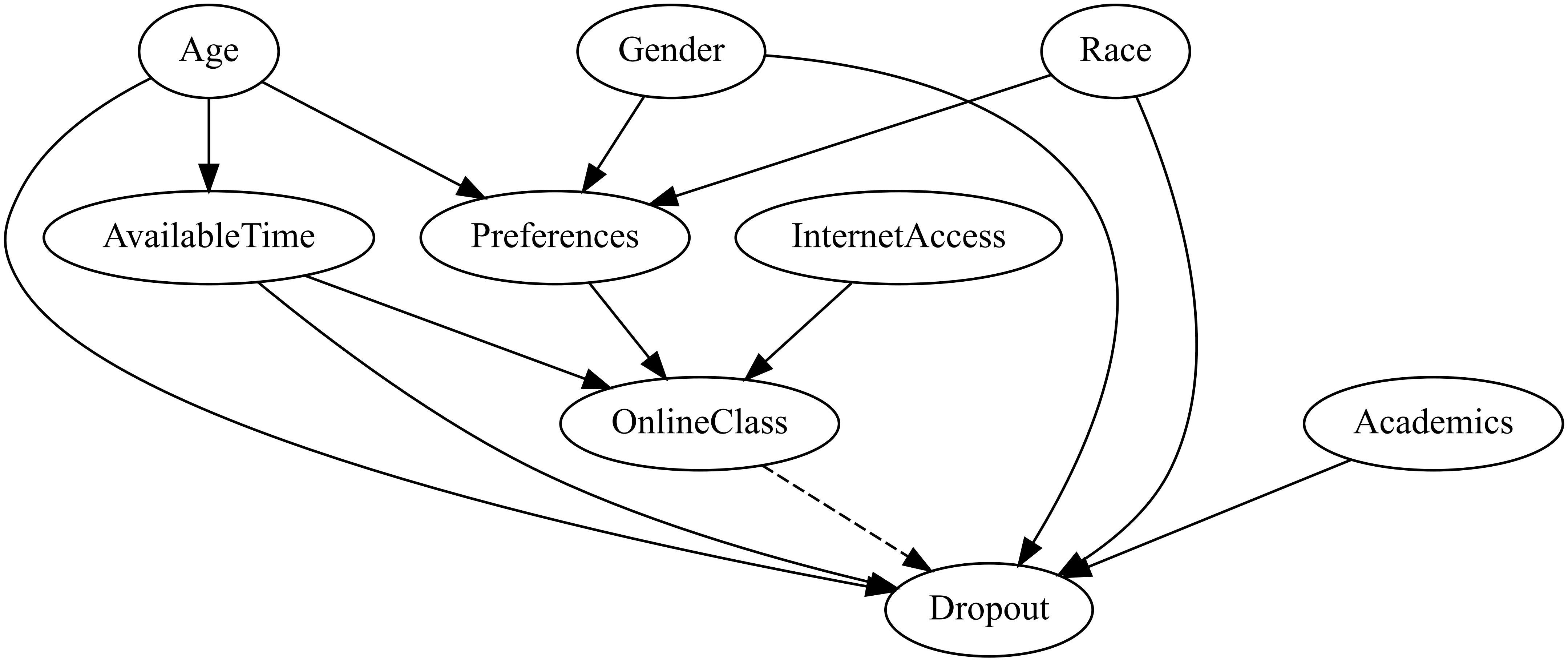
Building a Causal Graph

- Then there are some other variables that affect just `OnlineClass` and not `Dropout`



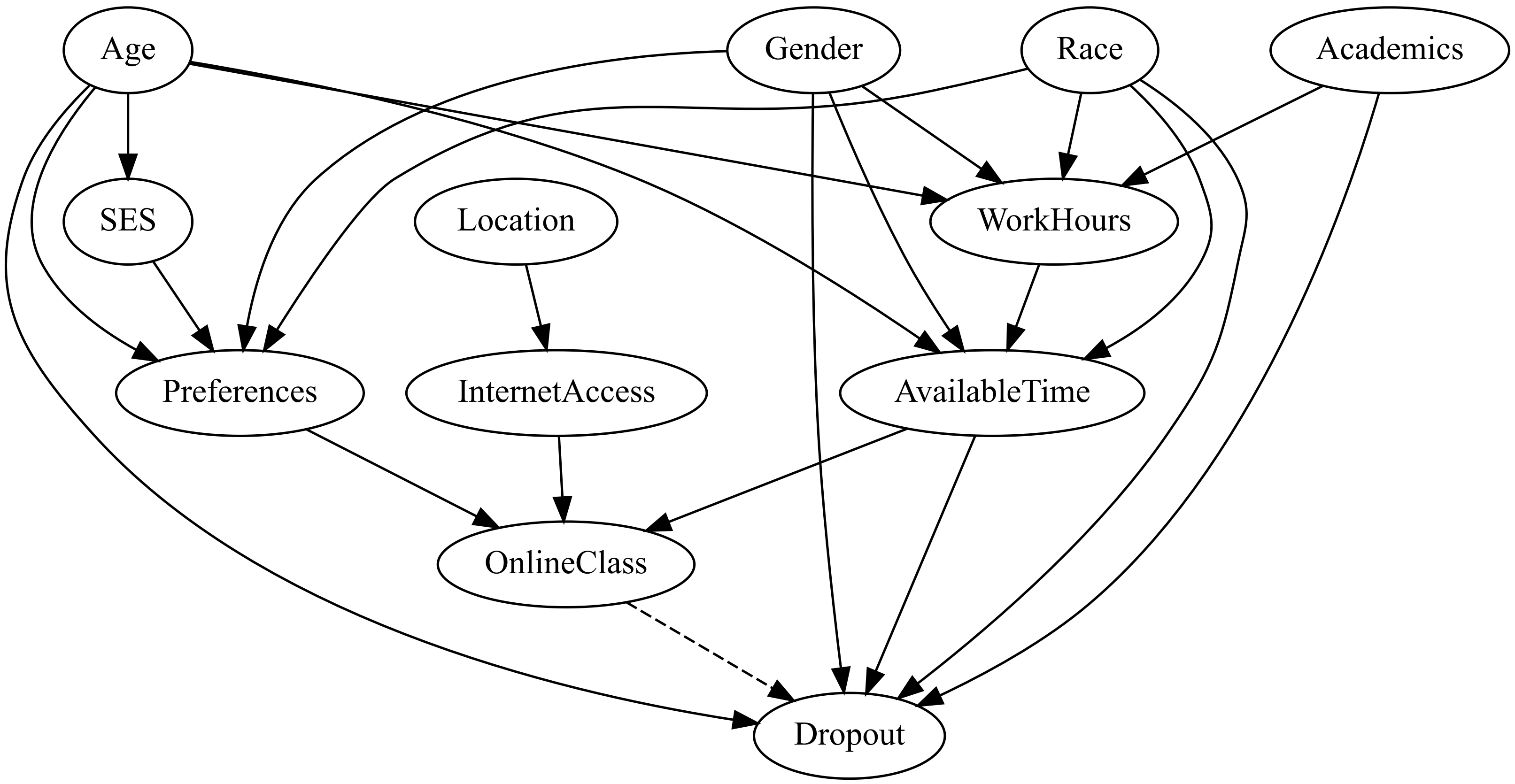
Building a Causal Graph

- There are relationships between some of these variables as well



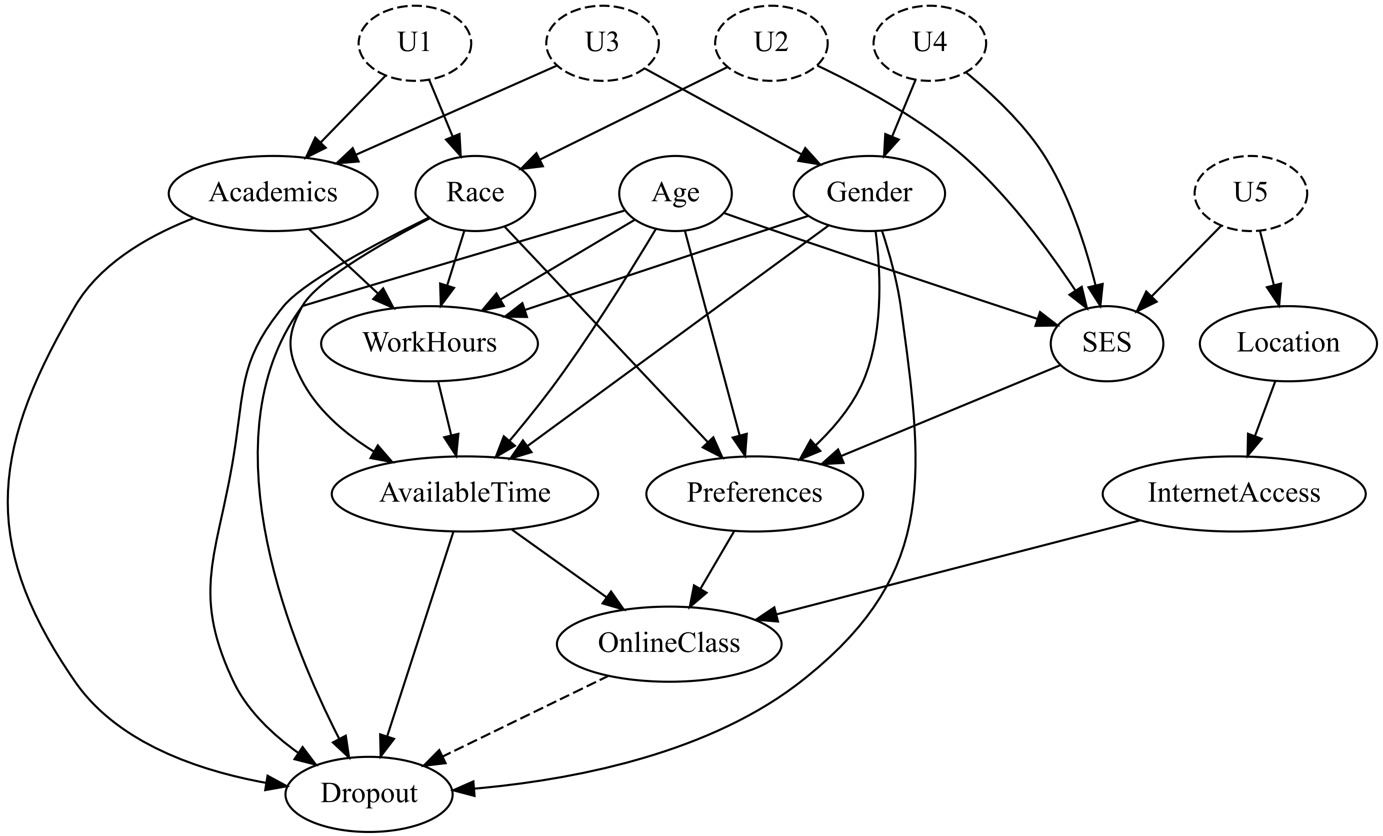
Building a Causal Graph

- Even if there is not a direct causal relationship between two variables, there might be an indirect relationship through a third variable

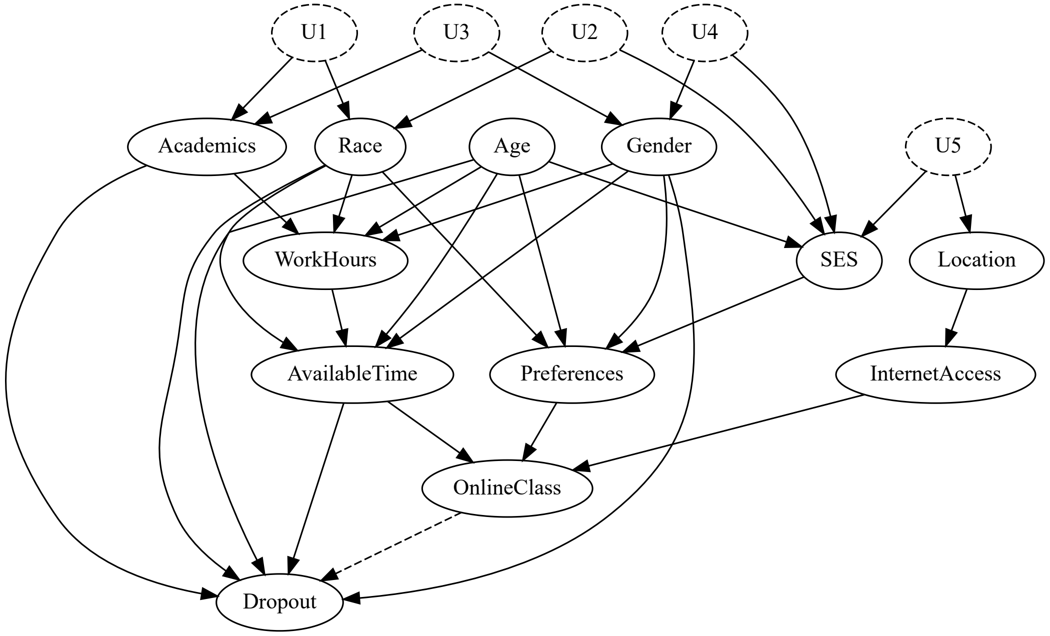


Building a Causal Graph

- And then there are some variables that we know are correlated, but due to some other combination of unknown factors

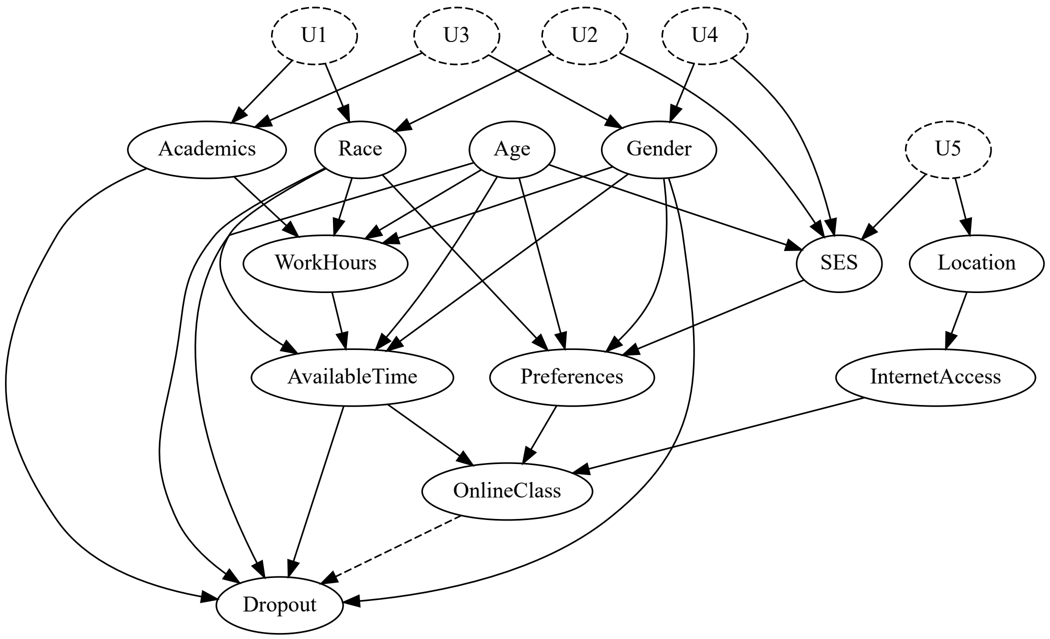


Building a Causal Graph



- As we can see, this graph got very complex, very quickly.
- It is clearly not capturing all of the relevant variables
- What else might be missing?

Building a Causal Graph



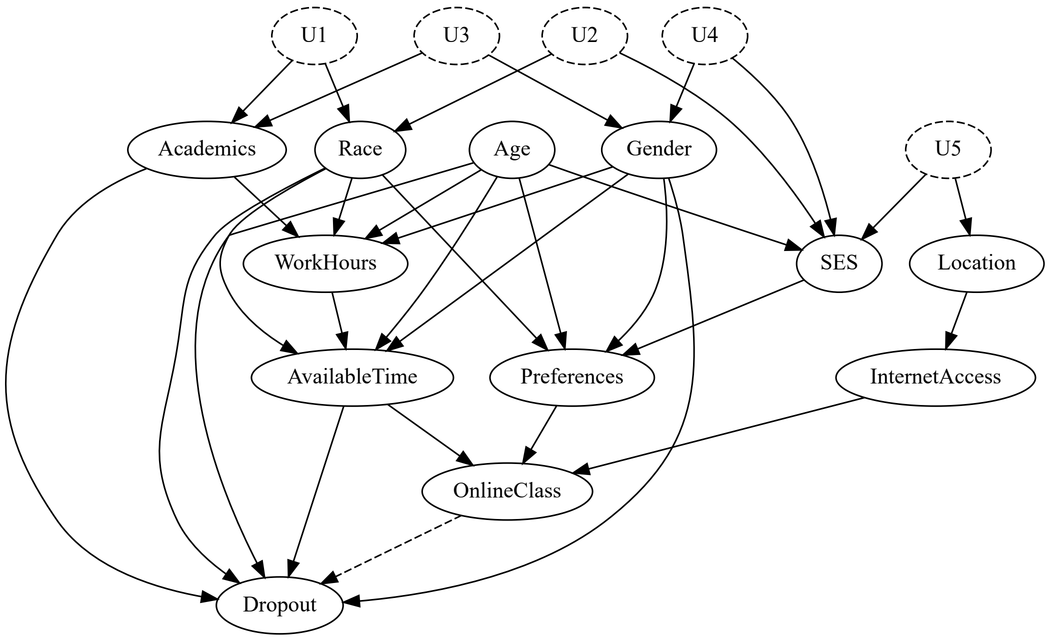
- As we can see, this graph got very complex, very quickly.
- It is clearly not capturing all of the relevant variables
- What else might be missing?
  - → CommunityCollege vs. University
  - → *Income*

Simplifying the Graph

- In practice, we will not be able to draw a graph that captures everything. How can we choose which relationships to include, and which to ignore?

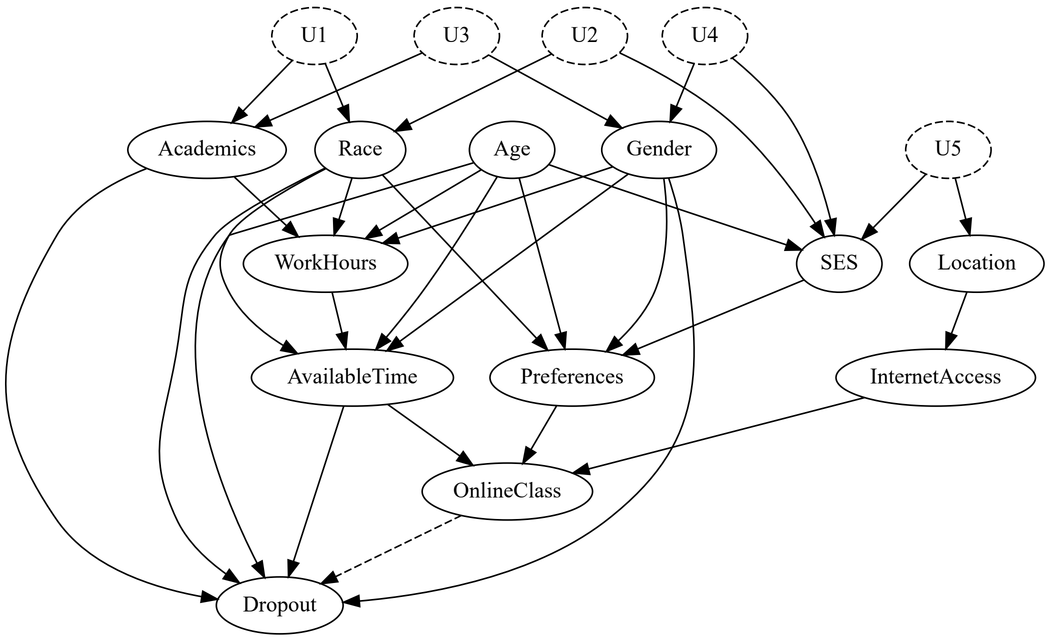
1. **Unimportance** - If the arrows coming in and out of a variable all represent small or negligible effects, we can ignore them.
2. **Redundancy** - If there are variables on the diagram that occupy the same space (i.e. they both have incoming and outgoing links from the same variables) then we can probably combine them into a single variable.
3. **Mediators** - If a variable is **only** included as a way to connect two other variables, we can probably remove it.
4. **Irrelevance** - If a variable is important to the DGP, but isn't part of a **dependence path** between the treatment and the outcome, we can ignore it.

Building a Causal Graph



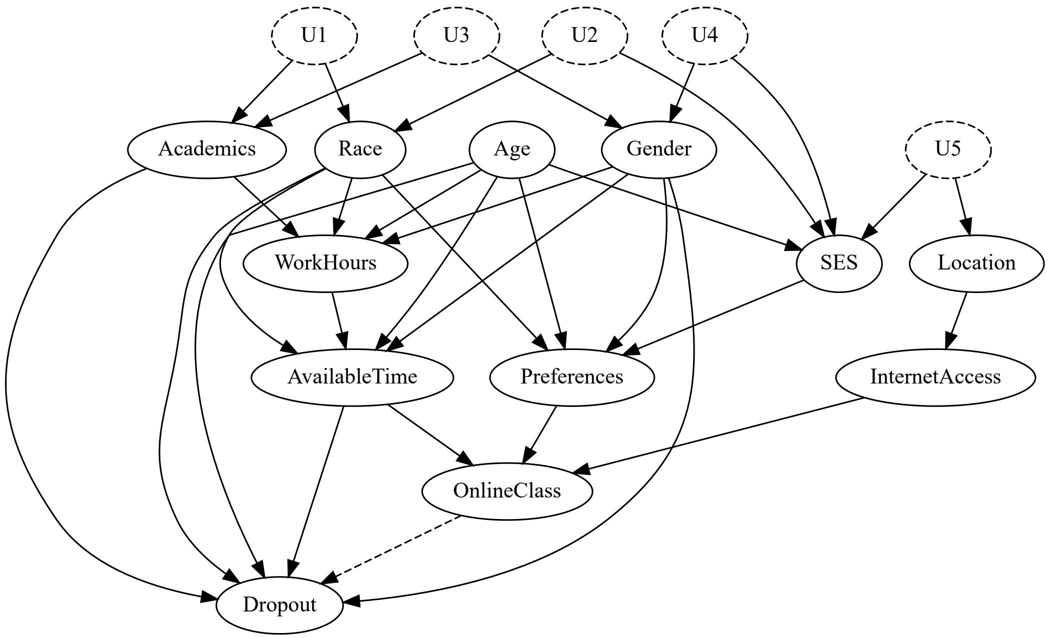
- **Unimportance** was already necessarily applied in the creation of the graph.
- We can also apply **redundancy** by combining `Race` and `Gender` into a single variable, `Demographics`

Building a Causal Graph



- There are also quite a few **Mediators**
  - → The most prevelant is `Preferences`, (we can just have `Demographics`, etc. directly affect `OnlineClass`).
  - → We can throw out `InternetAccess` without losing anything.
- There is one more. Can you find it?

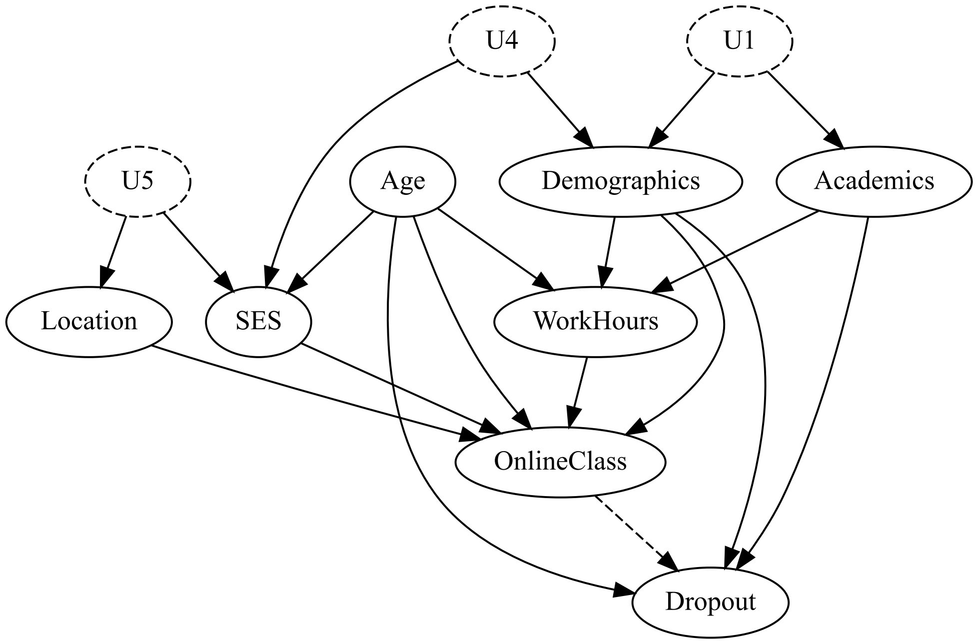
Building a Causal Graph



- Notice that all of the variables that cause `AvailableTime` are also causes of `WorkHours`.
- We can therefore throw out `AvailableTime`, and just have `WorkHours` directly affect `OnlineClass`, rather than through `AvailableTime`.
- `AvailableTime` is also a mediator.

Building a Causal Graph

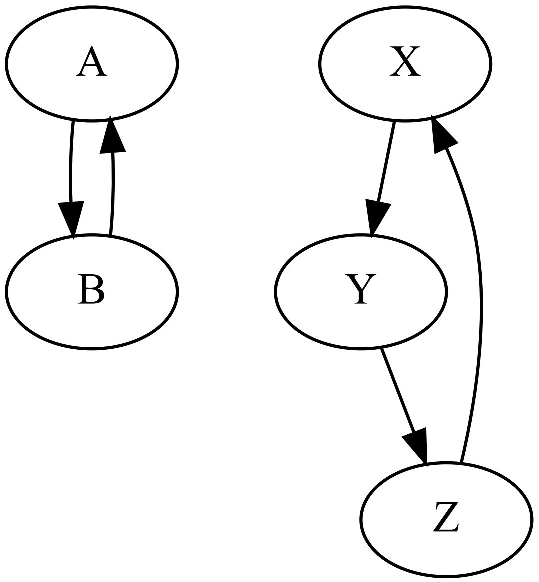
- We are left with a much simpler (although still messy) model, that still captures most of the relevant relationships.
- Even though the simplified model is nicer to work with, these rules are just heuristics and shouldn't be applied blindly.



Aside: Cycles

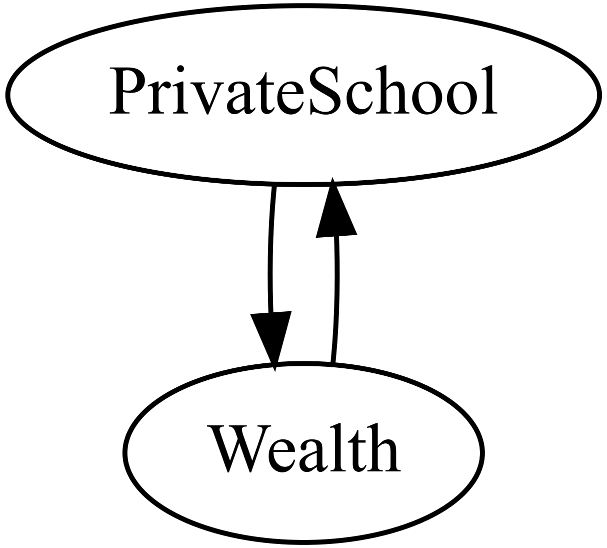
Cycles

- There is one thing that a causal diagram cannot have: a **cycle**.
- A cycle is a directed path that starts and ends at the same node.
  - → This would mean that a variable causes itself, which is impossible.
- The simplest cycles look like this:



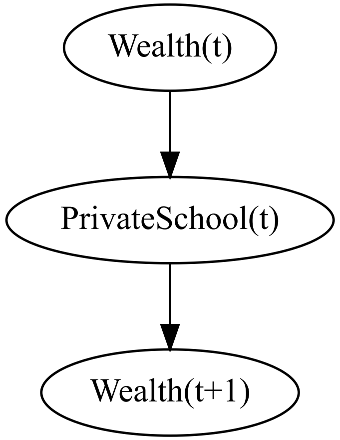
Cycles

- Why can't we have cycles? Surely there are feedback loops in the real world.



- If we have a cycle, then our causal problem is ill-specified.
  → We can't identify the effect of `PrivateSchool` on `Wealth`, because `Wealth` also causes `PrivateSchool`... or does it?

Cycles



- We can solve this problem by introducing the **time** dimension.
- By creating a new variable to represent the lagged value, we can break the cycle.
  - → This also loosely corresponds with the physicists view of causality, where the arrow of time is fundamental.
  - → Similar to the time-series concept of **Granger Causality**.

Credits

This lecture draws heavily from Causal Inference for the Brave and True: Chapter 04 - Graphical Causal Models by Matheus Facure.

There is also material from A Short Course on Graphical Models Chapter 2: Structured Representations by Mark Paskin.

As well as The Effect: Chapter 7 - Drawing Graphical Diagrams by Nick Huntington-Klein.