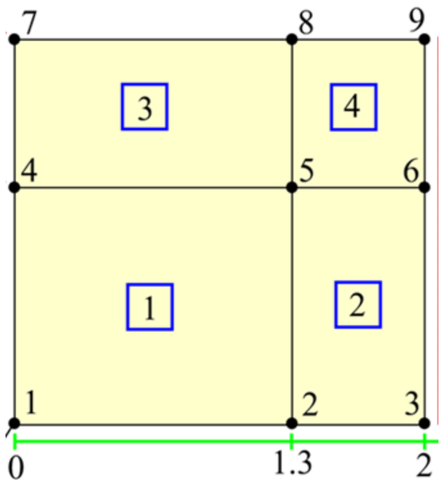
Etude d'une plaque métallique en régime transitoire

$\rho C_p \frac{\partial T}{\partial t} = \lambda \Delta T$ une plaque en béton carrée de 2 m de côté



Conditions aux limites :

$T_0(x, y) = 5xy$ température initiale

A l'instant 0+ on impose $T(0, y) = T(x, 0) = 20$°C avec

$\emptyset_n(2, y) = 10(T(2, y) - 20)$ et $\emptyset_n(x, 2) = -100\ w.m^{-2}$

Proposez les valeurs de λ, ρ et C_p

Ecrire le système différentiel total pour une approximation linéaire et résoudre itérativement par les 2 schémas implicite et explicite.

Refaire les calculs en affinant le maillage

NB : Une grande importance sera accordée à la clarté de la rédaction. Tout outil de calcul devra être précisé. Et tout élément de programmation pour la résolution sera mis en annexe.

Project Elements fini :

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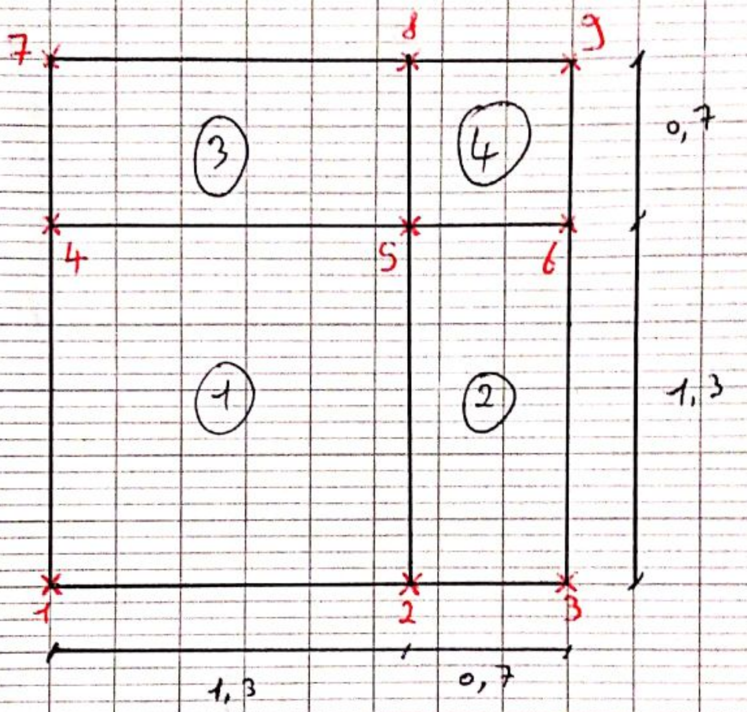
1) Données :

Bloc en béton :



$\lambda = 2,1 \ W/m\,°C$
$\rho = 2500 \ Kg/m^3$
$C_p = 0,75 \ KJ/Kg.K$
$\Rightarrow$ Propriétés du béton

$$\rho C_p \frac{\partial T}{\partial t} = \operatorname{div}(\lambda \, \overrightarrow{\operatorname{grad} T})$$

C.L : $T(0;y) = T(x;0) = 20°C$ $\quad (\Gamma_1) = \begin{cases}(0;y)\\(x;0)\end{cases}$

$\phi_n(2;y) = 10\,(T(2;y) - 20)$ $\quad (\Gamma_2) = \begin{cases}(2;y)\\(x;2)\end{cases}$

$\phi_n(x;2) = -100 \ W/m^2$

2) Formulation variationelle :

Soit $u_t(x;y) = T(x;y;t)$ à un temps fixé

Soit $\mathcal{V} = \{ v(x;y) \,;\, (x;y) \in \Omega \,/\, v/_{\Gamma_1} = 0 \}$

On definit $u_t^* = T(0;y) = T(x;0) = 20°C = 293\ K$

$$u_t(x;y) = u_t^*(x;y) + v \,.\, u(x;y)$$

$\forall v \in \mathcal{V}$:

$$\int_\Omega v \left(\rho c_p \frac{\partial u_t}{\partial t}\right) dS = \int_\Omega v \,.\, \operatorname{div}(\lambda \overrightarrow{\operatorname{grad}}\, u(x;y))\, dS$$

$$= \int_{\Gamma_1+\Gamma_2} \lambda \overrightarrow{\operatorname{grad}}\, u_t(x;y) \,.\, \vec{n} \,.\, v \, d\gamma$$

$$- \int_\Omega \lambda \overrightarrow{\operatorname{grad}}\, u_t(x;y)\, \overrightarrow{\operatorname{grad}}\, v(x;y)\, dS$$

$$\Rightarrow \int_\Omega \frac{\partial}{\partial t}(\rho c_p\, u_t\, v_t)\, dS = -\int_{\Gamma_2} \phi_n\, v_t(x;y)\, d\gamma$$

$$- \int_\Omega K\, \overrightarrow{\operatorname{grad}}\, u(x;y)\, \overrightarrow{\operatorname{grad}}\, v(x;y)\, dS$$

3) Approximation par Elements fini :

Soit v_h un sous espace de v; et $w_i(x;y)|_{i:1\to N}$

est une base de v_h.

$$u_{h t}(x;y) = u^*_t + \sum_{j=1}^{N} u_j(t)\, w_j(t)$$

$\forall i = 1 \to N$ on a :

$$\int_\Omega \frac{\partial}{\partial t}\left[\rho C_p \left(\sum_{j=1}^{N} u_j(t)\, w_j(t) + u^* t\right) w_i \, dS\right.$$

$$\underbrace{+ \int_\Omega \lambda\, \overrightarrow{grad}\, u^*_t\, \overrightarrow{grad}\, v \, dS} \underbrace{+ \int_\Omega \lambda\, \overrightarrow{grad}\, u_t \cdot v \cdot n\, d\gamma}$$

$$\int \lambda \left[\frac{\partial u^* t}{\partial x} \frac{\partial w_i}{\partial x} + \frac{\partial u^* t}{\partial y}\frac{\partial w_i}{\partial y}\right] dS \qquad \boxed{u^* t = cte}$$

$$\frac{\partial u^* t}{\partial x} = 0 \qquad \frac{\partial u^* t}{\partial y} = 0$$

$$\int \lambda \sum u_j \left[\frac{\partial w_j}{\partial x}\frac{\partial w_i}{\partial x} + \frac{\partial w_j}{\partial y} \cdot \frac{\partial w_i}{\partial y}\right]$$

$$\underbrace{\sum_{j=1}^{N} \left[\int \rho_C \, w_i\, w_j\, dS\right]}_{B_{ij}} \frac{\partial u_j(t)}{\partial t} + \underbrace{\sum_{j=1}^{N}\left[\lambda \int \frac{\partial w_j}{\partial x}\frac{\partial w_i}{\partial x} + \frac{\partial w_j}{\partial y}\frac{\partial w_i}{\partial y}\right]}_{A_{ij}} \times u_j(t)$$

$$= \underbrace{\int_{\Gamma_2} \phi_m\, w_i\, d\gamma}_{b_i}$$

$$B_{ij}\left[\frac{du_j(t)}{dt}\right] + A_{ij}\left[u_j(t)\right] = b_i$$

$$\Rightarrow B\frac{dU}{dt} + AU = b$$

4) Integration par rapport au temps :

Système : $B\frac{dU}{dt} + AU = b \qquad U(0) = U_0$

Appliquon la methode des différences finie :

On choisit un interval de temps de 2 s et avec $N = 4$; Alors $\boxed{\Delta T = 0,5\,s}$

$U(mt) = U_m \qquad mt = m \times \Delta T \qquad m : 0 \to 4$

$$U_0 = T_0(x;y) - u^*t = \begin{bmatrix} -20 \\ -20 \\ -20 \\ -20 \\ -11,55 \\ -7 \\ -20 \\ -7 \\ 0 \end{bmatrix}$$

a) Methode explicite :

$$B \cdot \frac{U_{m+1} - U_m}{\Delta T} + A\,U_m = b_{m+1}$$

$$\Rightarrow \frac{B}{\Delta T}U_{m+1} + \left(A - \frac{B}{\Delta T}\right)U_m = b_{m+1}$$

4) Méthode Implicite :

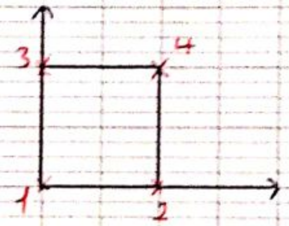
$$B\,\frac{U_{n+1}-U_n}{\Delta T} + A\,(U_{n+1}) = b_{n+1}$$

$$\Longrightarrow \left(\frac{B}{\Delta T}+A\right)U_{n+1} - \frac{B}{\Delta T}\,U_n = b_{n+1}$$

On doit maintenant Calculer A_{ij} ; B_{ij} et b_i pour continuer.

5) Maillage :

On va prendre celui de la figure donné dans La 1er page :



a) Maillage locale :

b) tableau de correspondance :

Numero de L'élement	1	2	3	4
1	1	2	4	5
2	2	3	5	6
3	4	5	7	8
4	5	6	8	9

c) Tableau de Reperage :

Noeud	x	y
1	0	0
2	1,3	0
3	2	0
4	0	1,3
5	1,3	1,3
6	2	1,3
7	0	2
8	1,3	2
9	2	2

b) Calcul de A :

$$A_{ij} = \lambda \int_{\Omega} \frac{\partial \omega_i}{\partial x} \frac{\partial \omega_j}{\partial x} + \frac{\partial \omega_i}{\partial y} \frac{\partial \omega_j}{\partial y}$$

$$= \Big[\lambda \int_{\Omega} \left(\frac{d\omega_i}{d\bar{x}} \frac{d\bar{x}}{dx} + \frac{d\omega_i}{d\bar{y}} \frac{d\bar{y}}{dx} \right) \times \left(\frac{d\omega_j}{d\bar{x}} \frac{d\bar{x}}{dx} + \frac{d\omega_j}{d\bar{y}} \frac{d\bar{y}}{dx} \right)$$

$$+ \left(\frac{d\omega_i}{d\bar{x}} \frac{d\bar{x}}{dy} + \frac{d\omega_i}{d\bar{y}} \frac{d\bar{y}}{dy} \right) \times \left(\frac{d\omega_j}{d\bar{x}} \frac{d\bar{x}}{dy} + \frac{d\omega_j}{d\bar{y}} \frac{d\bar{y}}{dy} \right) \Big]$$

$$\times J \underbrace{d\bar{x}\, d\bar{y}}_{dS}$$

$$\omega_1(\bar{x};\bar{y}) = 4\left(\frac{1}{2}-\bar{x}\right)\left(\frac{1}{2}-\bar{y}\right)$$
$$\omega_2(\bar{x};\bar{y}) = 4\,\bar{x}\left(\frac{1}{2}-\bar{y}\right)$$
$$\omega_3(\bar{x};\bar{y}) = 4\,\bar{x}\,\bar{y}$$
$$\omega_4(\bar{x};\bar{y}) = 4\,\bar{y}\left(\frac{1}{2}-\bar{x}\right)$$

$$\frac{\partial \omega_1}{\partial \bar{x}} = -4\left(\frac{1}{2}-\bar{y}\right) \qquad \frac{\partial \omega_1}{\partial \bar{y}} = 4\left(\bar{x}-\frac{1}{2}\right)$$

$$\frac{\partial \omega_2}{\partial \bar{x}} = 4\left(\frac{1}{2}-\bar{y}\right) \qquad \frac{\partial \omega_2}{\partial \bar{y}} = -4\,\bar{x}$$

$$\frac{\partial \omega_3}{\partial \bar{x}} = 4\,\bar{y} \qquad \frac{\partial \omega_3}{\partial \bar{y}} = 4\,\bar{x}$$

$$\frac{\partial \omega_4}{\partial \bar{x}} = -4\,\bar{y} \qquad \frac{\partial \omega_4}{\partial \bar{y}} = 4\left(\frac{1}{2}-\bar{x}\right)$$

$$x = \omega_1 x_i + \omega_2 x_j + \omega_3 x_K + \omega_4 x_L$$

$$y = \omega_1 y_i + \omega_2 y_j + \omega_3 y_K + \omega_4 y_L$$

$$\begin{pmatrix} \partial x \\ \partial y \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial \bar{x}} & \frac{\partial x}{\partial \bar{y}} \\ \frac{\partial y}{\partial \bar{x}} & \frac{\partial y}{\partial \bar{y}} \end{pmatrix} \begin{pmatrix} \partial \bar{x} \\ \partial \bar{y} \end{pmatrix}$$

$$J = \det \begin{pmatrix} \frac{\partial x}{\partial \bar{x}} & \frac{\partial x}{\partial \bar{y}} \\ \frac{\partial y}{\partial \bar{x}} & \frac{\partial y}{\partial \bar{y}} \end{pmatrix}$$

$$\frac{\partial x}{\partial \bar{x}} = -4\left(\frac{1}{2}-\bar{y}\right)x_i + 4\left(\frac{1}{2}-\bar{y}\right)x_j + 4\bar{y}\,x_k - 4\bar{y}\,x_l$$

$$\frac{\partial x}{\partial \bar{y}} = -4\left(\frac{1}{2}-\bar{x}\right)x_i - 4\bar{x}\,x_j + 4\bar{x}\,x_k + 4\left(\frac{1}{2}-\bar{x}\right)x_l$$

$$\frac{\partial y}{\partial \bar{x}} = -4\left(\frac{1}{2}-\bar{y}\right)y_i + 4\left(\frac{1}{2}-\bar{y}\right)y_j + 4\bar{y}\,y_k - 4\bar{y}\,y_l$$

$$\frac{\partial y}{\partial \bar{y}} = -4\left(\frac{1}{2}-\bar{x}\right)y_i - 4\bar{x}\,y_j + 4\bar{x}\,y_k + 4\left(\frac{1}{2}-\bar{x}\right)y_l$$

	x_i	y_i	x_j	y_j	x_k	y_k	x_l	y_l	$\frac{\partial x}{\partial \bar{x}}$	$\frac{\partial x}{\partial \bar{y}}$	$\frac{\partial y}{\partial \bar{x}}$	$\frac{\partial y}{\partial \bar{y}}$	J
element 1	0	0	1,3	0	0	1,3	1,3	1,3	$-10,4\bar{y}+2,6$	$-10,4\bar{x}+2,6$	0	2,6.	$-27,04\bar{y}+6,76$
element 2	1,3	0	2	0	1,3	1,3	2	1,3	$-5,6\bar{y}+1,4$	$-5,6\bar{x}+1,4$	0	2,6.	$-14,56\bar{y}+3,64$
element 3	0.	1,3	1,3	1,3	0	2	1,3	2	$-10,4\bar{y}+2,6$	$-10,4\bar{x}+2,6$	0	1,4	$-14,56\bar{y}+3,64$
element 4	1,3	1,3	2	1,3	1,3	2	2	2.	$-5,6\bar{y}+1,4$	$-5,6\bar{x}+1,4$	0	1,4	$-7,84\bar{y}+1,96$.

Pour chaque element on calcul:

$$A_{11} = \lambda \int_0^{1/2}\int_0^{1/2} \left[\left(-4\left(\tfrac{1}{2}-\bar{y}\right) \times \frac{d\bar{x}}{dx} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dx}\right)^2 + \left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dy}\right)^2 \right] J \, d\bar{x}\, d\bar{y}$$

$$A_{12} = \lambda \int_0^{1/2}\int_0^{1/2} \left[\left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dx}\right) \cdot \left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} - 4\bar{x}\frac{d\bar{y}}{dx}\right) + \left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dy}\right) \times \left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} - 4\bar{x}\frac{d\bar{y}}{dy}\right) \right] J \, d\bar{x}\, d\bar{y}$$

$$A_{13} = \lambda \int_0^{1/2}\int_0^{1/2} \Big[\left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dx}\right)\cdot\left(4\bar{y}\,\frac{d\bar{x}}{dx} + 4\bar{x}\,\frac{d\bar{y}}{dx}\right) + \left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dy}\right) \times \left(4\bar{y}\,\frac{d\bar{x}}{dy} + 4\bar{x}\,\frac{d\bar{y}}{dy}\right)\Big] J\,d\bar{x}\,d\bar{y}$$

$$A_{14} = \lambda \int_0^{1/2}\int_0^{1/2} \Big[\left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dx}\right)\cdot\left(-4\bar{y}\frac{d\bar{x}}{dx} + 4\left(\tfrac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dx}\right) + \left(-4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} + 4\left(\bar{x}-\tfrac{1}{2}\right)\frac{d\bar{y}}{dy}\right) \times \left(-4\bar{y}\frac{d\bar{x}}{dy} + 4\left(\tfrac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dy}\right)\Big] J\,d\bar{x}\,d\bar{y}$$

$$A_{22} = \lambda \int_0^{1/2}\int_0^{1/2} \Big[\left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} - 4\bar{x}\,\frac{d\bar{y}}{dx}\right)^2 + \left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} - 4\bar{x}\,\frac{d\bar{y}}{dy}\right)^2\Big] J\,d\bar{x}\,d\bar{y}$$

$$A_{23} = \lambda \int_0^{1/2}\int_0^{1/2} \Big[\left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} - 4\bar{x}\frac{d\bar{y}}{dx}\right)\cdot\left(4\bar{y}\frac{d\bar{x}}{dx} - 4\bar{x}\,\frac{d\bar{y}}{dx}\right) + \left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} - 4\bar{x}\frac{d\bar{y}}{dy}\right)\cdot\left(4\bar{y}\frac{d\bar{x}}{dy} - 4\bar{x}\,\frac{d\bar{y}}{dy}\right)\Big] J\,d\bar{x}\,d\bar{y}$$

$$A_{24} = \lambda \int_0^{1/2}\int_0^{1/2} \Big[\left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dx} - 4\bar{x}\frac{d\bar{y}}{dx}\right)\cdot\left(-4\bar{y}\,\frac{d\bar{x}}{dx} + 4\left(\tfrac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dx}\right) + \left(4\left(\tfrac{1}{2}-\bar{y}\right)\frac{d\bar{x}}{dy} - 4\bar{x}\,\frac{d\bar{y}}{dy}\right)\cdot\left(-4\bar{y}\,\frac{d\bar{x}}{dy} + 4\left(\tfrac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dy}\right)\Big] \times J\,d\bar{x}\,d\bar{y}$$

$$A_{33} = \lambda \int_0^{1/2}\int_0^{1/2} \left[\left(4\bar{y}\frac{d\bar{x}}{dx} + 4\bar{x}\frac{d\bar{y}}{dx} \right)^2 + \left(4\bar{y}\frac{d\bar{x}}{dy} + 4\bar{x}\frac{d\bar{y}}{dy} \right)^2 \right] J\, d\bar{x}\, d\bar{y}$$

$$A_{34} = \lambda \int_0^{1/2}\int_0^{1/2} \left[\left(4\bar{y}\frac{d\bar{x}}{dx} + 4\bar{x}\frac{d\bar{y}}{dx} \right) \cdot \left(-4\bar{y}\frac{d\bar{x}}{dx} + 4\left(\frac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dx} \right) + \left(4\bar{y}\frac{d\bar{x}}{dy} + 4\bar{x}\frac{d\bar{y}}{dy} \right) \cdot \left(-4\bar{y}\frac{d\bar{x}}{dy} + 4\left(\frac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dy} \right) \right] J\, d\bar{x}\, d\bar{y}$$

$$A_{44} = \lambda \int_0^{1/2}\int_0^{1/2} \left[\left(-4\bar{y}\frac{d\bar{x}}{dx} + 4\left(\frac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dx} \right)^2 + \left(-4\bar{y}\frac{d\bar{x}}{dy} + 4\left(\frac{1}{2}-\bar{x}\right)\frac{d\bar{y}}{dy} \right)^2 \right] J\, d\bar{x}\, d\bar{y}.$$

Matrice A: sur MATLAB: (en utilisant la fonction integral2)

1er élément:

$$A^{①} = \lambda \begin{bmatrix} 7,1085 & -3,5543 & -2,5388 & -1,538\times10^{-14} \\ -3,5543 & 7,1085 & 2,5388 & 4,5698 \\ -2,5388 & 2,5388 & -7,1085 & 4,5698 \\ -1,538\times10^{-14} & 3,6\times10^{-16} & 4,5698 & -7,1085 \end{bmatrix}$$

2ème élément:

$$A^{②} = \lambda \begin{bmatrix} 1,1098 & -1,0305 & 0,1585 & 2,9273\times10^{-15} \\ -1,0305 & 1,1098 & 0,3964 & -1,648\times10^{-17} \\ 0,1585 & 0,3964 & -1,1098 & 0,7134 \\ 2,927\times10^{-15} & -1,648\times10^{-17} & 0,7134 & -1,1098 \end{bmatrix}$$

$3^{ème}$ élément:

$$A^{(3)} = \lambda \begin{bmatrix} 5,1627 & -1,6688 & -1,3304 & -1,2787 \\ -1,6688 & 5,1627 & 2,4204 & -0,3013 \\ -1,3304 & 2,4204 & -0,9153 & 1,3536 \\ -1,2787 & -0,3013 & 1,3536 & -0,9153 \end{bmatrix}$$

$$A^{(4)} = \lambda \begin{bmatrix} 0,5976 & -0,2988 & 0,0854 & -1,0504\times10^{-15} \\ -0,2988 & 0,5976 & 0,2134 & -1,387\times10^{-17} \\ 0,0854 & 0,2134 & -0,5976 & 0,3842 \\ -1,05\times10^{-15} & -1,387\times10^{-17} & 0,3842 & -0,5976 \end{bmatrix}$$

Assemblage de A

	1	2	3	4	5	6	7	8	9
1	7,1083	-3,5543	0	-2,5388	$-1,538\times10^{-14}$ +0,1584	0	0	0	0
2	-3,5543	7,1085 +1,1098	-1,0305	2,5388	4,5698	$2,927\times10^{-15}$	0	0	0
3	0	-1,0305	+1,1098	0	+0,3964	$-1,648\times10^{-17}$	0	0	0
4	-2,5388	2,5388	0	-7,1085 5,1627	4,56980 -1,6688	0	-1,3304	-1,2787	0
5	$-1,538\times10^{-14}$	4,5698 0,1585	0 0,3964	4,5698 -1,6688	-7,1085 -1,1098 +5,1627 +0,5976	0,7134 -0,2988	2,4904	-0,3013 0,0854	$-1,05\times10^{-15}$
6	0	$2,927\times10^{-15}$	$-1,648\times10^{-17}$	0	0,7104 -0,2988	-1,098 0,5976	0	0,2134	$-1,387\times10^{-17}$
7	0	0	0	-1,3304	2,4204	0	-0,9153	1,3536	0
8	0	0	0	-1,2787	-0,3013 0,0854	0 +0,2134	1,3536	-0,9153 0,5976	0,3842
9	0	0	0	0	$-1,0504\times10^{-15}$	$-1,387\times10^{-12}$	0	0,3842	0,5976

$$
\begin{bmatrix}
7,1083 & -3,5543 & 0 & -2,5388 & 0,1584 & 0 & 0 & 0 & 0 \\
-3,5543 & 8,2188 & -1,0305 & 2,5388 & 4,5678 & 2,927\times10^{-15} & 0 & 0 & 0 \\
0 & -1,0305 & 1,1098 & 0 & 0,3964 & -1,648\times10^{-17} & 0 & 0 & 0 \\
-2,5388 & 2,5388 & 0 & -1,9458 & 2,901 & 0 & -1,3304 & -1,2787 & 0 \\
-1,538\times10^{-14} & 4,7283 & 0,3964 & 2,901 & -2,458 & \overset{0,4146}{\cancel{-1,3304}} & 2,4204 & -0,2159 & -1,05\times10^{-15} \\
0 & 2,927\times10^{-15} & -1,648\times10^{-17} & 0 & 0,4116 & -0,5122 & 0 & 0,2134 & -1,387\times10^{-17} \\
0 & 0 & 0 & -1,3304 & 2,4204 & 0 & -0,953 & 1,3536 & 0 \\
0 & 0 & 0 & -1,2787 & -0,2159 & 0,2134 & 1,3536 & -0,3177 & 0,3849 \\
0 & 0 & 0 & 0 & -1,0504\times10^{-15} & -1,387\times10^{-12} & 0 & 0,3849 & 0,5976
\end{bmatrix}
$$

7) Matrice B – Calcul.

$$B_{ij} = \rho C_p \int_{\Omega} \omega_i(x,y)\, \omega_j(x,y)\, dx\, dy = \rho C_p \int_0^{1/2}\int_0^{1/2} \omega_i(\bar{x},\bar{y})\, \omega_j(\bar{x},\bar{y})\, J\, d\bar{x}\, d\bar{y}$$

élément	J
e1	$-27{,}04\,\bar{y} + 6{,}76$
e2	$-14{,}56\,\bar{y} + 3{,}64$
e3	$-14{,}56\,\bar{y} + 3{,}64$
e4	$-7{,}84\,\bar{y} + 1{,}96$

$$\boxed{\rho C_p = 1875000\ J/_{m^3\,{}^\circ K}}$$

$$B_{11} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\left(\tfrac{1}{2}-\bar{x}\right)\left(\tfrac{1}{2}-\bar{y}\right)\right]^2 J\, d\bar{x}\, d\bar{y}$$

$$B_{12} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\left(\tfrac{1}{2}-\bar{x}\right)4\bar{x}\left(\tfrac{1}{2}-\bar{y}\right)^2\right] J\, d\bar{x}\, d\bar{y}$$

$$B_{13} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\left(\tfrac{1}{2}-\bar{x}\right) 4\bar{x}\bar{y}\left(\tfrac{1}{2}-\bar{y}\right)\right] J\, d\bar{x}\, d\bar{y}$$

$$B_{14} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\left(\tfrac{1}{2}-\bar{x}\right)\left(\tfrac{1}{2}-\bar{y}\right)\times 4\left(\tfrac{1}{2}-\bar{x}\right)\bar{y}\right] J\, d\bar{x}\, d\bar{y}$$

$$B_{22} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\bar{x}\left(\tfrac{1}{2}-\bar{y}\right)\right]^2 J\, d\bar{x}\, d\bar{y}$$

$$B_{23} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\bar{x}\left(\tfrac{1}{2}-\bar{y}\right) 4\bar{x}\bar{y}\right] J\, d\bar{x}\, d\bar{y}$$

$$B_{24} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\bar{x}\left(\tfrac{1}{2}-\bar{y}\right)\times 4\left(\tfrac{1}{2}-\bar{x}\right)\bar{y}\right] J\, d\bar{x}\, d\bar{y}$$

$$B_{33} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\bar{x}\bar{y}\right]^2 J\, d\bar{x}\, d\bar{y}$$

$$B_{34} = \rho C_p \int_0^{1/2}\int_0^{1/2} (4\bar{x}\bar{y})\, 4\left(\frac{1}{2}-\bar{x}\right)\bar{y} \;\; J\, d\bar{x}\, d\bar{y}$$

$$B_{44} = \rho C_p \int_0^{1/2}\int_0^{1/2} \left[4\left(\frac{1}{2}-\bar{x}\right)\bar{y}\right]^2 J\, d\bar{x}\, d\bar{y}$$

Solution de la matrice B sur MATLAB :

fonction integral2

Resultats :

Element 1 :

$$B = \rho C_p \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0{,}0939 & 0{,}469 & 0 & 0 \\ 0{,}469 & 0{,}0939 & 0 & 0 \\ 0 & 0 & -0{,}0939 & -0{,}0469 \\ 0 & 0 & -0{,}0469 & -0{,}0939 \end{pmatrix} \end{array}$$

Element 2 et 3

$$B = \rho C_p \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0{,}0506 & 0{,}0253 & 0 & 0 \\ 0{,}0253 & 0{,}0506 & 0 & 0 \\ 0 & 0 & -0{,}0506 & -0{,}0253 \\ 0 & 0 & -0{,}0253 & -0{,}0506 \end{pmatrix} \end{array}$$

Element 4

$$B = \rho C_p \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0{,}0272 & 0{,}0136 & 0 & 0 \\ 0{,}0136 & 0{,}0272 & 0 & 0 \\ 0 & 0 & -0{,}0272 & -0{,}0136 \\ 0 & 0 & -0{,}0136 & -0{,}0272 \end{pmatrix} \end{array}$$

Assemblage de B 8

$B = \rho C_p$

	1	2	3	4	5	6	7	8	9
1	0,0939	0,469							
2	0,469	0,939 +0,0506	0,0253						
3		0,0253	0,0506						
4				-0,0939 +0,0506	-0,0469 +0,0253				
5				-0,0469 +0,0253	-0,0939 -0,0506 +0,0272 +0,0506	-0,0253 +0,0136			
6					-0,0253 +0,0136	-0,0506 0,0272			
7							-0,0506	-0,0253	
8							-0,0253	-0,0506 -0,0272	-0,0136
9								-0,0136	-0,0272

Scanned by CamScanner

$$\Rightarrow B = \rho C_p \begin{pmatrix} 0{,}0939 & 0{,}469 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0{,}0469 & 0{,}9896 & 0{,}0253 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0{,}0253 & 0{,}0506 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0{,}043 & -0{,}0216 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0{,}0216 & -0{,}0667 & -0{,}0117 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0{,}0117 & -0{,}0234 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0{,}0506 & -0{,}0253 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0{,}0253 & -0{,}077 & -0{,}0136 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0{,}0136 & -0{,}0272 \end{pmatrix}$$

```
A =

   14.9274   -7.4640         0   -5.3315    0.3326         0         0         0         0
   -7.4640   17.2595   -2.1641    5.3315    9.5966    0.0000         0         0         0
         0   -2.2481    2.3306         0    0.8324   -0.0000         0         0         0
   -5.3315    5.3315         0   -4.0862    6.0921         0   -2.7938   -2.6853         0
   -0.0000    9.9294    0.8324    6.0921   -5.1618    0.8707    5.0828   -0.4534   -0.0000
         0    0.0000   -0.0000         0    0.8644   -1.0756         0    0.4481   -0.0000
         0         0         0   -2.7938    5.0828         0   -2.0013    2.8426         0
         0         0         0   -2.6853   -0.4534    0.4481    2.8426   -0.6672    0.8068
         0         0         0         0   -0.0000   -0.0000         0    0.8068    1.2550

B =

   1.0e+06 *

    0.1761    0.8794         0         0         0         0         0         0         0
    0.0879    1.8555    0.0474         0         0         0         0         0         0
         0    0.0474    0.0949         0         0         0         0         0         0
         0         0         0   -0.0806   -0.0405         0         0         0         0
         0         0         0   -0.0405   -0.1251   -0.0219         0         0         0
         0         0         0         0   -0.0219   -0.0439         0         0         0
         0         0         0         0         0         0   -0.0949   -0.0474         0
         0         0         0         0         0         0   -0.0474   -0.1444   -0.0255
         0         0         0         0         0         0         0   -0.0255   -0.0510

b =

     1
     1
     1
     1
     1
     1
     1
     1

U0 =

  -20.0000
  -20.0000
  -20.0000
  -20.0000
  -11.5500
   -7.0000
  -20.0000
   -7.0000
         0
```

Explicite

```
U1 =

  -20.0005
  -19.9999
  -20.0000
  -19.9983
  -11.5523
   -6.9989
  -19.9999
   -7.0000
   -0.0001

U2 =

  -20.0010
  -19.9998
  -20.0000
  -19.9967
  -11.5545
   -6.9979
  -19.9998
   -7.0000
   -0.0001

U3 =

  -20.0014
  -19.9996
  -20.0000
  -19.9950
  -11.5568
   -6.9968
  -19.9997
   -7.0000
   -0.0002
```

Implicite

```
U1 =

   -1.6939
   -2.5377
  -10.1494
    2.4077
   -2.4638
    3.8771
   -1.8440
   -1.5622
    1.0043

U2 =

   -0.0642
    1.0217
   -2.6668
   -1.4151
   -1.9670
   -4.9946
   -1.4489
    0.4576
    0.5061

U3 =

   -0.0688
   -0.6142
   -1.8612
    0.7011
    0.3484
    4.6732
   -0.2229
   -0.6006
    0.7894
```