

1. Show that a matrix is unitarily diagonalizable if and only if it is normal.
2. Show that a real matrix is orthogonally diagonalizable if and only if it is symmetric.
3. Show that if  $Q$  is an invertible  $n \times n$  complex matrix, then  $Q = UT$  for some unitary matrix  $U$  and upper-triangular matrix  $T$ .
4. Show that if a matrix is Hermitian then its eigenvalues are real.
5. Show that if a matrix is Hermitian then its eigenvectors with distinct eigenvalues are orthogonal.
6. Show that if matrix  $A$  is Hermitian with  $n$  distinct eigenvalues, then  $A$  is unitarily diagonalizable.