IT Learning

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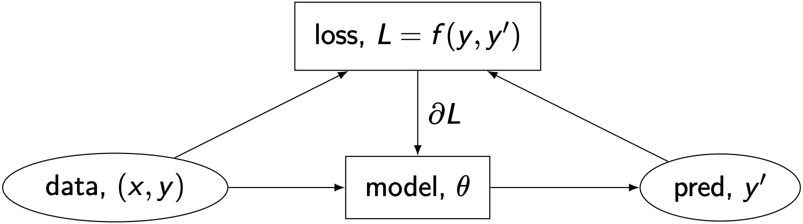
Bonus Model is generative for free! No need to train a GAN !

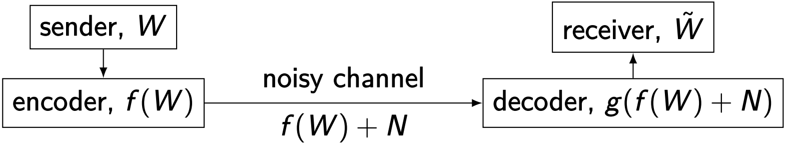
In short, learning is fast, less computationally expensive and theoretically with the best model.

Machine learning: overview



Why should the data be separated into x and y?

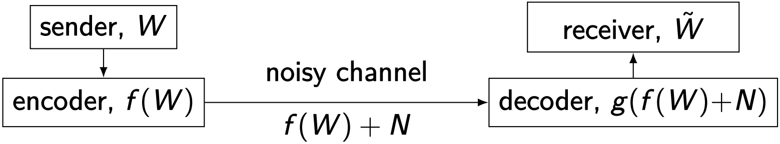
IT: Overview



Claude Shannon asked, "*How can we achieve perfect communication over an imperfect, noisy communication channel?*"

Unifying IT and ML

IT: How to predict the random variable W by observing the co-related random variable $Y = W + N$?



ML: How to the predict the true natural distribution of Y by sampling a co-related random variable X?

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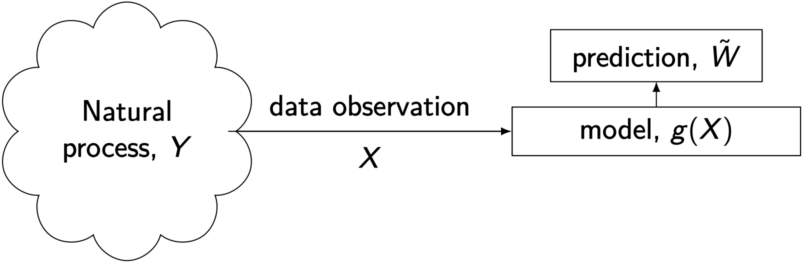

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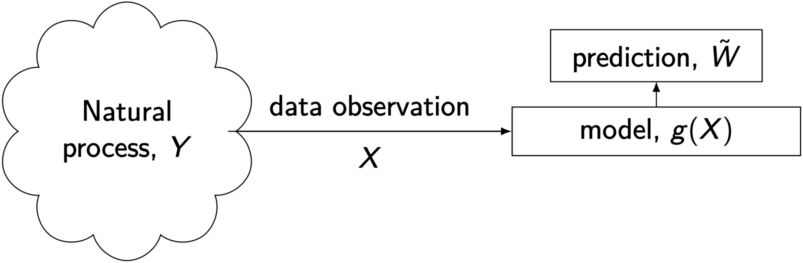

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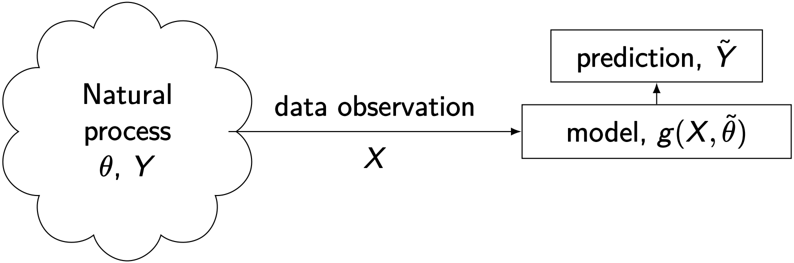
Probability theory: Definitions

Consider a process that we want to learn more about, i.e, predict and generate.

1. **Stochastic variables** These are the variables that take different values randomly.
2. **Non-stochastic variables**, (denoted by θ) are those which are fixed for a given system or process.
3. **Alphabet set**, $\mathcal{A}$, is a set of values that a given stochastic variable can take.
4. **Random variable**, X, is a mapping from the alphabet set, $\mathcal{A}$ to another measurable quantity.
5. **Distribution** of over a random variable X, $p(X)$ is defined as the probability of observing a given value of the variable.

Model fitting and prediction



In statistics (and ML), the process of model fitting, or the **maximum likelihood estimate** is done as,

$$\tilde{\theta} = \arg\max_{\theta} \Pr[\theta|X] \tag{1}$$

The techniques employed is mostly *gradient-based backprop*.

Probability theory: model fitting

The probability of the hypotheses θ, given the evidence X, is given by the Bayes rule,

$$p[\theta|X] = \frac{p[X|\theta] \cdot p[\theta]}{p[X]} \tag{2}$$

where

1. $p[\theta|X]$ is the **posterior** distribution after the model has seen the data.
2. $p[X|\theta]$ is the **likelihood** of seeing the data, if the hypotheses is true.
3. $p[\theta]$ is **prior** knowledge before seeing the data.
4. $p[X]$ is the probability of seeing the data.

Probability theory: model prediction and generation

In the context of probability theory, the model prediction can be computed by,

$$\Pr[x] = p(x \leq X \leq x + \epsilon|\theta) \tag{3}$$

As promised, the generative model is, as simple as computing the posterior predictive distribution,

$$p(\tilde{X}) = \int_{\Theta} d\theta \; p(\tilde{X}|\theta)p(\theta) \tag{4}$$

and sampling from it.

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But what is the prior distribution?

Bayes rule is subjective!

Bayes rule says,

$$p(\theta|X) = c \cdot p(X|\theta) \cdot p(\theta) \tag{5}$$

where, c is the **prior predictive** given as,

$$c = \frac{1}{p(X)} = \int_{\Theta} d\theta \; p(X|\theta) \cdot p(\theta) \tag{6}$$

And the likelihood is given by, our choice of our model.

- What model should we choose?
- What should be the prior?

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- What should be the prior?

The answer was provided by **information theory**.

Shannon's information content

For any given distribution, $p(X)$ over a random variable, the **information content** or simply, *information* of any observation the random variable, $X = x$ is given by,

$$h(x) = \frac{1}{\log(x)} \tag{7}$$

where, the information is measured in *bits* or *shannon*.

Over the entire distribution, the expected information, called **entropy** is,

$$H(x) = \int_{x \in X(\mathcal{A})} dx\ p(x) \frac{1}{\log p(x)} \tag{8}$$

MaxEnt: Maximum Entropy Principle

- Without prior knowledge, use *principle of indifference* i.e, there must be no reason for suspecting one outcome over another.
- The distribution with maximum entropy has the least amount of information about the process, i.e, it is indifferent to any outcome.
- Use the distribution with MaxEnt for prior distribution.

Standby for demonstration !

Numerical approximation

If the posterior, $p(\theta|X)$ is in the same distribution family as the prior distribution $p(\theta)$, the prior and posterior are called **conjugate distributions** and the prior is called **conjugate prior**.

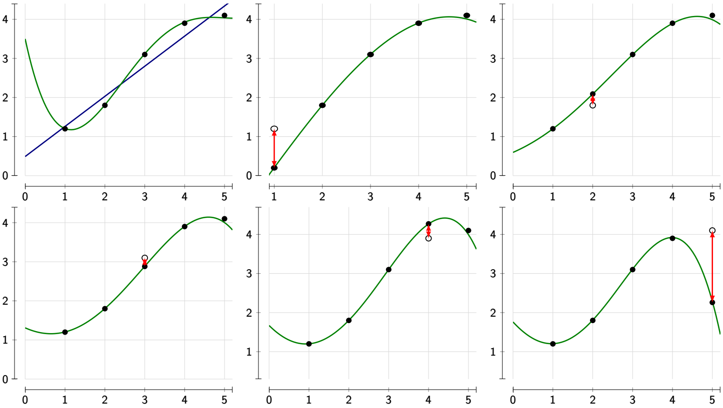
Example Beta distribution is a conjugate for binomial distribution.

$$B(r; N, f) = \binom{N}{r} f^r (1-f)^{N-r}, \quad B(m, n; f) = \frac{(1-f)^{m-1} f^n}{\mathrm{B}(m, n)} \tag{9}$$

- Grid approximation
- Laplace's (quadratic) approximation
- Metropolis-Hastings (MCMC with Gibbs sampling)
- Rejection sampling

Model selection

Leave-one-out cross-validation Drop every data point at a time and calculate the "out-of-sample" accuracy.



Widely Applicable Information Criteria

The KL divergence of two distribution, p and q is given as,

$$D(p||q) = \int_{-\infty}^{\infty} dx\ p(x) \log \frac{q(x)}{p(x)} \tag{10}$$

- Information criteria estimates the relative out-of-sample KL divergence.
- WAIC makes no assumptions about the posterior and it converges to the cross-validation in large samples.

Probabilitistic Programming Languages

- R
- OpenBUGS (Open-source WinBUGS)
- STAN (Andrew Gelman and team)
- PyMC (Uses, now deprecated, Theano backend)
- Pyro (Uses Facebook's Torch backend)
- Numpyro (Uses Google's JAX backend)

Questions?

Thank you!

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