Adaptive Expected Shortfall

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Is the Value at Risk good enough?

Definition

$\mathrm{VaR}_{1-\alpha}(L) = F_L^{-1}(1-\alpha) = inf\{l \in \mathbb{R} : F_L(l) \geq 1-\alpha\}$

- The Value at Risk is the $(1-\alpha)$-quantile of the distibution of the loss F_L (positive losses)
- risk measure
- used in the standard formula in Sonvency II
- not subadditive $\Rightarrow$ not coherent

Coherent Risk Measure $\rho(L)$

Definition

a monotonous : $L_1 \leq L_2 \Rightarrow \rho(L_1) \leq \rho(L_2)$

b sub-additive: $\rho(L_1 + L_2) \leq \rho(L_1) + \rho(L_2)$

c positively homogeneous: $\rho(hL) = h\rho(L)$

d translation invariant: $\rho(L + a) = \rho(L) - a$

- subadditivity $\rightarrow$ diversification is beneficial for portfolio
- VaR in general not sub-additive risk measure, but for elliptically distributed losses it is

Outline of my seminar paper

1. Motivation
2. Expected Shortfall
    2.1 Quantile (Value at Risk)
    2.2 Expectile
3. TERES
4. ICARE
    4.1 Adaptivity
5. Method (combine two papers)
6. Data (equity: Bloomberg; simulate?)
7. Application of Method
    7.1 Results
    7.2 Testing
8. Conclusion

Quantile-based Expected Shortfall

Definition

$\mathrm{ES}_{1-\alpha}(L) = \frac{1}{1-\alpha} \int_{1-\alpha}^{1} VaR_\gamma(L)\, d\gamma$

for continous loss function:

$ES_{1-\alpha}(L) = \mathbb{E}[L | L \geq VaR_{1-\alpha}]$

- coherent risk measure
- more sensitive to the shape of the tail of the loss distributen than VaR
- also called
  - conditional value at risk
  - average value at risk
  - expected tail loss
  - superquantile
- not elicitable

Expectile-based Expected Shortfall

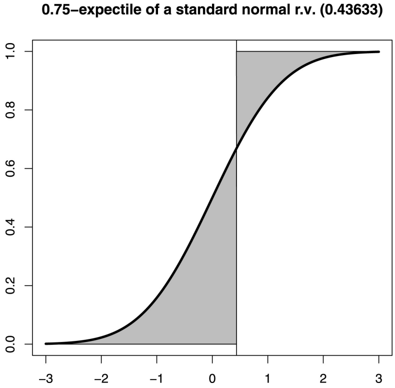
Definition

$$e_{1-\alpha}(L) = \underset{l \in \mathbb{R}}{\operatorname{argmin}}\,(1-\alpha)\mathbb{E}[(L-l)_+^2] + \alpha\mathbb{E}[(L-l)_-^2]$$

with

$(L-l)_+^2 := \max((L-l)^2, 0)$ and $(L-l)_-^2 := \max(-(L-l)^2, 0)$

- one-parameter family of coherent and elicitable risk measure
- minimizers of an asymmetric quadratic loss
- for $1-\alpha = \frac{1}{2}$: $e_{1-\alpha}(L) = \mathbb{E}[L]$
- an asymmetric generalisation of the mean
- "expectiles" = "expectation" & "quantiles"



- thick solid line: cdf of a standard normal random variable
- vertical line: 75%-expectile $e = 0.43633$
- shaded left area is $\frac{0{,}75}{1-0{,}75} = 3$ times the shaded right area

TERES: Tail Event Risk Expectile Shortfall

- generalized risk measure for expectile-based expected shortfall estimation
- generalization: mixture of Gaussian and Laplace densities
- there is an analytical relationship between expectiles and the expected shortfall
- a plug-in estimator is derived
- investigate the sensitivity and robustness

Mixture of Gaussian and Laplace density

- normalily assumption ($\varphi_{\theta_1}(y)$): underestimate risk for modeling heavy-tailed data
- add heavier-tailed standardized Laplace distribution ($h_{\theta_2}(y)$)

$f(y|\alpha) = (1-\alpha)\varphi_{\theta_1}(y) + \alpha h_{\theta_2}(y)$

ICARE - localizing Conditional AutoRegressive Expectiles

- conditional expectile based value at risk (EVaR) model
- EVaR is more sensitive to the magnitude of losses than the quantile-based Value at Risk (QVaR)
- additionally the time-varying parameter properties are considered
- tail risk dynamics are qantified

Combination of TERES and ICare

- take the generalized risk measure for expectile-based expected shortfall estimation (TERES)
- apply adaptive model (ICARE)
- quantify tail risk dynamics
- (compare my results to ICARE results?)

My choise of data

- equity
  - Bloomberg
    - DAX30
    - FTSE100
    - S&P500

Real data allows comparing results with ICARE:

- daily index returns (obtained from Datastream)
- 03.01.2005 - 31.12.2014
- 2608 trading days

For Further Reading

A. Mihoci, W. Härdle and C. Yi-Hsuan Chen
TERES: Tail Event Risk Expectile Shortfall
Quantitative Finance, 2021, Vol. 21, No. 3, 449-460

X. Xu, A. Mihoci and W. Härdle
lCARE - localizing Conditional AutoRegressive Expectiles
SFB 649, ECONOMIC RISK BERLIN, Discussion Paper, 2015-052

For Further Reading

- F. Bellini and E. Di Bernardino
  Risk Management with Expectiles
  European Journal of Finance, May 2015, DOI: 10.1080/1351847X.2015.1052150

- I. Cascos and M. Ochoa
  Expectile depth: Theory and computation for bivariate datasets
  Journal of Multivariate Analysis, Volume 184, July 2021, 104757