

## Numerical Introductory Seminar - Prime Factorization

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Motivation:	The cryptosystem RSA multiplies two large primes to create one part of the keys
Example:	Trial Division (inefficient for large prime factors)
Modification:	Test in Trial Division only primes
Sieve of Eratosthenes:	Creates list of primes (and zeros)

### Code in Mathematica (Sieve of Eratosthenes)

```
SoE[n]:=Module[{v = Table[t, {t, 1, n}], y = 2, k},  
v = ReplacePart[v, 0, 1];  
While[y * y ≤ n, k = y + y;  
  While[k ≤ n,  
    v = ReplacePart[v, 0, k];  
    k = k + y];  
y = y + 1];  
Return[v]]
```

### Pollard (p-1) factorization method

Theoretical base: Fermat's little theorem  
Previous algorithm: Pollard rho method

### Algorithm (Pollard (p-1) factorization method)

1. Input:  $n \geq 2$
2. choose  $a$  with  $1 \leq a \leq n - 1$  and arbitrary  $B \in \mathbb{N}$
3.  $\forall q \in \mathbb{P}, q \leq B$ :
  - $a := a^q \pmod n$
  - $p := \gcd(a - 1, n)$
  - if  $p \mid n$  break
  - else select new  $a$  and go to step 3
4. return  $p$