ag2388
ORIE 5380

Cargo Operations of Express Air

Executive summary

The purpose of this study is to develop an optimization model that determines the optimal weekly aircraft schedule that has a repeatable 1-week cycle (Mon-Fri). The model tries to minimize the cost of repositioning empty aircrafts and holding cargo at an airport, while making sure that all cargo is delivered within 5 days (excluding weekends). The results show that there is an imbalance of incoming vs. outgoing cargo at airport B which is causing the majority of the cost increase. Management is recommended to purchase additional cargo capacity (depending on the price of the aircrafts), as this will drive down the cost. It is also recommended that management come up with a marketing plan to promote more delivery of cargo from airport B to airports A and C. Lastly, if adding additional capacity is not an option, management should do competitive research on cheaper ways to store the cargo and/or devise a pricing scheme that penalizes late deliveries less.

Problem overview

The purpose of this study is to develop an optimization model that determines the optimal weekly aircraft schedule that has a repeatable 1-week cycle (Mon-Fri).

There are 3 airports, and on each weekday each airport has an incoming amount of cargo that it needs to deliver to the other 2 airports. The total capacity across all airports is fixed, but the starting capacities at each individual airports are to be determined by the model.

All cargo has to be delivered to its destination airport within 5 days (excluding weekends) of when it first arrives into the system. Due to the certainty of all cargo being delivered, the revenue received from delivering the cargo as well as the cost associated with transporting the cargo along every route are fixed. Conversely, there is a cost associated with repositioning an empty aircraft and leaving cargo at an airport rather than delivering it (see Data description), which depends on the finalized schedule determined by the model.

Therefore, assuming the total capacity of the cargo is infinitely divisible, the goal of the model is to find the best schedule for the movement of the aircrafts that minimizes the cost of repositioning and leaving behind cargo.

Data description

- 3 Airports: A, B, C
- a schedule of days: Monday thru Friday
- Total amount of cargo carrying capacity: 120,000 (divisible at will)

- cost of 1 ton of undelivered cargo (holding it on the ground rather than flying it to its destination) is 10 per day (cost of holding over the weekend has been ignored)
- cost of empty repositioning shown in Figure 1 below

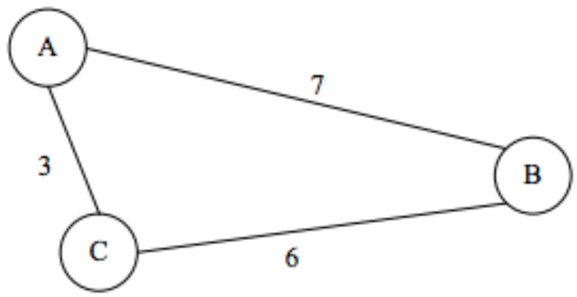

Figure 1: Empty repositioning costs among different airports

Origin-destination \ Day	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	10	20	10	40	30
A-C	5	5	5	5	5
B-A	2.5	2.5	2.5	2.5	2.5
B-C	2.5	2.5	2.5	2.5	2.5
C-A	4	4	4	4	4
C-B	40	20	30	20	40

Table 1: Amounts of cargo (in 1,000 tons) arriving into the system on each day that need to be carried between each origin-destination airport

Table 1 above shows the cargo that must be delivered from origin to destination airport, the cargo must be delivered within 5 days (excluding weekends).

Procedure & Model

The model was created with Gurobi within a python file. Below are the Decision Variables, Objective Function, and Constraints:

Decision Variables:

Where *i* is the origin airport (A, B, C), *j* is the destination airport (A, B, C), and *t* is the time period (1 – 5)

- x_{ijt} – amt. of cargo delivered from airport i to airport j at time t
- y_{ijt} – empty cargo repositioned from airport i to airport j at time t
- y_{iit} – amt. of empty cargo that is left at the origin airport i at time t

- u_{ijt} – cumulative undelivered amt. of cargo at time t that needs to be delivered from airport i to airport j

Other Variables:

- d_{ijt} – amt. of cargo coming in at time t that needs to be transported from airport i to airport j

Objective Function:

MIN $\left(3(y_{act} + y_{cat}) + 7(y_{abt} + y_{bat}) + 6(y_{cbt} + y_{bct}) + 10(u_{abt} + u_{bat} + u_{act} + u_{cat} + u_{bct} + u_{cbt})\right)$

$$\forall\, t \;\in [1, 2, 3, 4, 5]$$

Minimize cost of repositioning and holding cargo at the airport

Constraints:

- Amount shipped

$x_{ijt} \leq u_{ijt-1} + d_{ijt}$

$\forall\, t \;\in [1, 2, 3, 4, 5]$
$\forall\, i \;\in [A, B, C]$
$\forall\, j \;\in [A, B, C]$

- amount of accumulated undelivered cargo

$d_{ijt} + u_{ijt-1} - x_{ijt} = \; u_{ijt}$

$\forall\, t \;\in [1, 2, 3, 4, 5]$
$\forall\, i \;\in [A, B, C]$
$\forall\, j \;\in [A, B, C]$

- flow balance constraints*

$$\sum_{i=A}^{C} x_{ijt-1} + \sum_{i=A}^{C} y_{ijt-1} + \sum_{i=A}^{C} y_{jjt-1} = \sum_{i=A}^{C} x_{jit} + \sum_{i=A}^{C} y_{jit} + \sum_{i\,=A}^{C} y_{jjt}$$

$\forall\, j \;\in [A, B, C]$
$\forall\, t \;\in [1, 2, 3, 4, 5]$

* the model was constructed in such a way that in the equation above, if i = j for x_{ijt} and y_{ijt}, the corresponding coefficient will be 0

- Capacity constraint:

$\sum_{i=A}^{C} x_{ijt} + \sum_{i=A}^{C} y_{ijt} + \sum_{i=A}^{C} y_{jjt}$ = 120,000

$\forall\, t \in [1, 2, 3, 4, 5]$
$\forall\, j \in [A, B, C]$

- All variables have to be non-negative

Results & Analysis

Results:

Objective Value: **1,792,500**
Values for Decision Variables:

x0_1_0	20000	x1_0_0	2500	x2_0_0	4000
x0_1_1	29000	x1_0_1	2500	x2_0_1	4000
x0_1_2	10000	x1_0_2	2500	x2_0_2	4000
x0_1_3	40000	x1_0_3	2500	x2_0_3	4000
x0_1_4	11000	x1_0_4	2500	x2_0_4	4000
x0_2_0	5000	x1_2_0	2500	x2_1_0	40000
x0_2_1	5000	x1_2_1	2500	x2_1_1	20000
x0_2_2	5000	x1_2_2	2500	x2_1_2	30000
x0_2_3	5000	x1_2_3	2500	x2_1_3	20000
x0_2_4	5000	x1_2_4	2500	x2_1_4	40000

Figure 2: full cargo shipped out from airport i to airport j at day t

y0_1_0	0	y1_0_0	27500	y2_0_0	0
y0_1_1	0	y1_0_1	8500	y2_0_1	0
y0_1_2	0	y1_0_2	48000	y2_0_2	0
y0_1_3	0	y1_0_3	0	y2_0_3	0
y0_1_4	0	y1_0_4	18500	y2_0_4	0
y0_2_0	0	y1_2_0	18500	y2_1_0	0
y0_2_1	0	y1_2_1	24500	y2_1_1	0
y0_2_2	0	y1_2_2	18000	y2_1_2	0
y0_2_3	0	y1_2_3	35000	y2_1_3	0
y0_2_4	0	y1_2_4	36500	y2_1_4	0

Figure 3: empty cargo shipped out from airport i to airport j at day t (repositioning of aircrafts)

u0_1_0	9000	u1_0_0	0	u2_0_0	0
u0_1_1	0	u1_0_1	0	u2_0_1	0
u0_1_2	0	u1_0_2	0	u2_0_2	0
u0_1_3	0	u1_0_3	0	u2_0_3	0
u0_1_4	19000	u1_0_4	0	u2_0_4	0
u0_2_0	0	u1_2_0	0	u2_1_0	0
u0_2_1	0	u1_2_1	0	u2_1_1	0
u0_2_2	0	u1_2_2	0	u2_1_2	0
u0_2_3	0	u1_2_3	0	u2_1_3	0
u0_2_4	0	u1_2_4	0	u2_1_4	0

Figure 4: empty cargo shipped out from airport i to airport j at day t

stay0_0_0	0	stay1_1_0	0	stay2_2_0	0
stay0_0_1	0	stay1_1_1	22000	stay2_2_1	2000
stay0_0_2	0	stay1_1_2	0	stay2_2_2	0
stay0_0_3	9500	stay1_1_3	0	stay2_2_3	1500
stay0_0_4	0	stay1_1_4	0	stay2_2_4	0

Figure 5: capacity that stays at each airport for a given day t

The values in Figure 3 (how much empty cargo will be shipped from i to j) show us that empty airplanes are exclusively repositioned out of airport B to airports A and C for all time periods other than one.

This is coherent with the information of the demand table (see Table 1 above), as it is visible that the greatest imbalance between incoming and outgoing cargo shipments takes place at airport B. This is due to outgoing cargo being the lowest of all values in the table (consistently 2.5 from both airports A and B across all time periods), and the sum of incoming cargo across all time periods for airports A and B being the highest sums of all rows in the table. There is not enough cargo to transport from airport B to the other 2 airports, but demand at A and B are high, so airport B needs to send empty aircrafts back (which contributes to the cost). Figure 4 simply shows that, at times, cargo that is destined for airport B accumulates at airport A due to insufficient capacity (this also contributes to cost).

There is a second way the model deals with this imbalance at airport B. When looking at Figure 5, it is visible that the model recommends keeping 22,000 aircrafts at airport B, again because there is no cargo to ship out to other airports.

One positive is that despite the imbalance, the demand out of airport B is extremely consistent, and in fact when looking at the second column in Figure 2, the amount of cargo that is shipped out from airport B is 2.5 tons across all time periods and destination airports. This consistency is good for operations and staffing at airport B.

From the results it follows that if the amounts of cargo coming into the system were more evenly distributed, operations overall would be smoother and cost would be lower. In the Analysis section, different demand patterns will be analyzed. However, because demand is difficult to control, other parameters that are easier to manipulate will be taken into consideration as well, such as capacity.

ag2388
ORIE 5380

Analysis:

Capacity

Consider increasing the total cargo carrying capacity in increments of 5,000 (these larger increments seem to make the most sense from a purchasing perspective). The total cost can be seen in the figure below:

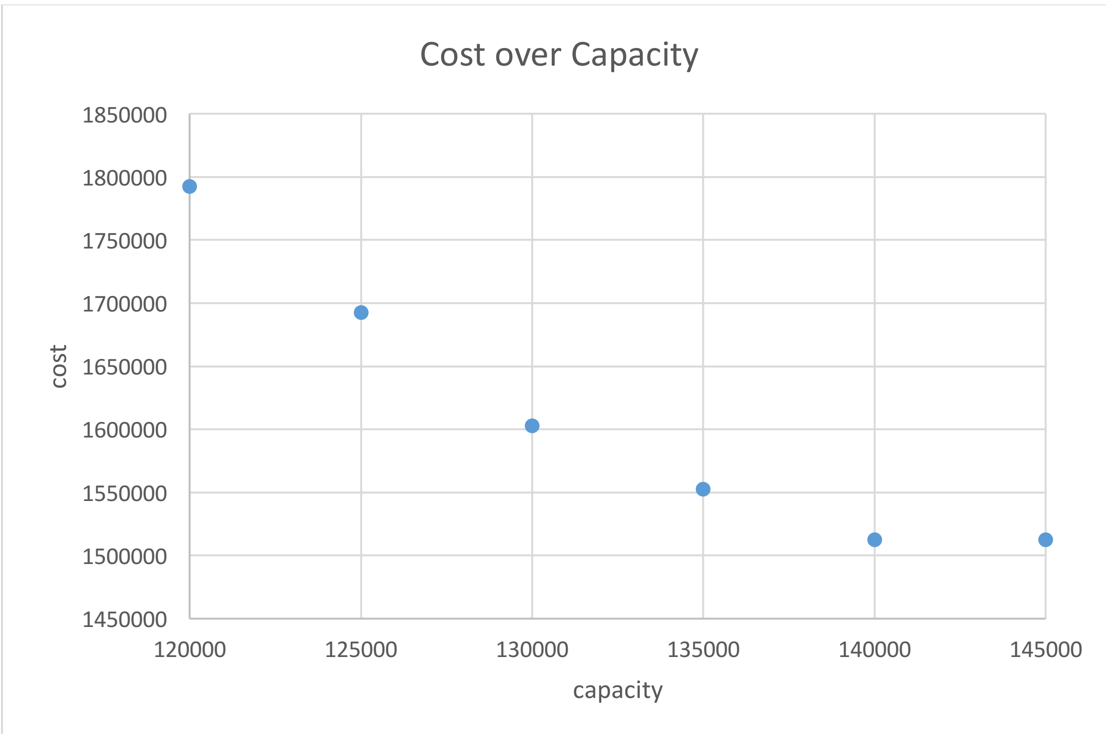

Figure 6: the effect that increasing capacity has on cost

As is visible in the figure above, increasing capacity by 5,000 units of cargo capacity between 120,000 and 130,000 decreases the cost by ~100,000 each time. This is consistent if we look at the **dual variable** for the capacity constraint, which is **-20** (meaning that for every additional unit of capacity purchased, the price goes down by 20). The decrease in cost then slows to ~50,000 currency for every 5,000 units of cargo capacity added, and ultimately plateaus at a cargo capacity of 140,000, at which the cost remains at 1512500. Therefore, adding any further capacity beyond 140,000 at this level of demand would not make financial sense.

Demand

When increasing the demand from airport B to airports A and C, there is a significant decrease in the total cost associated with the aircraft schedule. For instance, if we increase the amount of cargo shipped out from 2,500 to 10,000 for both routes B-A and B-C, the total cost decreases from 1,792,500 to 1,305,000. Below is a graph:

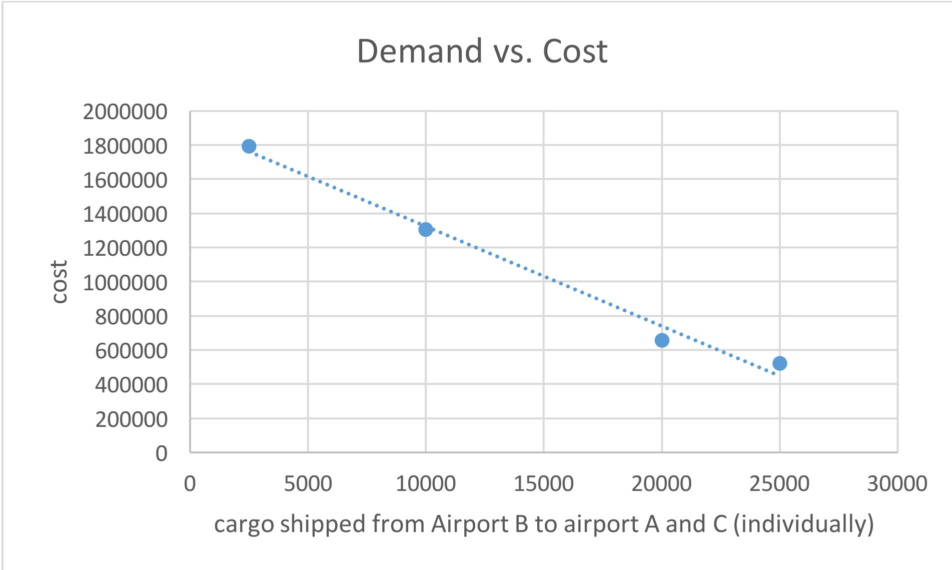

Figure 6: the effect that increasing demand from airport B to airports A and C has on cost

The model breaks when increase in cargo climbs much further than 27000 per destination from airport B, however it is unlikely that there would be such a strong change in demand in such a short amount of time that management cannot plan for it, and over a longer period of time other variables would change as well (capacity, costs, etc.).

Accumulation

Unsurprisingly, when the cost of storage is reduced, the objective value decreases and less money is spent. One insight is that by doing so, the importance of capacity is reduced, as the dual variable associated with overall aircraft capacity decreases from -20 to -16 if the cost of storage per day decreases from 10 to 8, and from -20 to -10 if the cost of storage decreases to 5. There seems to be a linear relationship, and while overall total cost is reduced when decreasing the cost of accumulation, the savings experienced when adding aircraft capacity is larger. Of course, research needs to be done whether increasing capacity or decreasing cost of storage is more expensive.

Recommendation

Based on the results displayed in Figure 6, It is highly recommended that management purchase more aircrafts **provided that** the cost of purchasing 5,000 units of cargo capacity is less than the amount that would be consequently saved. So if the total cost of purchasing 5,000 units of cargo capacity is less than 100,000, at the current level of demand, it would be recommendable to purchase up to 10,000 units of cargo capacity. If the cost of purchasing 5,000 units of capacity is less than 40,000 units of currency, then it would be recommendable to purchase up to 20,000 units of additional cargo capacity. Anything beyond that in terms of purchasing additional capacity does not make sense for reasons explained in the analysis section.

ag2388
ORIE 5380

Due to the significant decrease in cost when more cargo is shipped out of airport B to airports B and C, management should invest in marketing to promote their services as much as possible for mailing cargo out of airport B.

At 10/day, accumulation is the costliest expense, so management should do competitive research on cheaper ways to store the cargo or again devise a pricing scheme that penalizes late deliveries less. Management should also research whether reducing holding cost or increasing aircraft capacity is a cheaper investment and focus on one approach. However, it seems more advisable to focus on increasing capacity instead, especially if management predicts an increase in demand and customer satisfaction and timeliness are a priority.