

STA260 Tutorial 12 Question 4

Question 4

Let X_1, X_2, \dots, X_n be a random sample of size n from a Gamma distribution with mean $\alpha\theta$ and variance $\alpha\theta^2$. Use the method of moments to find estimates of α and θ .

$$E(X) = \alpha\theta \stackrel{\text{set}}{=} \bar{y} \Rightarrow \alpha = \bar{y}/\theta$$

$$V(X) = \alpha\theta^2 \stackrel{\text{set}}{=} s^2 \Rightarrow \alpha = s^2/\theta^2$$

$$\text{Solve for } \theta \text{ first: } \bar{y}/\theta \stackrel{\text{set}}{=} s^2/\theta^2$$

$$\Rightarrow \theta \bar{y} = s^2 \Rightarrow \theta = s^2/\bar{y}$$

$$\text{thus } \alpha = \frac{\bar{y}}{s^2/\bar{y}} = \frac{\bar{y}^2}{s^2}$$

$$\text{Hence } \theta = s^2/\bar{y} \text{ and } \alpha = \bar{y}^2/s^2$$