

STA260 Tutorial 8 Question 3

Question 3

Let Y_1, \dots, Y_n be a random sample with the following common probability density function:

$$f(y) = \begin{cases} \frac{1}{\theta^2} y e^{-\frac{y}{\theta}} & y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Here $\theta > 0$. Prove that $\sum_{i=1}^n Y_i$ is a complete sufficient statistic for θ .

$$f(y) = \frac{1}{\theta^2} y e^{-y/\theta} = e^{-y/\theta + \ln(y) + \ln(\theta^{-2})}$$

$$f(y|\theta) = e^{p(\theta)u(y) + q(\theta) + s(y)} \quad U = \sum_{i=1}^n u(y_i)$$

$$p(\theta) = -1/\theta \quad u(y) = y \quad q(\theta) = \ln(\theta^{-2}) \quad s(y) = \ln(y)$$

hence, $U = \sum_{i=1}^n y_i$ is sufficient and complete

as it is part of the exponential family.