STA260 Tutorial 1 Question 3

Question 3

Let Y_1, Y_2, Y_3, Y_4 be a random sample of size 4 from a normal population with mean 0 and variance 9. Let $\bar{Y} = \frac{1}{4}\sum_{i=1}^{n} Y_i$. Find the distribution of the following random variables.

(a) $\frac{Y_1^2}{9}$

(b) $\sum_{i=1}^{4} \frac{Y_i^2}{9}$

(c) $\sum_{i=1}^{4} \frac{(Y_i - \bar{Y})^2}{9}$

a) $\quad \frac{Y_1 - \mu}{\sigma} = \frac{Y_1 - 0}{3} \sim N(0,1)$

$$\Rightarrow \left(\frac{Y_1 - 0}{3}\right)^2 \sim \chi^2_{(1)}$$

$$\Rightarrow \quad \frac{Y_1^2}{9} \sim \boxed{\chi^2_{(1)}}$$

b) Since $Y_1, Y_2, \ldots Y_4$ are $\underline{indep}$ and

$\frac{Y_i^2}{9} \sim \chi^2_{(1)}$ then $\sum_{i=1}^{4} \frac{Y_i^2}{9} \sim \boxed{\chi^2_{(4)}}$

c) Recall: $\sum_{i=1}^{n} \frac{(Y_i - \bar{Y})^2}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$

thus $\sum_{i=1}^{4} \frac{(Y_i - \bar{Y})^2}{9} \sim \chi^2_{(4-1)} = \boxed{\chi^2_{(3)}}$