

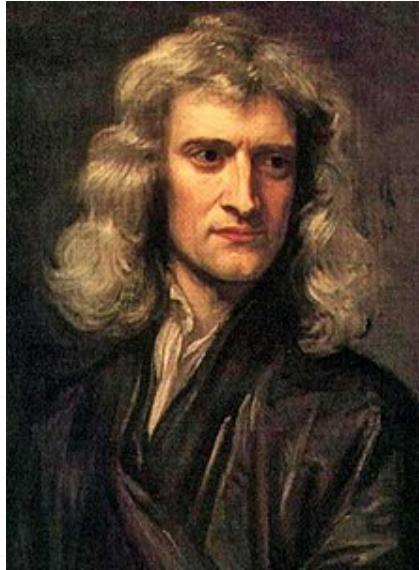


# INTRODUCTION TO CALCULUS

# ETYMOLOGY

The word **calculus** comes from Latin and means « *a small pebble or stone used for counting* »

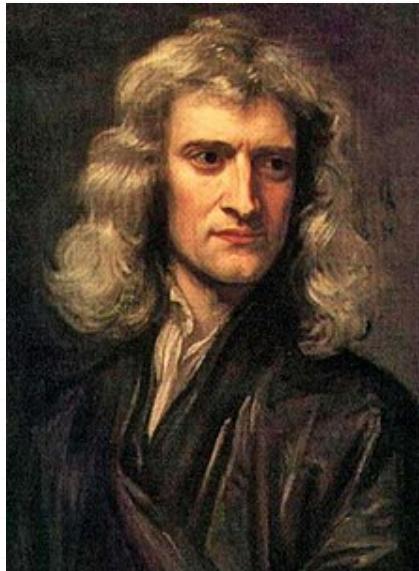




# BRIEF HISTORY

Modern calculus was developed in the 17th century by Newton and Leibniz independently of each other.



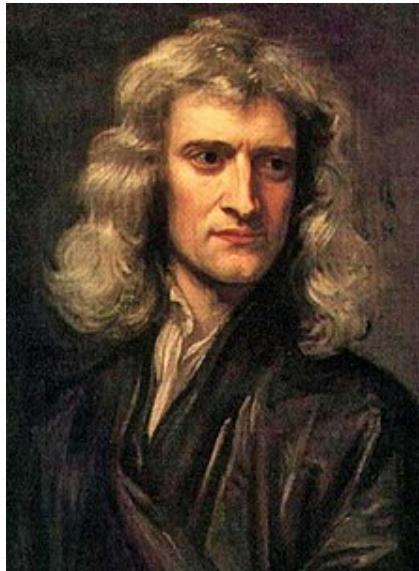


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**Leibniz:** developed much of the notation used in calculus today

# What does calculus study?

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**Example:** Motion of an object along a fixed path

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- What does it change in this example?  
The position varies with time.
- And how does the position change with time?  
This depends on the *velocity* of the object.

# Average velocity...



Sam and Alex are traveling in the car ... but the speedometer is broken.

Alex: "Hey Sam! How fast are we going now?"

Sam: "Wait a minute ..."

"Well in the last minute we went 1,2 km, so we are going:"

$$1,2 \text{ km per minute} \times 60 \text{ minutes in an hour} = \mathbf{72 \text{ km/h}}$$

Alex: "No, Sam! Not our **average** for the last minute, or even the last second, I want to know our speed RIGHT NOW."

# ... vs instantaneous velocity

Sam: "OK, let us measure it up here ... at this road sign... NOW!"



"OK, we were AT the sign for **zero seconds**, and the distance was ... **zero meters!**"

The speed is  $0\text{m} / 0\text{s} = 0/0 = \mathbf{I\ Don't\ Know!}$

"I can't calculate it Sam! I need to know **some** distance over **some** time, and you are saying the time should be zero? Can't be done."

# ... vs instantaneous velocity

Sam: "OK, let us measure it up here ... at this road sign... NOW!"

Here we  
need limits!!



"OK, we were AT the sign for **zero seconds**, and the distance was ... **zero meters!**"

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# Two problems

- 1) Find the instantaneous velocity given a position (we will see that more in general this corresponds to compute the *derivative* of a function)



**DIFFERENTIAL CALCULUS**

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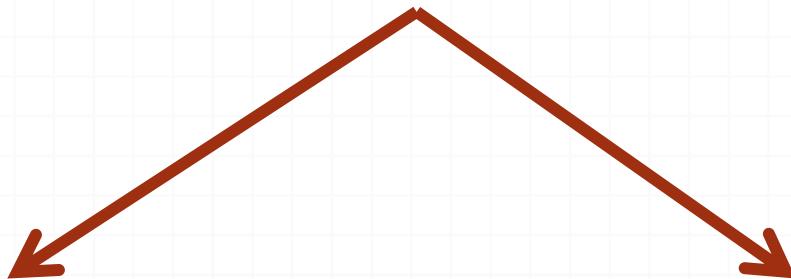
- 2) Find the position function by knowing the instantaneous velocity at all time (or, more in general, find the function by knowing its derivative).



**INTEGRAL CALCULUS**

# **CALCULUS**

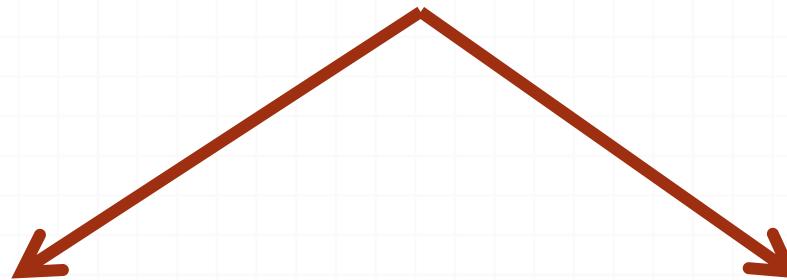
# **CALCULUS**



**DIFFERENTIAL  
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**FUNDAMENTAL THEOREM  
OF CALCULUS**