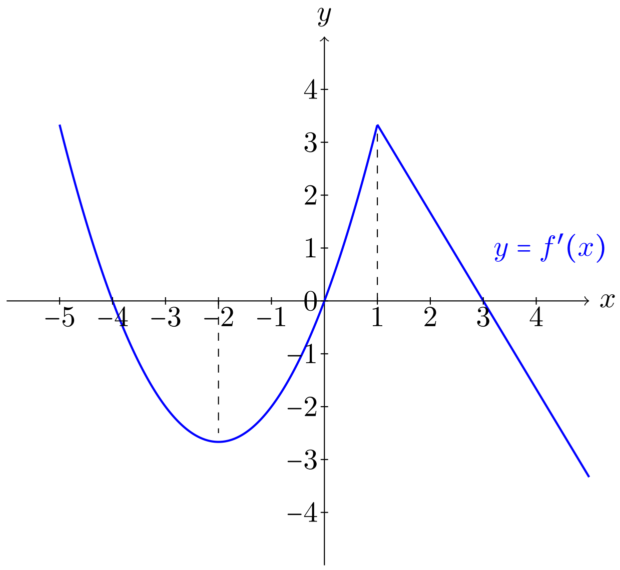
Calculus I - MAC 2311 - Section 003

Quiz 6 - Solutions

10/31/2018

Instructions: The total number of points of this quiz is 10. You will get an extra point if you solve correctly the last exercise.

1) [5 points] The graph of the derivative f' of a function f is shown below.



a) What are the critical numbers of f?

Since the function f' is defined everywhere (i.e. f is differentiable), then c is a critical number if and only if $f'(c) = 0$. Hence the critical numbers of f are the x-coordinates of the points at which the graph of f' crosses the x-axis:

$$\text{critical numbers}: x = -4,\ x = 0,\ x = 3.$$

b) Over which intervals is the function f increasing/decreasing?

We have $f'(x) > 0$ on $(-\infty, -4) \cup (0, 3)$ and $f'(x) < 0$ on $(-4, 0) \cup (3, \infty)$. Then f is increasing on $(-\infty, -4) \cup (0, 3)$ and decreasing on $(-4, 0) \cup (3, \infty)$:

		-4		0		3	
$f'(x)$	+		−		+		−
$f(x)$	↗		↘		↗		↘

c) At what numbers does f have a local minimum/maximum value?

From (b) we get that f has a local minimum value at $x = 0$, and a local maximum value at $x = -4$ and $x = 3$.

d) Over which intervals is f concave down/up?

We have $f''(x) > 0$ on $(-2, 1)$ and $f''(x) < 0$ on $(-\infty, -2) \cup (1, \infty)$. Then f is concave up on $(-2, 1)$ and concave down on $(-\infty, -2) \cup (1, \infty)$.

		-2		1	
$f''(x)$	–		–		–
$f(x)$	DOWN		UP		DOWN

e) What are the x-coordinates of the inflection points?

Since $f''(x)$ changes sign at $x = -2$ and at $x = 1$ and f is continuous everywhere, then $x = -2$ and $x = 1$ are the coordinates of the two inflection points.

2) [5 points] Sketch the graph of **a** function f that satisfies **all of the given conditions**:

a) f is continuous on $(-\infty, \infty)$;

b) $f(-4) = f(4) = -3$;

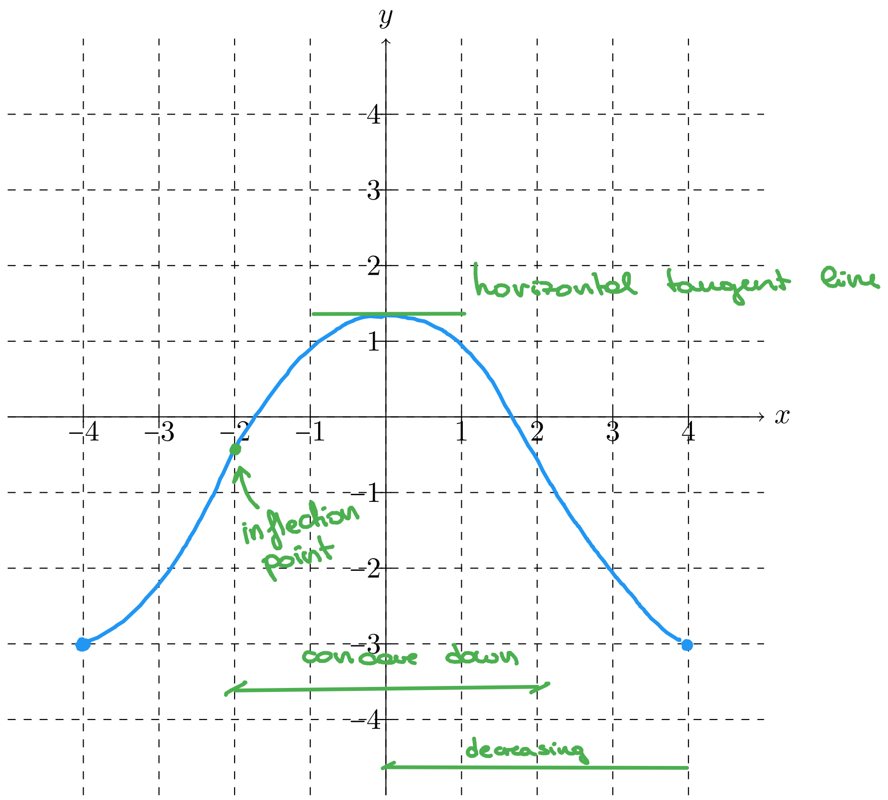
c) f has an inflection point at $(-2, 0)$; $\rightarrow$ f changes concavity at $x = -2$.

d) $f''(x) < 0$ on $(-2, 2)$; $\rightarrow$ f is concave down on $(-2,2)$

e) $f'(0) = 0$; $\rightarrow$ horizontal tangent at $(0, f(0))$

f) $f'(x) < 0$ on $(0, \infty)$. $\rightarrow$ f decreasing on $(0,\infty)$

Make sure that your graph is the graph of a function, i.e. it passes the vertical line test.



3) Let f be a function such that $f'(x_0) = 0$ and $f''(x) > 0$ near x_0. Show that f has a local minimum at x_0.

First of all, notice that f is continuous at x_0 (since f is differentiable at x_0). Since $f''(x) > 0$ near x_0, then $f'(x)$ is increasing near x_0. As $f'(x_0) = 0$ this implies that $f'(x_0) < 0$ before x_0 and $f'(x_0) > 0$ after x_0, so that f is decreasing before x_0 and increasing after x_0. Then f has a local minimum at x_0.