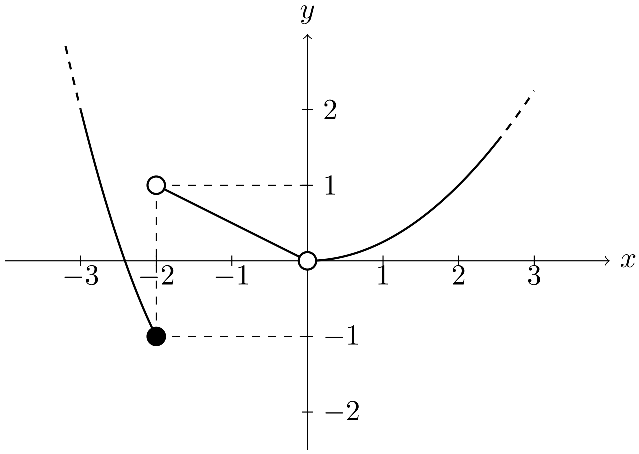
Calculus I - MAC 2311 - Section 001

Quiz 1 - Solutions

01/17/2018

1) [5 points] The graph of a function f is given.



State the value of each quantity. If a quantity does not exist or is undefined **explain why**.

a) $\lim_{x\to -2^-} f(x) = -1$

b) $\lim_{x\to -2^+} f(x) = 1$

c) $\lim_{x\to -2} f(x)$ does not exist because $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$.

d) $f(-2) = -1$

e) $\lim_{x\to 0^-} f(x) = 0$

f) $\lim_{x\to 0^+} f(x) = 0$

g) $\lim_{x\to 0} f(x) = 0$

h) $f(0)$ is undefined because 0 does not belong to the domain of f.

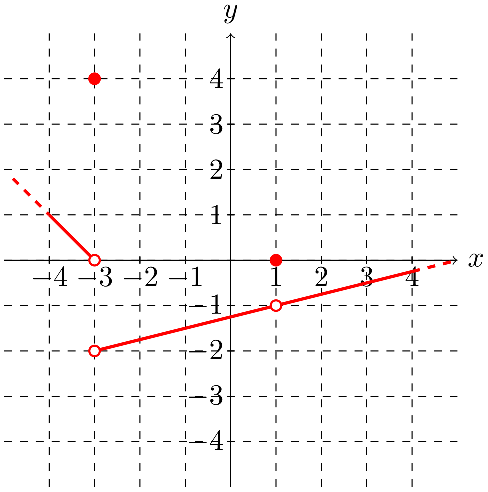
2) [5 points] Sketch the graph of **a** function f that satisfies **all of the given conditions**:

$$\lim_{x\to -3^-} f(x) = 0, \qquad f(-3) = 4, \qquad \lim_{x\to -3^+} f(x) = -2,$$
$$\lim_{x\to 1} f(x) = -1, \qquad f(1) = 0.$$

Make sure that your graph is the graph of a function, i.e. it passes the vertical line test.

Solution:

There are **infinitely many** functions that satisfy all these conditions (this is why we used the indefinite article "a" in the text of the exercise). Here below is an example of graph of one of the "easiest" functions with those properties.



We can also describe algebraically the selected function as a piecewise function:

$$f(x) = \begin{cases} -x-3, & \text{if } x < -3 \\ 4, & \text{if } x = -3 \\ \frac{1}{4}x - \frac{5}{4}, & \text{if } -3 < x < -1 \text{ or } x > 1 \\ 0 & \text{if } x = 1 \end{cases}$$

We say that this function is "easy" because roughly speaking it is an union of linear functions (lines).

3) [Bonus] A student says:

"***If*** *f is a function such that $f(1) = 2$* ***then*** $\lim_{x\to 1} f(x) = 2$."

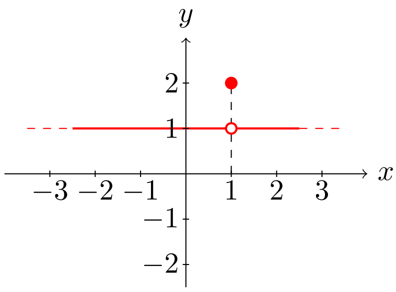
Do you agree or disagree? If you agree explain why, otherwise show (algebraically or visually with a graph) a **counterexample**, i.e. an example of function such that $f(1) = 2$ and $\lim_{x\to 1} f(x) \neq 2$.

Solution:
I disagree. Indeed an example of a function such that $f(1) = 2$ and $\lim_{x\to 1} f(x) \neq 2$ (in this case $\lim_{x\to 1} f(x) = 1$) is the following one:

Algebraically

$$f(x) = \begin{cases} 1, & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$$

Visually



Conclusion: The student should actually review the definition of the limit of a function at a number...