Calculus I - MAC 2311 - Section 003
Homework 1

Instructions: Solve the following exercises in a **separate sheet of paper**. Be tidy and organized! You can work on the exercises with your friends (or enemies!) but the final editing has to be yours. The homework has to be returned **by Wednesday September 12, 12:30 pm**. The total number for this homework is 110 (there are 10 extra points). The grade you will receive for this homework will count as a part of *Quizzes and handwritten homework* component of the total grade (15%).

Ex 1. (40 points) Compute the following limits and show all your work:

a) $\lim_{x\to 2} \dfrac{\sin(\pi x)}{x+1}$

b) $\lim_{t\to 3} \dfrac{t^2-2t-3}{2t-6}$

c) $\lim_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

d) $\lim_{x\to\infty} \dfrac{\pi x^7+2x-1}{-3x^7+x^5}$

e) $\lim_{u\to-\infty} \dfrac{-u^3+3u}{u+1}$

f) $\lim_{t\to\infty} \dfrac{t+5}{2t^5-3t^3-1}$

g) $\lim_{\alpha\to 0} \dfrac{\sin(2018\alpha)}{2019\alpha}$

h) $\lim_{\theta\to\frac{\pi}{2}+} \dfrac{\cos x-1}{\cos x}$

i) $\lim_{x\to -1} \dfrac{x^2}{x+1}$

j) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{when } x<2 \\ \sqrt{x+2}+2 & \text{when } x\geq 2 \end{cases}$

Ex 2. (25 points) Sketch the graph of a function f which satisfies simultaneously the following conditions:

a) $\lim_{x\to-\infty} f(x) = 0$,

b) f has a jump discontinuity at $x = -2$,

c) $f(-2) = 3$,

d) $\lim_{x\to(-2)^+} f(x) = 3$,

e) $\lim_{x\to 0^-} f(x) = -\infty$,

f) $x = 0$ is a solution for the equation $f(x) = 2$,

g) The line $y = 2$ is a horizontal asymptote.

Ex 3. (25 points) Let f be the function defined as:

$$f(x) = \begin{cases} c^2 \cdot \cos(x+1) + 2c, & \text{when } x \leq -1 \\ \frac{c}{x+3} & \text{when } x \geq -1 \end{cases},$$

where c is a constant (i.e. a real number).

a) Compute $\lim_{x\to(-1)^-} f(x)$, $\lim_{x\to(-1)^+} f(x)$ and $f(-1)$.

b) Find the value(s) of c what make f continuous at $x = -1$.

c) If c is one of the values found in (b), is f continuous for all real numbers?

Ex 4. (20 points) Which statements are True/False? Justify your answers.

a) The function $f(x) = \frac{x^2-9}{x+3}$ has a vertical asymptote at $x = -3$.

b) Let f be a function which is continuous at $x = 2$. If $\lim_{x\to 2} f(x) = 3$, then $f(2) = 3$.

c) If f is a continuous function on $[a, b]$ such that $f(a) < 0$ and $f(b) > 0$ then the equation $f(x) = 0$ has at least a solution.

d) There exists a rational function that has 2 different horizontal asymptotes.