

Bridge - MGF 3301 - Section 001

Quiz 1 - Solution

01/22/2020

Instructions: The total number of points of this quiz is 10. You will get an extra point if you solve correctly the last exercise. Calculators are not allowed (and actually not needed).

EXERCISE 1 (10 points)

- (1) (6 points) Recall the following definition:

Definition

Two propositional forms are **equivalent** if they have the same truth tables.

Prove that the following propositional forms are equivalent:

$$P \wedge (Q \vee R) \quad \text{and} \quad (P \wedge Q) \vee (P \wedge R).$$

- (2) (4 points) Determine the truth value of the above propositional forms, when P , Q and R are the following propositions:

- $P :=$ “Today is Wednesday January 23, 2020”;
- $Q :=$ “ $x = 1$ is a solution of the equation $x^2 - 3x + 1 = 0$ ”;
- $R :=$ “A triangle has three sides”.

Explain your answer fully and concisely.

Solution

- (1) By definition, in order to prove that two propositional forms are equivalent we have to write down their truth tables, i.e. we have to study their truth value for each combination of truth values of their components.

We have:

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

From the above table we can see that, for any combination of truth values of P , Q and R , the propositional forms $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$ have the same truth values. Therefore they are equivalent.

(2) For the given propositions, it is easy to check that:

- P is **False**,
- Q is **False**,
- R is **True**.

So in this case the truth value of the propositional form $P \wedge (Q \vee R)$ can be obtained by looking at the 7th row of the previous table:

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
F	F	T	T	F

The truth value of $(P \wedge Q) \vee (P \wedge R)$ will also be **False**, since the two propositional forms are equivalent.

EXERCISE 2

(Bonus - 1 point)

Anna says to Vanessa:

“If 7 is even, then I’ll give you \$1,000”.

Which of the following is true? (Check the correct box.)

- ☐ Anna would keep her promise only in the case where she gives \$1,000 to Vanessa.
- ☐ Anna would keep her promise only in the case where she does not give \$1,000 to Vanessa.
- ☒ Anna would keep her promise if she gives \$1,000 to Vanessa or if she does not give \$1,000 to Vanessa.