Ferrers Potential in galpy

1. Variables and Units

amp – Amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or $G \times$ mass
a – Scale radius (can be Quantity)
n – Power of Ferrers density ($n > 0$), not necessarilly integer, calculations generally fail for $n > 4$
b – y-to-x axis ratio of the density, usually $b < 1$
c – z-to-x axis ratio of the density, usually $c < b$
Ω_b – pattern speed of the bar, $\vec{\Omega}_b = \hat{e}_z \, \Omega_b$
pa – If set, the initial position angle of the bar in the xy plane measured from x (rad or Quantity)
normalize – if True, normalize such that $v_c(1., 0.) = 1.$, or, if given as a number, such that the force is this fraction of the force necessary to make $v_c(1., 0.) = 1.$.
r_0, v_0 – Distance and velocity scales for translation into internal units (default from configuration file)
Variables α appearing in the code in form _α2 carry the value of α^2 initialized in the begining of the class __init__ function.

2. Functions

...implemented for rotating potential in a frame of reference in which the bar lies aligned with x axis.)

2.1. Potential

Purpose: Evaluation of the bar potential as a function of cartesian coordinates in corotating frame of reference.

$$\Phi(\vec{x}) = \frac{-\mathrm{amp}\, b\, c}{4(n+1)} \int_\lambda^\infty A^{n+1}(\tau)\, \mathrm{d}\tau \tag{1}$$

where

$$A^\nu(\tau) = \frac{\left(1 - \sum_{i=1}^{3} \frac{x_i^2}{\tau + a_i^2}\right)^\nu}{[(\tau + a^2)(\tau + a^2 b^2)(\tau + a^2 c^2)]^{\frac{1}{2}}} \tag{2}$$

is part of an integrand which is repeatedly used in following functions. Lower limit for the integral based on definition of the density distribution is given by relation:

$$\lambda = \begin{cases} \text{unique positive solution of}\, m^2(\lambda) = 1, & \text{for } m \geq 1 \\ \lambda = 0\,, & \text{for } m < 1 \end{cases} \tag{3}$$

where $m^2(\lambda) = \sum_{i=1}^{3} \frac{x_i^2}{\lambda + a_i^2}$.

2.2. Density

Purpose: Evaluation of the density as a function of (x,y,z) in the aligned coordinate frame from cylindrical coordinates given as input

$$\rho(m^2) = \begin{cases} \frac{\mathrm{amp}}{4\,\pi\,a^3}\,(1 - (m/a)^2)^n, & \text{for}\, m < a \\ 0, & \text{for}\, m \geq a \end{cases}$$

where

$$m^2 = x^2 + \frac{y^2}{b^2} + \frac{z^2}{c^2} \tag{4}$$

2.3. Force

Evaluation of the x component of the force as a function of (x,y,z) in the aligned coordinate frame, which is used for evaluation of the force in cylindrical coordinates and then in orbit integration; does not take into account bar's rotation or initial position and therefore shall not be used directly.

$$F_i = \frac{-\mathrm{amp}\, b\, c}{2} \int_{\lambda}^{\infty} \frac{x_i}{a_i^2 + \tau} A^n(\tau)\, \mathrm{d}\tau \tag{5}$$

where

$$a_1 = a,\ a_2 = ab,\ a_3 = ac$$

2.4. General Second Derivative

General 2nd derivative of the potential as a function of (x,y,z) in the aligned coordinate frame

$$\Phi_{\mathrm{ij}} = -\frac{1}{4}\, b\, c\, \Phi'_{\mathrm{ij}} \tag{6}$$

2.5. Integration for Second Derivative

Integral that gives the 2nd derivative of the potential in x,y,z

The derivative is generally $\frac{\partial^2 \Phi}{\partial x_i \partial x_j}$; for $i = j$ the integral to be evaluated is:

$$\Phi'_{\mathrm{ii}} = \int_{\lambda}^{\infty} \frac{4\, n\, x_i^2}{(\tau + a_i^2)^2}\, A^{\nu-1}(\tau) - \frac{2}{\tau + a_i^2} A^{\nu}(\tau) \mathrm{d}\, \tau \tag{7}$$

In all other cases, the integral has this form:

$$\Phi'_{\mathrm{ij}} = \int_{\lambda}^{\infty} \frac{4\, n\, x_i\, x_j}{(\tau + a_i^2)(\tau + a_j^2)} A^{\nu-1}(\tau) \mathrm{d}\tau \tag{8}$$

2.6. Second Derivative in Nonrotating FoR

Transformation of the second derivative into the frame of reference which is corotating with the potential

$$\Phi_{x'x'} = \cos^2(\Omega_b t)\Phi_{xx} - 2\sin(\Omega_b t)\cos(\Omega_b t)\Phi_{xy} + \sin^2(\Omega_b t)\Phi_{yy} \tag{9}$$

$$\Phi_{y'y'} = \cos^2(\Omega_b t)\Phi_{yy} + 2\sin(\Omega_b t)\cos(\Omega_b t)\Phi_{xy} + \sin^2(\Omega_b t)\Phi_{xx} \tag{10}$$

$$\Phi_{z'z'} = \Phi_{zz} \tag{11}$$

$$\Phi_{x'y'} = [\cos^2(\Omega_b t) - \sin^2(\Omega_b t)]\Phi_{xy} + \sin(\Omega_b t)\cos(\Omega_b t)[\Phi_{xx} - \Phi_{yy}] \tag{12}$$

$$\Phi_{x'z'} = \cos(\Omega_b t)\Phi_{xz} - \sin(\Omega_b t)\Phi_{yz} \tag{13}$$

$$\Phi_{y'z'} = \cos(\Omega_b t)\Phi_{xz} + \sin(\Omega_b t)\Phi_{yz} \tag{14}$$

3. Milky Way Bar Potential

Values of parameters used for further work are set as the final state values in [1], that is:

$$a = 8\,kpc,\ b = 0.35,\ c = 0.2375,\ \text{amp} = 3.3 \times 10^{10}\,M_{\odot},\ \Omega_b = 10\,km/s/kpc.$$

References

[1] R. E. G. Machado and T. Manos. Chaotic motion and the evolution of morphological components in a time-dependent model of a barred galaxy within a dark matter halo. MNRAS, 458:3578–3591, June 2016.

[2] J. Binney and S. Tremaine. *Galactic Dynamics*. Princeton University Press, 2nd edition, 2008.

[3] J. Bovy. galpy: A python Library for Galactic Dynamics. ApJS, 216:29, February 2015.