

Ferrers Potential in galpy

1. Variables and Units

amp – Amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or $G \times \text{mass}$

a – Scale radius (can be Quantity)

n – Power of Ferrers density ($n > 0$), not necessarily integer, calculations generally fail for $n > 4$

b – y-to-x axis ratio of the density, usually $b < a$

c – z-to-x axis ratio of the density, usually $c < b$

Ω_b – pattern speed of the bar, $\vec{\Omega}_b = \hat{e}_z \Omega_b$

pa – If set, the initial position angle of the bar in the xy plane measured from x (rad or Quantity)

normalize – if True, normalize such that $v_c(1., 0.) = 1.$, or, if given as a number, such that the force is this fraction of the force necessary to make $v_c(1., 0.) = 1.$.

r_0, v_0 – Distance and velocity scales for translation into internal units (default from configuration file)

Variables α appearing in the code in form $_a2$ carry the value of α^2 initialized in the beginning of the class `_init_` function.

2. Functions

(... those I have written so far, except for potential evaluating function itself implemented for rotating potential in an inertial frame of reference in which a bar lies aligned with x axis.)

2.1. Potential

Purpose: Evaluation of the bar potential as a function of cartesian coordinates in inertial frame.

$$\Phi(\vec{x}) = \frac{-\text{amp } b c}{4(n+1)} \int_{\lambda}^{\infty} A^{n+1}(\tau) d\tau \quad (1)$$

where

$$A^{\nu}(\tau) = \frac{\left(1 - \sum_{i=1}^3 \frac{x_i^2}{\tau + a_i^2}\right)^{\nu}}{[(\tau + a^2)(\tau + a^2 b^2)(\tau + a^2 c^2)]^{\frac{1}{2}}} \quad (2)$$

is part of an integrand which is repeatedly used in following functions. Lower limit for the integral based on definition of the density distribution is given by relation:

$$\lambda = \begin{cases} \text{unique positive solution of } m^2(\lambda) = 1, & \text{for } m \geq 1 \\ \lambda = 0, & \text{for } m < 1 \end{cases} \quad (3)$$

where $m^2(\lambda) = \sum_{i=1}^3 \frac{x_i^2}{\lambda + a_i^2}$.

2.2. Density

Purpose: Evaluation of the density as a function of (x,y,z) in the aligned coordinate frame from cylindrical coordinates given as input

$$\rho(m^2) = \begin{cases} \frac{\text{amp}}{4\pi a^3} (1 - (m/a)^2)^n, & \text{for } m < a \\ 0, & \text{for } m \geq a \end{cases}$$

where

$$m^2 = x'^2 + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} \quad (4)$$

2.3. Force

Purpose: Evaluation of the x component of the force as a function of (x,y,z) in the aligned coordinate frame, which is used for evaluation of the force in cylindrical coordinates and then in orbit integration; does not take into account bar's rotation or initial position and therefore shall not be used directly.

$$F_i = \frac{-\text{amp } b c}{2} \int_{\lambda}^{\infty} \frac{x_i}{a_i^2 + \tau} A^n(\tau) d\tau \quad (5)$$

where

$$a_1 = a, \quad a_2 = ab, \quad a_3 = ac$$

2.4. General Second Derivative

Purpose: General 2nd derivative of the potential as a function of (x,y,z) in the aligned coordinate frame

$$\Phi_{ij} = -\frac{1}{4} b c \Phi'_{ij} \quad (6)$$

2.5. Integration for Second Derivative

Purpose: Integral that gives the 2nd derivative of the potential in x,y,z

The derivative is generally $\frac{\partial^2 \Phi}{\partial x_i \partial x_j}$; for $i = j$ the integral to be evaluated is:

$$\Phi'_{ii} = \int_{\lambda}^{\infty} \frac{4 n x_i^2}{(\tau + a_i^2)^2} A^{\nu-1}(\tau) - \frac{2}{\tau + a_i^2} A^{\nu}(\tau) d\tau \quad (7)$$

In all other cases, the integral has this form:

$$\Phi'_{ij} = \int_{\lambda}^{\infty} \frac{4 n x_i x_j}{(\tau + a_i^2)(\tau + a_j^2)} A^{\nu-1}(\tau) d\tau \quad (8)$$

3. Milky Way Bar Potential

Values of parameters used for further work are set as the final state values in [3], that is:

$$a = 8 \text{ kpc}, \quad b = 0.35, \quad c = 0.2375, \quad \text{amp} = 3.3 \times 10^{10} M_{\odot}, \quad \Omega_b = 10 \text{ km/s/kpc}.$$

References

- [1] J. Binney and S. Tremaine. *Galactic Dynamics*. Princeton University Press, 2nd edition, 2008.
- [2] J. Bovy. galpy: A python Library for Galactic Dynamics. *ApJS*, 216:29, February 2015.
- [3] R. E. G. Machado and T. Manos. Chaotic motion and the evolution of morphological components in a time-dependent model of a barred galaxy within a dark matter halo. *MNRAS*, 458:3578–3591, June 2016.