Calculating an indexed loan

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Fixed amortization (real terms)

Consider N period loan with an initial principal of X_0 with a fixed real period rate r. Inflation is measured as the percentage change of CPI, $\pi_t = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}$.

Amortization in real terms: $A_t^r = \frac{X_0}{N}$

Amortization in nominal terms: $A_t^n = A_t^r \frac{CPI_t}{CPI_0} = \frac{X_0}{N} \frac{CPI_t}{CPI_0} = \frac{X_0}{N} \prod_{j=1}^{t} (1 + \pi_j)$

The per (end of) period Principal in real terms: $X_t^r = X_0 - \frac{X_0}{N} t = X_0 \left(1 - \frac{t}{N}\right)$

The per period Principal in nominal terms: $X_t^n = X_0 \left(1 - \frac{t}{N}\right) \prod_{j=1}^{t} (1 + \pi_j)$

Interest payments in real terms (payed at end of period): $I_t^r = r X_{t-1}^r$

Interest payments in nominal terms (payed at end of period): $I_t^n = r (1 + \pi_t) X_{t-1}^n$

Total payments per period:

$$\begin{aligned} P_t &= A_t^n + I_t^n = \frac{X_0}{N} \prod_{j=1}^{t} (1 + \pi_j) + r (1 + \pi_t) X_{t-1}^n \\ &= \frac{X_0}{N} \prod_{j=1}^{t} (1 + \pi_j) + r X_t^n \end{aligned}$$

Fixed payment (annuity in real terms)

First you need to find the fixed payment in real terms, using the standard equation of annuities

$$P^r = \frac{r}{X_0} \frac{1}{1 - \frac{1}{(1+r)^N}}.$$

Payment in nominal terms:[1] $= P_t^n = P^r \frac{CPI_t}{CPI_0} = P^r \prod_{j=1}^{t} (1 + \pi_j)$.

The rest is calculated recursively:

Nominal interest payment: $I_t^n = r (1 + \pi_t) X_{t-1}^n$

Nominal amortization: $A_t^n = P_t^n - I_t^n$

Nominal principal: $X_t^n = (1 + \pi_t) X_{t-1}^n - A_t^n$,

[1] An alternative method, often used when the principal changes for various reasons is to recalculate P^r based on the updated principal and the number of periods left. The outcome is identical if payments are regular.