112b Midterm II
Wed Apr 9, 2008
90 minutes

Please answer all 7 parts below, in your blue books. **Show all your work** *and* **justify clearly** *where appropriate. You may not use your books, notes or calculators on this exam. Good luck!*

1. *(16 pts.)* Find the derivatives of the following functions:

   (a) $x^3 + 3x^2$ (b) $3\tan^{-1} x$

   (c) $\sin x + \cos x$ (d) $\dfrac{1}{x} + \ln x$

2. *(16 pts.)* Find the derivatives of the following functions:

   (a) $\sin\left(\dfrac{1}{\sqrt{1+x^2}}\right)$ (b) $x^2 e^{\sin x}$

   (c) $x(1+x^2)^{500}$ (d) $\sqrt{\ln x}$

3. *(14 pts.)* A 10 foot ladder is standing vertically with its foot fixed to the ground by a bolt. The sun is shining directly overhead. The painter slips and the ladder begins to fall, rotating around its foot toward the ground. When the ladder makes an angle of 30° with the ground, it is rotating at 1/2 revolution per second. The painter has time to wonder: How fast is its shadow growing?

4. *(13 pts.)* Use linear approximation to estimate the cube root of 27.1. You may write your final answer as a fraction.

5. *(13 pts.)* Find the equation of the tangent line to the graph of
$$x^4 + xy^2 + y^3 = 13$$
   at the point $(1, 2)$.

6. *(14 pts.)* Find the absolute maximum and minimum values of the function
$$f(x) = x \sin x + \cos x$$
   on the interval $[-\pi, \pi]$.

7. *(14 pts.)* I have a function f which is differentiable everywhere, and which satisfies

   (a) $f(0) = 5$, $f(2) = 1$, and $f(3) = 6$.
   (b) f' is an increasing function. (Recall this means for any x, y, if $x < y$ then $f'(x) < f'(y)$.)

   Prove carefully that

   (i) There must be a point a in $(0, 3)$ such that $f'(a) = 0$.
   (ii) There is a point b in $[0, 3]$ such that $f(b) = 5.5$.
   (iii) There is *only one* point b in $[0, 3]$ such that $f(b) = 5.5$.