Math 112a - Calculus of Functions of One Variable I

Exam 2

11/10/2011
90 minutes

(1) (16 pts) Find the derivatives of the following functions (do not simplify):

(a) $2x^4 + 3$ (b) $\frac{1}{2}\sin^{-1} x$

(c) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (d) $3^x + \tan x$

(2) (16 pts) Find the derivatives of the following functions (do not simplify):

(a) $\frac{\sin(\cos x)}{\cos(\sin x)}$ (b) $\log_5(\ln x)$

(c) $\sin^4 x + 2\sin^2 x \cos^2 x + \cos^4 x$ (d) $x^{e^2}\, e^{-x^2}$

(3) (10 pts) Find an equation of the tangent line to

$$y = x\, e^{-y}$$

at the point $(x, y) = (e, 1)$.

(4) (14 pts) Let $f(x) = (x+1)^5 - 5x - 3$. Find the absolute minimum and maximum of $f(x)$ on $[-3, 0]$.

(5) (16 pts) The volume of the Antarctic ice sheet at any time can be approximated by $V = \frac{3\pi}{4} h r^2$, where $h = h(t)$ is the mean thickness and $r = r(t)$ is the mean radius. At some point in time, $h = 3\,\text{km}$, $r = 2000\,\text{km}$, the mean radius r is decreasing at a rate of $30\,\text{km/yr}$, and the volume V is decreasing at a rate of $3\pi \cdot 10^5\,\text{km}^3/\text{yr}$. How quickly is the mean thickness h decreasing? Derive the formula for $\frac{dh}{dt}$ first, then plug in the numbers.

(6) (14 pts)
- (a) State the Intermediate Value Theorem.
- (b) Show that the equation $\sin(\sin x) + 2x = 2$ has *at least* one real solution.
- (c) State the Mean Value Theorem.
- (d) Show that the equation $\sin(\sin x) + 2x = 2$ has *exactly* one real solution.

(7) (14 pts) Consider the function $f(x) = x^x$ for $x > \frac{1}{1000}$.
- (a) For which values of x is $f(x)$ increasing and decreasing?
- (b) Find the local minima and maxima of $f(x)$.