

112b Midterm I

Wed Feb 20, 2008

90 minutes

Please answer all the questions below, in your blue books. Show all your work and justify clearly where appropriate. You may not use your books, notes or calculators on this exam.

Good luck!

1. (15 pts.) For each function described below, decide if it is one-to-one on the indicated domain. If it is, find the inverse function and its domain.

(a) $g(x) = \sqrt{1-x}$, $\{-1 < x < 1\}$

(b) $h(x) = x^2 - 6x + 9$, $\{2 < x < 4\}$

(c) $f(x) = (\sin x)^2$, $\{0 < x < \pi/2\}$

2. (15 pts.) Find the following limits, or determine if they do not exist.

(a) $\lim_{x \rightarrow 2} (x-1)^2$

(b) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1}$

(c) $\lim_{x \rightarrow 1} \frac{|x-1|}{x^2 - 1}$

3. (15 pts.) Find the following limits at ∞ , or determine if they do not exist.

(a) $\lim_{x \rightarrow \infty} \frac{x^3 - x^2}{1 + 10x^2 - 3x^3}$

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{x+1}}{\sqrt{x+2}}$

(c) $\lim_{x \rightarrow \infty} \sqrt{x+1} - \sqrt{x}$

4. (10 pts.) State the definition of the derivative of a function f at a point a , as completely and precisely as you can.

5. (15 pts.) Use the Intermediate Value Theorem to find an integer n such that between n and $n+1$ lies a solution of

$$x^3 = 7 + x.$$

6. (15 pts.) List the following quantities, associated to the graph below, in increasing order. (You need not evaluate them numerically).

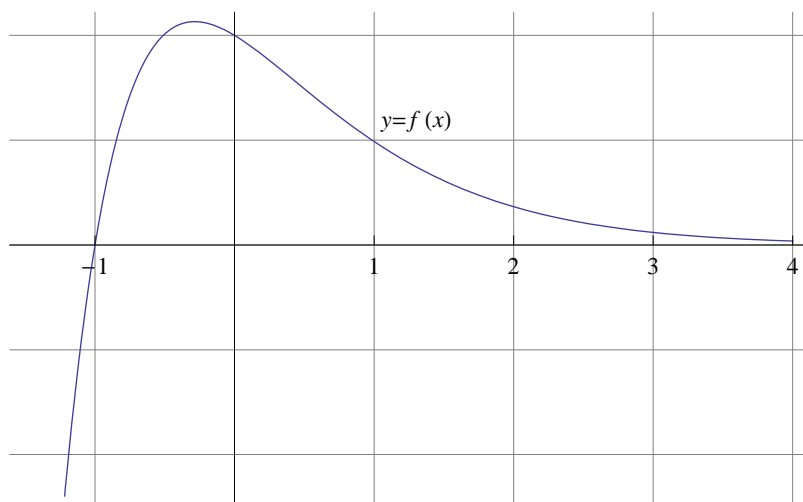
(a) $f'(-1)$

(b) $f'(1)$

(c) $f'(3)$

(d) 0

(e) $\frac{1}{2}(f(1) - f(-1))$



7. (15 pts.) Use the definition of the derivative to compute each indicated derivative, or show that it does not exist.

(a) $f(x) = (x+1)^2$. $f'(1)$

(b) $g(x) = 1/x$. $g'(2)$

(c) $h(x) = x^{1/3}$, $h'(0)$