Solutions to another practice midterm 1

1. (a) -1; (b) $\frac{1}{6}$.

2. $2\sqrt{2}$.

3. (a) $f(-1) = -5$, $f(0) = 1$, $f(1) = -1$, $f(4) = 5$. (b) The function is continuous everywhere (polynomial), and $f(-1) = -5 < 0$, $f(0) = 1 > 0$. Therefore, by IVT, there is a solution to the equation $f(x) = 0$ inside the interval $(-1, 0)$. By a similar argument, there are solutions inside the intervals $(0, 1)$ and $(1, 4)$. Since the intervals do not intersect, and each contains a solution, there are 3 solutions to the given equation.

4. $f'(-\frac{1}{2}) < 0 < f'(\frac{7}{2})$; $f'(-4) \approx 0 > f'(-1)$; $f'(1) \geq 0 > f(1)$; $f'(10) < 1 < 10$.

5. (a) $f(x)$, $g(x)$ and $h(x)$ have a vertical asymptote at $x = -1$. (b) $f(x)$ and $h(x)$ have a horizontal asymptote at $y = 1$. (c) $g(x)$ has a horizontal asymptote at $y = 0$. In (c), Squeeze theorem can be used to compute the limit.

6. (a) $-\frac{3}{2}$; (b) $\frac{\pi}{4}$; (c) ∞.

7. (a) $\frac{1}{3}$; (b) Hint: by the definition, $\lim_{x\to 0^+} \frac{g(x) - g(0)}{x} = 1$; $\lim_{x\to 0^-} \frac{g(x) - g(0)}{x} = -1$. So $g'(0)$ does not exist; Similarly, $h'(1)$ in (c) does not exist.