Solutions of practice Midterm 1

1. (a) $g^{-1}(x) = 1 - x^2, \quad \{0 < x < \sqrt{2}\}$;

(b) $h(x)$ is not one-to-one, for $h(2.5) = h(3.5)$;

(c)$f^{-1}(x) = \sin^{-1}(\sqrt{x}), \quad \{0 < x < 1\}$.

2. (a) $\lim_{x\to 2}(x-1)^2 = 1$;

(b)$\lim_{x\to 1} \frac{x^2+2x-3}{x-1} = 4$;

(c)$\lim_{x\to 1} \frac{|x-1|}{x^2-1}$ DNE, for $\lim_{x\to 1^+} \frac{|x-1|}{x^2-1} = \frac{1}{2}$, but $\lim_{x\to 1^-} \frac{|x-1|}{x^2-1} = -\frac{1}{2}$.

3. (a) $\lim_{x\to\infty} \frac{x^3-x^2}{1+10x^2-3x^3} = -\frac{1}{3}$;

(b) $\lim_{x\to\infty} \frac{\sqrt{x+1}}{\sqrt{x+2}} = 1$;

(c) $\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x} = 0$.

4. The derivative of a function f at a point a is the limit

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h},$$

if this limit exists and is finite.

5. Let $f(x) = x^3 - x - 7$. Then $f(2) = -1$ and $f(3) = 17$. By IVT, there is a solution to the equation $f(x) = 0$ inside the interval $(2, 3)$. Hence $n = 2$ is the integer we want.

6. $f'(1) < f'(3) < 0 < \frac{1}{2}(f(1) - f(-1)) < f'(-1)$.

7. (a) $f'(1) = 4$;

(b)$g'(2) = -\frac{1}{4}$;

(c)$h'(0)$ does not exist for $\lim_{h\to 0} \frac{h^{\frac{1}{3}}}{h} = \infty$.