(1) (10 pts) Find the following limits, or show that they do not exist.

(a) $\lim_{x\to 1} \cos(\pi x)$

(b) $\lim_{x\to 1} \dfrac{x-1}{x^2+4x-5}$

(2) (15 pts) Find

$$\cot\left(\sin^{-1}\left(\frac{1}{3}\right)\right).$$

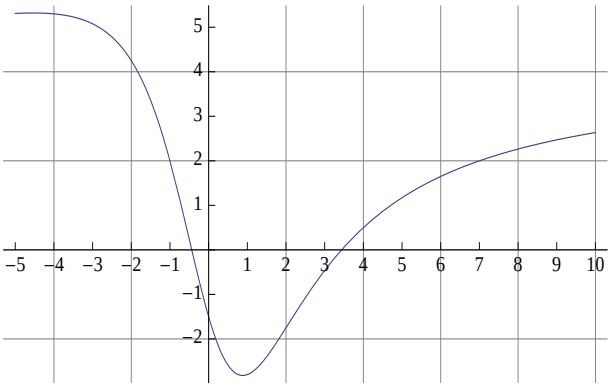
(*Hint:* You may use the identity $1+\cot^2 x = \frac{1}{\sin^2 x}$.)

(3) (15 pts) Let $f(x) = x^3 - 4x^2 + x + 1$. It is known and you may assume that any cubic equation can have one, two or three distinct solutions.

(a) Evaluate $f(-1)$, $f(0)$, $f(1)$ and $f(4)$.

(b) Use the Intermediate Value Theorem to determine how many solutions there are to the equation $f(x) = 0$. Justify your answer.

(4) (12 pts) Consider the graph of the function $f(x)$:



For each of the following pairs of numbers, associated to the graph of $f(x)$, decide which is greater. Justify your answer.

(a) $f'(-\frac{1}{2})$ and $f'(\frac{7}{2})$;

(b) $f'(-4)$ and $f'(-1)$;

(c) $f'(1)$ and $f(1)$;

(d) $f'(10)$ and 10.

(5) (15 pts) Consider three functions:

$$f(x) = \frac{x^2+1}{(x+1)^2}, \quad g(x) = \frac{x^2\sin x}{x^3+1}, \quad h(x) = \frac{x^2+x+1}{x^2-1}.$$

Which of these functions

(a) Have a vertical asymptote at $x = -1$;

(b) Have a horizontal asymptote at $y = 1$;

(c) Have a horizontal asymptote at $y = 0$?

Justify your answer.

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(6) (18 pts) Find the following limits, or show that they do not exist.

(a) $\lim_{x\to\infty} \dfrac{3x^3+4x+1}{3+x+5x^2-2x^3}$

(b) $\lim_{x\to\infty} \tan^{-1}\left(\dfrac{3x+1}{3x-2}\right)$

(c) $\lim_{x\to\infty} (\sqrt{9x+1} - \sqrt{x-3})$

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(7) (15 pts). Use the definition of the derivative to compute the derivative of each function at the given point, or show that it does not exist.

(a)

$$f(x) = \sqrt{2x+3}, \quad \text{at } x = 3.$$

(b)

$$g(x) = \sqrt{x^2}, \qquad \text{at } x = 0.$$

(c)

$$h(x) = \begin{cases} x^2, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases} \qquad \text{at } x = 1.$$