Modeling adstock using Weibull transformations

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Introduction

In MMM analyses (i.e. 'media mix modeling' or 'marketing mix modeling') adstock refers to the delayed - additional - impact of advertising. Advertising, which is aired or otherwise shown to consumers, does not necessarily lead to an impact on sales or store visits on the same day or even in the following week. It may take some time for a consumer to be triggered by an ad because a store visit needs to be planned, or because the product being sold is not an impulse purchase. Whatever the reason for the delayed impact, analysts need to take this reality into account by transforming their original media variable such that media aired on day *i* can have some impact on day *i+1* through day *i+x*. In order to model adstock, analysts tend to rely on exponential decay transformations or more ad hoc solutions. As an example, with exponential decay of 50%, a TV investment of 100 GRPs will have 50 GRPs impact on the second day, 25 GRPs on the third, 12,5 GRPs on the fourth, and so forth (see Figure 1).

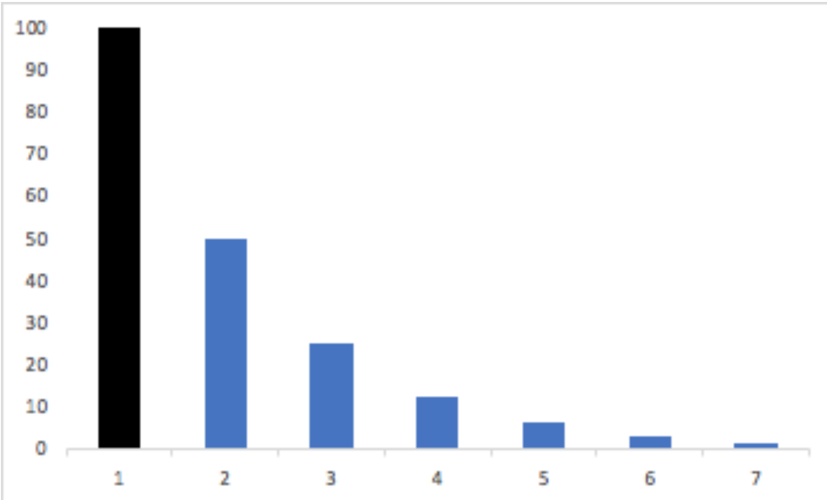

Figure 1. An example of an exponential decay transformation of 50% for adstock based on an initial investment of 100 TV GRPs on day 1.

Implicit in this approach is the assumption that the bulk of the impact of advertising occurs on the day that the ad is seen by consumers. Depending on the industry or the product being advertised and on the media channel, this assumption may be more or less tenable. The impact of a media investment may sometimes be as great or greater on the following day or later (see Figure 2 and 3).

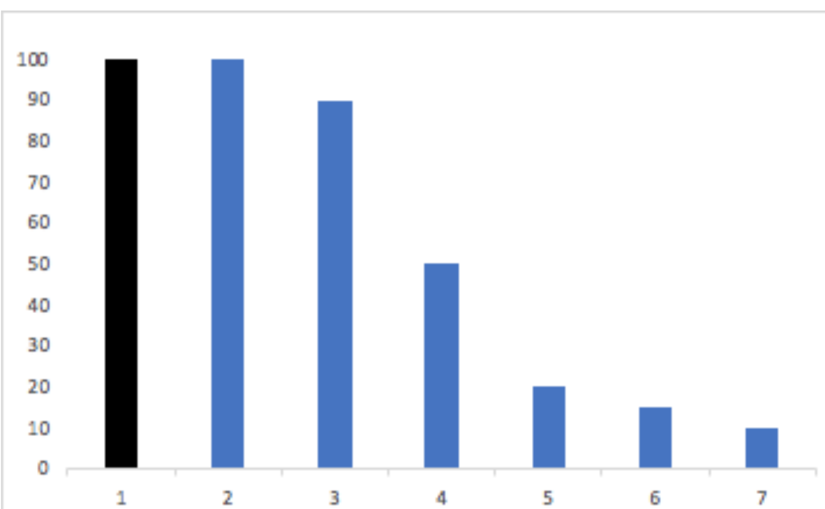

Figure 2. An example of an alternative adstock transformation with longer lasting impact based on an initial investment of 100 TV GRPs on day 1.

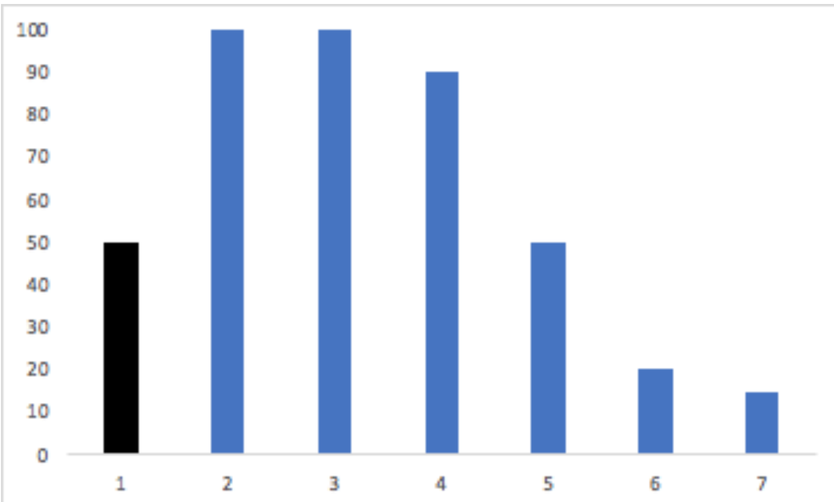

Figure 3. An example of an alternative adstock transformation with a bigger impact on the second day based on an initial investment of 100 TV GRPs on day 1.

One way to formalise the shape of an alternative adstock transformation is to use a Weibull distribution. In this paper we examine whether the accuracy of MMM analyses may be improved by modeling adstock using a Weibull distribution, allowing for an alternative allocation of both direct and delayed media impact. We assume that improvement from a Weibull distribution transformation will depend on the industry and on the media channel. In this study we will look at two industries, namely retail (department store) and informal eating out.

Method

Data

For this adstock modeling study we used daily sales and daily marketing data from a department store chain and an informal eating out restaurant chain. Both the sales and marketing data are focussed on the Netherlands only.

Dependent variables

For the department store the dependent variable in the analysis was the number of online transactions per day, measured by their e-commerce platform. The data does not include transactions made in the stores. For the informal eating out restaurant we examined gross sales revenue per day for the restaurants in the Netherlands.

Independent variables - both advertisers

TV and radio data is included in the models for both advertisers as the number of gross rating points (GRPs). This data is collected from Media Buying Systems (MBS), which is the Dutch TV and radio inventory buying tool. Data on Facebook advertising is extracted using Facebook's MMM feed, which is made available to Annalect as part of the MMM partnership.

In addition to media data, we include daily amount of precipitation for both advertisers, as earlier research has shown that this impacts revenue for both companies. The data on daily amount of precipitation is collected from KNMI, the Royal Dutch Meteorological Institute. Competitor spend information is gathered from Nielsen for both companies and included as the total gross spend by competitors.

Independent variables - Department store

In the department store analyses we use the number of regular and Google shopping paid search impressions. We also include Google display advertising impressions from affiliate marketing partners, social media impressions, as well as out-of-home (OOH) advertising spend. To account for multicollinearity, social media and other display impressions were aggregated together as: Digital Non-search. See the “Multicollinearity” heading below for more details.

The department store also distributes door drop advertising folders (brochures) across the Netherlands. In the analysis we include the number of pages that were delivered. Zero pages refers to no brochure being distributed. As brochures are delivered weekly rather than daily, we give each day in a week the same value for folders.

Independent variables - Informal eating out restaurant

Search advertising for the informal eating out restaurant was included by using reporting data from Google’s Search Ads 360 platform. We included the number of paid search impressions in the analysis. Similarly, Out-of-home (OOH) advertising spend was included and data on Google display advertising impressions were collected from Google’s Campaign Manager platform. Lastly, Snapchat impressions were included by utilising Snapchat Business Manager. Important to note is that for Snapchat, Google display, tv and radio, variables were split on campaign level to seven product categories. Finally, to account for multicollinearity, Snap paid impressions, Google display impressions and Facebook total video impressions were aggregated together as: Digital Non-search. See the “Multicollinearity” heading below for more details.

Multicollinearity

When it comes to running multicollinearity tests after and in parallel to data preparation, specifically the Farrar-Glauber test (F-G test) and Theil’s Method suggested that overall collinearity was present in both the department store and informal eating out data. In addition to overall collinearity diagnostics we used the following individual multicollinearity diagnostics as well: Variance Inflation Factors (VIF) and tolerance levels[1]. To account for multicollinearity, thresholds of VIF values equal or greater than 2.5 and tolerance levels equal or below 0.4 were used. As a consequence, four product categories were not included for the Google display data in the final informal eating out models, and the aggregated “Digital Non-search” regressor was used for both brands.

Modeling

For the development of the MMM models we used the package Prophet[2] for the R programming language. The Prophet procedure is an additive regression model which includes yearly and weekly seasonalities plus holiday effects. It uses the ‘RStan’ package, which enables Bayesian modeling implementations.

Adstock transformations

As discussed in the introduction, we look at different adstock transformations in this study. The different adstock transformations were created by varying the following properties:

- Decay type and strength: Weibull or traditional exponential decay
- Window: Length of the adstock effect in days
- First day correction: Decreased ad effect on the first day or not

Weibull transformations

We used the following formulas to calculate the weibull adstock transformations. These were derived from the Weibull cumulative distribution function[3]. The *k* and *window*

[1] “Tolerance Level / Tolerance Statistics: Definition / Examples” https://www.statisticshowto.datasciencecentral.com/tolerance-level-statistics/. Accessed 27 Aug 2019.

[2] "Prophet | Prophet is a forecasting procedure implemented in R and ...." http://facebook.github.io/prophet/. Accessed 6 Aug. 2019.

[3] "Weibull distribution - Wikipedia." https://en.wikipedia.org/wiki/Weibull_distribution. Accessed 28 Aug. 2019.

parameters define the resulting adstocks. See the results in figure 1. *Lag* represents the amount of days after the ad was published.

$$\lambda = \frac{window}{(-ln(0.001))^{1/k}}$$

$$adstock = e^{-(lag/\lambda)^k}$$

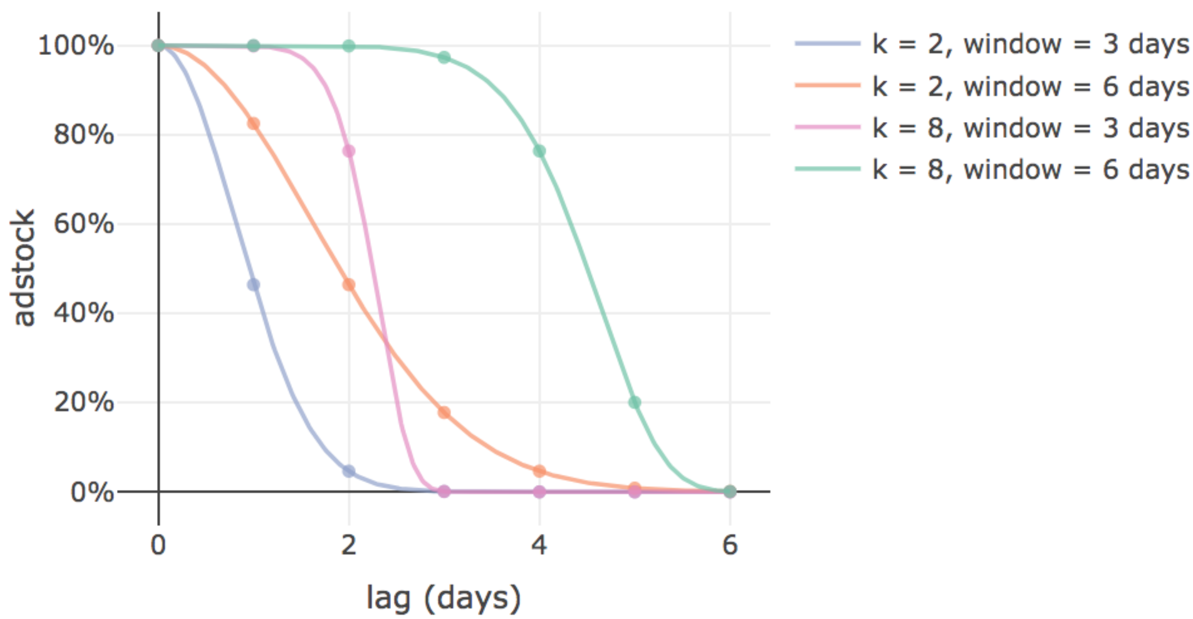

Figure 4. Weibull adstock transformations are shown. The k parameter defines the shape of the curve and the window parameter defines on which day the adstock will wear out to nearly zero (0.1% of the ad effect). As such, a window of 3 days means that the ad will have worn out 4 days after the ad was aired.

S-curve transformation

In reality, most advertising will have a non-linear impact on sales. Initial spend might have limited impact until a certain threshold is reached. On the other hand the impact tends to diminish as the ad-spend reaches a point of saturation. We used the following commonly used sigmoid equation[4] to account for this s-curve effect, as well as normalisation of the adstock data (Figure 5).

$$scale = mean(adstock\ vector)$$

$$s\text{-}curve\ adstock = \frac{1}{1 + b * (\frac{adstock}{scale})^{c}}$$

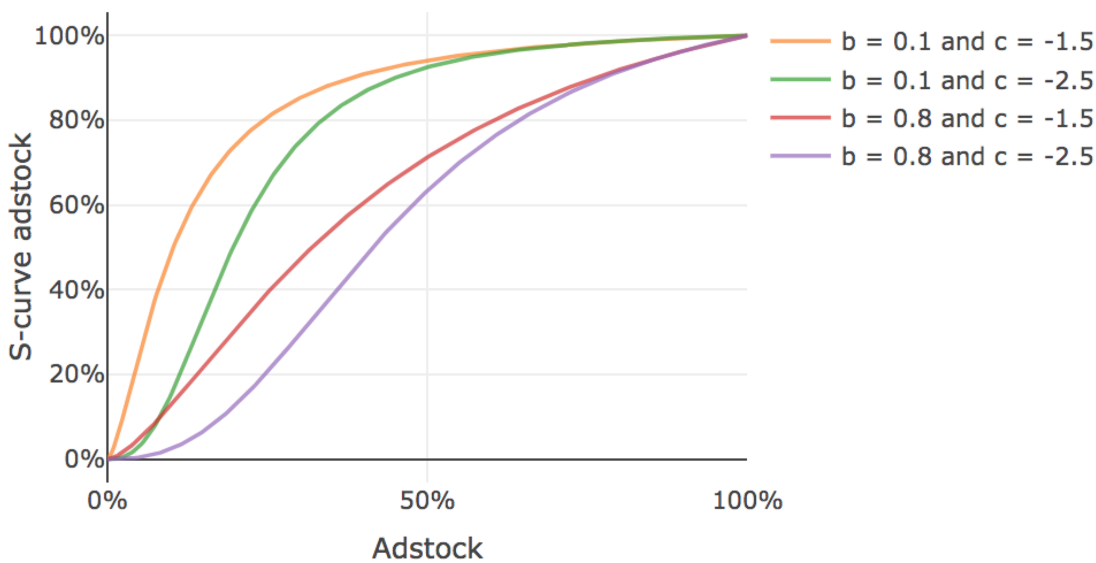

Figure 5. S-curve transformations. The b parameter mostly affects the saturation, whereas the c parameter determines the diminishing effect on low adstock values.

[4] "Sigmoid function - Wikipedia." https://en.wikipedia.org/wiki/Sigmoid_function. Accessed 2 Sep. 2019.

Evaluation of Weibull adstock

To investigate (Weibull) transformation contributions we created a so called 'adstock grid', containing all unique corresponding transformation combinations. The fit of these transformations was assessed using the explained variance (R^2) and the Mean Absolute Percentage Error (MAPE) of the overall model fit. The options of the parameters we included in the transformation grid were as follows. Traditional exponential decay rate: 0.25, 0.5, 0.75, 0.9, 1; Window: 2, 4, 6, 8; Weibull k parameter: 2, 3, 5, 8; first day correction: 50%, 100%; s-curve b parameter: 0.1, 0.3, 0.8; s-curve c parameter: -1.5.

Our aim is to evaluate whether the usage of Weibull adstock results in a more accurate model than using traditional adstock transformations. Therefore we first optimised the transformation parameters for each regressor using only traditional adstock transformations without Weibull transformations (baseline model). Next, we re-optimised the regressor transformations for only 1 regressor at a time, this time including traditional adstock and Weibull adstock in the adstock grid. As such, we assessed per regressor whether Weibull adstock results in a more accurate model than traditional adstock. Finally, we also fitted an extra model that included the optimal Weibull transformation for all regressors that were shown to improve the R^2 value in the previous step.

Furthermore, as the prophet package uses an additive model, it also returns the contribution of each regressor when a fitted model is used to predict the dependent variable values. Therefore, we also investigated how the Weibull transformations affected the contributions of each regressor to the sales related dependent variables.

Results

Department store data

We first optimised regressor transformations for the department store data, using only traditional adstock decay rates. See Table 1 for the optimised parameters for these baseline models. Next, we re-optimised transformations separately per regressor, this time including Weibull transformations. We found that Weibull transformations only increased model accuracy (R^2) for the regressor: digital search. More specifically, the optimal Weibull parameters we found for digital search were: k = 2 and window = 2, with no first day correction (100%), see Table 2 and Figure 6. This weibull transformation model for digital search only performed slightly better (R^2 = 0.79133, MAPE = 10.038%) than the traditional non-Weibull model (R^2 = 0.7911, MAPE = 10.047%). So, when reflecting on explained variance and MAPE for both models, adding a Weibull regressor slightly improved explained variance and reduced MAPE compared to using only traditional adstock transformations. The actuals, forecast and residuals are shown in Figure 7.

	Exponential decay rate	Window (days)	S-curve b parameter
Department store			
Digital search	75 %	6	0.8
Digital non-search	50 %	6	0.3
Folders	100 %	6	0.1
Competitor spend	100 %	8	0.1
TV	100 %	8	0.8
Radio	75 %	8	0.1
Informal eating out			
Digital non-search	25 %	2	0.8
Digital search	25 %	2	0.3
OOH	25 %	4	0.1
Competitor spend	25 %	4	0.8
TV	100 %	6	0.1
Radio	25 %	2	0.8

Table 1. Regressor transformations for the Baseline models (traditional adstock).

	Weibull k parameter	Window (days)	First day correction	S-curve b parameter
Department store				
Digital search	2	2	100%	0.8
informal eating out				
Digital non-search	3	2	50%	0.8
Digital search	2	2	100%	0.8
OOH	2	2	100%	0.1
Competitor spend	5	2	50%	0.8
TV	8	8	50%	0.1

Table 2. Weibull adstock transformation parameters per model.

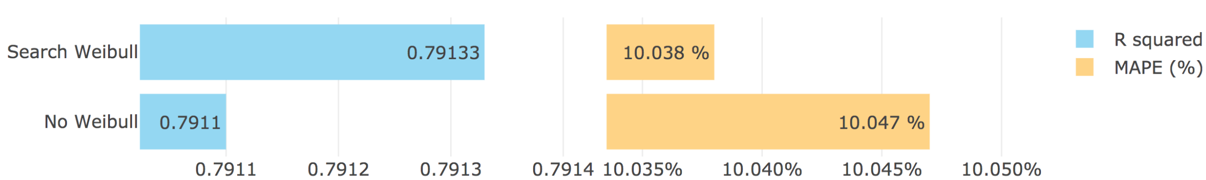

Figure 6. Prediction accuracy measures R^2 and MAPE shown for the 2 tested department store models: No Weibull (using only traditional adstock transformations) and Search Weibull (using Weibull adstock transformation for the digital search regressor and traditional adstock transformations for the other ad regressors).

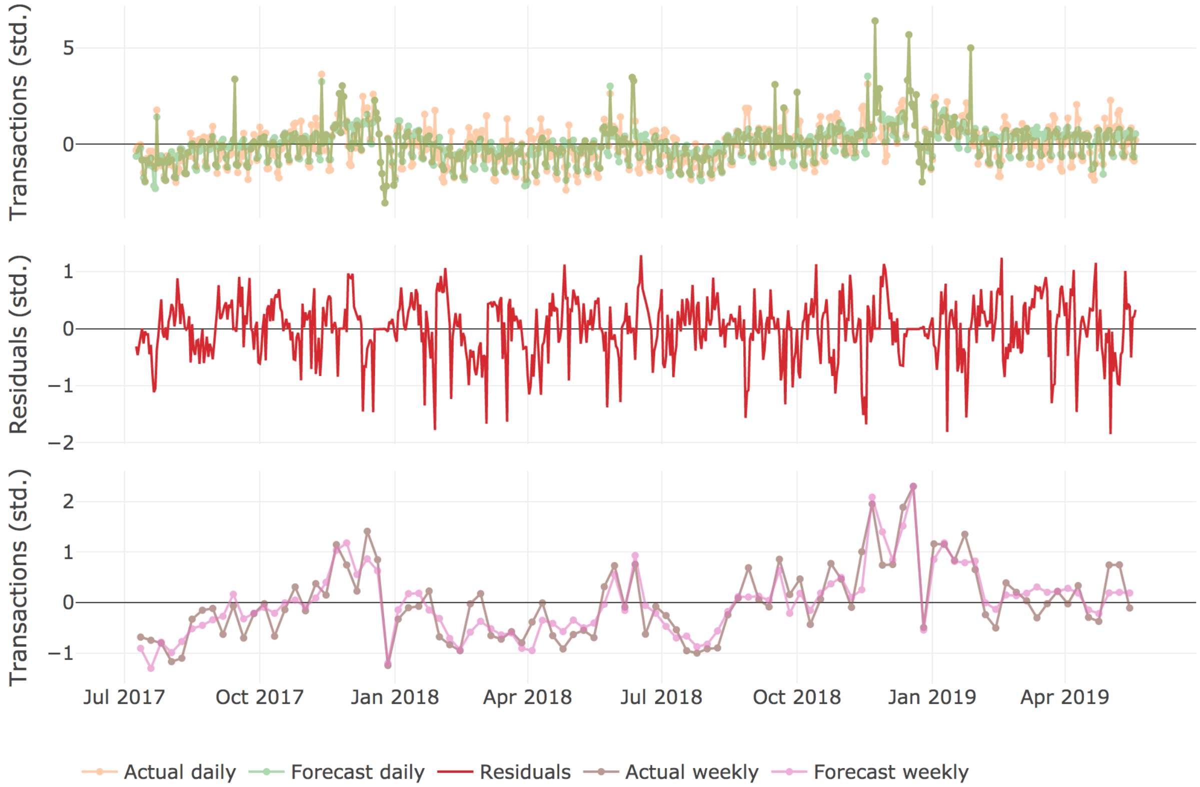

Figure 7. Standardised actuals, forecast and residuals of the department store model with Weibull adstock for digital search. We standardised these time series by subtracting the average and dividing by the standard deviation of the forecasted transactions.

We also examined how the introduction of Weibull transformations affects the contribution of each regressor to the predicted number of transactions. In Figure 8 these contributions are compared between the no-Weibull model and the model with Weibull for digital search. For digital search the contribution in the Weibull model is more than halved (2.27%) compared to the no-Weibull model (4.76%). For the other regressors the difference in contribution between the models is very small.

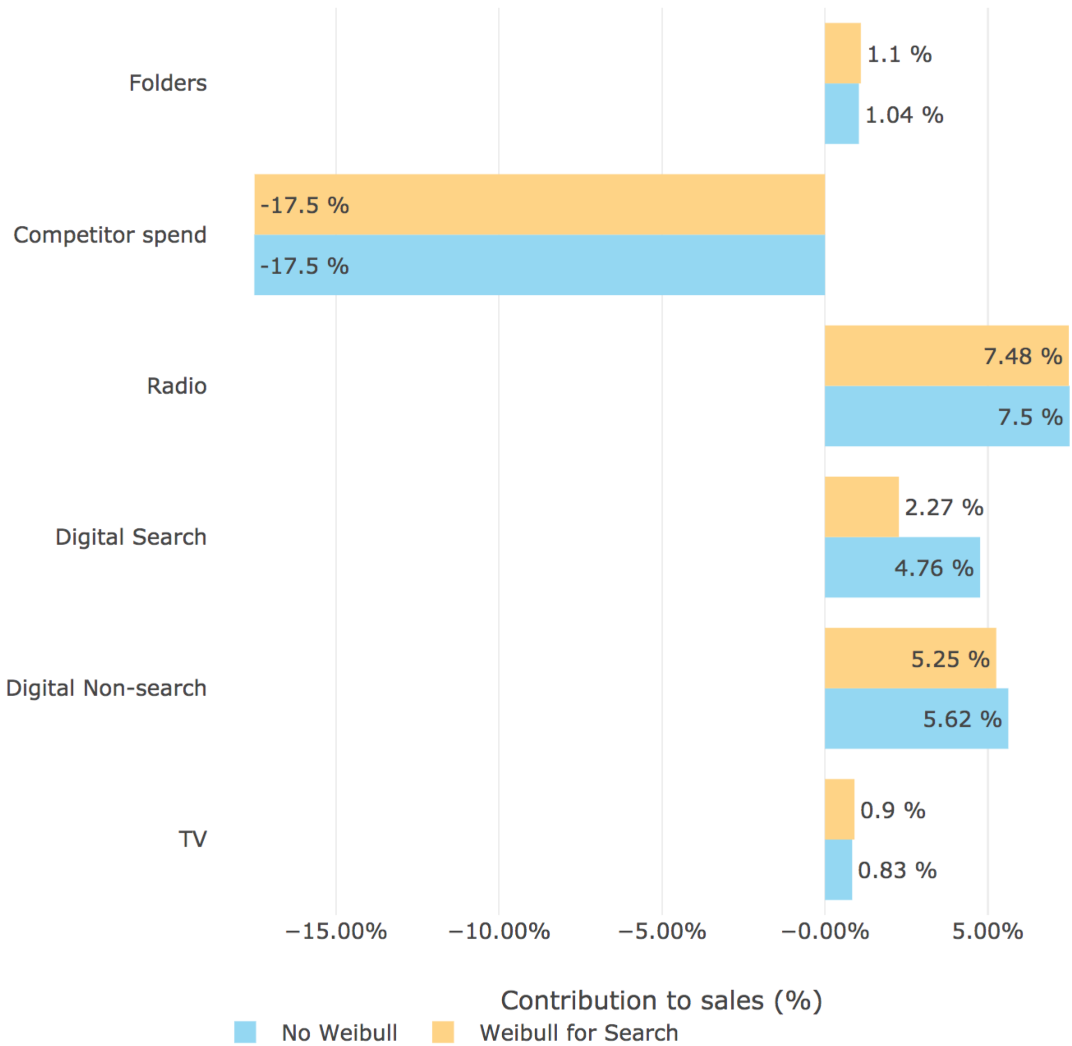

Figure 8. Ad contributions are compared for the department store data between the model without Weibull adstock transformations, and the model with Weibull adstock transformation for digital search. The base of both models include the weekly and yearly seasonalities, and weather regressors (temperature and precipitation). For the "no weibull" model the base had a positive contribution of 97.75%. For the "Weibull for search" model the base had a positive contribution of 100.5%.

Informal eating out restaurant data

As with the department store regressors, we first optimised the regressor transformations using only traditional adstock transformations (Table 1). Next, we optimised regressor transformations per ad channel including traditional and Weibull transformations. We found a slightly improved model performance (higher R^2 and lower MAPE) with Weibull transformations for digital search, digital non-search, OOH, competitor spend and TV. Only for radio no model performance improvement was found using Weibull transformations (Table 2, Figure 9). The best model performance was found when the optimised weibull transformations for digital search, digital non-search, OOH, competitor spend and TV were all included in the model.

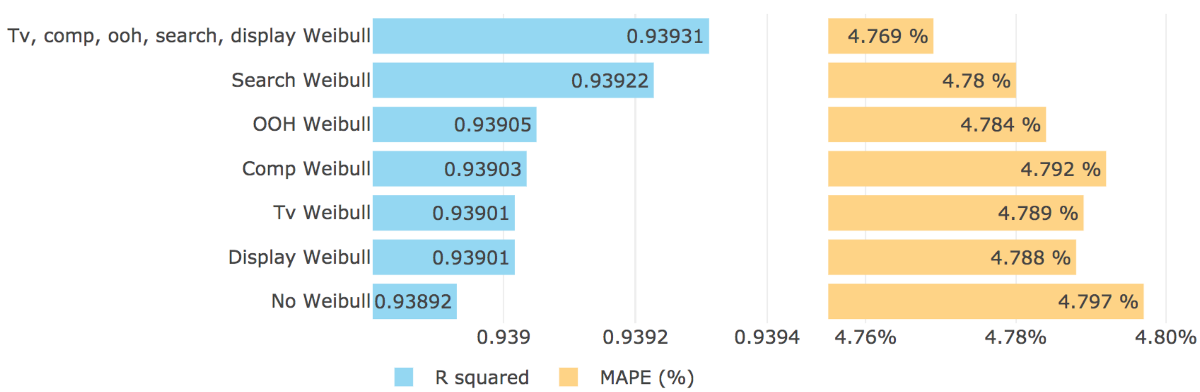

Figure 9. Prediction accuracy measures R^2 and MAPE shown for the 7 tested informal eating out models - note: 'comp' refers to competitor spend.

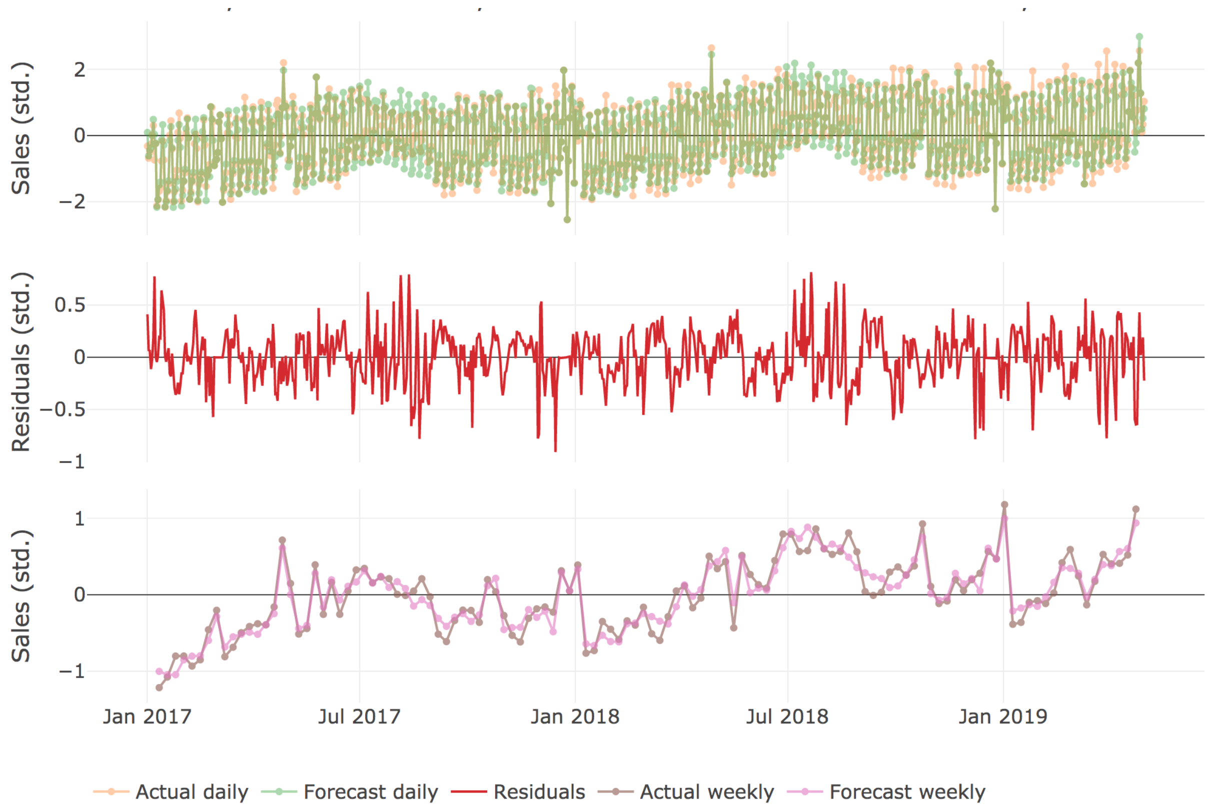

Figure 10. Standardised actuals, forecast and residuals of the informal eating out model with Weibull adstock for TV GRPs, competitor spend, OOH spend, digital search impressions and digital non-search impressions. We standardised by subtracting the average and dividing by the standard deviation of the forecasted sales.

When we compared regressor contributions for informal eating out between the no-Weibull and the best Weibull model (Figure 11), we found that negative contribution signs for competitor spend were amplified in the Weibull model. Conversely, tv and digital non-search showed a less positive contribution in this model. On the other hand, radio, OOH and digital search showed an increased contribution to sales in the Weibull model.

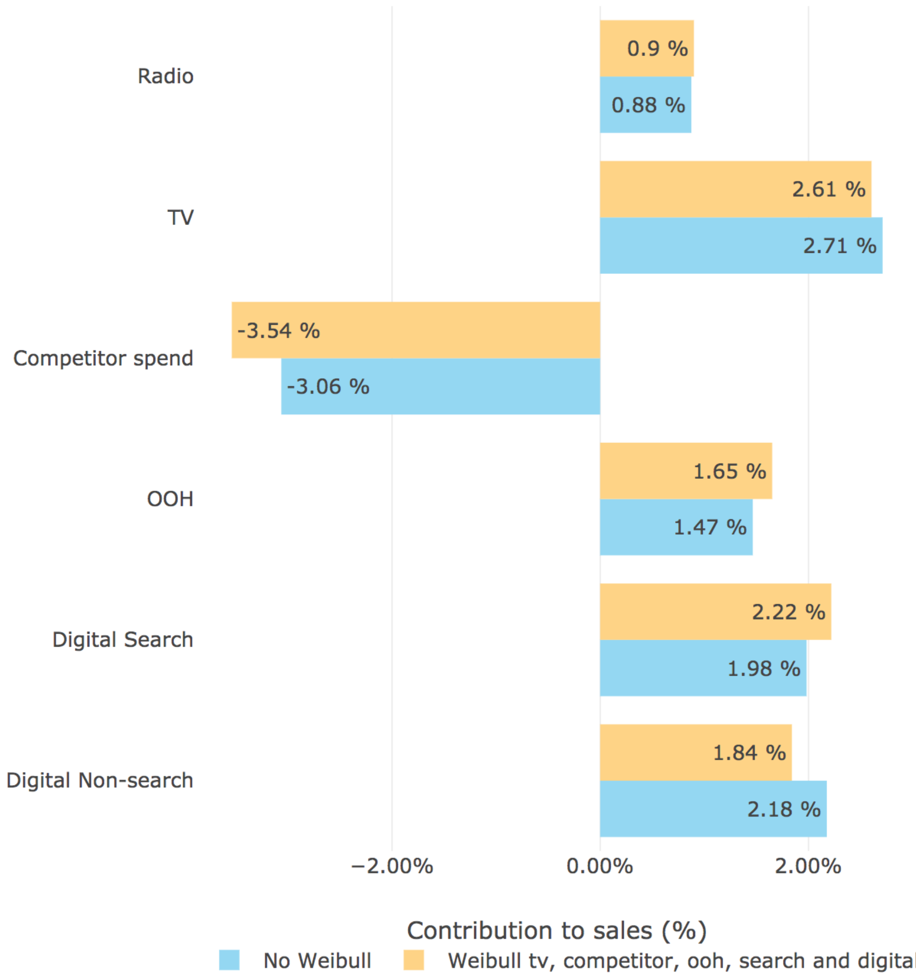

Figure 11. Ad contributions are compared between the informal eating out model without, and the model with (tv, competitor, OOH, digital search and digital non-search) Weibull adstock transformations. The base of both models include the weekly and yearly seasonalities, and weather regressors (temperature and precipitation). For the "no weibull" model the base had a positive contribution of 93.84%. For the Weibull model the base had a positive contribution of 94.32%.

Weibull transformations

As already outlined in the company specific result sections, Weibull transformations improved model performance for one or more regressors. For each of those regressors we show how the Weibull parameters k and window affect the R^2 of the resulting model fit (Figure 12 and 13). We found a pattern where optimal Weibull transformations tend to be in lower regions of the window parameter for digital search, competitor spend, digital non-search and OOH. However, for TV the best R^2 was found for a window of 8 days and a k-value of 8.

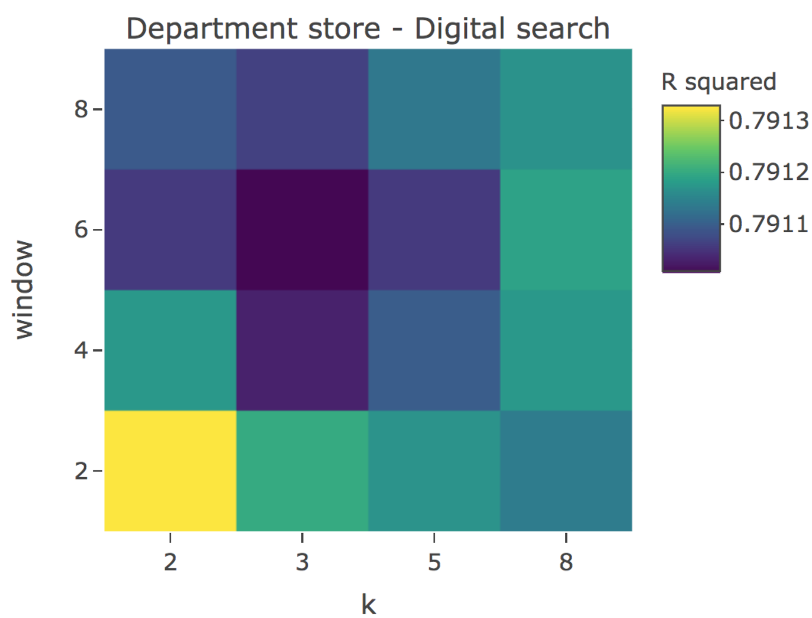

Figure 12. Heatmap of the Weibull parameters ‘k’ and ‘window’ for the department store regressor digital search.

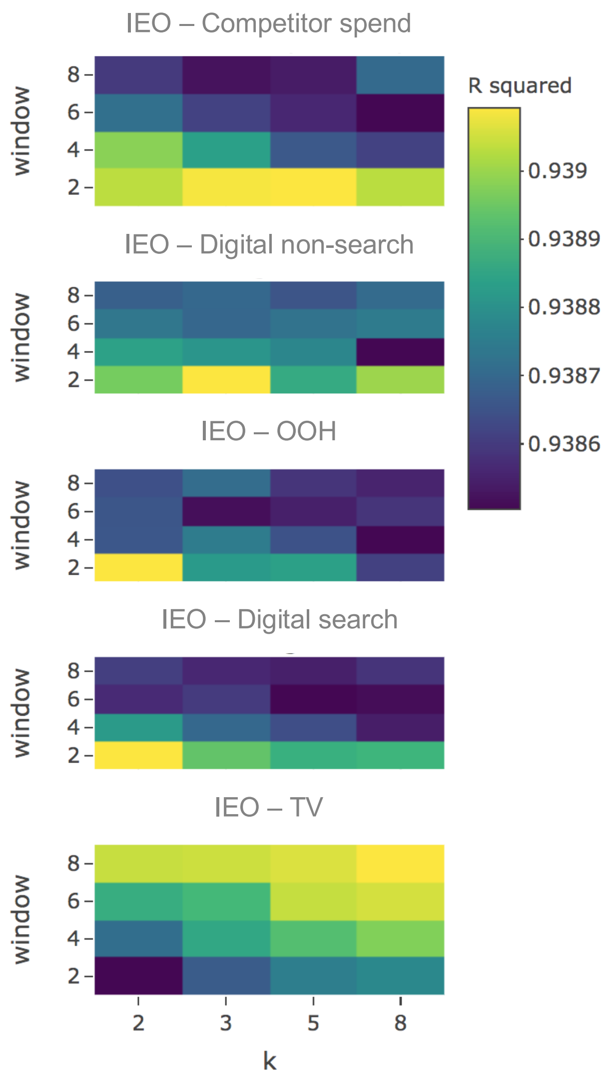

Figure 13. Heatmaps Weibull parameters ‘k’ and ‘window’ per informal eating out (IEO) regressor.

Discussion and Conclusions

Overall, minor improvements were found for executing Weibull adstock transformations on media variables, when using the R package Prophet for Marketing Mix Modeling. In this study we specifically zoomed in on applying these transformations for two different companies: a department store company and an informal eating out restaurant company.

These brands were not only different in terms of industry, but also in the context of modeling dependent variables; i.e. online sales for the department store and offline sales for the informal eating out restaurant.

In order to adequately measure whether utilising Weibull transformations results in better model performance, we specifically looked into explained variance (R^2) and Mean Average Prediction Errors (MAPE) as model accuracy metrics. To this end we first created a baseline model using only traditional decay rate adstock transformations. We subsequently evaluated whether Weibull transformations improve model performance, separately for each ad regressor. We finally created 1 extra model using Weibull transformations for all regressors that showed improvements in the previous step. This process was repeated for each brand.

We found that applying Weibull transformations for the department store only resulted in improvements for the digital search regressor. However, applying Weibull transformations for the informal eating out company improved performance for five out of the six considered regressors: TV, competitor spend, OOH, digital search and digital non-search.

Transformation parameters: Baseline vs. Weibull models

The adstock transformations that were found for the baseline models (without Weibull transformations) are shown in Table 1 and the Weibull parameters for the Weibull models are shown in Table 2 and Figures 12 and 13. For the department store Weibull model the digital search regressor saw a fairly large change compared to the baseline model, as the window decreased from 6 to 2 days. This means that the ad effect of digital search wears out after 3 instead of 7 days. For the informal eating out data we found less dramatic changes in the window for the Weibull models compared to the baseline model. For OOH and competitor spend the window changed from 4 to 2, TV changed from 6 to 8 and digital search and digital non-search remained unchanged at 2. So for digital search the Weibull transformation parameters were the same for informal eating out as for the department store. In addition, Figures 12 and 13 show that for the Weibull models OOH and the online ad channels (digital search and digital non-search) have a relatively shorter ad effect compared to TV, as the optimal k value and window are smaller. The same pattern can be found for the baseline models (Table 1), based on the decay rates and windows. For the department store this distinction is less pronounced, however the online channels (digital search and digital non-search) still have the shortest overall effects.

Furthermore, a first day correction (50%) was only found to improve the digital non-search, competitor spend and TV regressors in the informal eating out model. The department store Weibull model did not benefit from first day corrections. This could be related to the different sales variable that was used for this company, which was online transactions. For informal eating out offline sales, one might expect a delayed advertising effect, which could result in a higher effect on sales on the day after the ad exposure. However for department store online transactions one would not expect as much of a delayed effect as these sales do not depend on a customer visiting a physical store.

Lastly, it is noteworthy that for the department store the Weibull transformation for digital search resulted in much lower contribution for this regressor (2.27%) than in the baseline model (4.76%), while the other regressors were hardly affected (Figure 8). For informal eating out the effects on regressor contribution where much less dramatic (Figure 11).

Conclusion

Utilising Weibull adstock transformations and first day corrections slightly improve marketing mix modeling performance (R^2 and MAPE) for informal eating out (offline) and department store (online) sales, compared to traditional exponential adstock decay transformations. However, regressor contributions might see more substantial changes due to altered adstock transformations. Moreover, Weibull transformations only improved modeling performance for TV, competitor spend, OOH, digital search and digital non-search ad regressors for informal eating out (offline sales). For the department store (online sales) Weibull adstock only improved modeling performance for the digital search ad regressor. First day corrections where only found to improve modeling performance for the digital non-search, competitor spend and TV ad regressors for informal eating out. For department store online sales, first day corrections did not improve modeling. Mainly for marketing mix modeling of offline sales data one could consider to use first day corrections and Weibull adstock transformations to improve modeling performance.

Future research

When setting up a future study, a more ideal situation would be to include more brands per sector. This would allow for more insight into market segments as a whole.

Secondly, it was likely that some degree of selection bias was present. E.g. for paid search impressions, these individuals have already showed interest in a particular topic or product via a related query on a search engine. In other words, customers that already have an increased intent to buy something from a certain online shop, often first search for the webshop on Google and are exposed to the paid search impression. As such, an increase in online sales is automatically related to an increase in paid search impressions. Therefore one would expect its calculated contribution to sales to be biased as well. To statistically correct for selection bias, so called ‘selection sample models' could be used; e.g. Heckman's two-step selection model[5].

Furthermore, in this study we built an “adstock grid” (a grid containing all unique combinations of the included adstock parameters) to find the best variable transformations. In future research a wider range of parameter values might be included to allow for a more fine grained parameter optimisation. Moreover, instead of this brute force approach of trying all combinations, it might be fruitful to look into more flexible approximations like random hyperparameter search or more contemporary methods such as genetic or evolutionary algorithms[6]. These optimisation methods are based on natural

[5] “Models with sample selection” https://rpubs.com/wsundstrom/t_selection . Accessed 16 Sept 2019.

[6] “Introduction to Evolutionary Algorithms” https://towardsdatascience.com/introduction-to-evolutionary-algorithms-a8594b484ac . Accessed 16 Sept 2019.

selection in biology, where the “fittest” parameter combinations are combined to produce offspring for further optimisations. Both from an effectiveness as well as efficiency perspective these approaches could be fruitful.