VE492 Final Recitation Class

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Probability - Outline

- Random Variables
- Joint and Marginal Distributions
- Conditional Distribution
- Product Rule, Chain Rule, Bayes' Rule
- Inference
- (Conditional) Independence

Probability - Background knowledge

- X, Y independent if and only if $\forall x, y : P(x, y) = P(x)P(y)$
- X, Y are conditionally independent given Z if and only if:
  $X \perp\!\!\!\perp Y|Z, \quad \forall x, y, z : P(x, y|z) = P(x|z)P(y|z)$
- Conditional Probability: $P(x|y) = P(x, y)/P(y)$
- Product rule: $P(x, y) = P(x|y)P(y)$
- Chain rule: $P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i|X_1, ..., X_{n-1})$
- Sum rule (marginalization): $p(X) = \sum_y P(X, y)$
- Variant of sum rule: $p(X) = \sum_y P(X|y)P(y)$
- Bayes rule: $P(y|x) = p(x|y)p(y)/P(x)$

Quick Example: Bayes Rule

- $P(Z) = 0.02$ (zebra in 2% of images)
- $P(O|Z) = 0.8$ (true positive)
- $P(O|Z) = 0.1$ (false positive)
- We want to calculate: $p(Z|O)$
- Apply Bayes rule!

$$p(Z|O) = \frac{p(O|Z)p(Z)}{p(O)} = \frac{p(o|z)\,p(z)}{\sum_z p(o|z)\,p(z)}$$

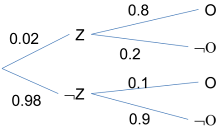
Conditioning

$$= \frac{p(O|Z)p(Z)}{p(O|Z)p(Z) + p(O|\neg Z)p(\neg Z)}$$

$$= \frac{0.8 \times 0.2}{0.8 \times 0.02 + 0.1 \times 0.98}$$

$$= \frac{0.016}{0.016 + 0.098}$$

$$= 0.1404$$



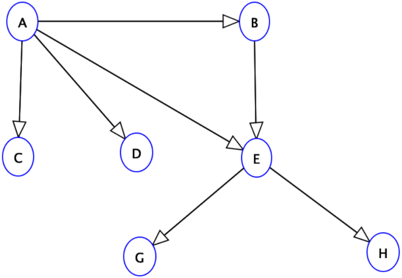
Bayes Nets - Outline

- Representation
- Conditional Independence
  - D-separation
  - Active / Inactive Paths
- Probabilistic Inference
  - Enumeration
  - Variable elimination
  - Probabilistic inference
  - Sampling

Bayes Nets - Sample Questions

- How to get formula of joint distribution from BN graphs?
- How to count the degree of freedom of BN graphs?
- How to run variable elimination? What is the best ordering for VE? What is the largest generated factor? What is the cutset?
- What are the difference of the four sampling methods? What is the time complexity?

Example: Small Bayes Net



- Provide the formula of the joint distribution over all the variables given by the Bayes net. Solution: $P(A|C)P(B)P(C)P(D|C)P(E|B,D)P(F|D,E)P(H|B)$
- Provide the number of degrees of freedom of the BN.
- Run VE to compute to compute $P(A|H=h)$. Provide the list of the sizes (i.e., number of variables) of the factors obtained at the end of each iteration in VE.

Hidden Markov Models - Outline

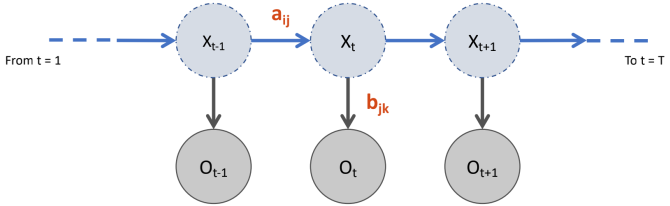
- Markov Models and Hidden Markov Models
- Forward algorithm
- Viterbi algorithm
- Particle filtering
- Dynamic Bayesian Network

Hidden Markov Models - Sample Questions

- How to compute stationary distribution?
- How to do filtering / prediction / smoothing / explanation?
- What is particle filtering?
- How to distinguish Dynamic Bayes Nets and Bayes Nets?

HMM Terminology

Time instants	t *in* {1,2 . . . T}
Hidden States / States / Emitters	X_t
Outputs / Emissions / Observations / Visible States	O_t
All possible states / states set	X_t *in* {1,2 . . . N}
All possible emissions / emissions set	O_t *in* {1,2 . . . K}
Initial state distribution / Initial state probabilities	p_i in q *or* π_i in π
Transition probabilities / State transition probabilities	a_{ij} *in* row-stochastic matrix A
Emission probabilities / Observation probabilities	b_{jk} *in* row-stochastic matrix B

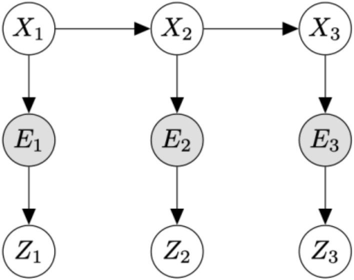


Example - Adapt the Forward Algorithm

Filtering Algorithm

$$P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1}) \sum_{x_t} P(x_t|e_{1:t}) P(X_{t+1}|x_t)$$

- Adapt the forward algorithm to this variant of HMM.
- Step 1. Predict step
- Step 2. Update



Example - Adapt the Forward Algorithm

- 1. Predict step

$$P(x_{t+1}|e_{1:t}) = \sum_{x_t} P(x_{t+1}|x_t)P(x_t|e_{1:t})$$

- 2. Update

$$\begin{aligned} P(x_{t+1}|e_{1:t+1}) &\propto \sum_{z_{t+1}} P(x_{t+1}|e_{1:t})P(e_{t+1}|x_{t+1})P(z_{t+1}|e_{t+1}) \\ &= \alpha P(x_{t+1}|e_{1:t})P(e_{t+1}|x_{t+1}) \end{aligned}$$

Hidden Markov Models - Background knowledge

Forward Algorithm (rewrite)

$$\alpha_t(i) = [\sum_{j=1}^{N} \alpha_{t-1}(j)a_{ji}]b_i(O_t)$$

Viterbi Algorithm (rewrite)

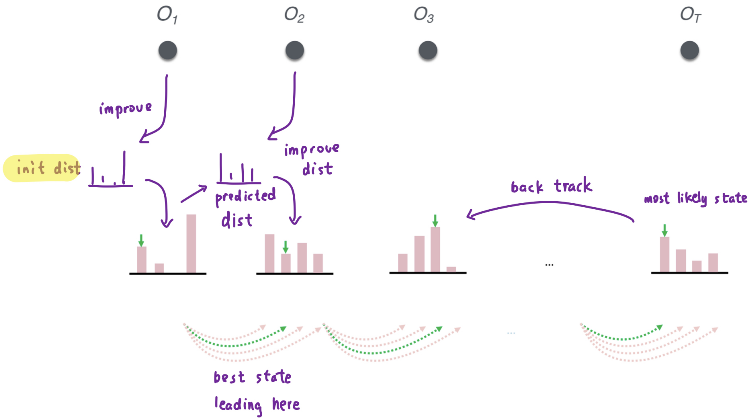
$$\delta_t(i) = max_{j\in\{1,\dots,N\}}[\delta_{t-1}(j)a_{ji}b_i(O_t)]$$

- a_{ij}: state transition probabilities, b_{jk}: emission probabilities
- Forward Algorithm can be used to predict the current state given all of the current and past evidence.
- Viterbi algorithm can be used to calculate the most likely state sequence, namely $argmax_{X_{1:T}} P(X_{1:T}|O_{1:T}, \lambda)$, where $\lambda = \{A, B, \pi\}$.

Please refer to the lecture slides if this leads to any confusion!

Example: Viterbi Algorithm

- Viterbi: calculate most likely state sequence



Example: Viterbi Algorithm

A: $a_{ji} = P(X_{t+1} = j \mid X_t = i)$

$X_t \mid X_{t+1}$	A	B	H	S
A	0.6	0.1	0.1	0.2
B	0.0	03	0.2	0.5
H	0.8	0.1	0.1	0.1
S	0.2	0.0	0.1	0.7

B: $b_{ik} = P(O_t = k \mid X_t = i)$

$X_t \mid O_t$	p	e	b	l
A	0.6	0.2	0.1	0.1
B	0.1	0.4	0.1	0.4
H	0.0	0.0	0.7	0.3
S	0.0	0.0	0.1	0.9

$\pi = P(X_1 = i)$:

A	B	H	S
0.5	0.0	0.0	0.5

Observations :

$o_{1:4} = \{ b, p, l, e \}$

Find:

Most likely hidden state sequence: $X^*_{1:4}$

Naive Bayes model for classification

- Naive Bayes assumes all features are independent effects of the label.
- Naive Bayes for text: $P(Y, W_1, ..., W_n) = P(Y)\prod_i P(W_i|Y)$, where W_i is the word at position i, Y $\in$ {spam, ham}.

Maximum likelihood estimation

- Given the observed set D, find θ to maximize the probability of D
- $\theta = \underset{\theta}{argmax} P(D|\theta)$
- Set the derivative of $P(D|\theta)$ with respect to θ to zero, and solve for θ.

Laplace smoothing

- pretend that we have seen every outcome k extra times.
- $P_{Lap,k} = \frac{c(x)+k}{N+k|X|}$

Discriminative Learning

Linear classifiers

feature values(inputs), weights (learned), activation (sum, $\text{activation}_w(x) = \sum_i w_i \phi_i(x)$)

Binary perceptron learning process

- start with weights=0
- for each training instance (x,y*): classify with current weights, no change if correct else adjust the weight vector by adding or subtracting the feature vector (subtract if y*=-1).

Discriminative Learning

Multiclass perceptron learning process

- start with weights=0
- pick up training examples one by one
- predict with current weights $\hat{y} = argmax_y(w_y \cdot \phi(x))$, no change if correct. Otherwise, lower score of wrong answer and raise score of right answer $w_{\hat{y}} = w_{\hat{y}} - \phi(x)$, $w_{y*} = w_{y*} + \phi(x)$

Probabilistic Perceptron

- softmax function

Discriminative Learning

Learning by gradient descent

initialize w (e.g., randomly)

repeat for K iterations:

for each example (x_i, y_i):

compute gradient $\Delta_i = -\nabla_w \log P_w(y_i|x_i)$

compute gradient $\nabla_w \mathcal{L} = \sum_i \Delta_i$

$w \leftarrow w - \alpha \nabla_w \mathcal{L}$

$$\frac{d}{dw_y} \log P_w(y_i|x_i) = x_i(I(y = y_i) - P(y|x_i))$$

- α: learning rate —- hyperparameter that needs to be chosen carefully
- How? Try multiple choices
  - Crude rule of thumb: update should change w by about 0.1-1%

Propositional Logic

Propositional Logic

- Which of the following are correct?
  - False $\models$ True
  - True $\models$ False
  - $(A \land B) \models (A \Leftrightarrow B)$
  - $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$
  - $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable
  - $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable
  - $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C

Except for the second sentence, which is incorrect, others are all correct.

Propositional Logic

$(A \wedge B) \models (A \Leftrightarrow B)$

$(A \wedge B) \models (A \Leftrightarrow B)$ is incorrect if $(A \wedge B)$ is true and $(A \Leftrightarrow B)$ is false. $(A \wedge B)$ is true when both A and B are true while $(A \Leftrightarrow B)$ is false when (A is true, B is false) or (B is true, A is false). Therefore, $(A \wedge B) \models (A \Leftrightarrow B)$ is correct.

This forth sentence can be proved to be correct using the same method as the third sentence.

$(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.

When A=True, B=False, the sentence is correct.

$(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.

When both A and B is true or false, the sentence is correct.

Propositional Logic

$(A \Leftrightarrow B) \Leftrightarrow C$ is correct when (A,B,C)=(T,T,T),(F,F,T),(T,F,F),(F,T,F).
$(A \Leftrightarrow B)$ is correct when (A,B,C)=(T,T,T),(T,T,F),(F,F,T),(F,F,F).
There are 4 models for each sentence.

Propositional Logic

Application: Propositional Logic

- Minesweeper: Let $X_{i,j}$ be true iff square $[i,j]$ contains a mine.
  - Write down the assertion that exactly two mines are adjacent to [1,1] as a sentence involving some logical combination of $X_{i,j}$ propositions.
  - Generalize your assertion by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines
  - Explain precisely how an agent can use DPLL to prove that a given square does (or does not) contain a mine.

Exactly two mines are adjacent to [1,1]: $X_{1,2} \wedge X_{2,1}$

The CNF sentence is a conjunction of all disjunctions, each of which indicates that there are exactly k neighbors containing mines.

Write the sentence into conjunctive normal form (CNF). Use the DPLL to solve the satisfiability of the CNF.

Advice for Final Exam

It's likely that you'll be asked to translate sentences in first-order logic using some given predicates.

The End