

# VG441 Problem Set 2

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## Problem 1

From the question, we could conclude that

$$\begin{aligned} \lambda &= 500 \text{ tons/day} \\ K &= 2250 \text{ per order} \\ i &= 0.25/365 \text{ per day} \end{aligned} \quad c = \begin{cases} \$1490 \text{ per ton,} & Q < 1200 \\ \$1220 \text{ per ton,} & 1200 \leq Q < 2400 \\ \$1100 \text{ per ton,} & Q \geq 2400 \end{cases} \quad (1)$$

## All-Units Discount

For this structure, we could generate the  $g(Q)$ , and get that:

$$\begin{aligned} g_0(Q) &= 1490 * 500 + 2250 * 500/Q + 0.25/365 * 1490/2 * Q \\ g_1(Q) &= 1220 * 500 + 2250 * 500/Q + 0.25/365 * 1220/2 * Q \\ g_2(Q) &= 1100 * 500 + 2250 * 500/Q + 0.25/365 * 1100/2 * Q \end{aligned} \quad (2)$$

And we could draw the graph like: and the  $Q_j^*$  is:

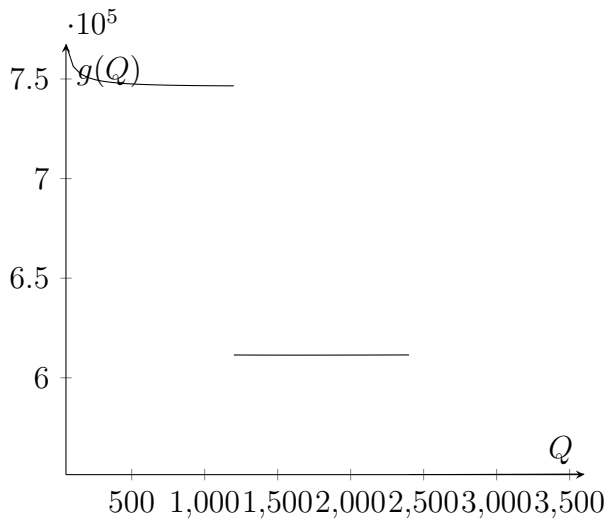


Fig. 1: Total cost for all-units quantity discount structure

$$\begin{aligned} Q_0^* &= 1484.8 \\ Q_1^* &= 1640.9 \\ Q_2^* &= 1728.1 \end{aligned} \quad (3)$$

Among these value,  $Q_1^*$  is feasible, which is:

$$g_1(1640.9) = 611371$$

Then we calculate the cost of breakpoints to the right of  $Q_1^*$  and get:

$$g_2(2400) = 551372$$

Therefore, the optimal order quantity is  $Q = 2400$ , which incurs a purchase cost of 1100 and a total daily cost of \$551372

## Incremental Discount

For this structure, we could calculate that :

$$\begin{aligned}\bar{c}_1 &= 1490 * 1200 - 1220 * 1200 = 324000 \\ \bar{c}_2 &= 1490 * 1200 + 1220 * 1200 - 1100 * 2400 = 612000 \\ g_0(Q) &= 1490 * 500 + 2250 * 500/Q + 0.25/365 * 1490/2 * Q \\ g_1(Q) &= 1220 * 500 + 0.25/365 * 324000/2 + (2250 + 324000) * 500/Q + 0.25/365 * 1220/2 * Q \\ g_2(Q) &= 1100 * 500 + 0.25/365 * 612000/2 + (2250 + 612000) * 500/Q + 0.25/365 * 1100/2 * Q\end{aligned}\quad (4)$$

And we could draw the graph like: Next, we calculate the  $Q_j^*$  for each j:

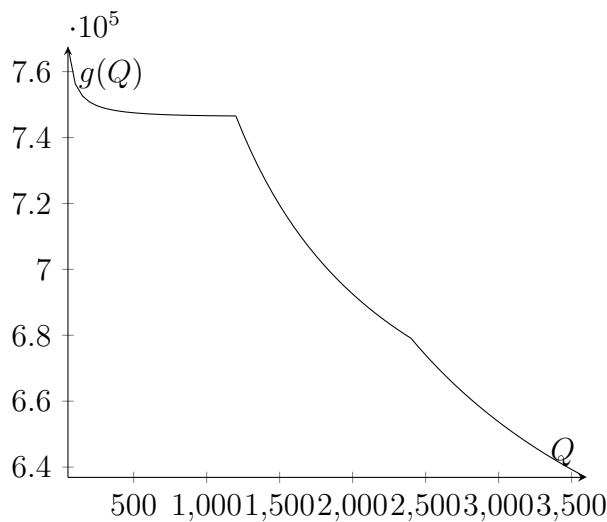


Fig. 2: Total cost for incremental discount structure

$$\begin{aligned}Q_0^* &= 1484.8 \\ Q_1^* &= 19759.3 \\ Q_2^* &= 28553\end{aligned}\quad (5)$$

Therefore, only  $Q_2^*$  is available.

$$g_2(28553) = 571722.169$$

Since there are no breakpoints to the right, therefore, the optimal order quantity is  $Q = 28553$ , which incurs a total amount daily cost of \$571722