VG441 Problem Set 3

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Problem 1

1. Formulate the set cover problem as a MILP

Decision Variables:
Our choices of sets: $x_i \in \{0, 1\}, \quad i \in \{1, 2, ..., m\}$.
elements and sets: $s_{mn} \in \{0, 1\}$, if set m has element n of V, then $s_{mn} = 1$, otherwise $s_{mn} = 0$
Objective:
Minimize $\sum_m^1 x_i$
Constraints:
$(SX)_n \geq 1$ for $\forall n$
$\sum_1^m x_i \geq 1$

2. Solve the problem on Page 4 of LEC015 using Gurobi

After running the gurobi codes, we get the solution that:

```
Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (linux64)
Thread count: 4 physical cores, 8 logical processors,
using up to 8 threads
Optimize a model with 8 rows, 5 columns and 13 nonzeros
Model fingerprint: 0x8fe9a7d4
Coefficient statistics:
Matrix range     [1e+00, 1e+00]
Objective range  [1e+00, 1e+00]
Bounds range     [0e+00, 0e+00]
RHS range        [1e+00, 1e+00]
Presolve removed 8 rows and 5 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
0     4.0000000e+00   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.00 seconds
Optimal objective  4.000000000e+00

Variable            X
```

```
————————————————————————————————
decision var[0]          1
decision var[2]          1
decision var[3]          1
decision var[4]          1

Process finished with exit code 0
```

Therefore, the solution is: we choose set 1, 3, 4, 5

Problem 2

We want to prove that greedy algorithm provides the optimal solution for the Fractional Knaspack Problem.
Suppose there are n items, each item i has a value v_i and size s_i
The capacity of backpack is B
We could use contradiction to prove:
Assume that there exists a solution SOL of Fractional Knaspack Algorithm which is not optimal,
Suppose SOL = $\{l_1, l_2, ..., l_n\}$, and the optimal solution is $\{o_1, o_2, ..., o_n\}$, s_i and o_i mean whether the ith item is chosen, and the items are ordered by $\frac{\text{value}}{\text{size}}$
According to the definition of optimal solution:

$$\Rightarrow \sum_{i=1}^{n} l_i v_i < \sum_{i=1}^{n} o_i v_i$$

According to the principal of greedy algorithm, there exists a certain a .For all $i \geq a$, $l_i \geq o_i$.

$$\begin{aligned} \text{Value}(SOL) - \text{Value}(Optimal) &= \sum_{i=1}^{n} (l_i - o_i)(\frac{v_i}{s_i})s_i & (1) \\ &= \sum_{i=1}^{a} (1 - o_i)(\frac{v_i}{s_i})s_i + \sum_{i=a+1}^{n} (-o_i)(\frac{v_i}{s_i})s_i & (2) \end{aligned}$$

Since for roughly equal size, value of every unit of size of optimal solution is less than the value of SOL,if SOL is not equal to the optimal solution, than the value of SOL is greater than the value of optimal solution, which contradicts. Therefore, optimal solution must be equal to the greedy algorithm solution.