VG441 Problem Set 2

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Problem 1

From the question, we could conclude that

$$\begin{array}{rcl} \lambda & = & 500 \text{ tons/day} \\ K & = & 2250 \text{ per order} \\ i & = & 0.25/365 \text{ per day} \end{array} \quad c = \begin{cases} \$1490 \text{ per ton}, & Q < 1200 \\ \$1220 \text{ per ton}, & 1200 \leq Q < 2400 \\ \$1100 \text{ per ton}, & Q \geq 2400 \end{cases} \tag{1}$$

All-Units Discount

For this structure, we could generate the $g(Q)$, and get that:

$$\begin{array}{rcl} g_0(Q) & = & 1490 * 500 + 2250 * 500/Q + 0.25/365 * 1490/2 * Q \\ g_1(Q) & = & 1220 * 500 + 2250 * 500/Q + 0.25/365 * 1220/2 * Q \\ g_2(Q) & = & 1100 * 500 + 2250 * 500/Q + 0.25/365 * 1100/2 * Q \end{array} \tag{2}$$

And we could draw the graph like: and the Q_j^* is:

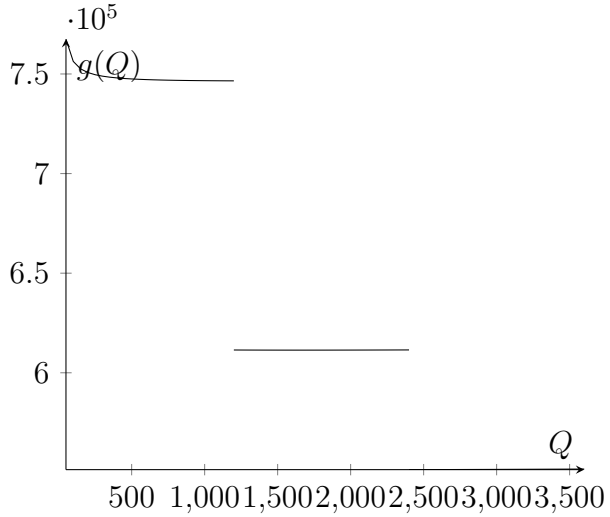

Fig. 1: Total cost for all-units quantity discount structure

$$\begin{array}{rcl} Q_0^* & = & 1484.8 \\ Q_1^* & = & 1640.9 \\ Q_2^* & = & 1728.1 \end{array} \tag{3}$$

Among these value, Q_1^* is feasible, which is:

$$g_1(1640.9) = 611371$$

Then we caculare the cost of breakpoints to the right of $Q_1^*$and get:

$$g_2(2400) = 551372$$

Therefore, the optimal orfer quantity is $Q = 2400$, which incurs a purchase cost of 1100 and a total daily cost pf $551372

Incremental Discount

For this instructure, we could calculate that :

$$\begin{aligned} \bar{c_1} &= 1490 * 1200 - 1220 * 1200 = 324000 \\ \bar{c_2} &= 1490 * 1200 + 1220 * 1200 - 1100 * 2400 = 612000 \\ g_0(Q) &= 1490 * 500 + 2250 * 500/Q + 0.25/365 * 1490/2 * Q \\ g_1(Q) &= 1220 * 500 + 0.25/365 * 324000/2 + (2250 + 324000) * 500/Q + 0.25/365 * 1220/2 * Q \\ g_2(Q) &= 1100 * 500 + 0.25/365 * 612000/2 + (2250 + 612000) * 500/Q + 0.25/365 * 1100/2 * Q \end{aligned} \tag{4}$$

And we could draw the graph like: Next, we caculate the Q_j^* for each j:

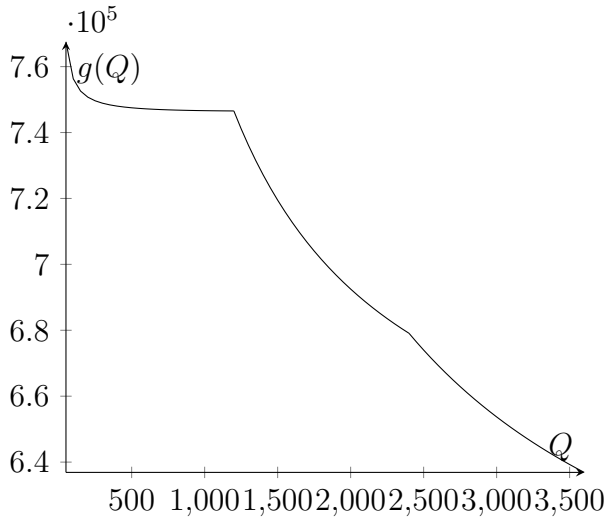

Fig. 2: Total cost for incremental discount structure

$$\begin{aligned} Q_0^* &= 1484.8 \\ Q_1^* &= 19759.3 \\ Q_2^* &= 28553 \end{aligned} \tag{5}$$

Therefore, only Q_2^* is available.

$$g_2(28553) = 571722.169$$

SInce there are no breakpoints to the right, therefore, the optimal order quantity if $Q = 28553$, which incurs a total amount daily cost is $571722