

VG441 Problem Set 3

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Problem 1

1. Formulate the set cover problem as a MILP

Decision Variables:

Our choices of sets: $x_i \in \{0, 1\}$, $i \in \{1, 2, \dots, m\}$.

elements and sets: $s_{mn} \in \{0, 1\}$, if set m has element n of V , then $s_{mn} = 1$, otherwise $s_{mn} = 0$

Objective:

Minimize $\sum_m x_i$

Constraints:

$(SX)_n \geq 1$ for $\forall n$

$\sum_1^m x_i \geq 1$

2. Solve the problem on Page 4 of LEC015 using Gurobi

After running the gurobi codes, we get the solution that:

```
1 Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
2 Optimize a model with 8 rows, 5 columns and 13 nonzeros
3 Model fingerprint: 0x74c2bfc5
4 Variable types: 0 continuous, 5 integer (5 binary)
5 Coefficient statistics:
6 Matrix range      [1e+00, 1e+00]
7 Objective range   [1e+00, 1e+00]
8 Bounds range      [1e+00, 1e+00]
9 RHS range         [1e+00, 1e+00]
10 Found heuristic solution: objective 4.0000000
11 Presolve removed 8 rows and 5 columns
12 Presolve time: 0.00s
13 Presolve: All rows and columns removed
14
15 Explored 0 nodes (0 simplex iterations) in 0.00 seconds
16 Thread count was 1 (of 8 available processors)
17
18 Solution count 1: 4
19
20 Optimal solution found (tolerance 1.00e-04)
```

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21 Best objective 4.000000000000e+00, best bound 4.000000000000e+00, gap 0.000
22
23 Variable          X
24 -----
25 decision var[1]    1
26 decision var[2]    1
27 decision var[3]    1
28 decision var[4]    1
29
30 Process finished with exit code 0

```

Therefore, the solution is: we choose set 1, 3, 4, 5

Problem 2

We want to prove that greedy algorithm provides the optimal solution for the Fractional Knapsack Problem.

Suppose there are n items, each item i has a value v_i and size s_i

The capacity of backpack is B

We could use contradiction to prove:

Assume that there exists a solution SOL of Fractional Knapsack Algorithm which is not optimal,

Suppose $SOL = \{l_1, l_2, \dots, l_n\}$, and the optimal solution is $\{o_1, o_2, \dots, o_n\}$, s_i and o_i mean whether the i th item is chosen, and the items are ordered by $\frac{\text{value}}{\text{size}}$

According to the definition of optimal solution:

$$\Rightarrow \sum_{i=1}^n l_i v_i < \sum_{i=1}^n o_i v_i$$

According to the principal of greedy algorithm, there exists a certain a . For all $i \geq a$, $l_i \geq o_i$.

$$\text{Value}(SOL) - \text{Value}(Optimal) = \sum_{i=1}^n (l_i - o_i) \left(\frac{v_i}{s_i} \right) s_i \quad (1)$$

$$= \sum_{i=1}^a (1 - o_i) \left(\frac{v_i}{s_i} \right) s_i + \sum_{i=a+1}^n (-o_i) \left(\frac{v_i}{s_i} \right) s_i \quad (2)$$

Since for roughly equal size, value of every unit of size of optimal solution is less than the value of SOL, if SOL is not equal to the optimal solution, then the value of SOL is greater than the value of optimal solution, which contradicts. Therefore, optimal solution must be equal to the greedy algorithm solution.