VG441 Final

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Problem 1 (Traveling Salesman Problem)

Task 1: Run double tree algorithm on paper.

1. we draw a graph based on the table, which isFig.1:

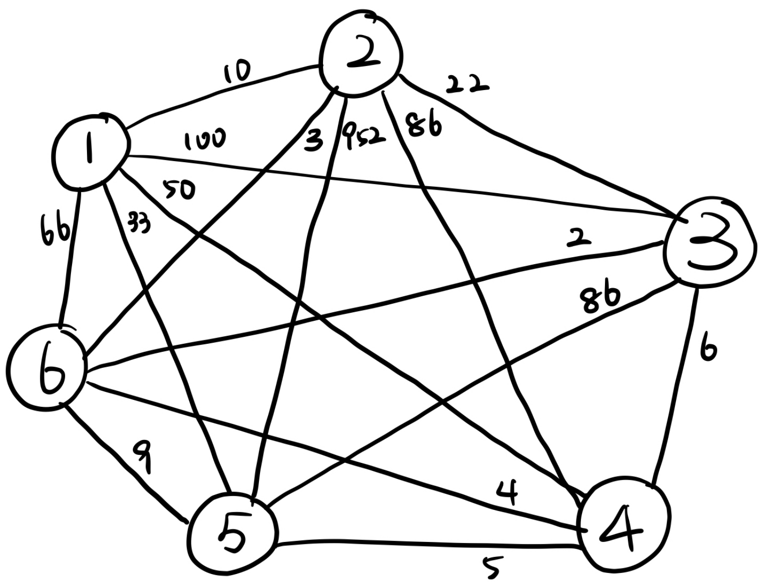

Fig. 1: Original Graph

2. We find an MST by kruskal on Fig.2:

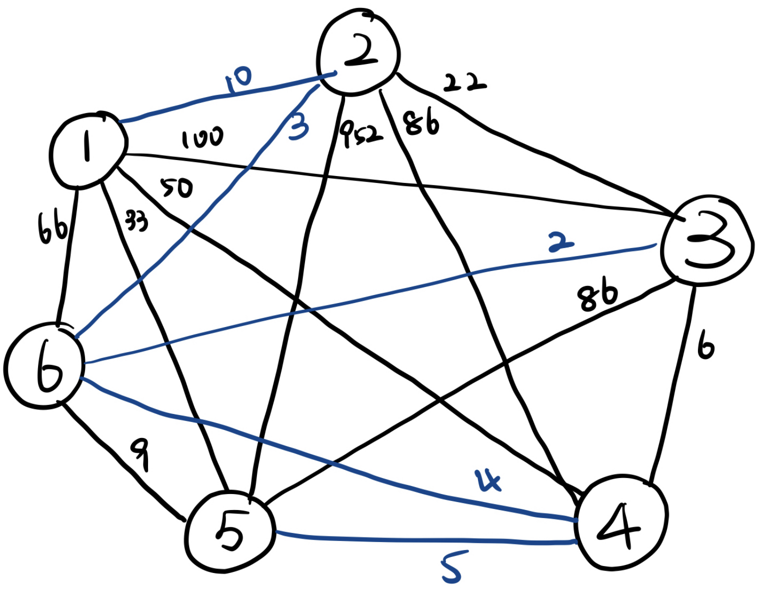

Fig. 2: MST

and if we use (edge, distance) to express, the MST is: (6-2,3),(6-3,2),(6-4,4),(2-1,10),(4-5,5)

3. Double edges of MST and convert it into a tree as fig.3

According to DFS, we could find a Eulerian path:

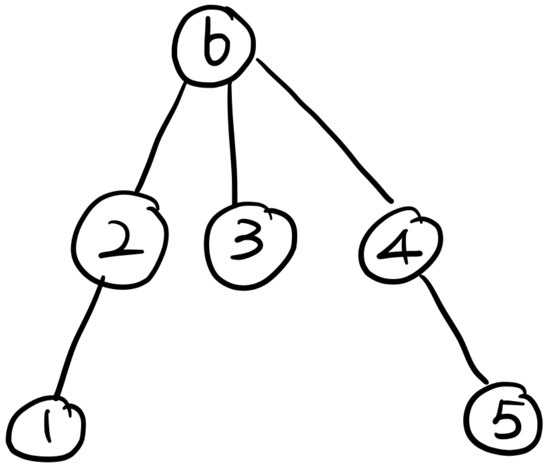

Fig. 3: tree

$$1 \to 2 \to 6 \to 3 \to 6 \to 4 \to 5 \to 4 \to 6 \to 2 \to 1$$

4. Then we shortcut visited nodes:

$$1 \to 2 \to 6 \to 3 \to 4 \to 5 \to 1$$

Therefore, the output is $1 \to 2 \to 6 \to 3 \to 4 \to 5 \to 1$ with cost 59

Task 2: Run Christofides' algorithm on paper. (Try eyeballing minimum cost matching solution.)

For the Christofides' algorithm, the first 2 steps are same to the double tree algorithm.
3. For the MST we find, the set of odd degree vertices is:

$$\{1, 3, 5, 6\}$$

Based on this, we find the min-weight-matching K like fig.4:

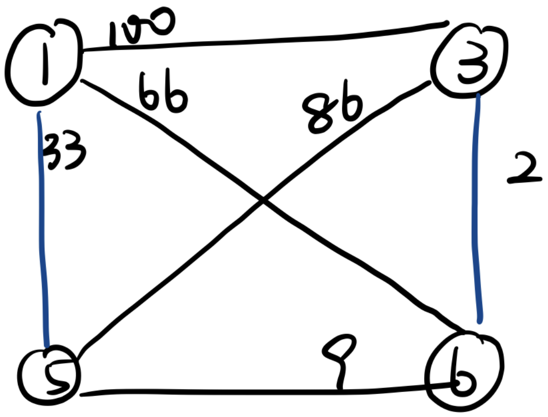

Fig. 4: Min-weight-matching

And we add K to M like fig.5:

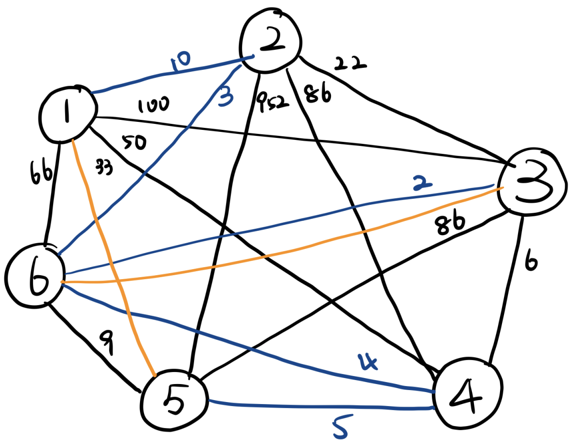

Fig. 5: M+K

4. On the basis of fig.5, we find Eulerian path:

$$3 \to 6 \to 2 \to 1 \to 5 \to 4 \to 6 \to 3$$

5. Then we shortcutting W:

$$3 \to 6 \to 2 \to 1 \to 5 \to 4 \to 3$$

This is the output, and the cost is 59

Problem 2 (Knapsack)

Task 1: Run the exact dynamic program (ExactKS) on paper to solve this problem.

Let T[i,w] be the min-size of subset $S \subseteq \{1, ..., i\}$
Let $v_{max} = 6$
$w \in \{0, 24\}$ Therefore, by iteration, we could build a table:

i	w	T
1	0	0
1	4	3
2	0	0
2	4	3
2	8	6
3	0	0
3	4	3
3	6	8
3	8	6
3	10	11
3	14	14
4	0	0
4	4	3
4	5	5
4	6	8
4	8	6
4	9	8
4	10	11
4	11	13
4	13	11
4	14	14
4	15	16
4	19	19

According to the table, since the bag size is 8, we could know that the best choice is {4,9,8}.
The maximum value of w is 9, which means we choose item 1 or 2 and item 4.

Task 2: Run the simple greedy algorithm (ranking via vi/si) and show that it is not optimal.

First, we rank the item with v/s:

$$\frac{v_1}{s_1} = 4/3, \frac{v_2}{s_2} = 4/3, \frac{v_4}{s_4} = 1, \frac{v_3}{s_3} = 3/4$$

And according to the greedy algorithm, we choose item 1 and item 2, in this way, the value is 8.
Since according to the dynamic program, there is a solution with value of 9, which is greater than 8, therefore, the greedy algorithm is not optimal.

Problem 3 (Minimum Cost Set Cover)

Task 1: Run this greedy algorithm on paper.

In the 1st iteration,

$$\text{ratio}_1 = \frac{6}{5} \quad \text{ratio}_2 = 3 \quad \text{ratio}_3 = 1$$

Therefore, we choose S_3 in this iteration
In the 2nd iteration,

$$\text{ratio}_1 = 2 \quad \text{ratio}_2 = 3$$

Therefore, we choose S_1 in this iteration
And in the 3rd iteration, only S_2 left, and the ratio is 7.5, so we choose S_2
Therefore, we will choose S_1,S_2,S_3 , the total cost is 28.

Task 2: Is your greedy solution optimal? Can you eyeball a better solution?

It is not optimal, because if I choose S_2,S_3, it could cover all of the set and the cost is only 22, which is smaller than the solution of greedy algorithm.