LEC006 Inventory Management I

VG441 SS2021

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Fundamental Tradeoff

Overstock

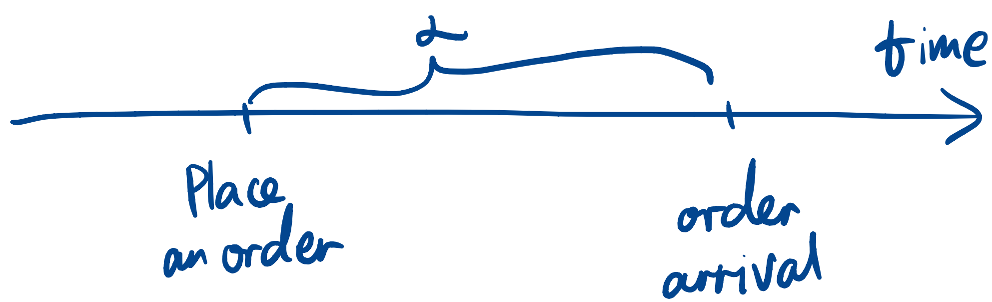
Understock

Inventory Turnover

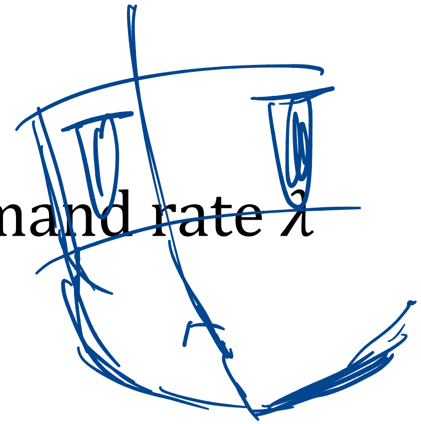
$$\text{Inventory Turnover Ratio Formula} = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory}}$$

Some Terminologies

- Demand rate
- Lead time important factor
- Quantity discount
- Review type
  - periodic-review
  - continuous-review
- Planning horizon length of planning season
- Stockout type
  - backlogging (back ordering)
  - lost-sales
- Service levels
- Fixed costs
- Perishability
- ……



Deterministic Inventory



INPUT:

- Constant deterministic demand rate λ
- No stockout is allowed
- Zero lead time
- Fixed cost K per order regarding the size of order } total ordering cost of q units
- Purchase cost c per unit
- Inventory hold cost h per unit per unit of time

存货

OUTPUT:

The optimal ordering strategy

Deterministic Inventory

INPUT:

- Constant deterministic demand rate λ
- No stockout is allowed
- Zero lead time
- Fixed cost K per order
- Purchase cost c per unit
- Inventory hold cost h per unit per unit of time

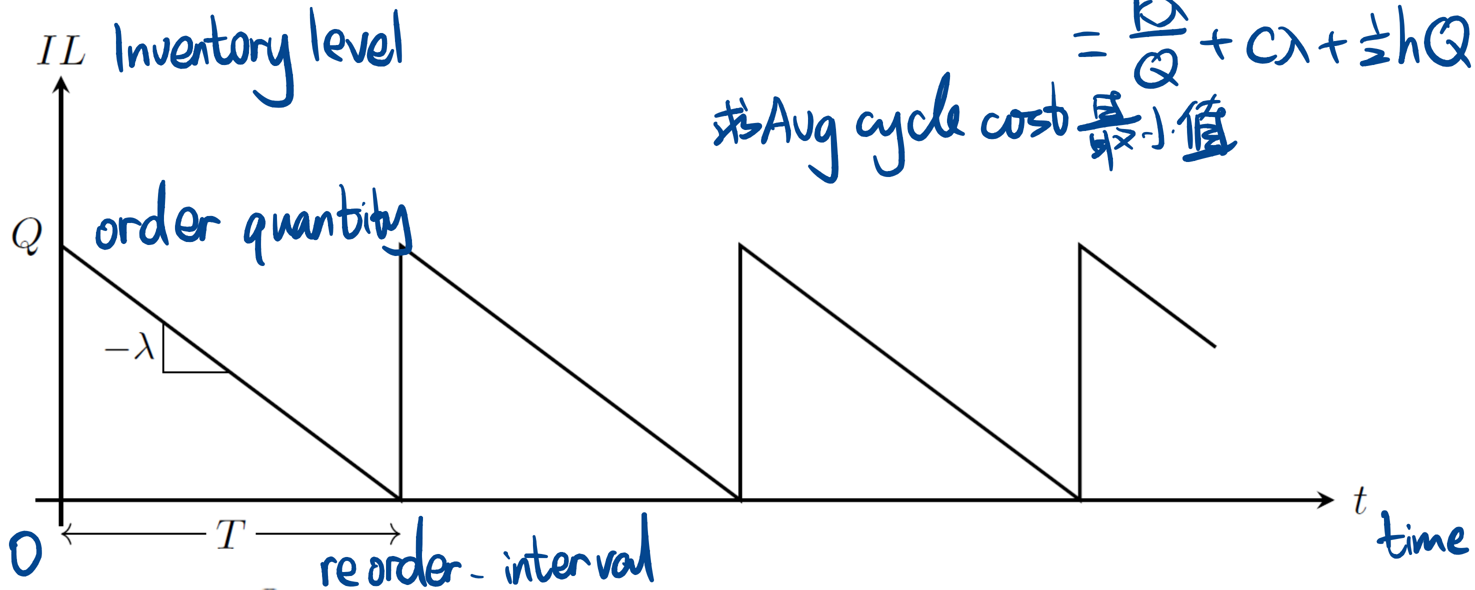
OUTPUT: The optimal ordering strategy

(1) It is only optimal to place a new order when IL hits 0

(2) Q should be constant

$$\text{Avg cycle cost} = \frac{\text{cycle cost}}{T} = \\ = \frac{K\lambda}{Q} + c\lambda + \frac{1}{2}hQ$$

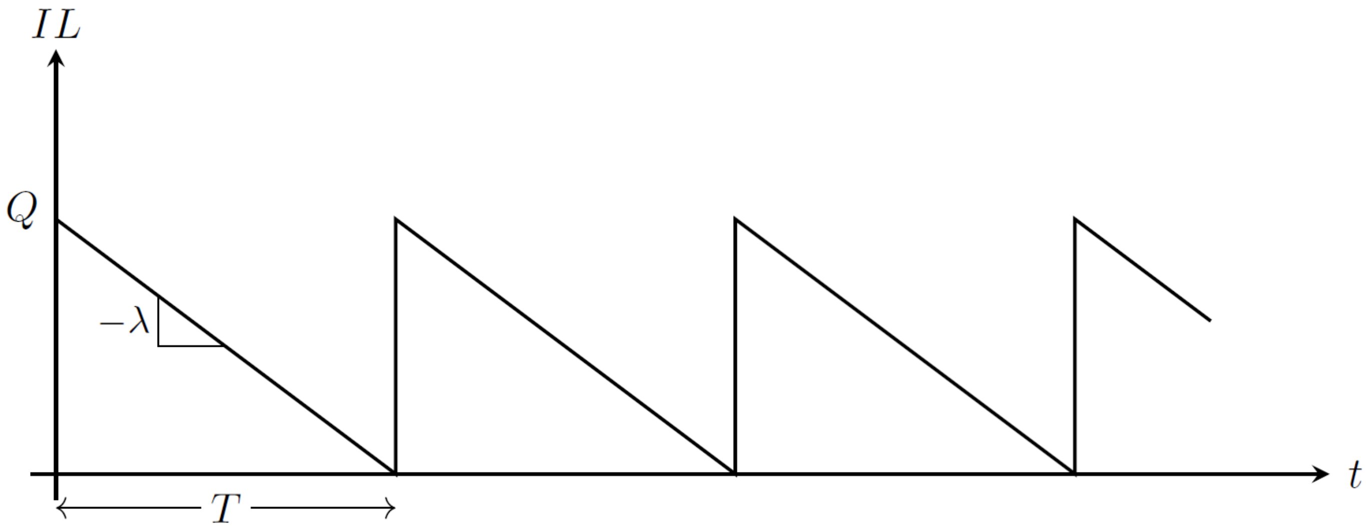
求Avg cycle cost 最小值



$T = \frac{Q}{\lambda}$.

Figure 3.2 EOQ inventory level curve.

cycle cost



$T = \dfrac{Q}{\lambda}$.

Figure 3.2 EOQ inventory level curve.

$$g(Q) = \frac{K\lambda}{Q} + \frac{hQ}{2}$$

EOQ economic order quantity

fixedcost

$$g(Q) = \frac{K\lambda}{Q} + \frac{hQ}{2}$$ holding cost

$Q^* \uparrow$ in K

如果K太高：订太多订单了

$Q^* \uparrow$ in λ

$Q^* \downarrow$ in h

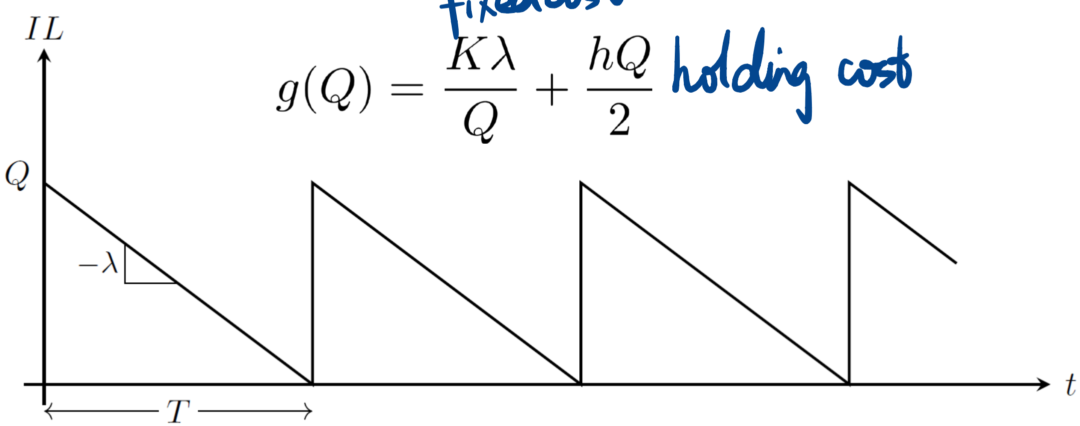


Figure 3.2 EOQ inventory level curve.

$$\frac{dg(Q)}{dQ} = -\frac{K\lambda}{Q^2} + \frac{h}{2} = 0$$

$$\Longrightarrow Q^2 = \frac{2K\lambda}{h}$$

$$\Longrightarrow Q^* = \sqrt{\frac{2K\lambda}{h}}$$

$$g(Q^*) == \sqrt{\frac{K\lambda h}{2}} + \sqrt{\frac{K\lambda h}{2}} = \sqrt{2K\lambda h}$$

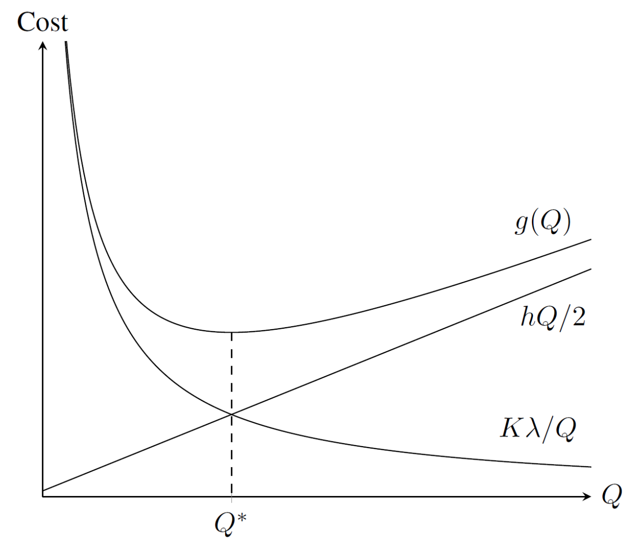


Figure 3.3 Fixed, holding, and total costs as a function of Q.

Adding a Lead Time L?

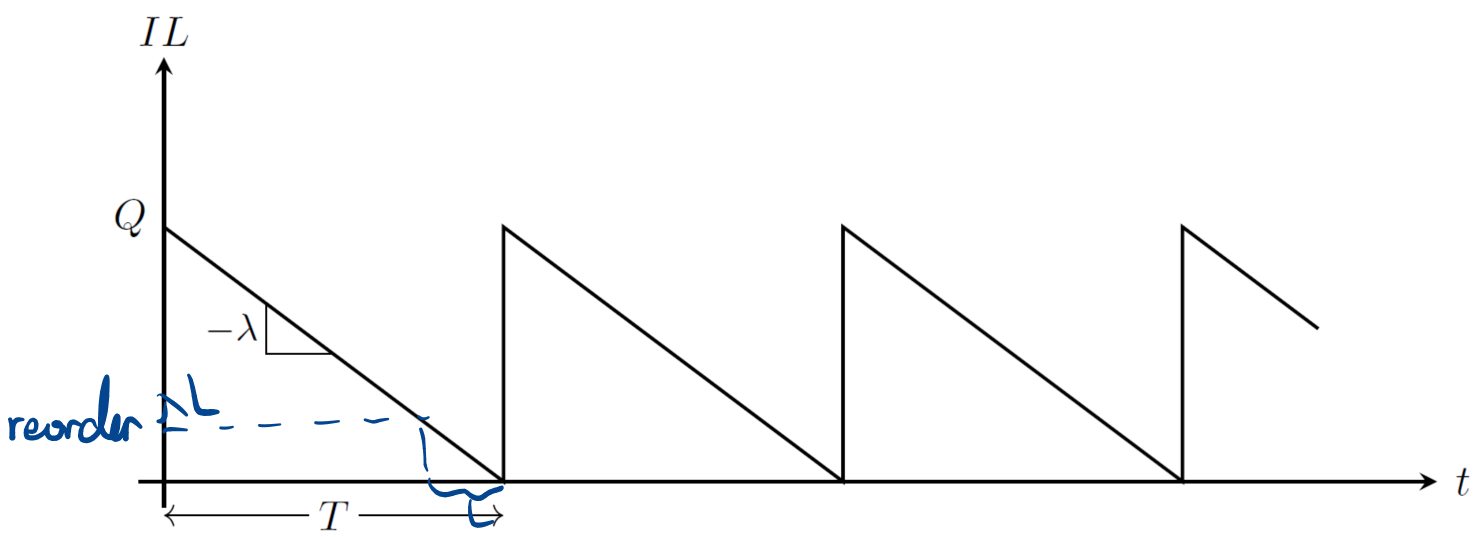


Figure 3.2 EOQ inventory level curve.

when reorder level is hit
it triggers a new order

Power of Two Polices important concept

EOQ highly ~~se~~ insensitive

- Sensitivity of order quantity

if Q differs from Q^*

how $g(Q)$ differs from $g(Q^*)$

$$\frac{g(Q)}{g\left(Q^*\right)}=\frac{1}{2}\left(\frac{Q^*}{Q}+\frac{Q}{Q^*}\right)$$

$Q^* = \sqrt{\frac{2k\lambda}{h}} \Rightarrow T^* = \frac{Q^*}{\lambda} = \sqrt{\frac{2k}{h\lambda}}$

reorder interval

power of 2 policy

- What if $T=T_B 2^k$ where k is some integer?

$$\frac{f(\hat{T})}{f\left(T^*\right)} \leq \frac{3}{2 \sqrt{2}} \approx 1.06$$

no more than 6%

why power of 2?

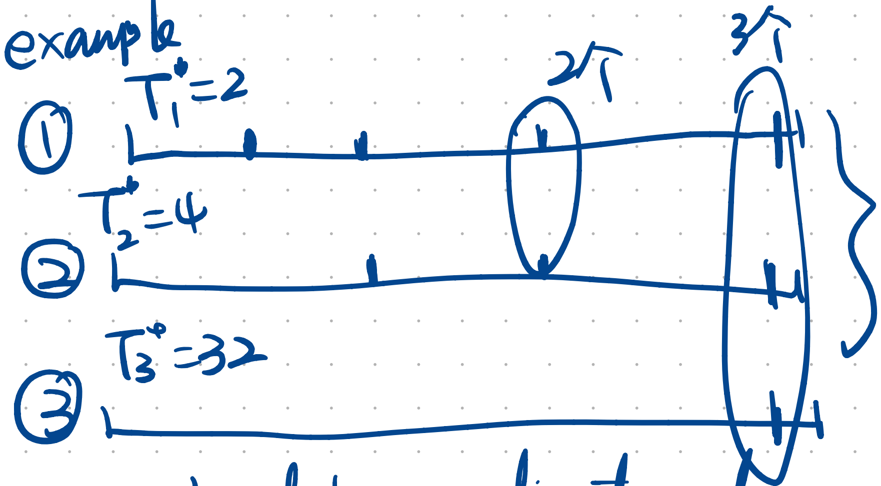
because mostly we need multi products

order

cost

synchronization

example



hard to coordinate orders

好处：可以放在一个track

同时间取货

EOQ with Backorders

p: per-unit of time backlogging cost

- Let x be the fraction of demand that is backordered

h: per-unit of time holding cost

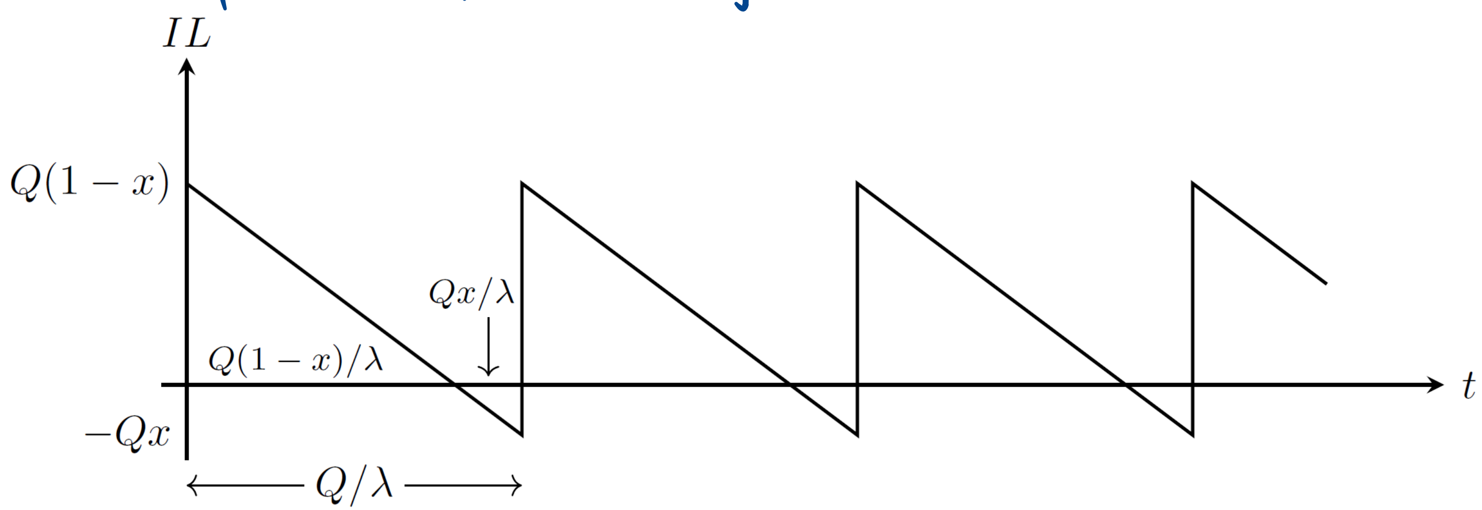


Figure 3.9 EOQB inventory curve.

average holding cost setup cost

$$g(Q,x) = \frac{hQ(1-x)^2}{2} + \frac{pQx^2}{2} + \frac{K\lambda}{Q}$$

goal: minimize the $g(Q,x) \Rightarrow$

EOQ with Backorders

→正常
sanity check
let $p \to \infty$, we could get $Q^* = \sqrt{\frac{2K\lambda}{h}}$

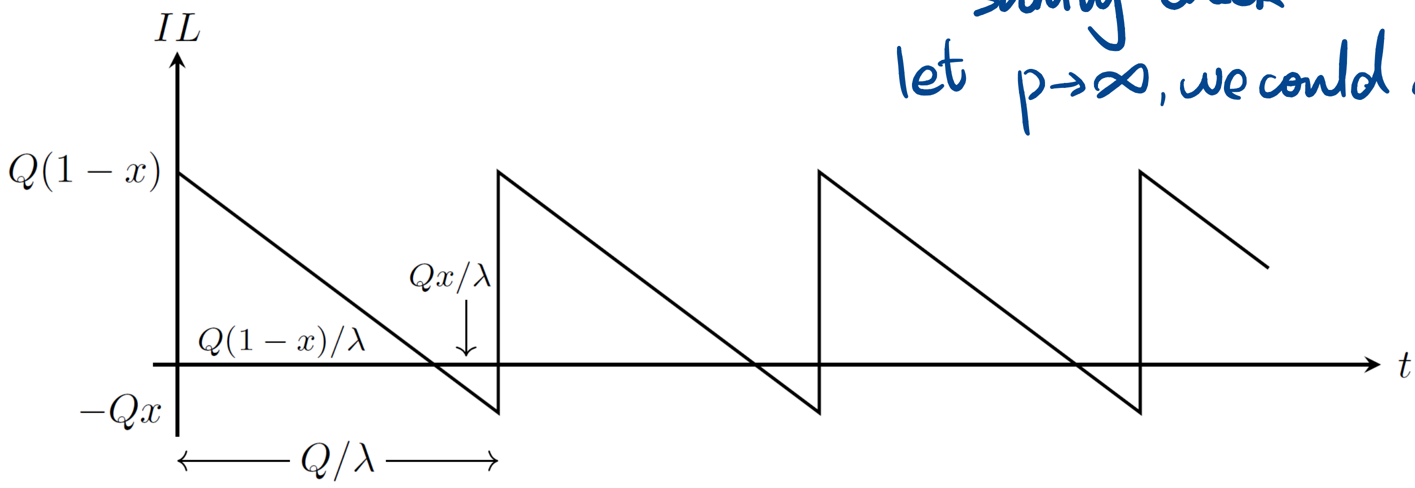


Figure 3.9 EOQB inventory curve.

如果关于x,Q都convex

$$g(Q,x) = \frac{hQ(1-x)^2}{2} + \frac{pQx^2}{2} + \frac{K\lambda}{Q}$$

$$\frac{\partial g}{\partial x} = -hQ(1-x) + pQx = 0$$

$$\frac{\partial g}{\partial Q} = \frac{h(1-x)^2}{2} + \frac{px^2}{2} - \frac{K\lambda}{Q^2} = 0$$

列联立求解

optimal solution

$$Q^* = \sqrt{\frac{2K\lambda(h+p)}{hp}}$$

$p \to 0$ ⇒ delay every one

how much demand to backlog

$$x^* = \frac{h}{h+p}$$

$$g(Q^*, x^*) = \sqrt{\frac{2K\lambda hp}{h+p}}$$

optimal object value

P larger → larger Q

可怕方程的解法

① check Hessian matrix，保证 positive semi-definite (psd)

EOQ with Backorders

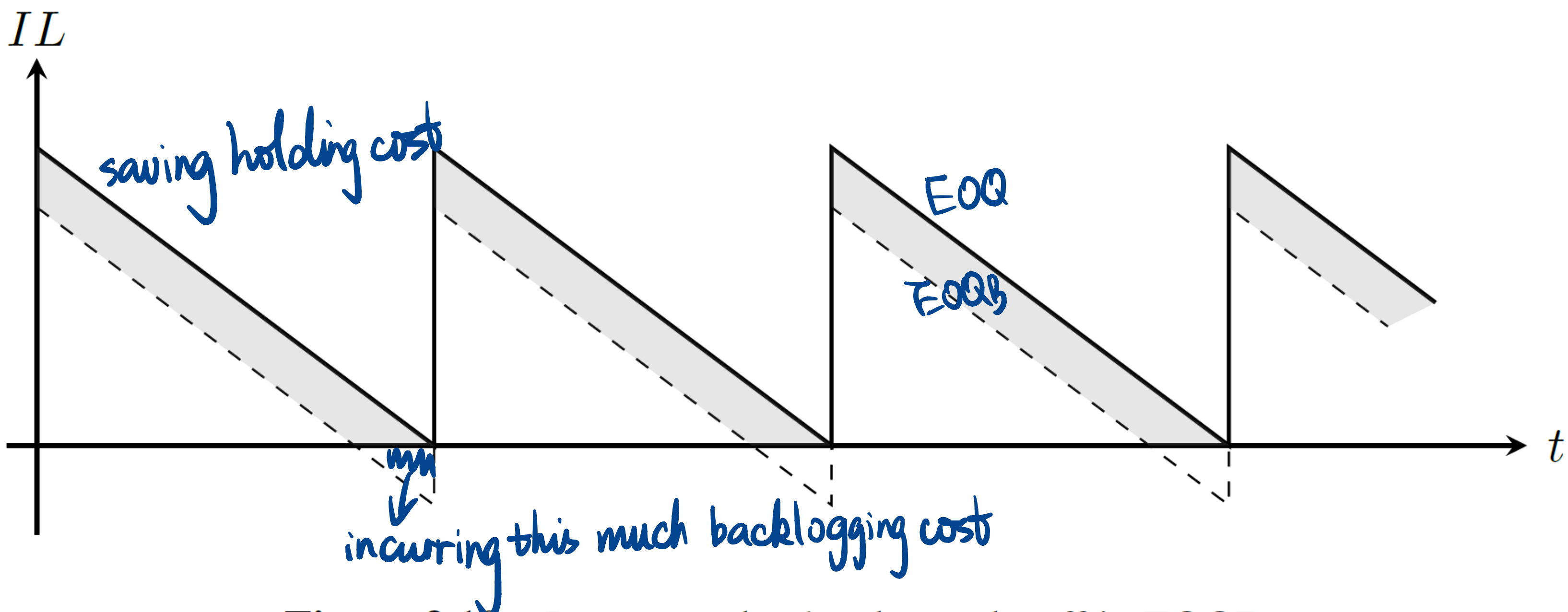


Figure 3.10 Inventory–backorder trade-off in EOQB.

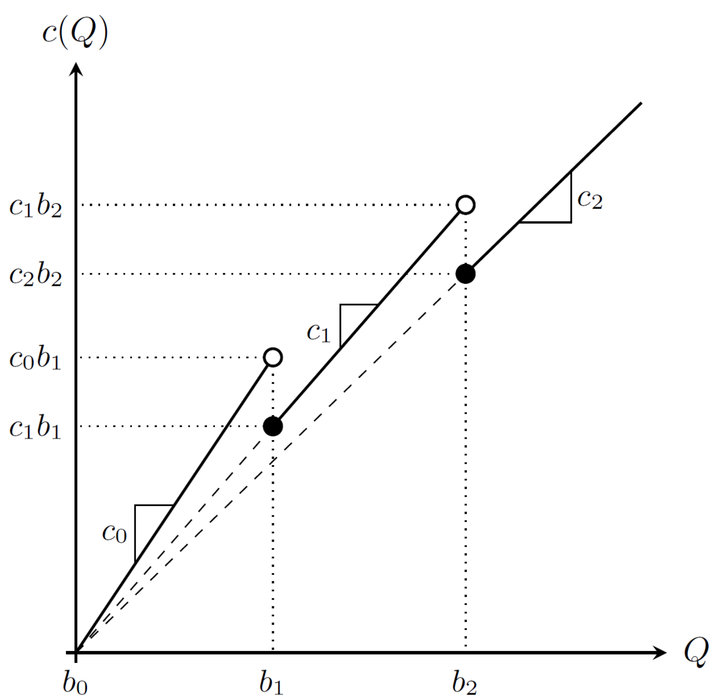
权衡

EOQ with Quantity Discount

the more you order, the less unit cost you pay

All-Unit Discount

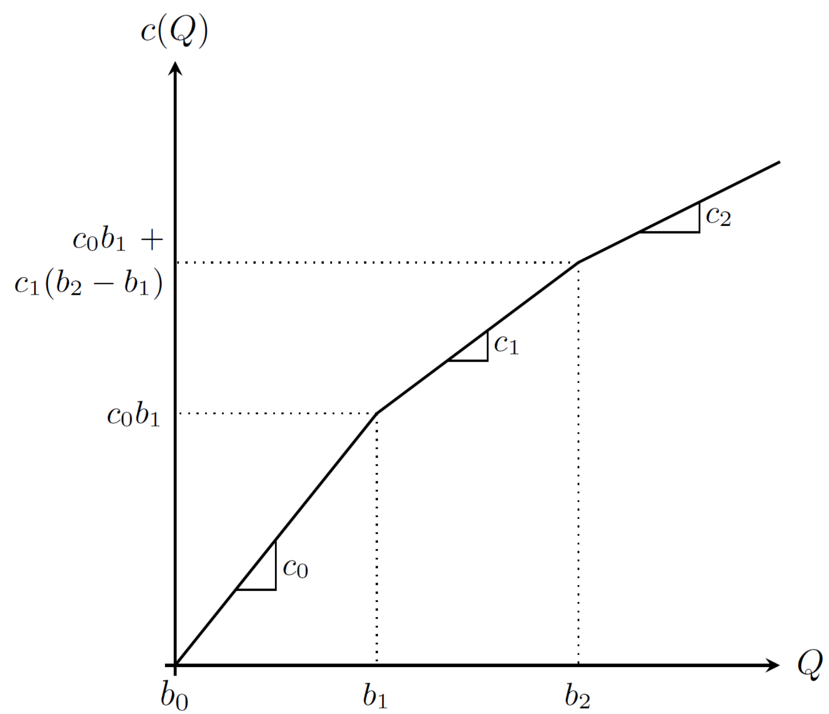
only care which interval you are in



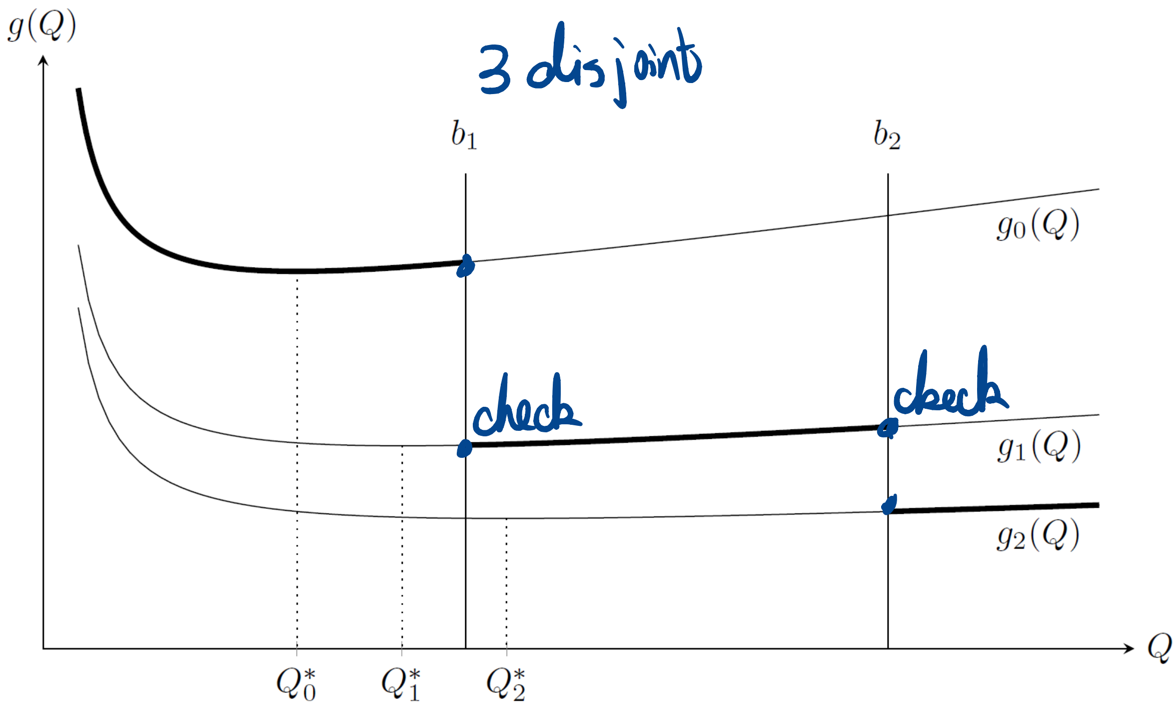
(a) All-units discounts.

Incremental Discount

more expensive than All unit



(b) Incremental discounts.



pre-set

$$g_j(Q) = c_j\lambda + \frac{K\lambda}{Q} + \frac{\boxed{ic_j}Q}{2}$$

$$Q_j^* = \sqrt{\frac{2K\lambda}{ic_j}}$$

h: express holding cost as a percentage of c_j

Figure 3.6 Total cost curves for all-units quantity discount structure.

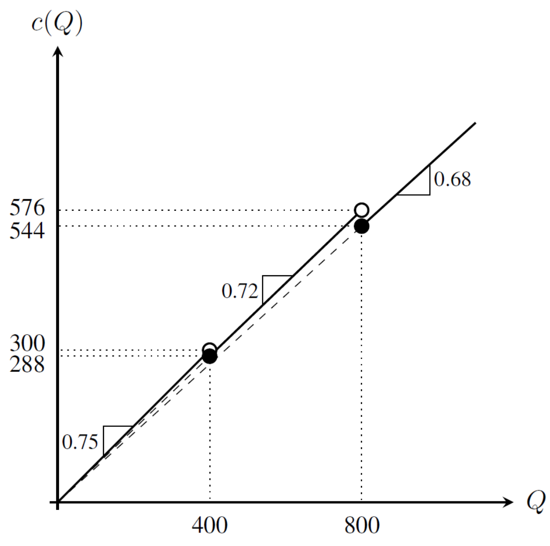
optimal min $(g_1(Q_1^*), g_1(b_1), g_2(b_2))$

Algorithm:

1. Calculate Q_j^* for each j
2. Check feasibility (or realizability)确认是哪3条实线要选择
3. Calculate the cost of break points to the right of the largest realizable Q_j^*

All-Unit Example

$\lambda = 1300, K = 8, i = 0.3$



(a) All-units discounts.

$$Q_j^* = \sqrt{\frac{2K\lambda}{ic_j}}$$

We first determine the largest realizable Q_j^* by working backward from segment 2:

$$Q_2^* = \sqrt{\frac{2 \cdot 8 \cdot 1300}{0.3 \cdot 0.68}} = 319.3$$

$$Q_1^* = \sqrt{\frac{2 \cdot 8 \cdot 1300}{0.3 \cdot 0.72}} = 310.3$$

$$Q_0^* = \sqrt{\frac{2 \cdot 8 \cdot 1300}{0.3 \cdot 0.75}} = 304.1$$

Only Q_0^* is realizable, and it has cost

$$0.75 \cdot 1300 + \sqrt{2 \cdot 8 \cdot 1300 \cdot 0.3 \cdot 0.75} = 1043.4.$$

Next, we calculate the cost of the breakpoints to the right of Q_0^*:

$$g_1(400) = 0.72 \cdot 1300 + \frac{8 \cdot 1300}{400} + \frac{0.3 \cdot 0.72 \cdot 400}{2} = 1005.2$$

$$g_2(800) = 0.68 \cdot 1300 + \frac{8 \cdot 1300}{800} + \frac{0.3 \cdot 0.68 \cdot 800}{2} = 978.6$$

Therefore, the optimal order quantity is $Q = 800$, which incurs a purchase cost of \$0.68 and a total annual cost of \$978.60. □

Incremental Discount easier

$$g_j(Q) = c_j\lambda + \frac{i\bar{c}_j}{2} + \frac{(K+\bar{c}_j)\lambda}{Q} + \frac{ic_jQ}{2}$$

$$\bar{c}_j = \sum_{i=0}^{j-1} c_i(b_{i+1} - b_i) - c_jb_j$$

$$Q_j^* = \sqrt{\frac{2(K+\bar{c}_j)\lambda}{ic_j}}$$

$$g_j(Q_j^*) = c_j\lambda + \frac{i\bar{c}_j}{2} + \sqrt{2(K+\bar{c}_j)\lambda ic_j}$$

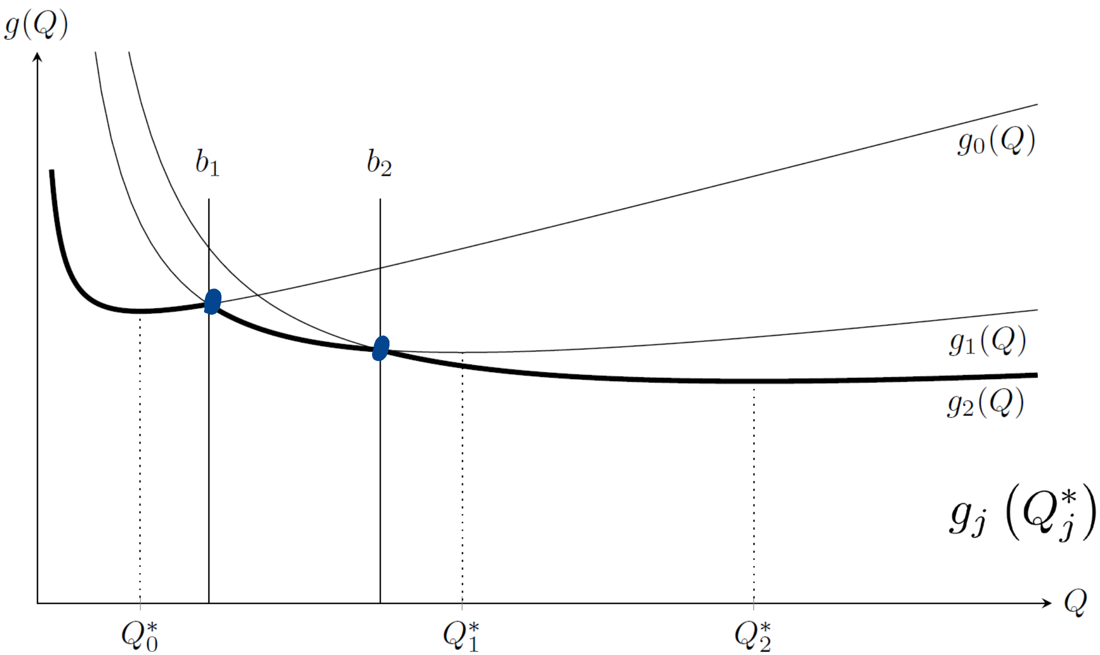
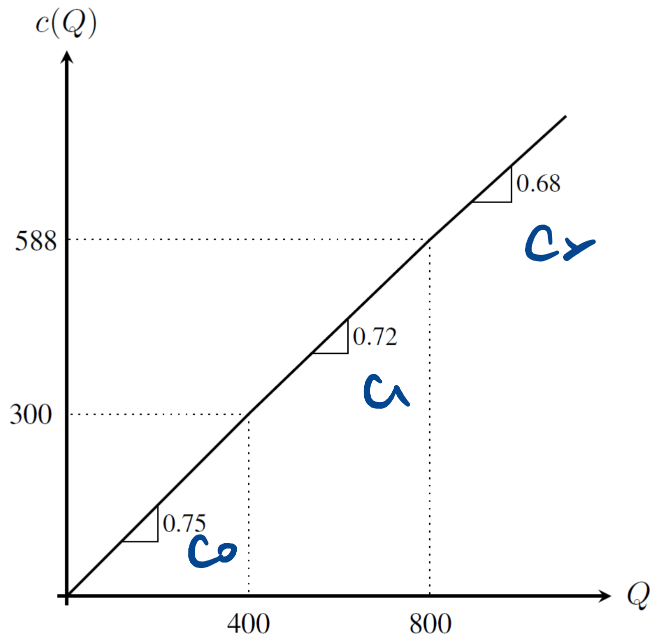


Figure 3.7 Total cost curves for incremental quantity discount structure.

Algorithm:
1. Calculate Q_j^* for each j
2. Check feasibility (or realizability) choose the lowest one

Incremental Example

$\lambda = 1300, K = 8, i = 0.3$



(b) Incremental discounts.

$$Q_j^* = \sqrt{\frac{2\left(K + \bar{c}_j\right)\lambda}{ic_j}}$$

$$\bar{c}_1 = 0.75 \cdot 400 - 0.72 \cdot 400 = 12$$
$$\bar{c}_2 = 0.75 \cdot 400 + 0.72 \cdot 400 - 0.68 \cdot 800 = 44$$

Next, we calculate Q_j^* for each j:

all feasible

$$Q_0^* = \sqrt{\frac{2(8+0)1300}{0.3 \cdot 0.75}} = 304.1$$
$$Q_1^* = \sqrt{\frac{2(8+12)1300}{0.3 \cdot 0.72}} = 490.7$$
$$Q_2^* = \sqrt{\frac{2(8+44)1300}{0.3 \cdot 0.68}} = 814.1$$

All three solutions are realizable. Using (3.22), these solutions have the following costs:

$$g_0(Q_0^*) = 0.75 \cdot 1300 + \frac{0.3 \cdot 0}{2} + \sqrt{2(8+0)1300 \cdot 0.3 \cdot 0.75} = 1043.4$$
$$g_1(Q_1^*) = 0.72 \cdot 1300 + \frac{0.3 \cdot 12}{2} + \sqrt{2(8+12)1300 \cdot 0.3 \cdot 0.72} = 1043.8$$
$$g_2(Q_2^*) = 0.68 \cdot 1300 + \frac{0.3 \cdot 44}{2} + \sqrt{2(8+44)1300 \cdot 0.3 \cdot 0.68} = 1056.7$$

Therefore, the optimal order quantity is $Q = 304.1$, which incurs a total annual cost of \$1043.40. □

Economic Production Quantity (EPQ)

rate of producing $= \mu$

rate of consumption $= \lambda$

assumption: $\mu > \lambda$

$\rho \triangleq \frac{\lambda}{\mu} < 0$: utilization factor

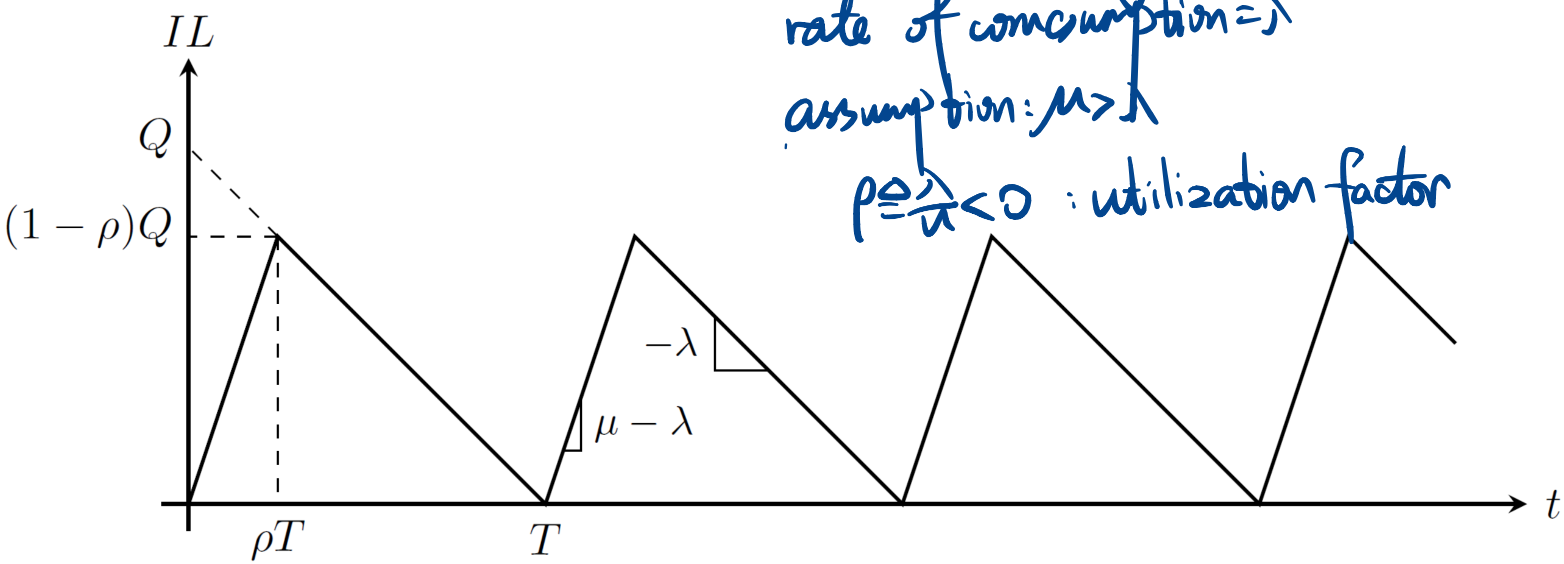


Figure 3.11 EPQ inventory level curve.

$$g(Q) = \underbrace{\frac{K\lambda}{Q}}_{\text{fixed cost}} + \underbrace{\frac{h(1-\rho)Q}{2}}_{\text{holding cost}} \quad \Rightarrow \quad Q^* = \sqrt{\frac{2K\lambda}{h(1-\rho)}}$$

$$g(Q^*) = \sqrt{2K\lambda h(1-\rho)}$$

Summary

- Basic Deterministic Inventory Models
  - EOQ
  - EOQ with backorders
  - EOQ with quantity discounts (all-unit and incremental)
  - EPQ

- Next Up: Wagner-Whitin Model