VG441 Midterm

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Problem 1(Forecasting)

(a) Scatter plot the demands against time (Figure 1).

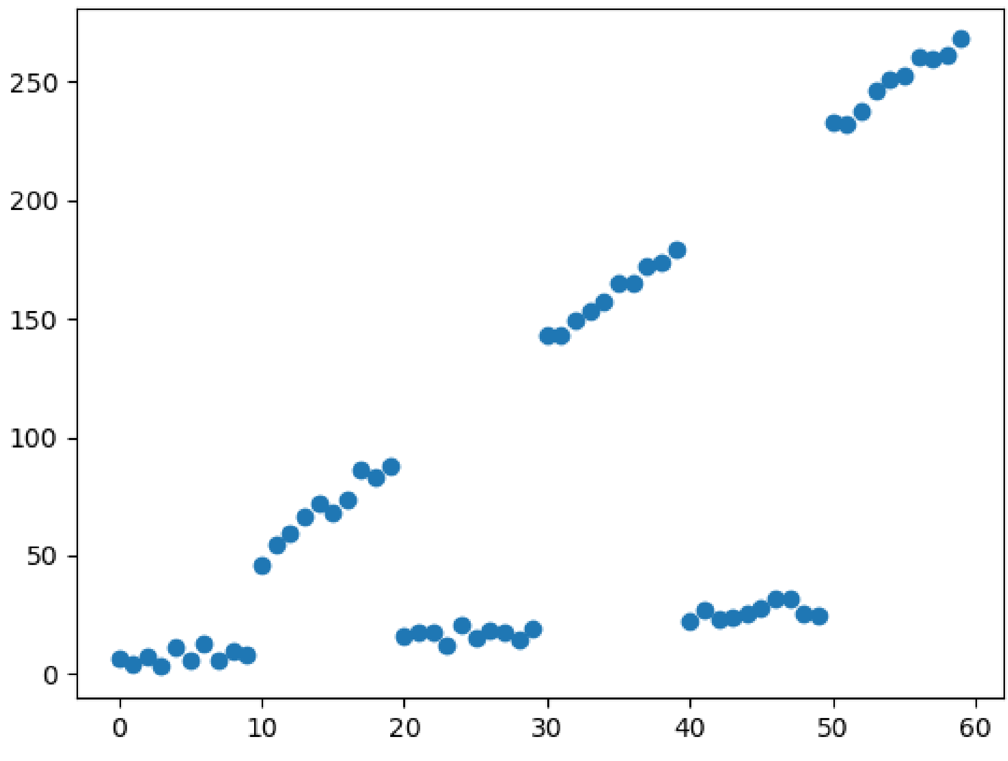

Fig. 1: demands of time

Figure 1 is the demands against time

(b) Run a simple regression and plot your results on top of scatter plot (Figure 2).

First, judging from the plot we got, we should use ARIMA model to forecast the data and get:

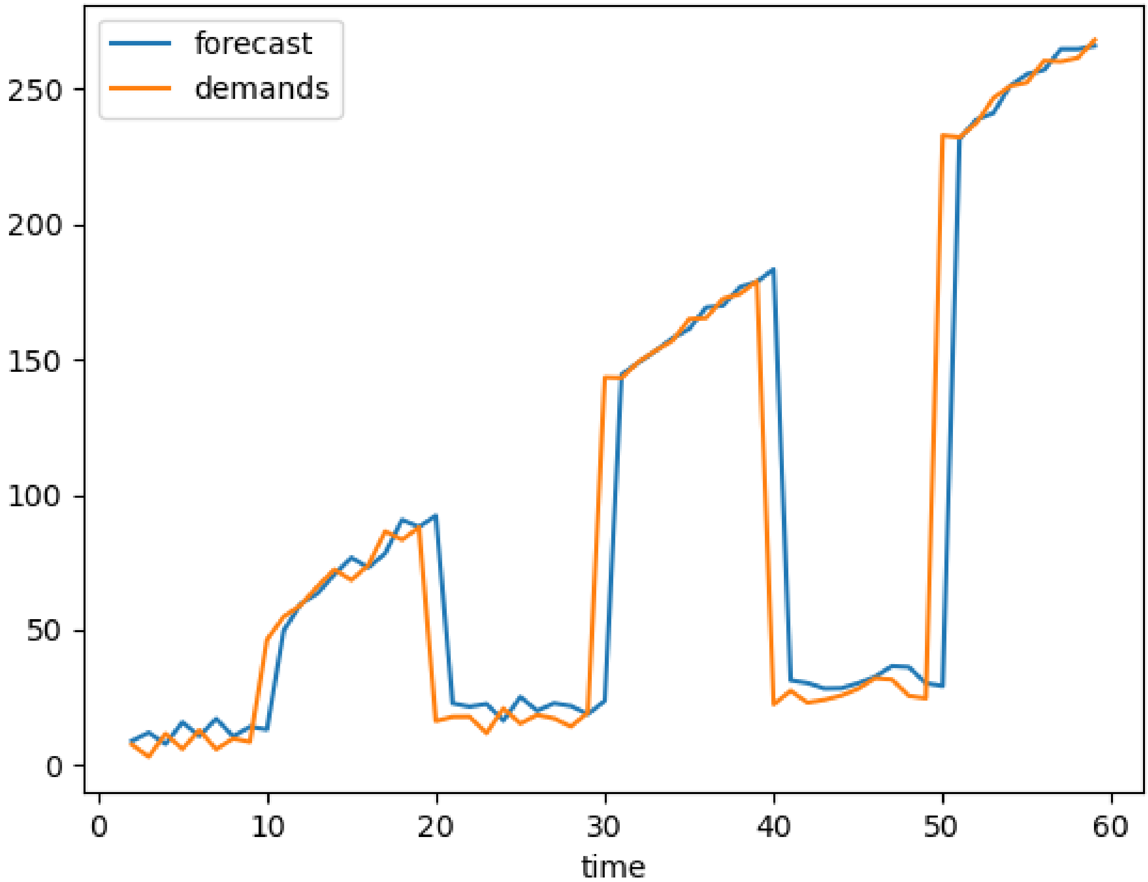

Fig. 2: ARIMA model of demands

(c) Run gradient boosting method with different number of trees:

1. params = 'n_estimators': 1, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'

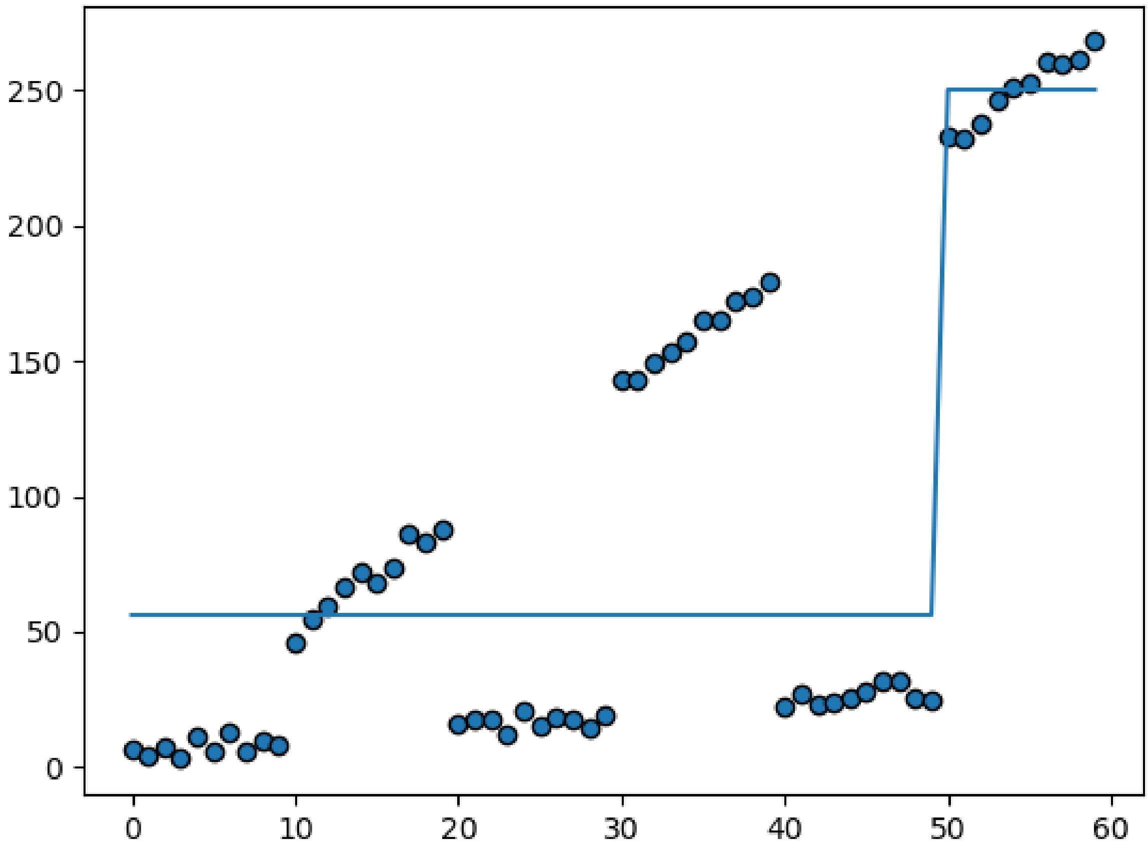

Fig. 3: results of param:1,1,1,'ls'

Just as Fig.4 shows, and R2 sq: 0.6959704096715094

2. params = 'n_estimators': 2, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'

Just as Fig.4 shows, and R2 sq: 0.7290384207566345

3. params = 'n_estimators': 5, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'

Just as Fig.5 shows, and R2 sq: 0.8860240765658012

4. params = 'n_estimators': 10, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'

Just as Fig.6 shows, and R2 sq: 0.9751642096479143

5. params = 'n_estimators': 20, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'

Just as Fig.7 shows, and R2 sq: 0.9875429979290639

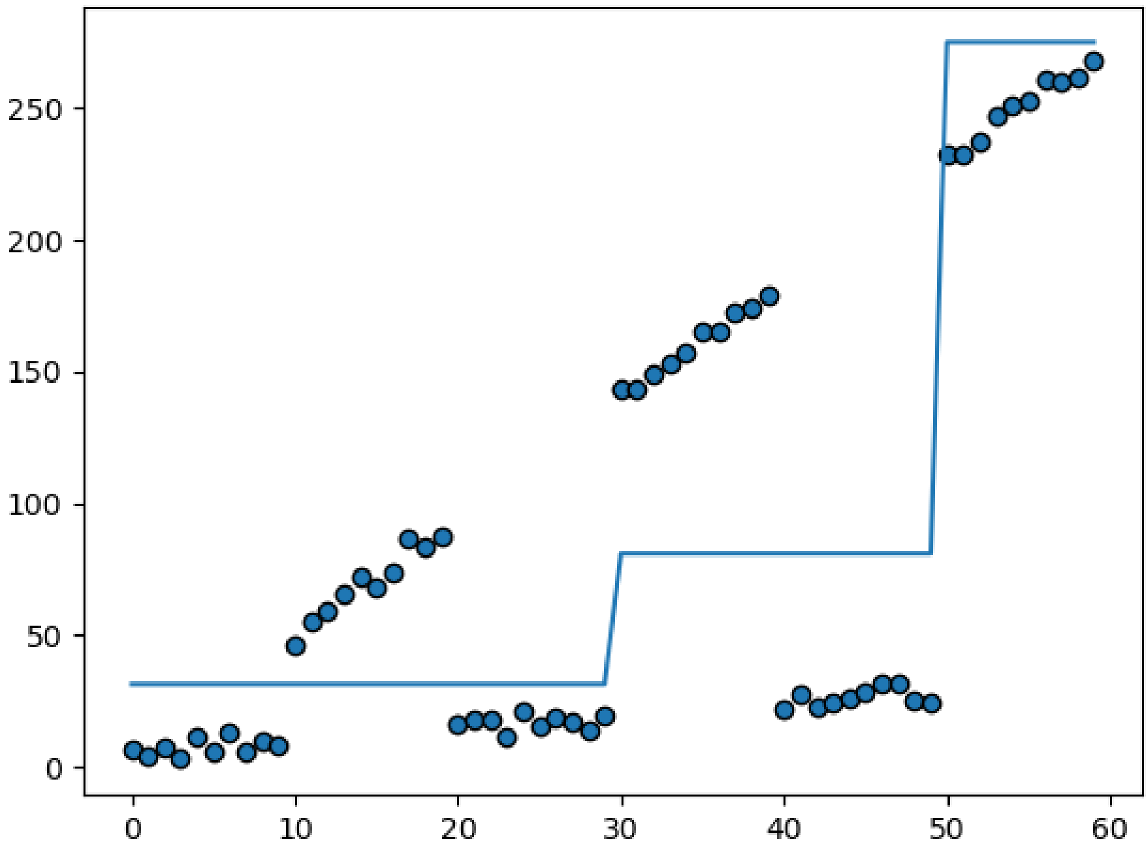

Fig. 4: results of param:2,1,1,'ls'

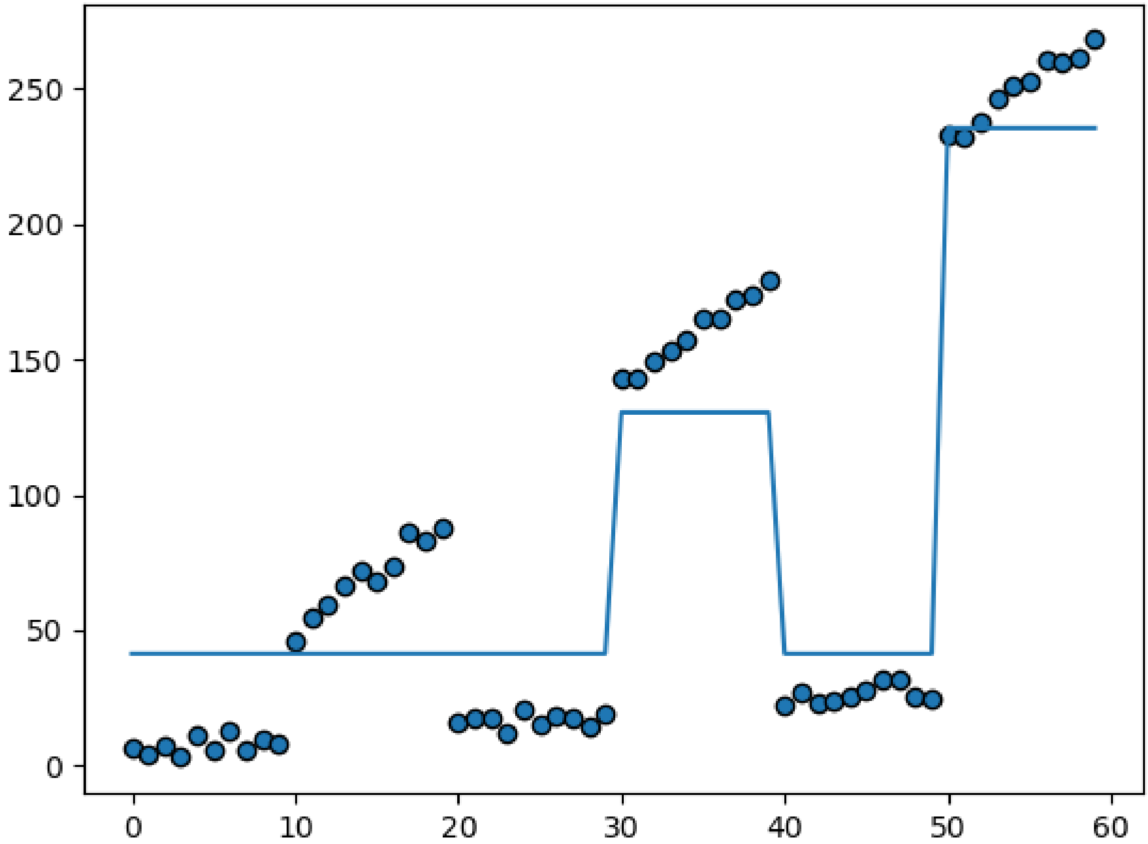

Fig. 5: results of param:5,1,1,'ls'

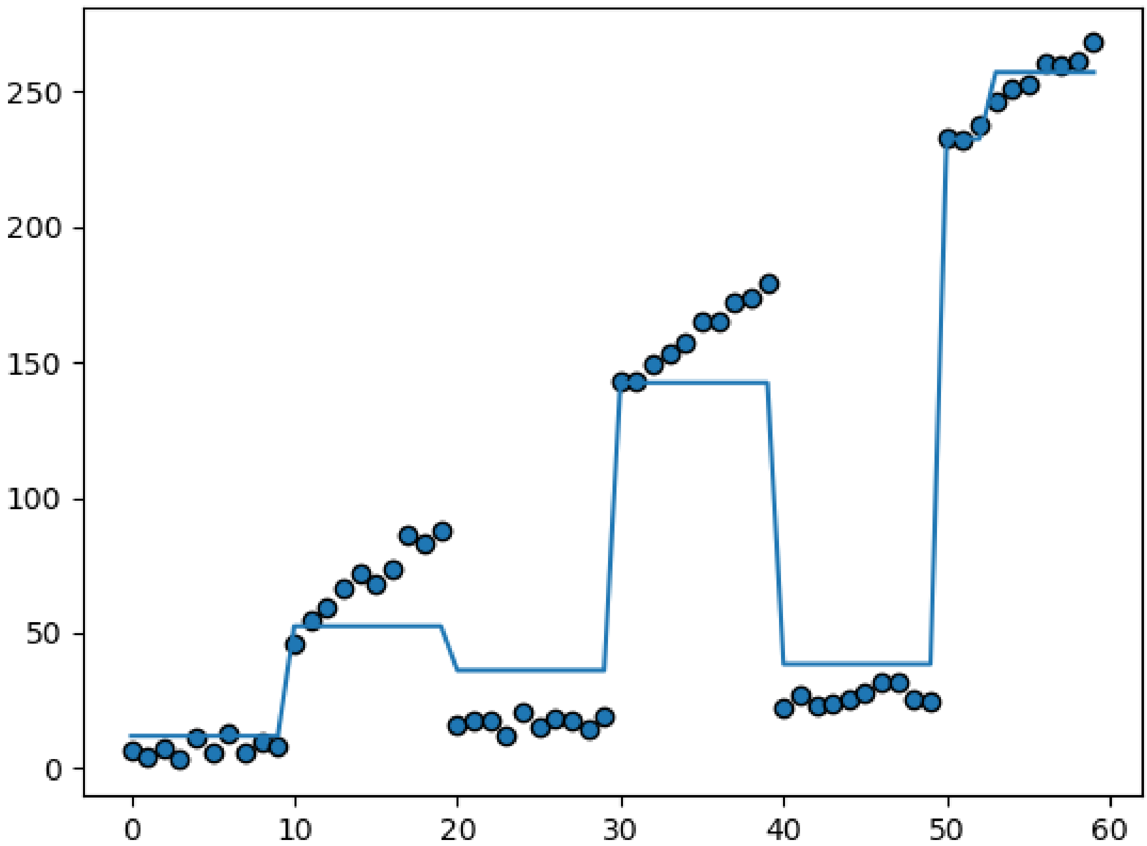

Fig. 6: results of param:10,1,1,'ls'

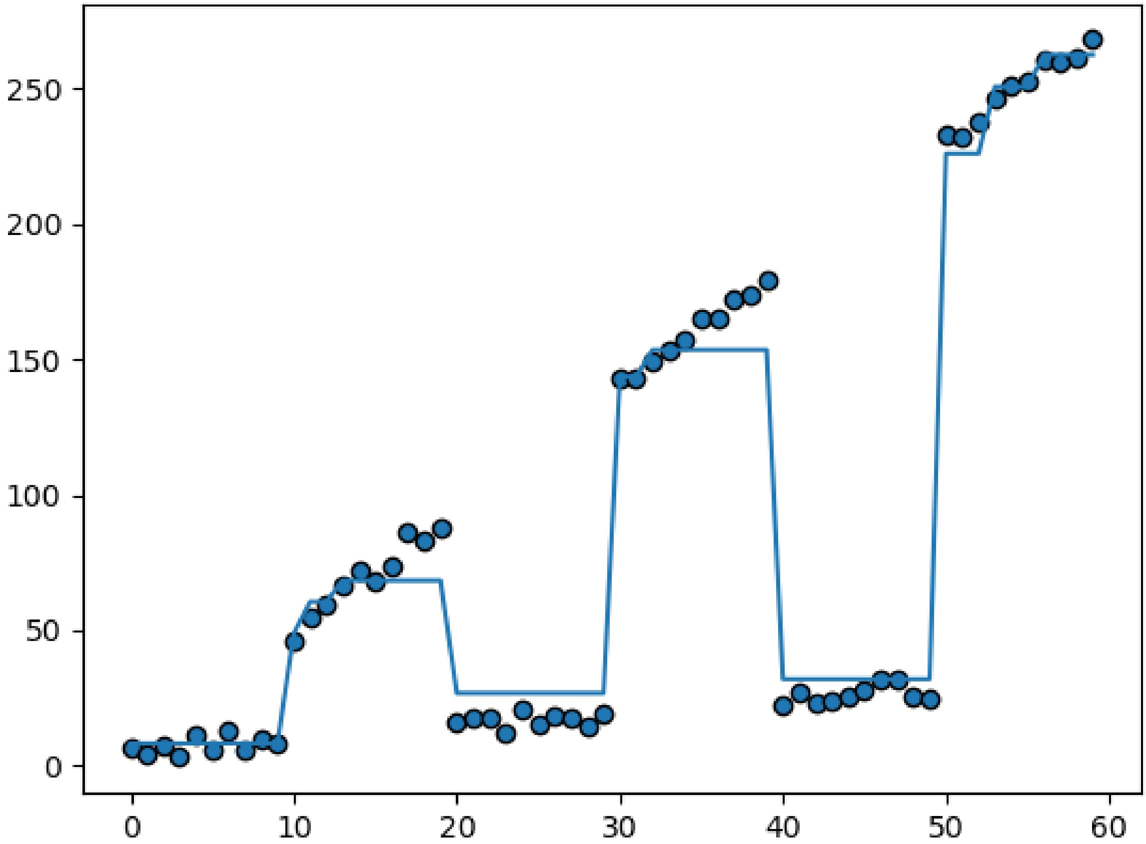

Fig. 7: results of param:20,1,1,'ls'

5. params = 'n_estimators': 50, 'max_depth': 1, 'learning_rate': 1, 'loss': 'ls'

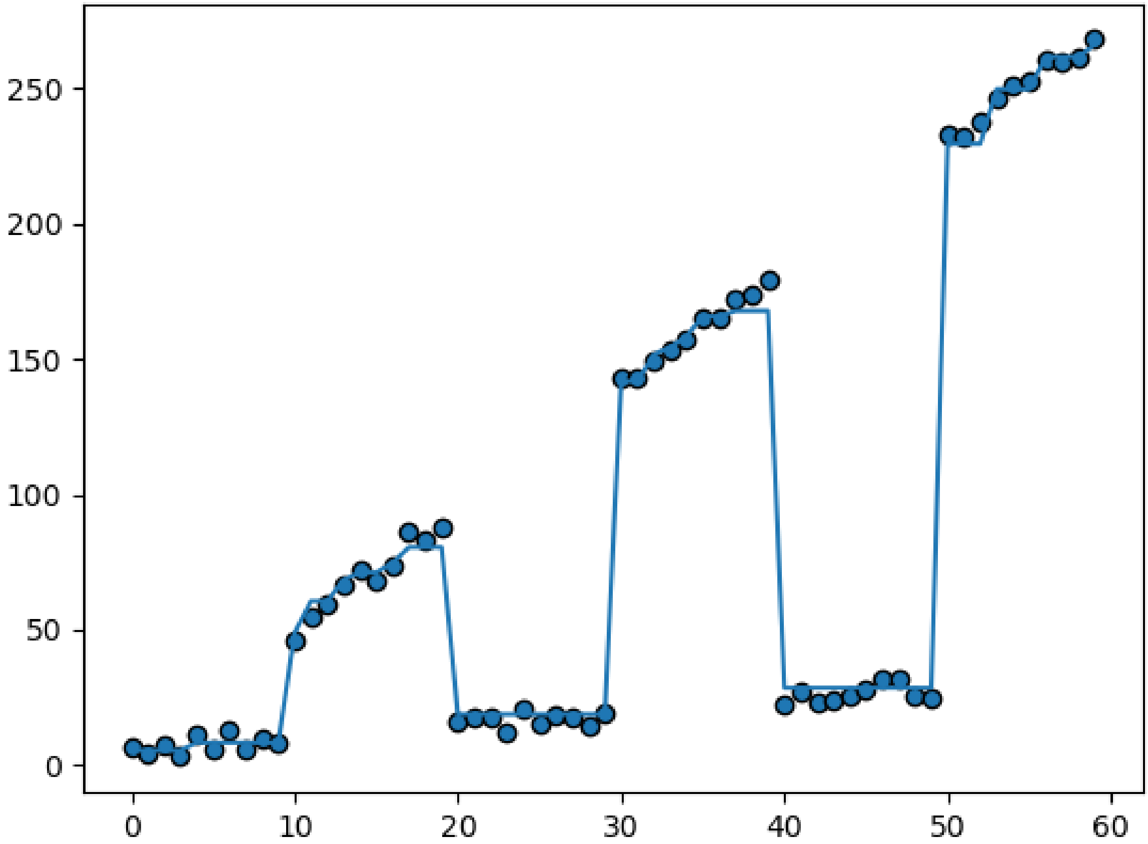

Fig. 8: results of param:50,1,1,'ls'

Just as Fig.8 shows, and R2 sq: 0.9981772426017594

Problem 2 (Quantity-Discount Model)

(a) What is the optimal ordering strategy?

From the question,

$$K = 50\$ \text{ /order} \quad h = 200/12\$ \text{ /(unit* month)} \quad \lambda = 50\text{units/month} \quad c = \begin{cases} 520 & , x < 12 \\ 510 & , 12 \le x \le 64 \\ 495 & , 65 \le x \le 128 \\ 485 & , x > 128 \end{cases} \tag{1}$$

Therefore, we use the All-unit Discount: For this structure, we could generate the $g(Q)$, and get that:

$$\begin{aligned} g_0(Q) &= 520 * 50 + 50 * 50/Q + 200/24 * Q \\ g_1(Q) &= 510 * 50 + 50 * 50/Q + 200/24 * Q \\ g_2(Q) &= 495 * 50 + 50 * 50/Q + 200/24 * Q \\ g_3(Q) &= 485 * 50 + 50 * 50/Q + 200/24 * Q \end{aligned} \tag{2}$$

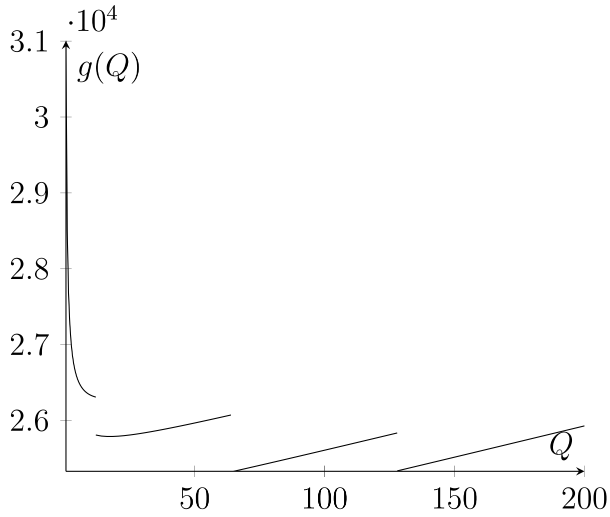

Fig. 9: Total cost for all-units quantity discount structure

And we could draw the graph like: and the Q_j^* is:

$$\begin{aligned} Q_0^* &= 17.32 \\ Q_1^* &= 17.32 \\ Q_2^* &= 17.32 \\ Q_3^* &= 17.32 \end{aligned} \tag{3}$$

Among these value, Q_1^* is feasible, which is:

$$g_1(17) = 25788.7$$

Then we caculare the cost of breakpoints to the left of $Q_1^*$and get:

$$g_2(65) = 25330.13$$

Therefore, the optimal orfer quantity is $Q = 65$, which incurs a total monthly cost of 25330.13\$

(b)The supplier has offered to be a drop shipper, i.e., they will ship directly to the customer. In exchange, they will increase the unit price to \$520 per computer, but not charge the ordering costs and all inventory will be held at the supplier. From a purely financial standpoint, should Zeus take them up on the offer?

According to the question, we could get this results:

$$\lambda = 50\text{units/month} \quad c = 520\$/unit \tag{4}$$

Therefore, we can list the equation that

$$\text{Average Cycle cost} = 26000\$/month \tag{5}$$

Therefore, Zeus should not take them up on the offer.

Problem 3 (Wagner-Whitin Model)

(a) Use dynamic programming to solve the problem (on paper by hands).

First, we read from question and get the equation that:

$$K = 1000 \quad h = 1.2 \tag{6}$$

Then we apply the dynamic programming and get:

$$
\begin{aligned}
\theta_8 &= 0 \\
\theta_7 &= 1000 + 1.2(0 \cdot d_7) + \theta_8 \\
&= 1000[\text{s(7)=8}] \\
\theta_6 &= \min\{1000 + 1.2(0 \cdot d_6) + \theta_7, 1000 + 1.2(0 \cdot d_6 + 1 \cdot d_7) + \theta_8\} \\
&= \min\{2000, 1348\} \\
&= 1348[\text{s(6)=8}] \\
\theta_5 &= \min\{1000 + 1.2(0 \cdot d_5) + \theta_6, 1000 + 1.2(0 \cdot d_5 + 1 \cdot d_6) + \theta_7, \\
&\ 1000 + 1.2(0 \cdot d_5 + 1 \cdot d_6 + 2 \cdot d_7) + \theta_8\} \\
&= \min\{2348, 2252, 1948\} \\
&= 1948[\text{s(5)=8}] \\
\theta_4 &= \min\{1000 + 1.2(0 \cdot d_4) + \theta_5, 1000 + 1.2(0 \cdot d_4 + 1 \cdot d_5) + \theta_6, \\
&\ 1000 + 1.2(0 \cdot d_4 + 1 \cdot d_5 + 2 \cdot d_6) + \theta_7, \\
&\ 1000 + 1.2(0 \cdot d_4 + 1 \cdot d_5 + 2 \cdot d_6 + 3 \cdot d_7) + \theta_8\} \\
&= \min\{2948, 2552, 2708, 2752\} \\
&= 2552[\text{s(4)=6}] \\
\theta_3 &= \min\{1000 + 1.2(0 \cdot d_3) + \theta_4, 1000 + 1.2(0 \cdot d_3 + 1 \cdot d_4) + \theta_5, \\
&\ 1000 + 1.2(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5) + \theta_6, \\
&\ 1000 + 1.2(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5 + 3 \cdot d_6) + \theta_7, \\
&\ 1000 + 1.2(0 \cdot d_3 + 1 \cdot d_4 + 2 \cdot d_5 + 3 \cdot d_6 + 4 \cdot d_7) + \theta_8\} \\
&= \min\{3552, 3056, 2864, 3272, 3664\} \\
&= 2864[\text{s(3)=6}] \\
\theta_2 &= \min\{1000 + 1.2(0 \cdot d_2) + \theta_3, 1000 + 1.2(0 \cdot d_2 + 1 \cdot d_3) + \theta_4, \\
&\ 1000 + 1.2(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4) + \theta_5, \\
&\ 1000 + 1.2(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5) + \theta_6, \\
&\ 1000 + 1.2(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5 + 4 \cdot d_6) + \theta_7, \\
&\ 1000 + 1.2(0 \cdot d_2 + 1 \cdot d_3 + 2 \cdot d_4 + 3 \cdot d_5 + 4 \cdot d_6 + 5 \cdot d_7) + \theta_8\} \\
&= \min\{3864, 3678, 3290, 3302, 3962, 4702\} \\
&= 3290[\text{s(2)=5}] \\
\theta_1 &= \min\{1000 + 1.2(0 \cdot d_1) + \theta_2, 1000 + 1.2(0 \cdot d_1 + 1 \cdot d_2) + \theta_3, \\
&\ 1000 + 1.2(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3) + \theta_4, \\
&\ 1000 + 1.2(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4) + \theta_5, \\
&\ 1000 + 1.2(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4 + 4 \cdot d_5) + \theta_6, \\
&\ 1000 + 1.2(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4 + 4 \cdot d_5 + 5 \cdot d_6) + \theta_7, \\
&\ 1000 + 1.2(0 \cdot d_1 + 1 \cdot d_2 + 2 \cdot d_3 + 3 \cdot d_4 + 4 \cdot d_5 + 5 \cdot d_6 + 6 \cdot d_7) + \theta_8\} \\
&= \{4290, 4050, 3990, 3710, 3926, 4838, 5926\} \\
&= 3710\text{s(1)=5}
\end{aligned}
\tag{7}
$$

Therefore, we could get that the best choice is:
Order 570 on Sunday, order 670 on Thursday

(b) Formulate as a shortest path problem and draw the corresponding diagram with nodes, edges, and edge costs. Solve using Dijkstra's algorithm (on paper by hands).

1 2 3 4 5 6 7 8

And we list the edge and edge cost as:

edge	edge cost
1-2	1000
1-3	1186
1-4	1438
1-5	1762
1-6	2578
1-7	3838
1-8	5926
2-3	1000
2-4	1126
2-5	1342
2-6	1954
2-7	2962
2-8	4702
3-4	1000
3-5	1108
3-6	1516
3-7	2272
3-8	3664
4-5	1000
4-6	1204
4-7	1708
4-8	2752
5-6	1000
5-7	1252
5-8	1948
6-7	1000
6-8	1348
7-8	1000

And by applying the Dikstra's algorithm, we could conclude that:

Number of steps	X	A[s]	B[s]
1	2,3,4,5,6,7,8	0	$\emptyset$
2	3,4,5,6,7,8	A[2]=1000	B[2]={1-2}
3	4,5,6,7,8	A[3]=1186	B[4]={1-3}
4	5,6,7,8	A[4]=1438	B[4]={1-4}
5	6,7,8	A[5]=1762	B[5]={1-5}
6	7,8	A[6]=2578	B[6]={1-6}
7	8	A[7]=3014	B[7]={1-5-7}
8	$\emptyset$	A[8]=3710	B[8]={1-5-8}

(c) Formulate the problem as a MILP on paper and solve it in Python.

for X, we use x_n to express the price of order when time n:
We want to minimize: $\sum_{1 \leq n \leq 8}$

Problem 4 (Linear Programming Duality)

(a) Please write down duality of the following linear programming problem.

$$\begin{aligned} min \quad & 24y_1 + 60y_2 \\ s.t. \quad & 3y_1 + y_2 \geq 6 \\ & 2y_1 + 2y_2 \leq 14 \\ & y_1 + 4y_2 = 13 \\ & y_1 \geq 0 \\ & y_2 \leq 0 \end{aligned} \tag{8}$$

(b) Bipartite graph is a special graph. Its vertices are divided into two separate sets and edges only exist between those two sets.

suppose we have n nodes in I and m nodes in J,

$$\begin{aligned} min \quad & \sum_{i:(i,j) \in E} y_i + \sum_{j:(i,j) \in E} y_{n+j} \\ s.t. \quad & \end{aligned} \tag{9}$$