Elliptic Curve Cryptography on Embedded Devices

Scalar Multiplication and Side-Channel Attacks

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NXP Semiconductors

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Outline

Outline

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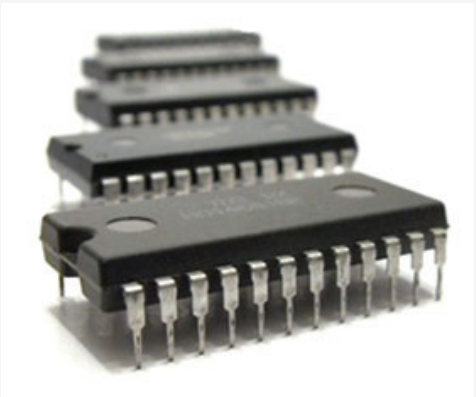
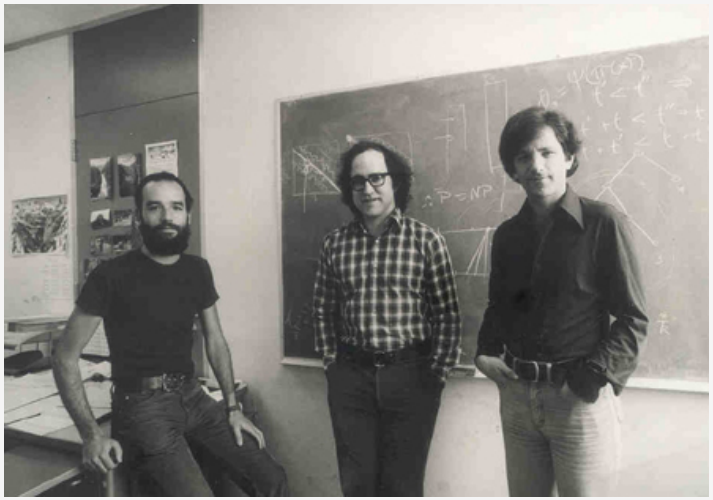
Cryptography



"Practice and study of techniques for secure communication in the presence of third parties."

- Symmetric cryptography
  - Known since the Antiquity
  - Secret key
  - Complex manipulations between the message and the key
  - Problem: key exchange
- Asymmetric cryptography
  - Invented during the 70's
  - Public key and private key
  - Rely on "difficult" mathematicals problems
  - Computations require more resources

RSA (Rivest-Shamir-Adleman)



A Method for Obtaining Digital Signatures and Public-Key Cryptosystems, 1978.

Key generation

- pick at random two primes p and q, and compute $N = p \times q$
- choose e and compute d such that: $e \times d = 1 \mod (p-1)(q-1)$

Public key

🔑 $= \{N, e\}$

Private key

🔑 $= \{p, q, d\}$

RSA (Rivest-Shamir-Adleman)

Encryption / Decryption

To encrypt a message m:

$$c = m^e \mod N$$

To decrypt c:

$$m = c^d \mod N$$

Security assumptions

1. Given 🔑 $= \{N, e\}$, recovering $d = e^{-1} \mod (p-1)(q-1)$ requires to factorize $N = p \times q$
2. Factorization is a difficult problem

Group structure

Set $\mathbb{G}$ provided with an internal operation $\oplus$ such that:

- $\oplus$ is associative: $(x \oplus y) \oplus z = x \oplus (y \oplus z)$
- there is an identity element $0_{\mathbb{G}}$: $\forall x \in \mathbb{G}, x \oplus 0_{\mathbb{G}} = 0_{\mathbb{G}} \oplus x = x$
- each element x has an inverse y in $\mathbb{G}$: $x \oplus y = 0_{\mathbb{G}}$

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Examples

- $(\mathbb{R}, +)$, identity element : 0
- $(\mathbb{R} \setminus \{0\}, \times)$, identity element : 1
- $(\mathbb{Z}/n\mathbb{Z}, +) \cong \{0, 1, 2, 3, ..., n-1\}$, identity element : 0
- $(\mathbb{Z}/p\mathbb{Z} \setminus \{0\}, \times) \cong \{1, 2, 3, ..., p-1\}$, identity element : 1

Discrete Logarithm Problem (DLP)

Statement

Let $(\mathbb{G}, \times)$ be a multiplicative group, $x \in \mathbb{G}$, $y \in \langle x \rangle$.
Given x and y, how to compute α s.t. $y = x^{\alpha}$?

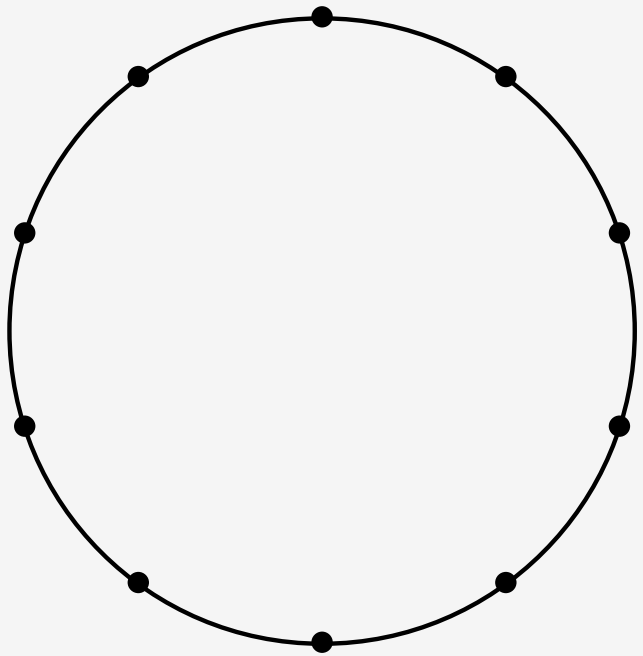
Computational complexity

Open problem: general case is NP-intermediate.

Discrete Logarithm Problem (DLP)

Example

$\mathbb{F}_p^* = \mathbb{Z}/p\mathbb{Z} \setminus \{0\}$
$p = 11$, $x = 8$



Discrete Logarithm Problem (DLP)

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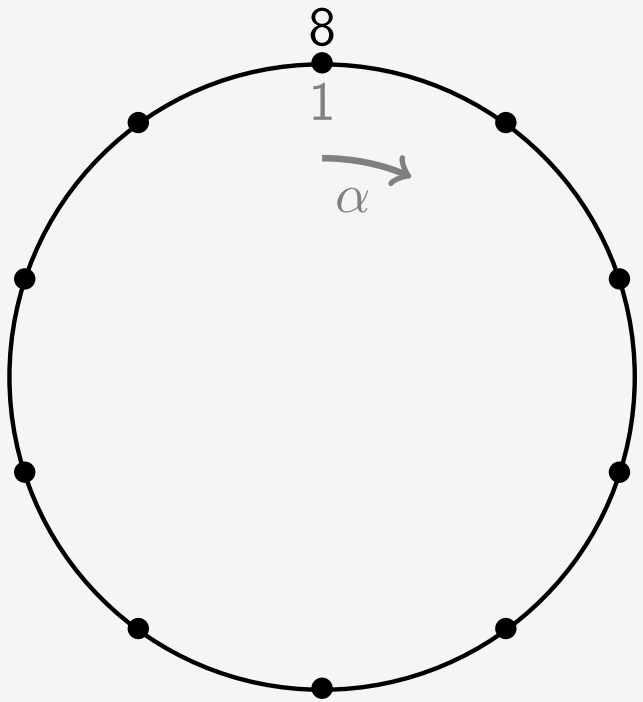

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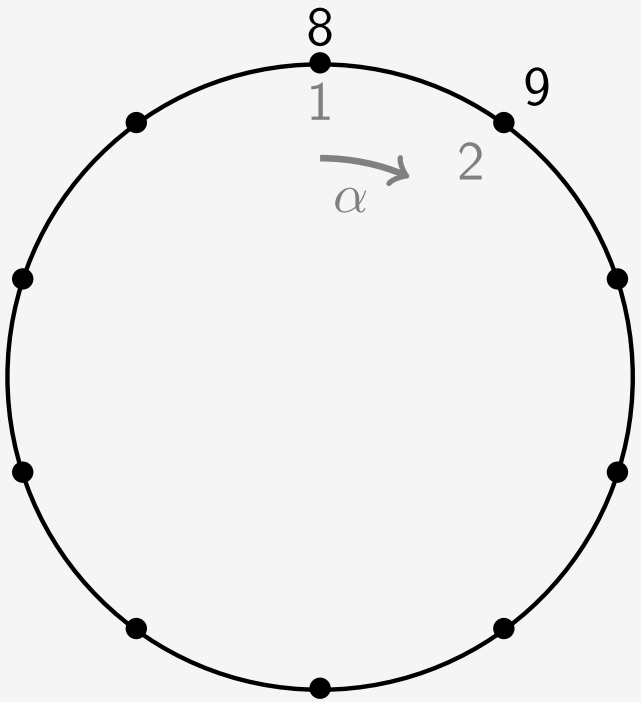
$p = 11$, $x = 8$

- $8^1 = 8 \mod 11$
- $8^2 = 9 \mod 11$



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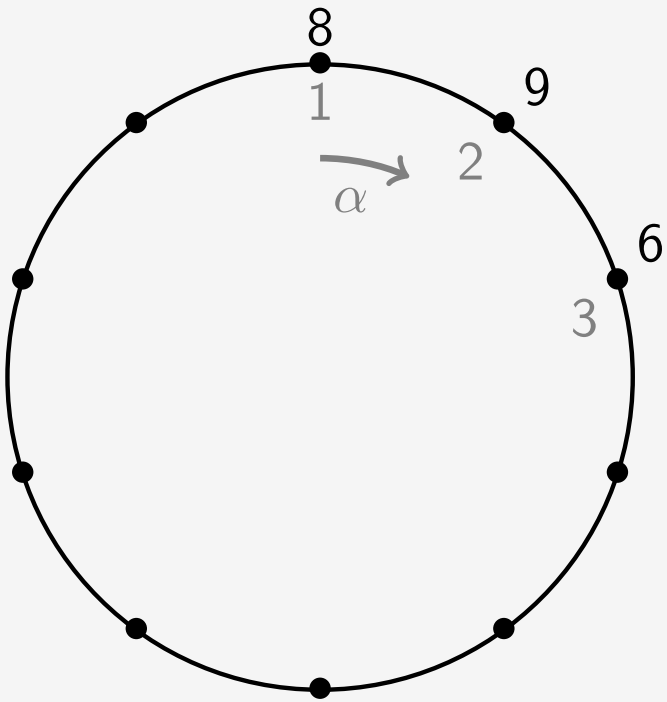

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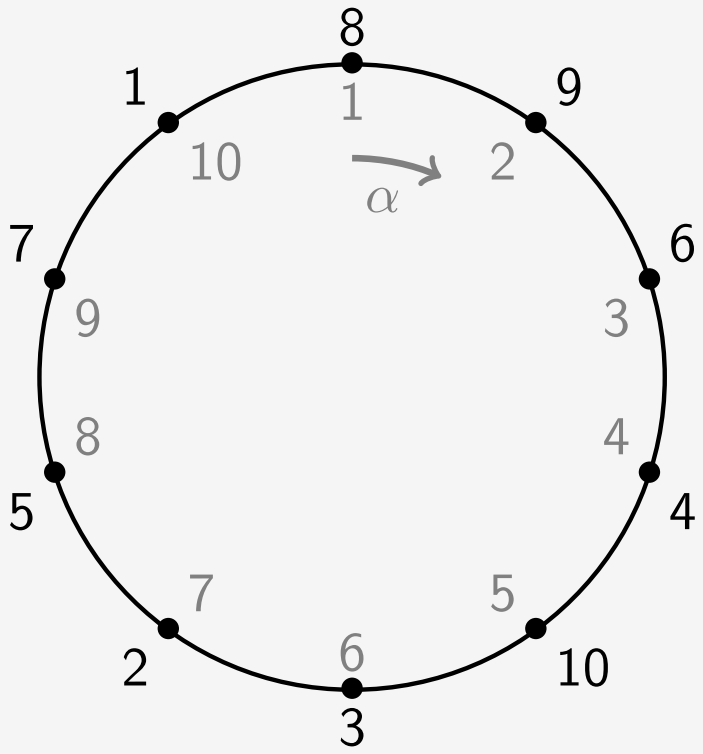
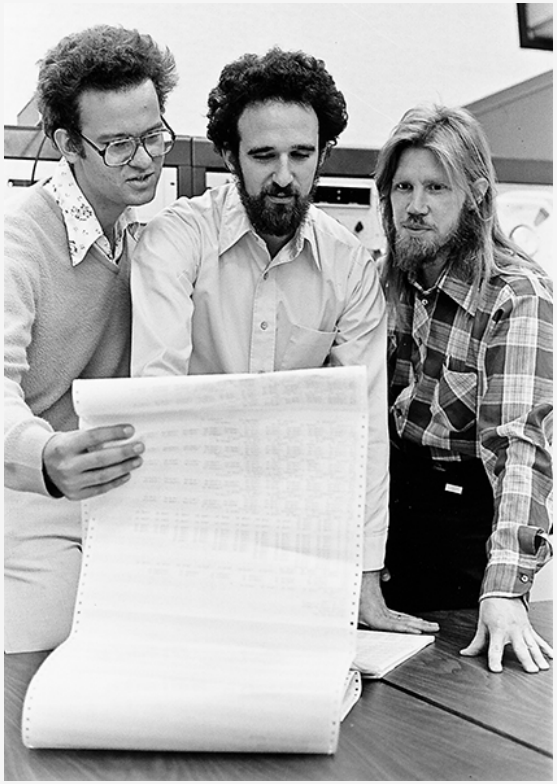
- $8^2 = 9 \mod 11$
- $8^3 = 6 \mod 11$
- $8^4 = 4 \mod 11$
- $8^5 = 10 \mod 11$
- $8^6 = 3 \mod 11$
- $8^7 = 2 \mod 11$
- $8^8 = 5 \mod 11$
- $8^9 = 7 \mod 11$
- $8^{10} = 1 \mod 11$
- $\mathbf{8^{\alpha+10} = 8^{\alpha} \mod 11}$



Diffie-Hellman(-Merkle) key exchange



New Directions in Cryptography, 1976.

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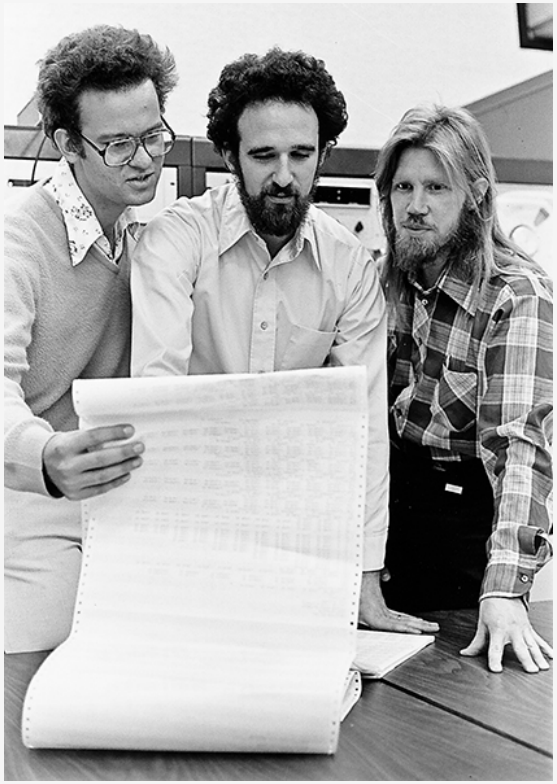

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Key exchange

Given $g \in \mathbb{F}_p^*$, $g \neq 1$:

Alice	Bob
$a = \text{rand}(1, p-1)$	$b = \text{rand}(1, p-1)$
$g_a = g^a \mod p$	$g_b = g^b \mod p$

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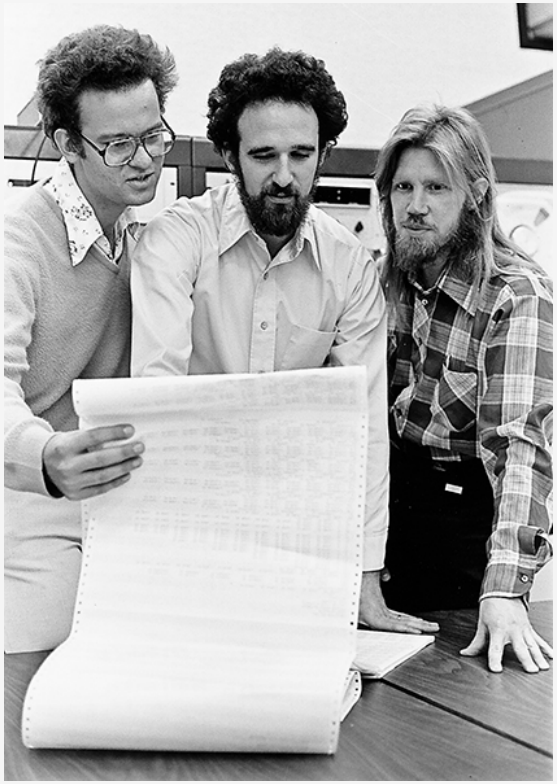

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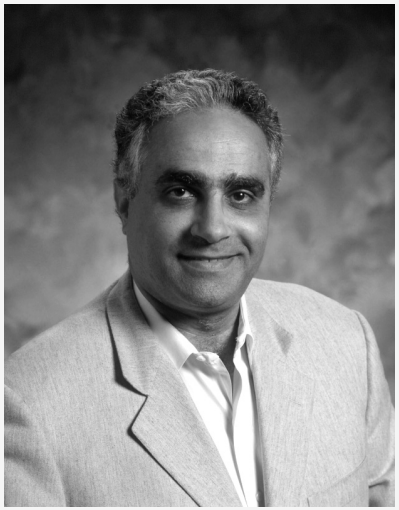
$a = \text{rand}(1, p-1)$		$b = \text{rand}(1, p-1)$
$g_a = g^a \mod p$		$g_b = g^b \mod p$
	$\xrightarrow{g_a}$	
	$\xleftarrow{g_b}$	
$g_{ab} = {g_b}^a \mod p$		$g_{ab} = {g_a}^b \mod p$

Alice and Bob now share the secret g_{ab}.

The Elgamal cryptosystem



A Public-Key Cryptosystem and a Signature Scheme Based on Discrete Logarithms, 1984.

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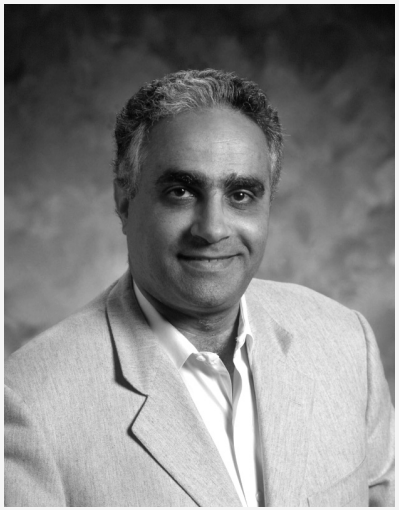

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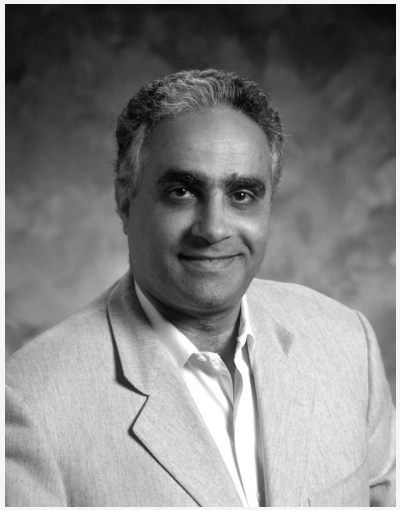

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Public key

 $= \{x\}$

Private key

 $= \{\alpha\}$

Outline

Elliptic Curve Cryptography

Independently introduced by Koblitz and Miller in 1985.

Field structure

Set $\mathbb{K}$ provided with two internal operations $\oplus, \otimes$ such that:

- $(\mathbb{K}, \oplus)$ is an additive group with identity elt. $0_{\mathbb{K}}$
- $(\mathbb{K} \setminus \{0_{\mathbb{K}}\}, \otimes)$ is a multiplicative group
- $\otimes$ is distributive over $\oplus$: $\forall a, b, c \in \mathbb{K}, \quad a \otimes (b \oplus c) = (a \otimes b) \oplus (a \otimes c)$

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Examples

- $(\mathbb{R}, +, \times)$
- $(\mathbb{Z}/n\mathbb{Z}, +, \times)$? Not in the general case !
- $(\mathbb{F}_p, +, \times)$

Elliptic Curve Equation

General equation

An elliptic curve over a field $\mathbb{K}$ has an equation:

$$\mathcal{E}/\mathbb{K} : y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$$

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Characteristic 2 (ordinary curves)

If $\mathbb{K} = \mathbb{F}_{2^n}$, $a_1 = 1$, $a_3 = a_4 = 0$:

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Large characteristic p

If $\mathbb{K} = \mathbb{F}_p$ with $p > 3$, $a_1 = a_2 = a_3 = 0$:

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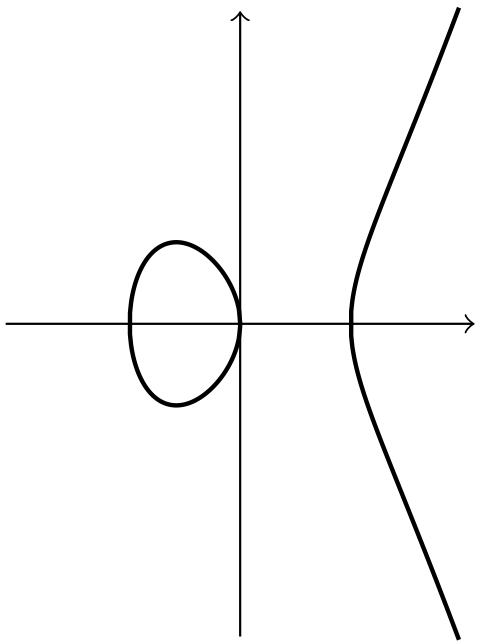
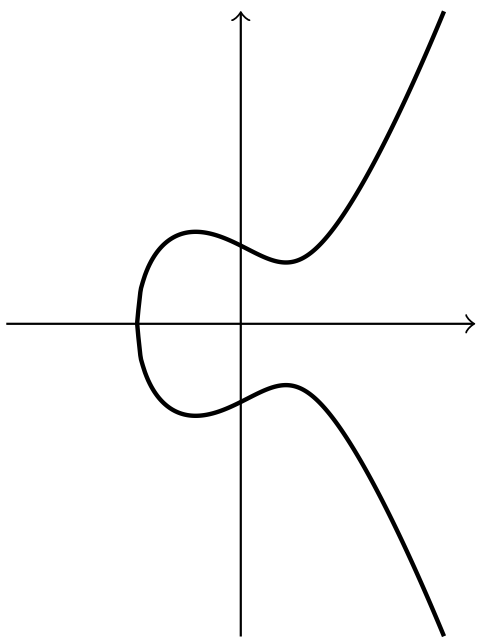
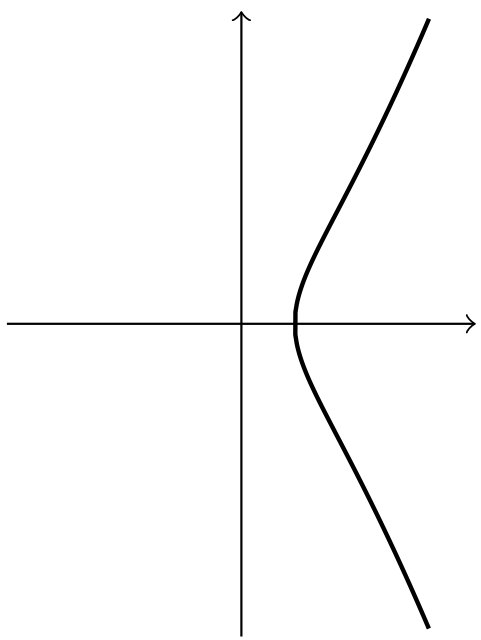
$$\mathbb{K} = \mathbb{R}$$



$y^2 = x^3 - 2x$



$y^2 = x^3 - x + 1$



$y^2 = x^3 + x - 1$

$$y^2 = x^3 + ax + b$$

Elliptic Curve Group Law

Group of points

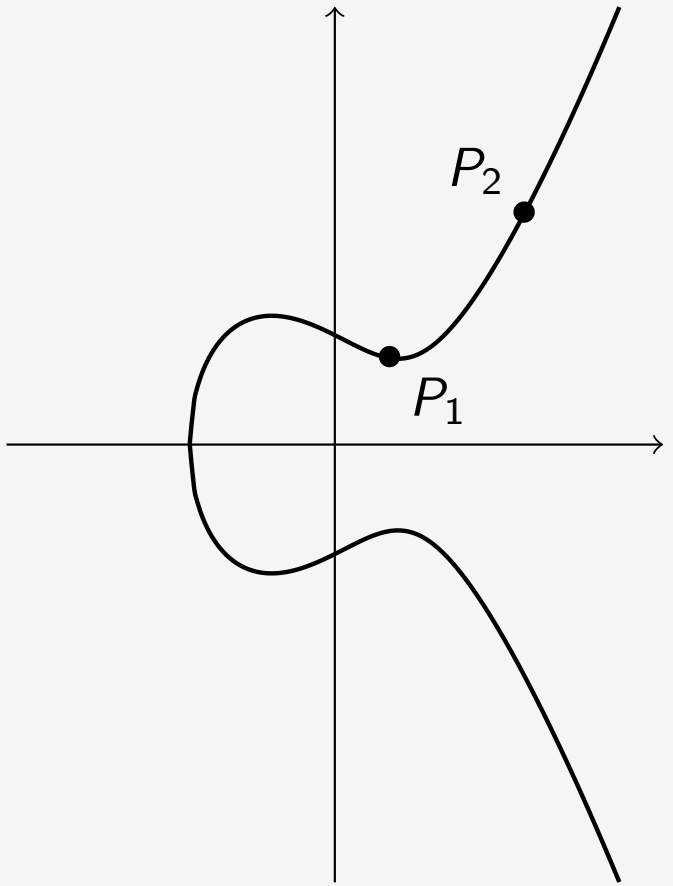
Let $\mathcal{E}(\mathbb{K})$ be the set of points $(x, y) \in \mathbb{K}^2$ satisfying the equation of $\mathcal{E}$ together with the *point at infinity* $\mathcal{O}$.

- $\mathcal{E}(\mathbb{K})$ is a group with neutral element $\mathcal{O}$
- Group law is denoted $+$

Elliptic Curve Group Law

$$\mathbb{K} = \mathbb{F}_p$$

Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, $P_1, P_2 \neq \mathcal{O}$.



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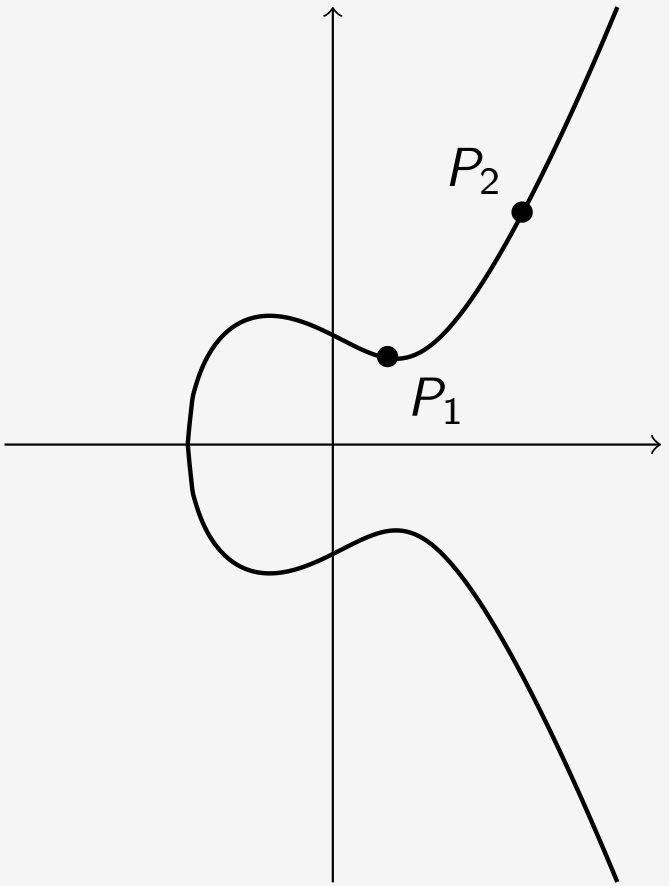

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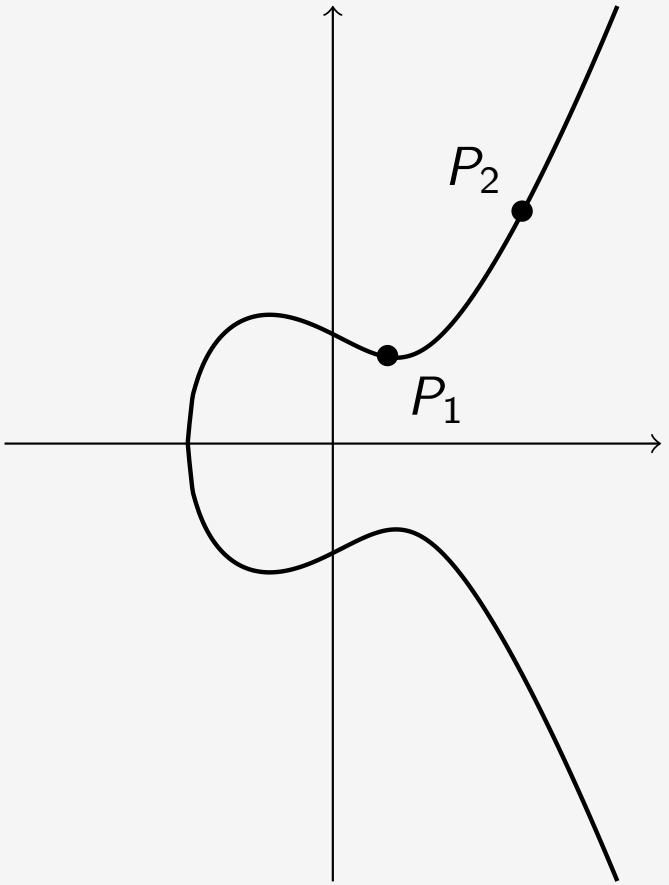

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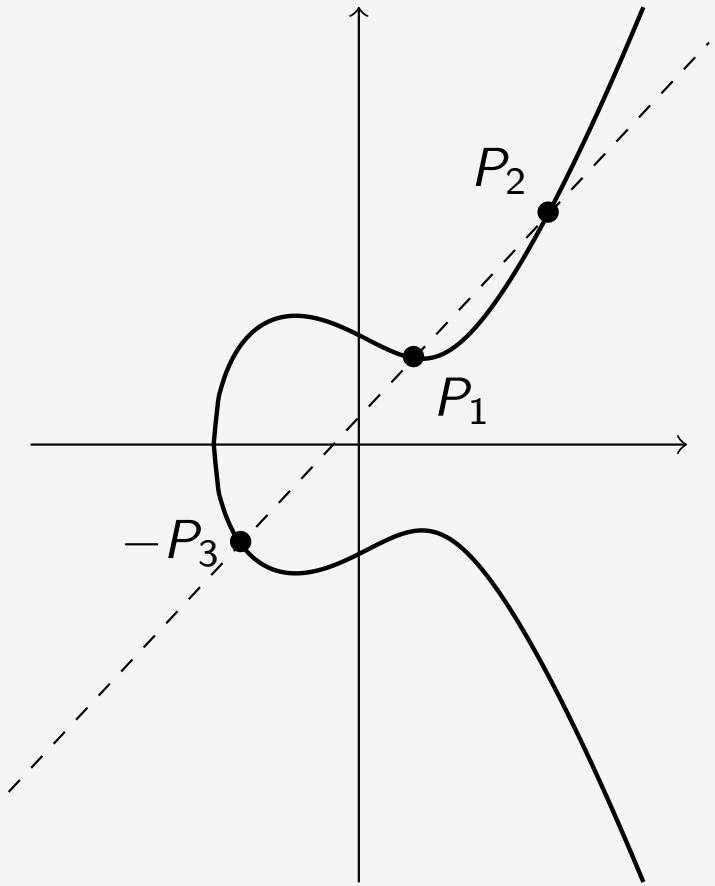

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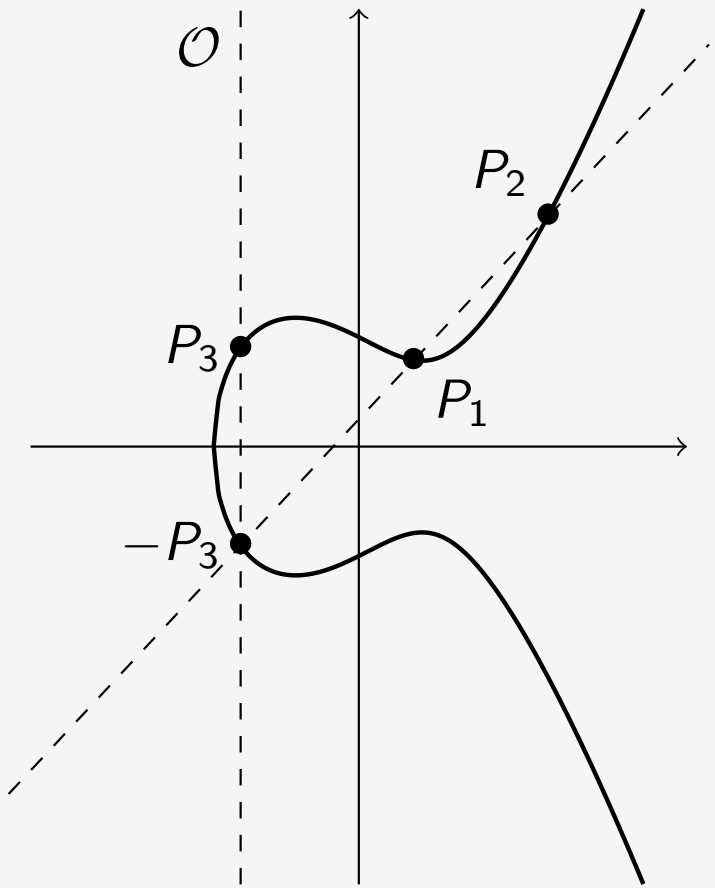

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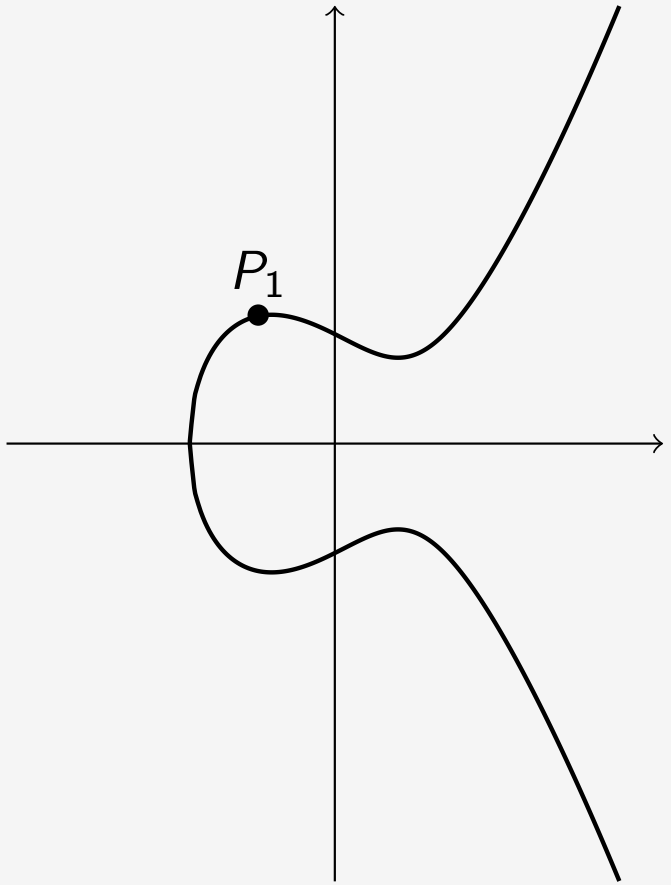

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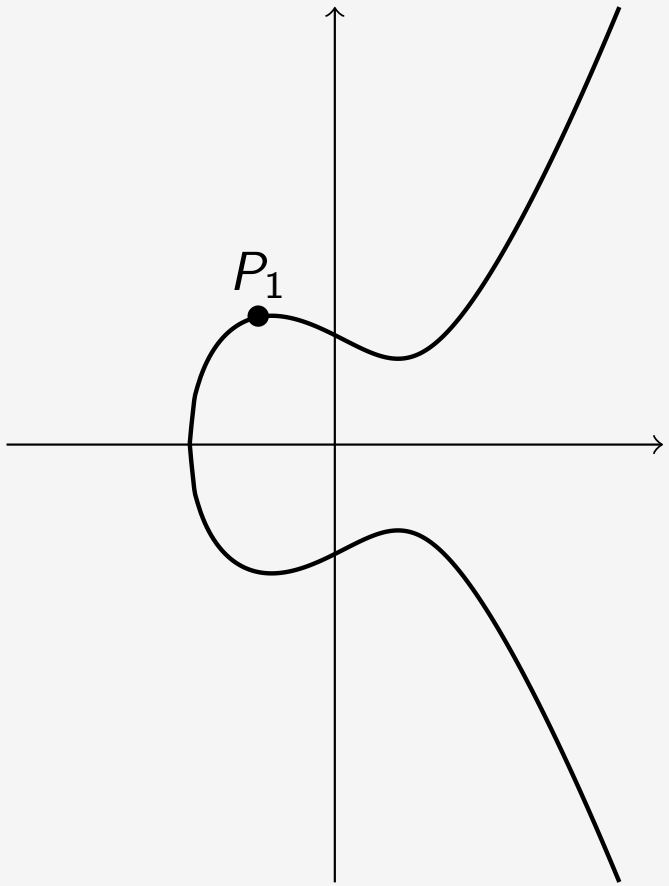

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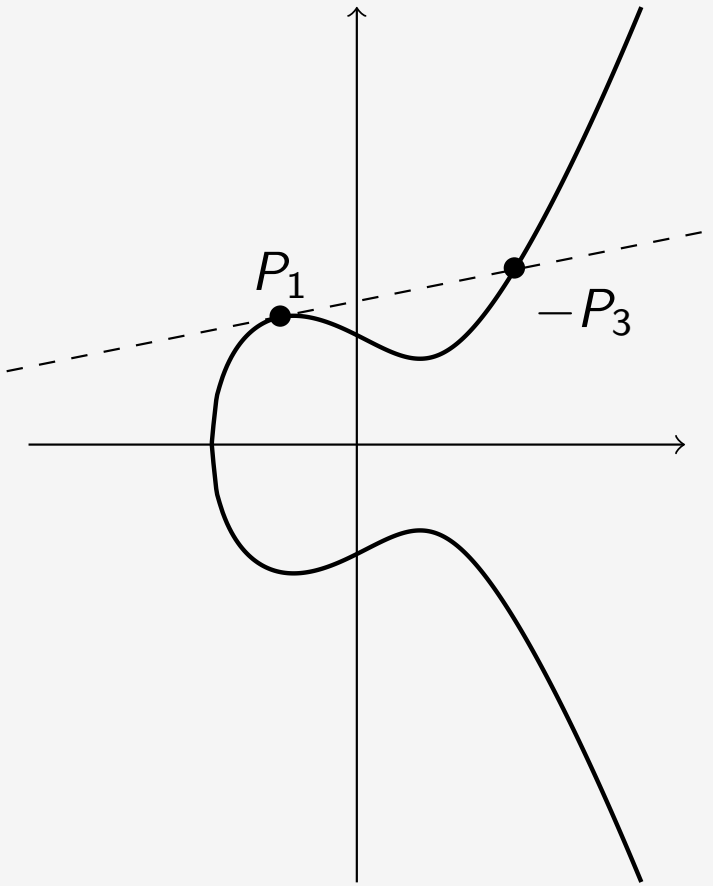
Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, $P_1, P_2 \neq \mathcal{O}$.

$P_3 = P_1 + P_2$ is given by:

$$\begin{cases} x_3 = m^2 - x_1 - x_2 \\ y_3 = m\,(x_1 - x_3) - y_1 \end{cases}$$

$$m = \frac{3{x_1}^2 + a}{2y_1} \quad \text{if} \quad P_1 = P_2$$



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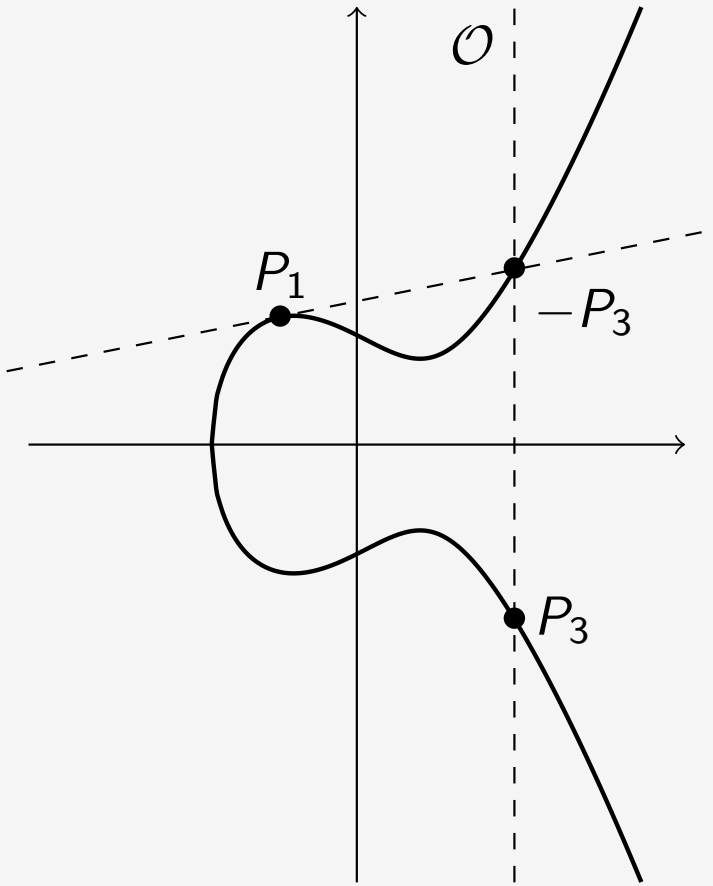

Scalar Multiplication

Given a point P in $\mathcal{E}(\mathbb{K})$ and an integer d,
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Given P in $\mathcal{E}(\mathbb{K})$ and $\alpha \cdot P$, $1 \leq \alpha \leq \#\mathcal{E}(\mathbb{K})$,
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Seems to be harder than DLP on finite fields or factorization.

Cryptosystems comparison

Diffie-H. Elgamal on GF

- multiplicative group $\mathbb{F}_p^* = \{1, 2, \ldots, p-1\}$
- DLP on $\mathbb{F}_p^*$
- keylength: 1024 to 16384 bits (eq. 2^{80} to 2^{256})

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ECC

- group of points $\mathcal{E}(\mathbb{K})$
- DLP on $\mathcal{E}(\mathbb{K})$
- keylength: 160 to 512 bits

Outline

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Exponentiation algorithms

Considering integers m, n, and $d = (d_{\ell-1} d_{\ell-2} \dots d_0)_2$,
how to compute $m^d \mod n$?

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Binary decomposition

- $m^d = m^{d_0} \times \left(m^{d_1} \times \left(\dots \left(m^{d_{\ell-1}} \right)^2 \dots \right)^2 \right)^2$
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Example with $d = 13 = (1101)_2$

$$m^{13} = m^1 \times \left(m^0 \times \left(m^1 \times \left(m^1 \right)^2 \right)^2 \right)^2$$

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Example with $d = 13 = (1101)_2$

$$\begin{aligned} m^{13} &= m^1 \times \left(m^0 \times \left(m^1 \times \left(m^1 \right)^2 \right)^2 \right)^2 \\ &= m^{1\times 8} \times m^{1\times 4} \times m^{0\times 2} \times m^{1\times 1} \end{aligned}$$

Left-to-right

$$m^d = m^{d_0} \times \left(m^{d_1} \times \left(\ldots \left(m^{d_{\ell-1}} \right)^2 \ldots \right)^2 \right)^2$$

Input: $m, n, d \in \mathbb{N}$
Output: $m^d \mod n$

```
a ← 1
for i = ℓ − 1 to 0 do
    a ← a^2 mod n
    if d_i = 1 then
        a ← a × m mod n
return a
```

Exponentiation algorithms

Square and multiply

Left-to-right

$$m^d = m^{d_0} \times \left(m^{d_1} \times \left(\ldots \left(m^{d_{\ell-1}} \right)^2 \ldots \right)^2 \right)^2$$

Input: $m, n, d \in \mathbb{N}$
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$a \leftarrow 1$
for $i = \ell - 1$ **to** 0 **do**
  $a \leftarrow a^2 \mod n$
  if $d_i = 1$ **then**
    $a \leftarrow a \times m \mod n$
return a

Right-to-left

$$m^d = m^{d_{\ell-1} 2^{\ell-1}} \times m^{d_{\ell-2} 2^{\ell-2}} \times \ldots \times m^{d_0}$$

Input: $m, n, d \in \mathbb{N}$
Output: $m^d \mod n$

$a \leftarrow 1$; $b \leftarrow m$
for $i = 0$ **to** $\ell - 1$ **do**
  if $d_i = 1$ **then**
    $a \leftarrow a \times b \mod n$
  $b \leftarrow b^2 \mod n$
return a

What about scalar multiplication ?

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- $d \cdot P = d_0 P + 2\left(d_1 P + 2\left(\dots + 2\left(d_{\ell-1} P\right)\dots\right)\right)$
- $d \cdot P = d_{\ell-1} 2^{\ell-1} P + d_{\ell-2} 2^{\ell-2} P + \dots + d_0 P$

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R ← O
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Scalar multiplication algorithms

Double and add

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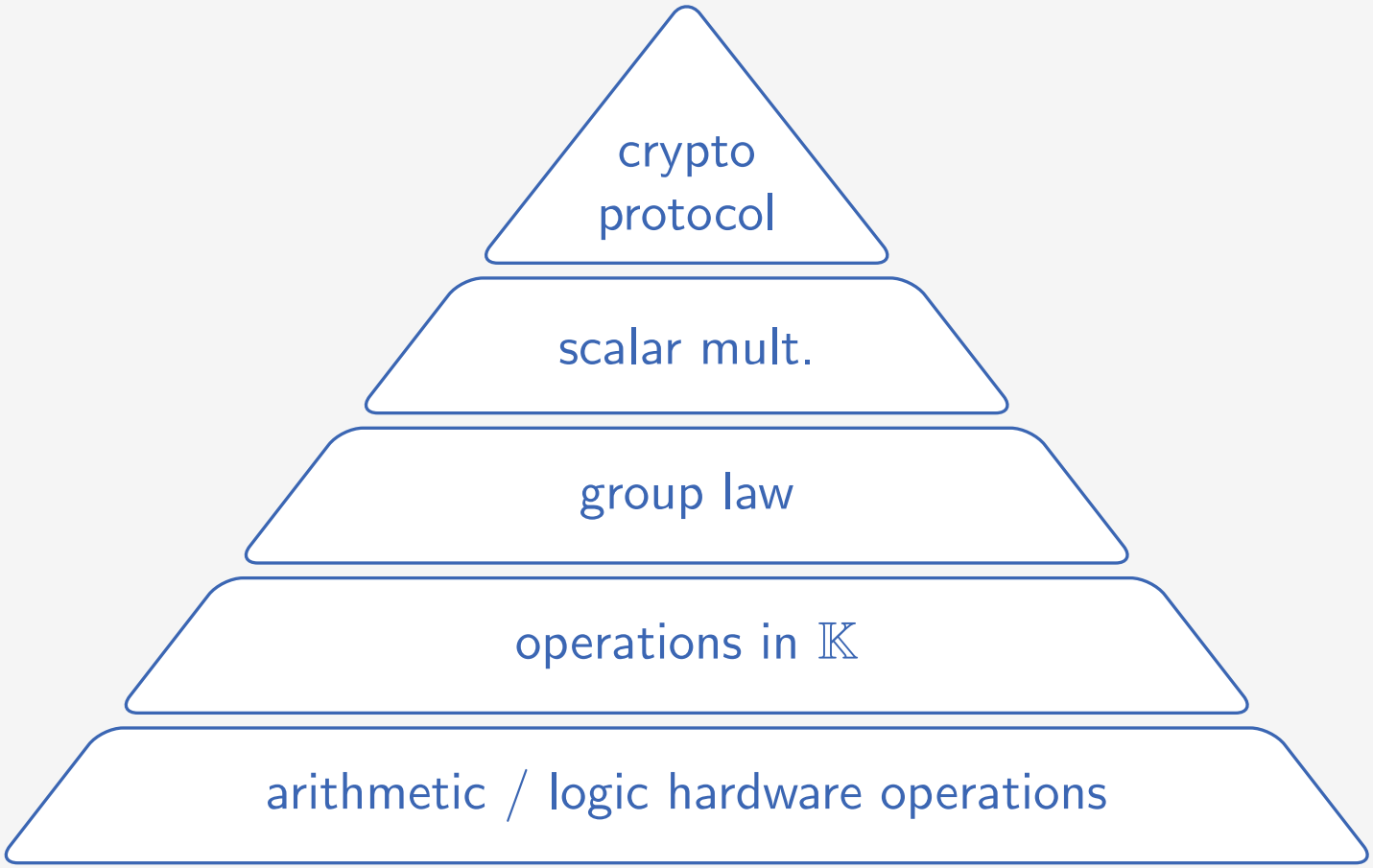

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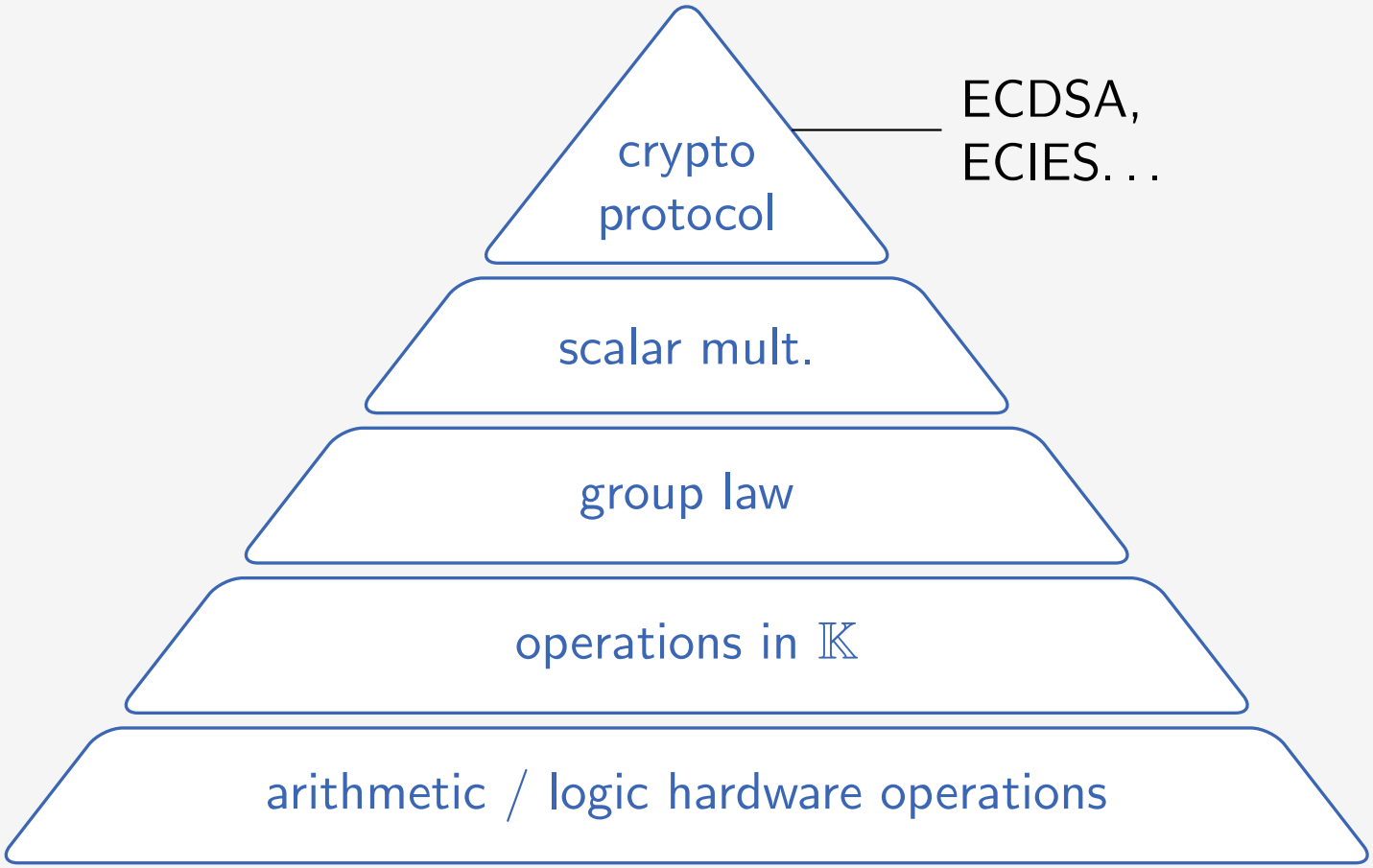

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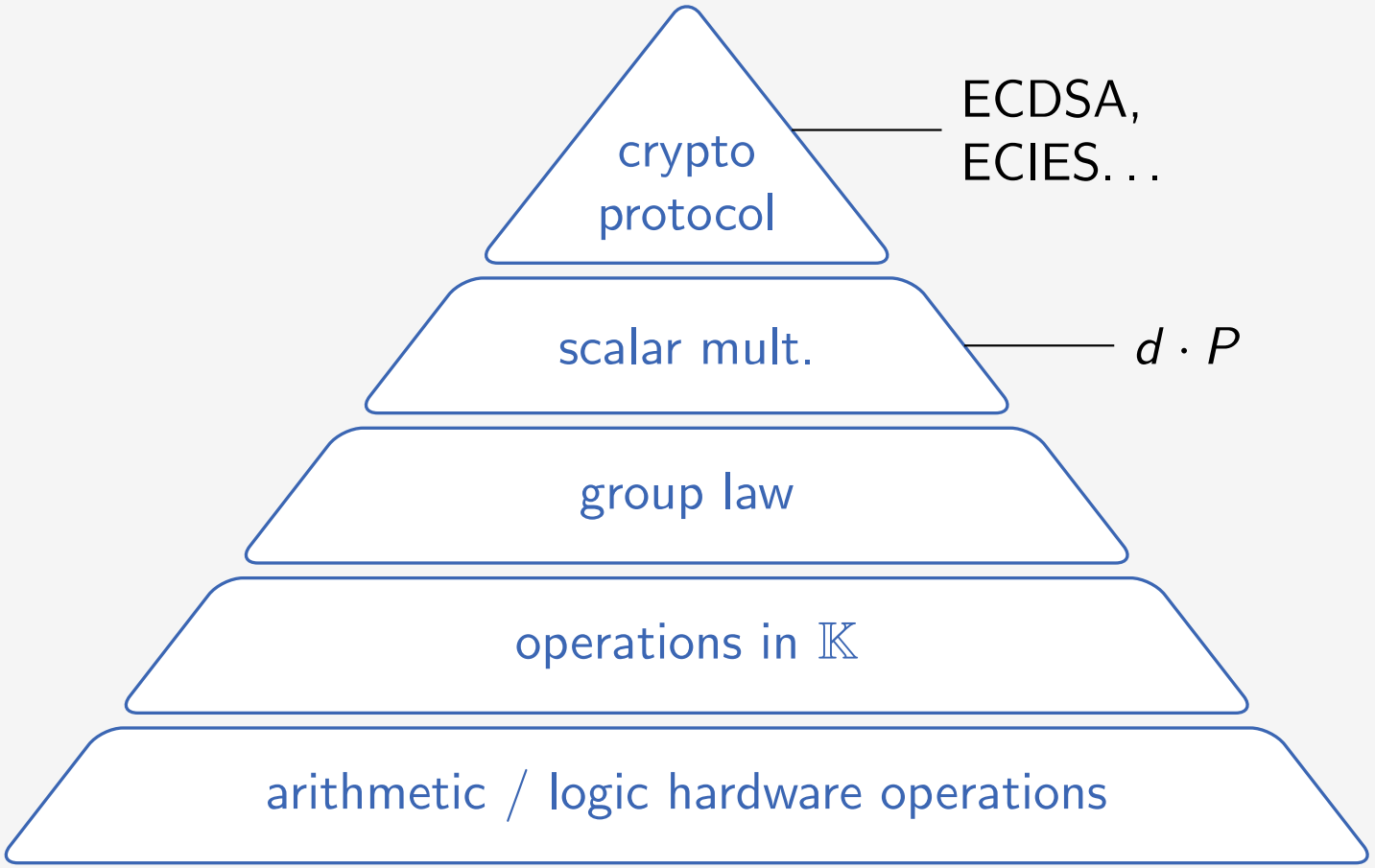

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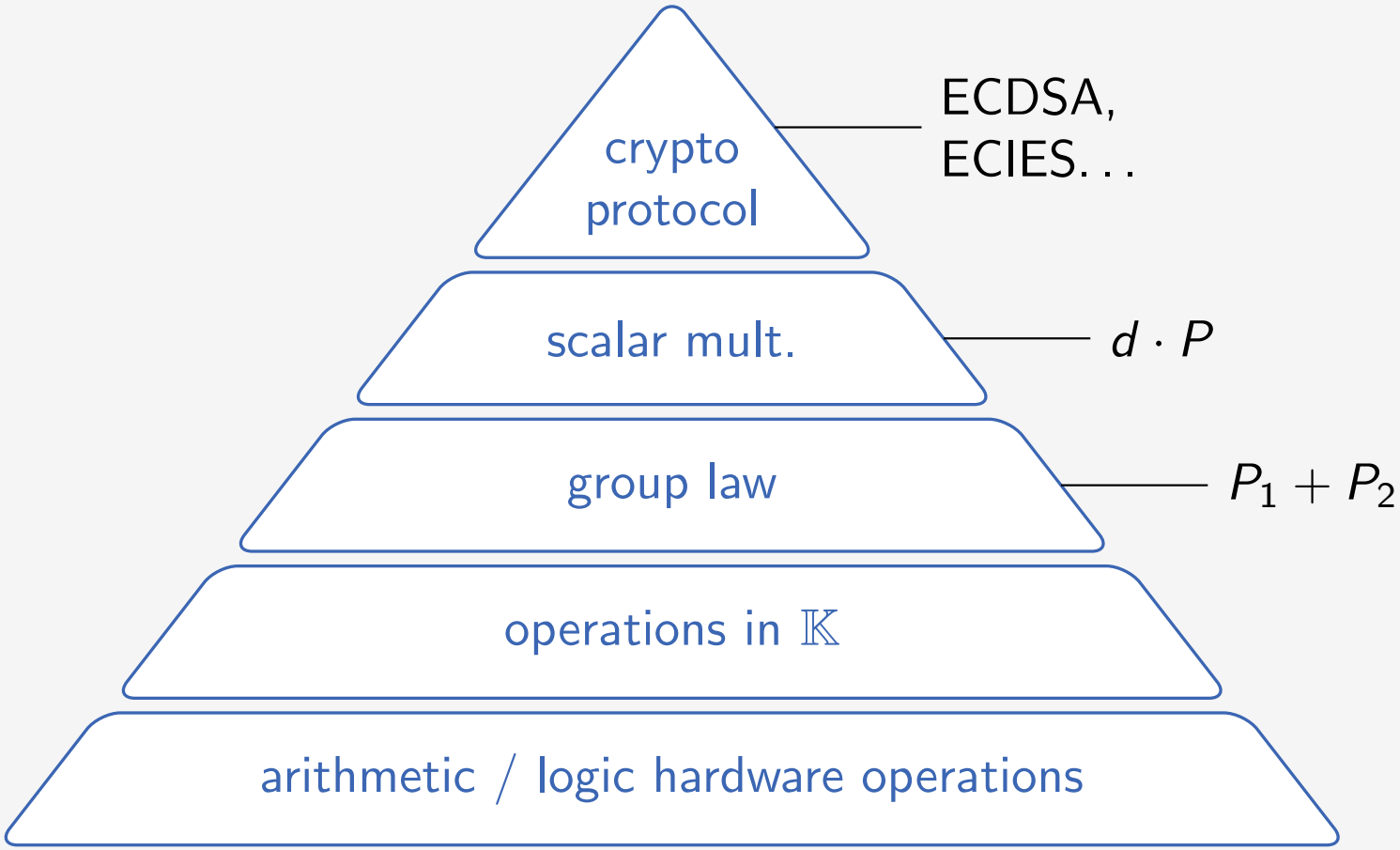

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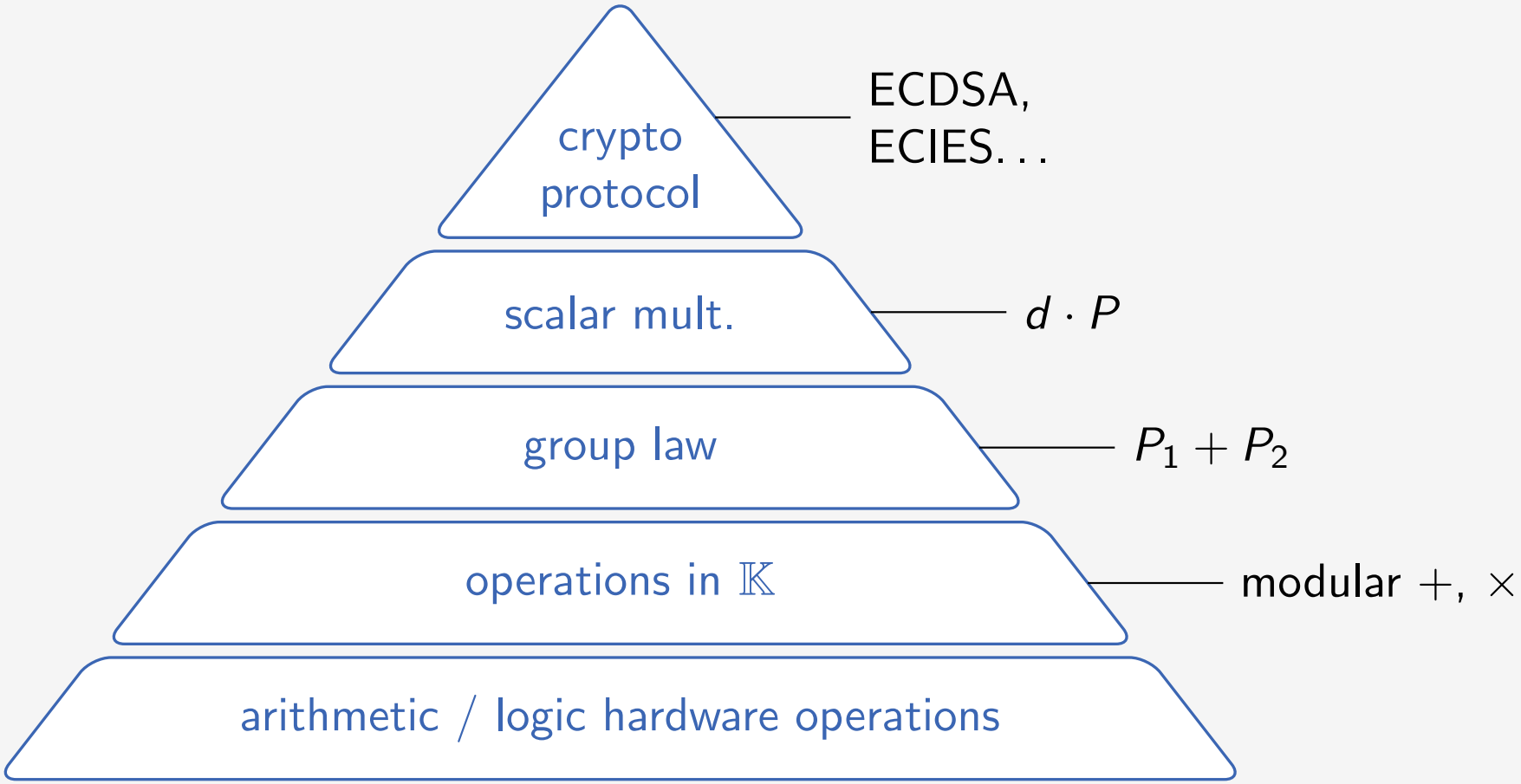

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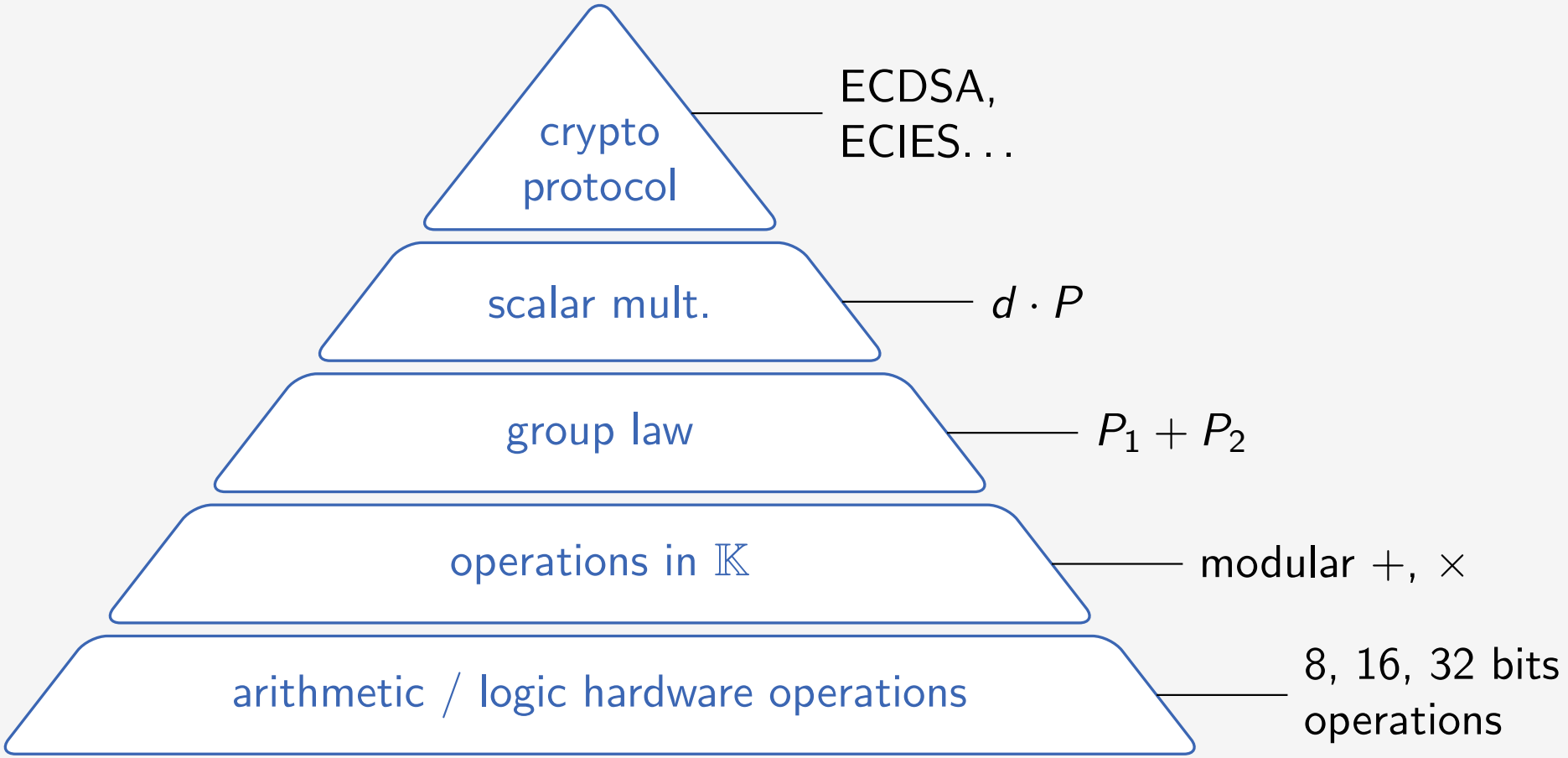

Outline

Embedded Devices Constraints

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- Most transactions have to take less than 500 ms

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A/M $\approx$ 0.2 on some smart cards

Affine Representation

A point of the curve $\mathcal{E} : y^2 = x^3 + ax + b$ is represented as (x, y).

No representation for $\mathcal{O}$

Add. : 1I + 2M + 1S, Doubl. : 1I + 2M + 2S

Homogeneous Projective Representation

A point is represented by an equivalence class $(X : Y : Z)$.

$(X : Y : Z)$ and $(\lambda X : \lambda Y : \lambda Z)$, $\lambda \neq 0$ represent the same point

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Add. : 12M + 2S, Doubl. : 6M + 6S

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Add. : 11M + 5S, Doubl. : 2M + 8S

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Faster doubling than Jacobian projective : 3M + 5S

But slower addition : 13M + 7S

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Output: $d \cdot P$

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On average :

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Interest

- Minimize the number of additions
- $P \to -P$ is cheap : $(X : Y : Z) \to (X : -Y : Z)$

NAF Multiplication

Right-to-Left

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Mixed coord. : 12,0M/bit

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Co-Z Addition

Euclidean Addition Chains [Meloni, WAIFI 2007]
Co-Z binary ladder [Goundar, Joye & Miyaji, CHES 2010]

Outline

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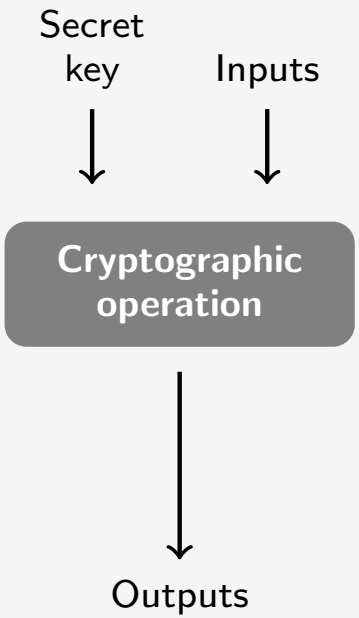
Physical Analysis Framework

Passive Attacks



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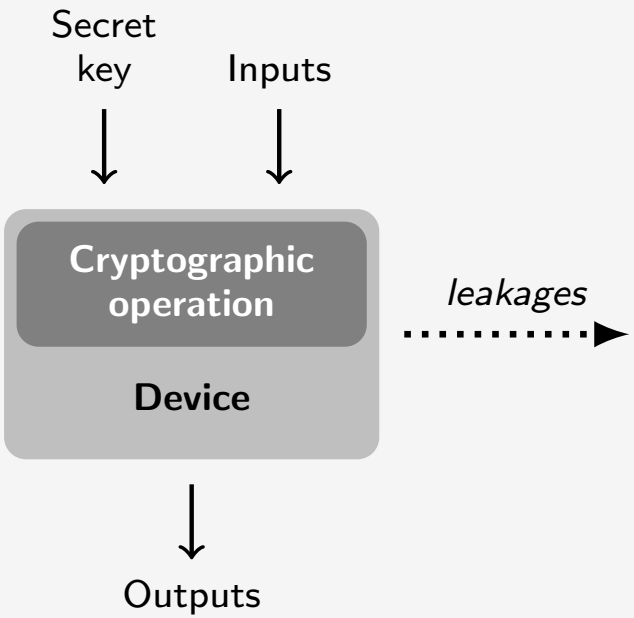

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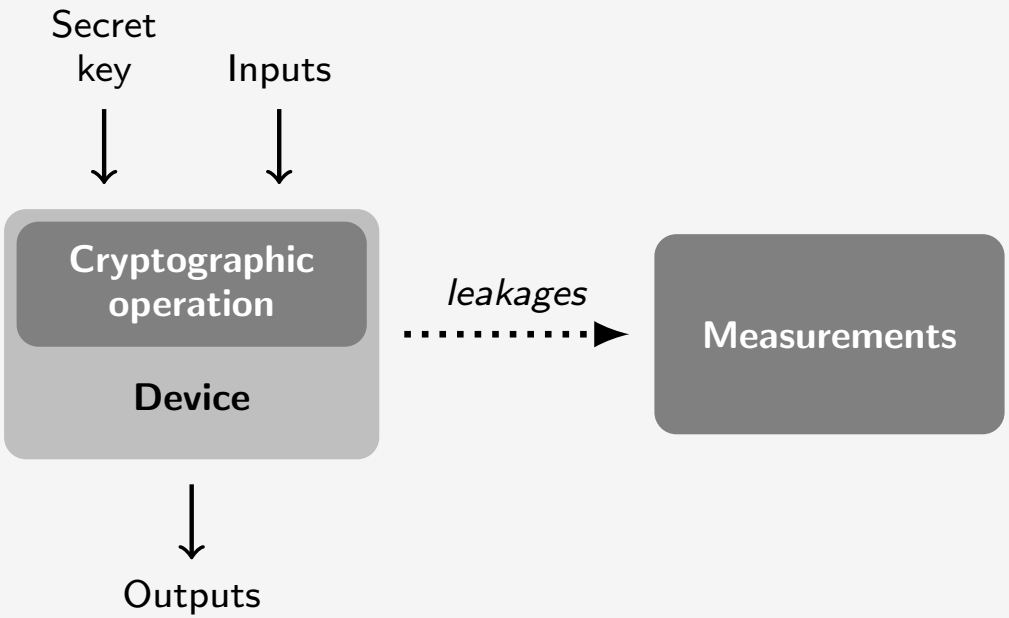

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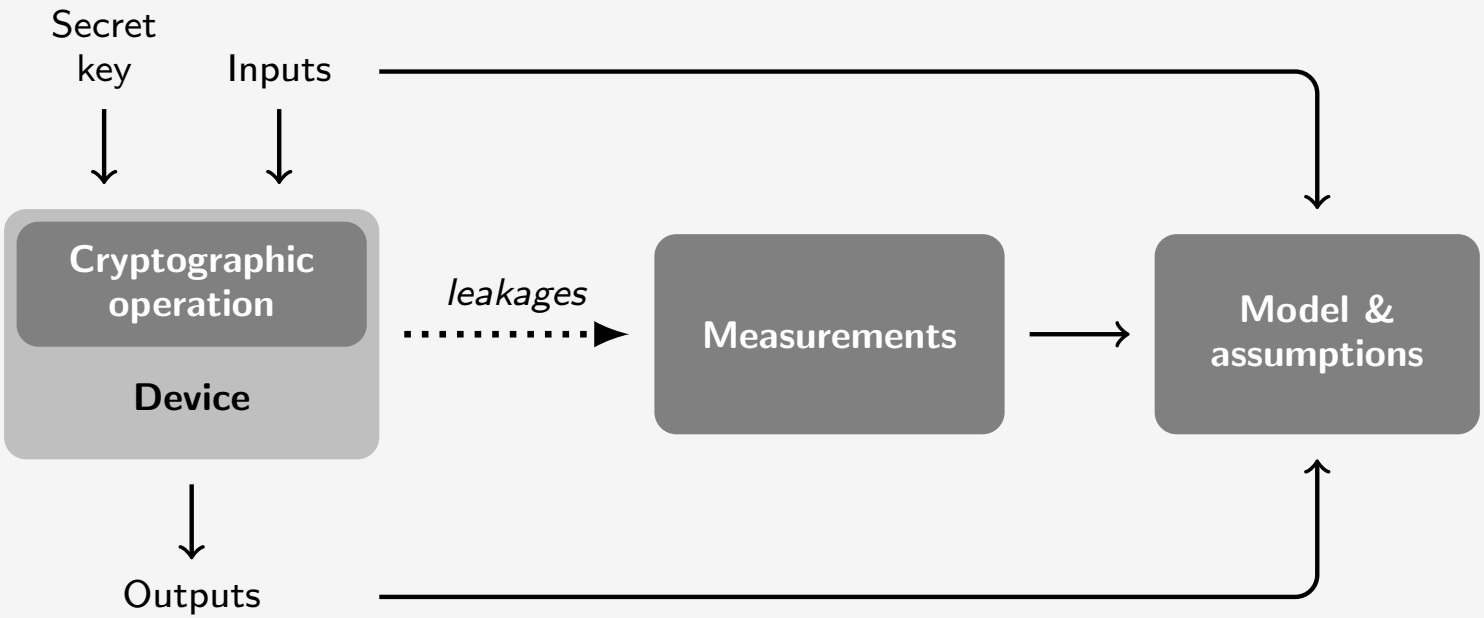

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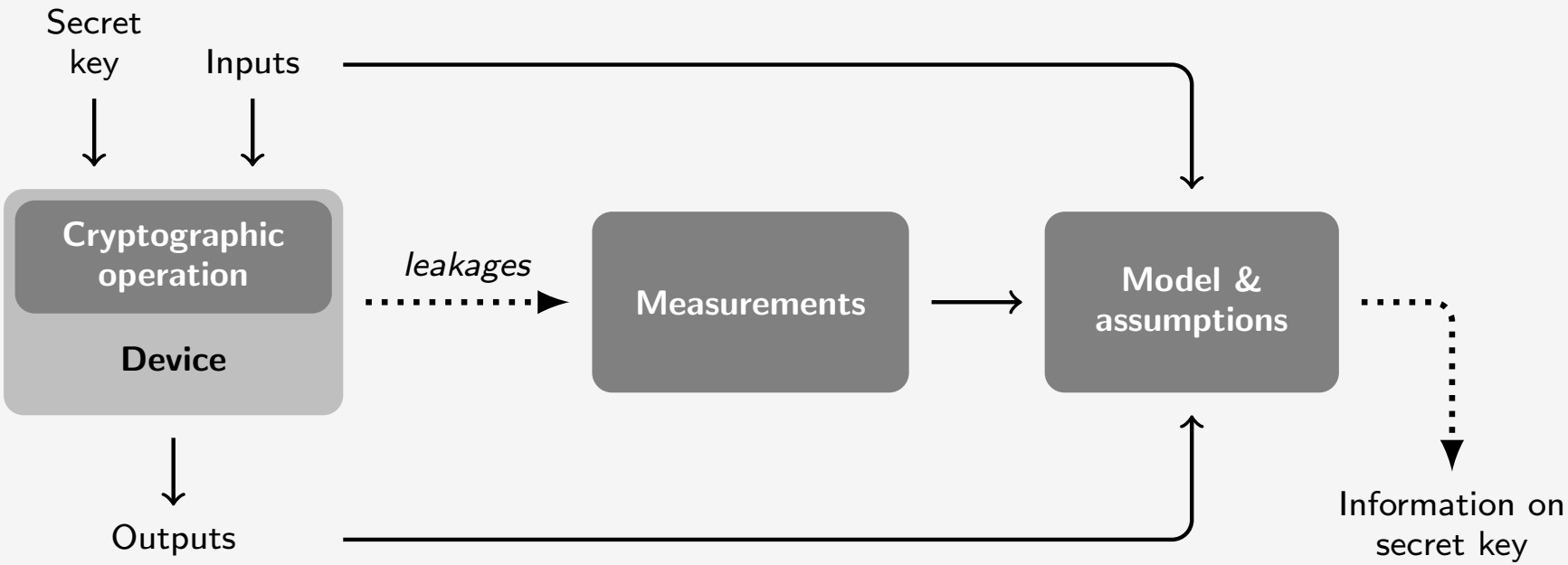
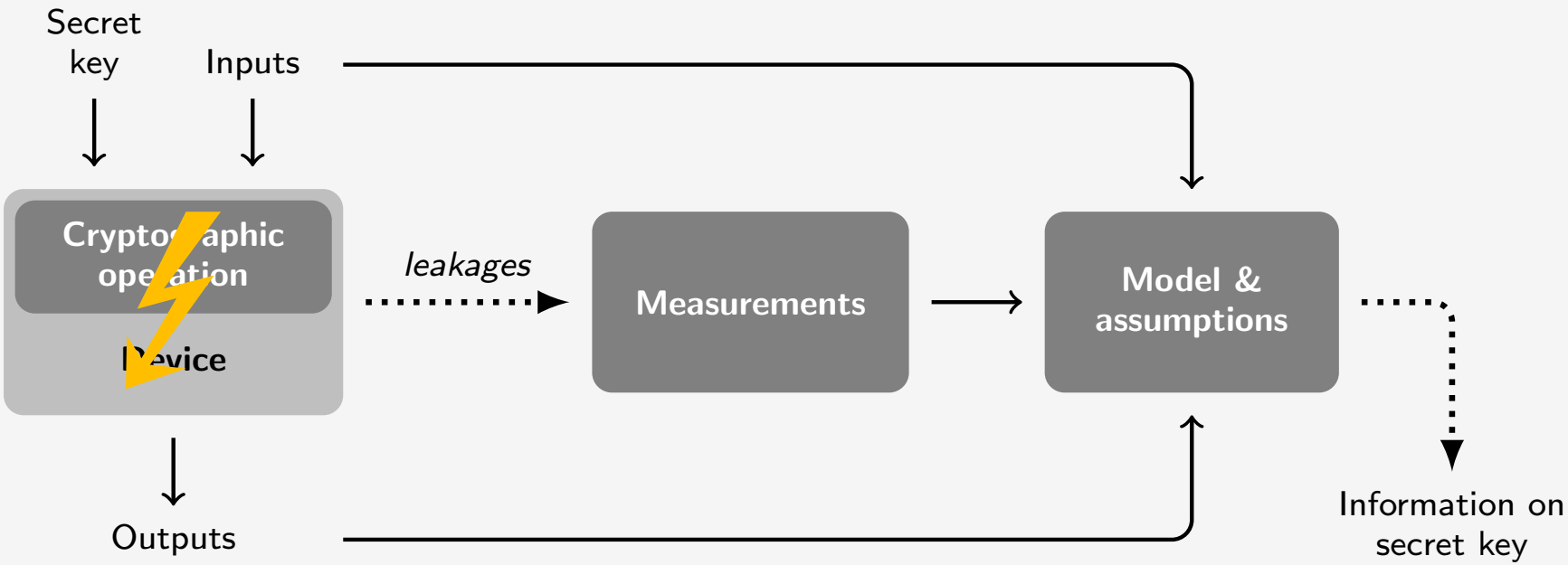


Physical Analysis Framework

Active Attacks

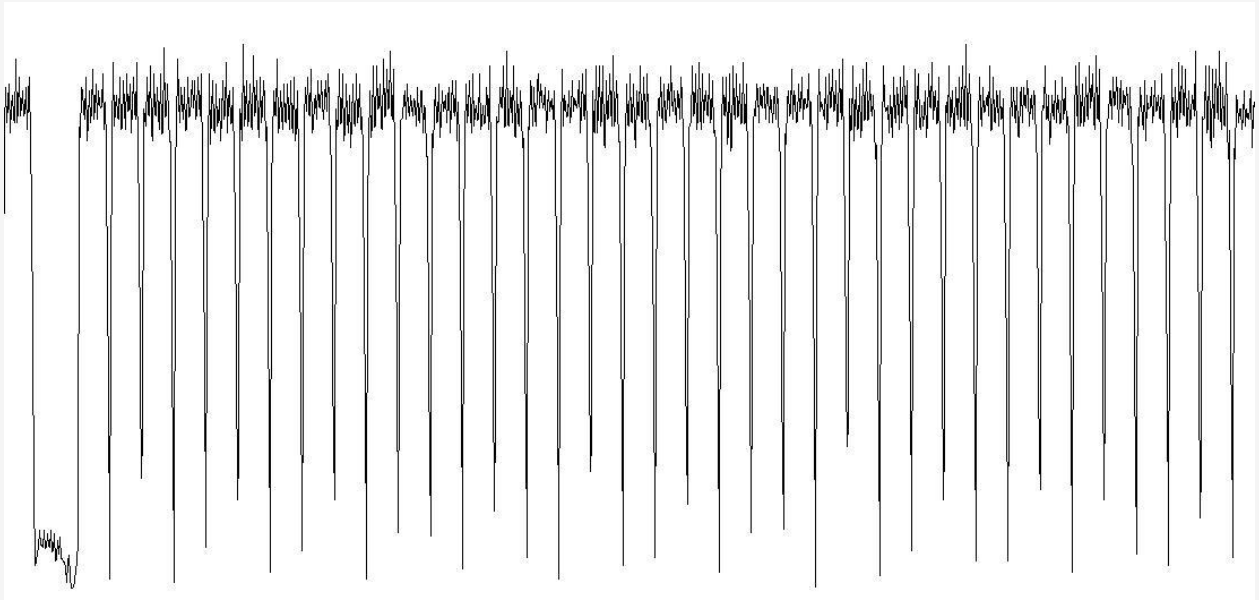
Physical Analysis Framework

Combined Attacks



Simple Analyse Example

Leakage on Performed Operations



RSA exponentiation

Simple Analyse Example

Leakage on Manipulated Data

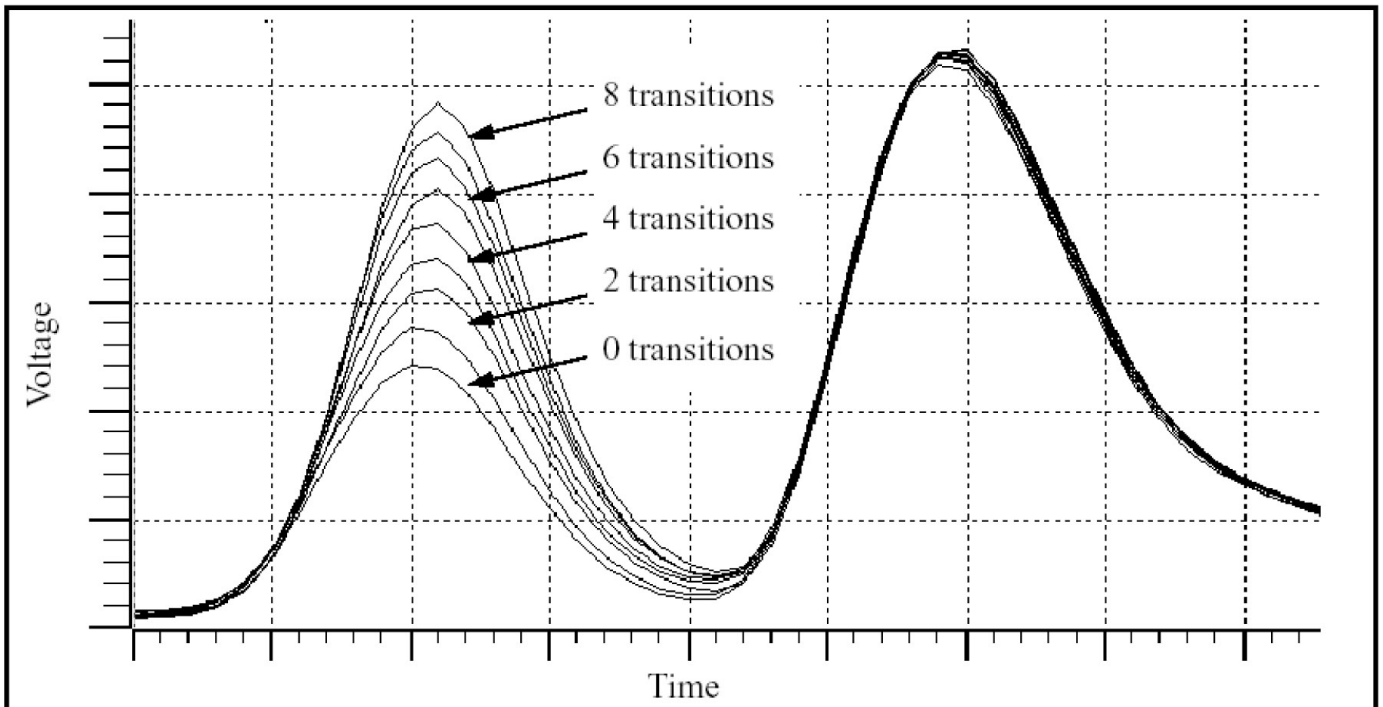


FIGURE 2. **Number of Bit Transitions versus Power Consumption**

These results show how the data effects the power levels. The nine overlayed waveforms correspond to the power traces of different data being accessed by an LDA instruction. These results were obtained by averaging the power signals across 500 samples in order to reduce the noise content. The difference in voltage between i transitions and i+1 transitions is about 6.5 mV.

Milestones

- Timing Attacks [Kocher, *Timing Attacks on Implementations of Diffie-Hellman, RSA, DSS, and Other Systems*, Crypto 1996]
- Fault Attacks [Boneh et al., *On the Importance of Checking Cryptographic Protocols for Faults*, Eurocrypt 1997]
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Outline

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Measure one side-channel leakage s function of t and consider the curve $s(t)$.

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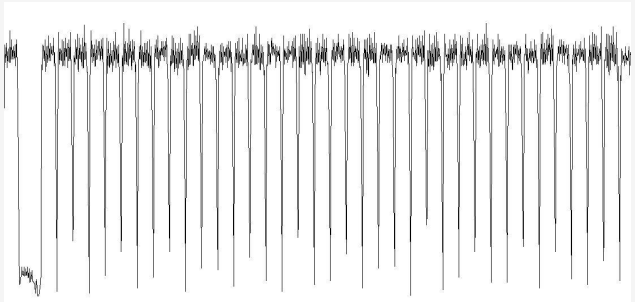

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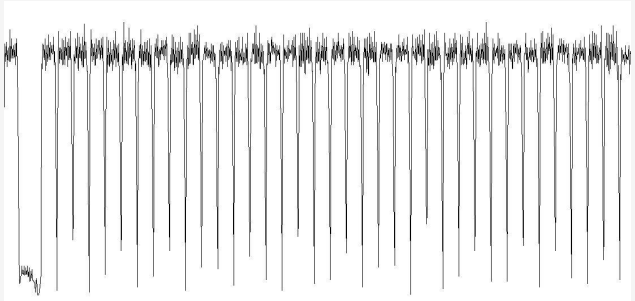

SPA/SEMA

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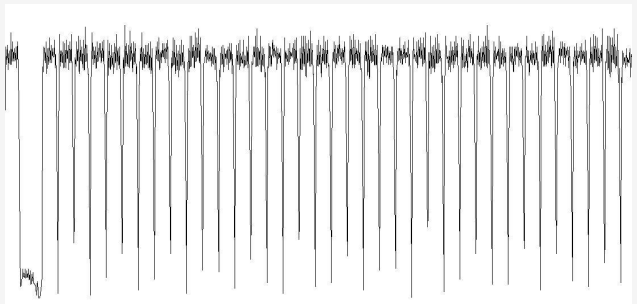

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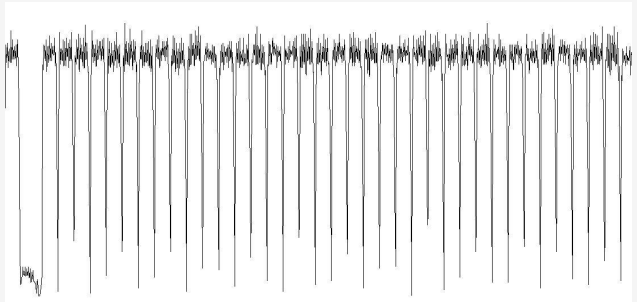

SPA/SEMA

- depicts the behavior of the chip depending on the performed operations / manipulated data
- each measure enables direct reading

Example

Left-to-Right Double and add Algorithm Analysis

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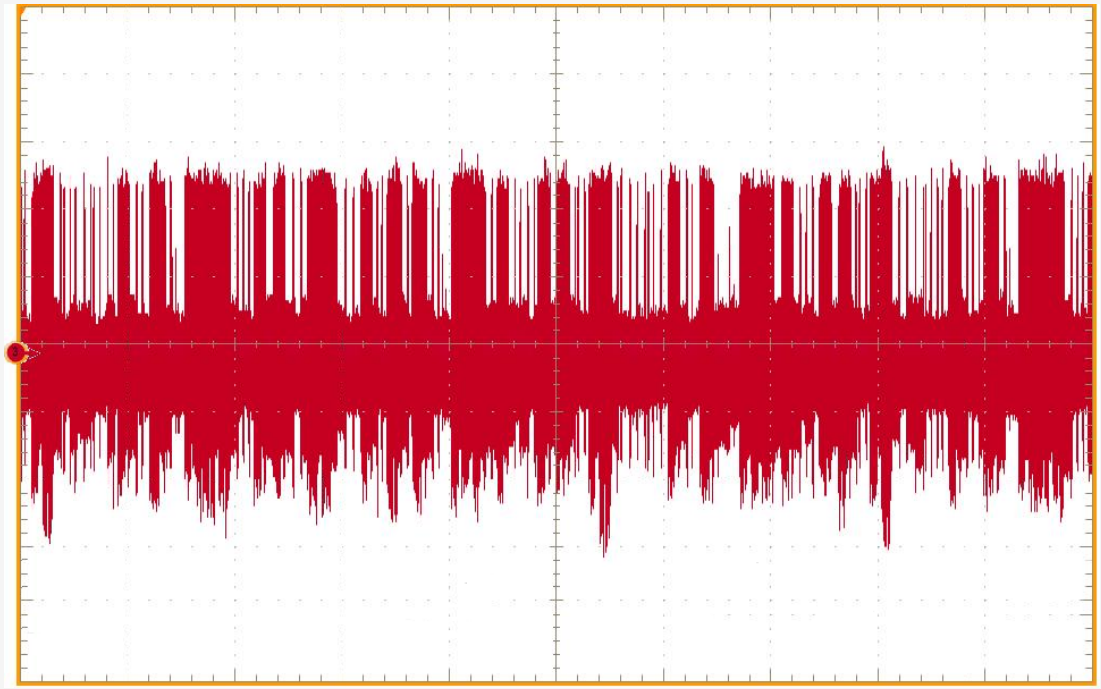

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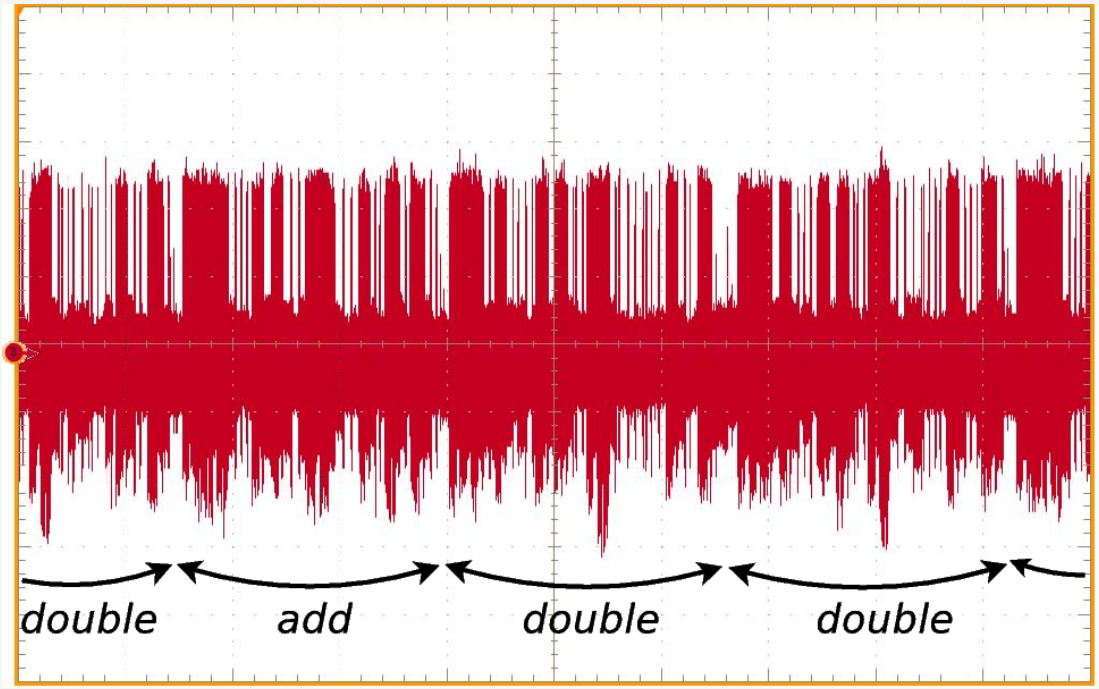

Outline

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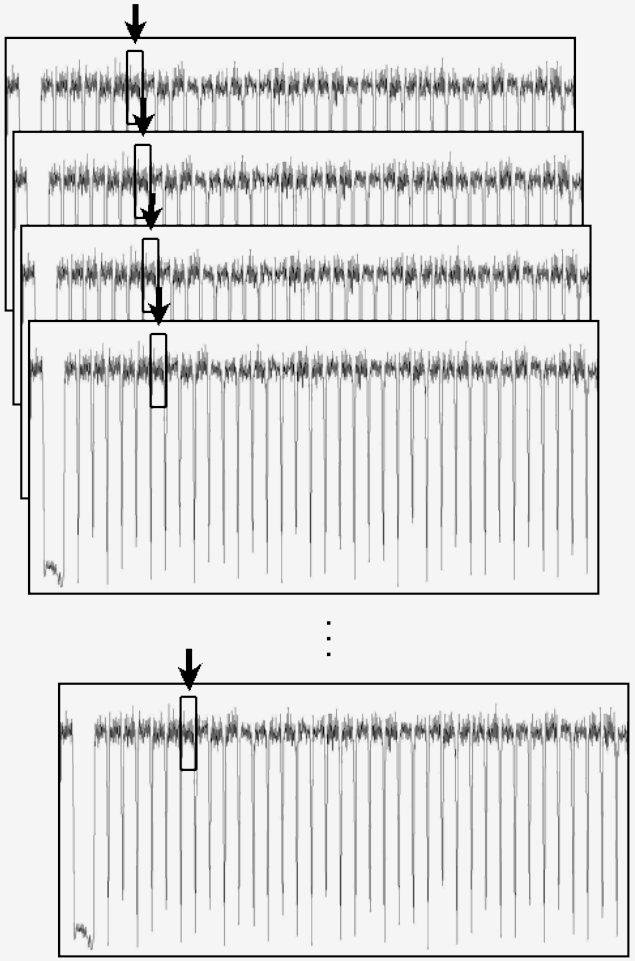

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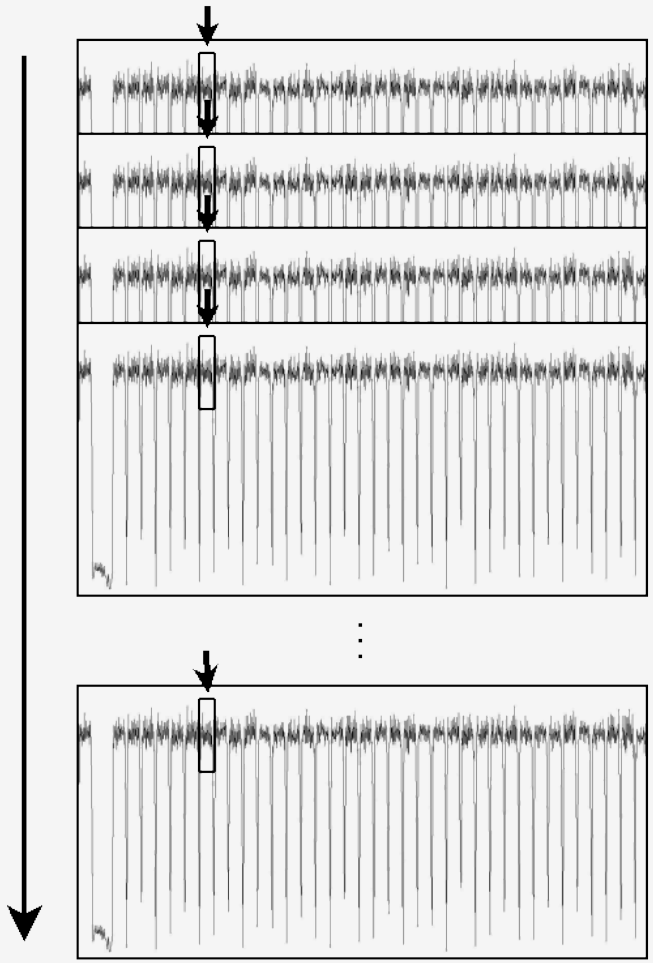

Differential Analysis

Statistical Treatment

Original DPA/DEMA

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Example

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$$C_1$$
$$C_2$$
$$\vdots$$
$$C_N$$

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$$\begin{matrix} C_1 & P_1 \\ C_2 & P_2 \\ \vdots & \vdots \\ C_N & P_N \end{matrix}$$

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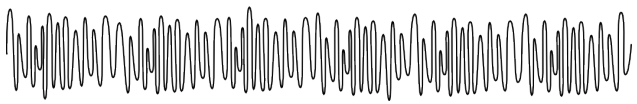
Compute $< S_0 > - < S_1 >$: 

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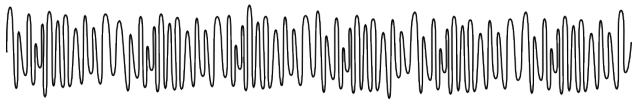
Compute $< S_0 > - < S_1 >$: 

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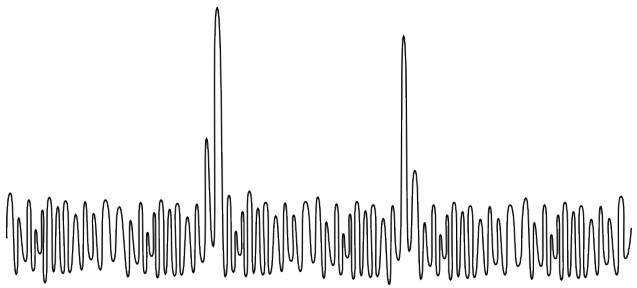

CPA/CEMA

Similar to the differential analysis, but uses the Pearson correlation factor :

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- Iterate and apply the chinese reminder theorem to recover k.

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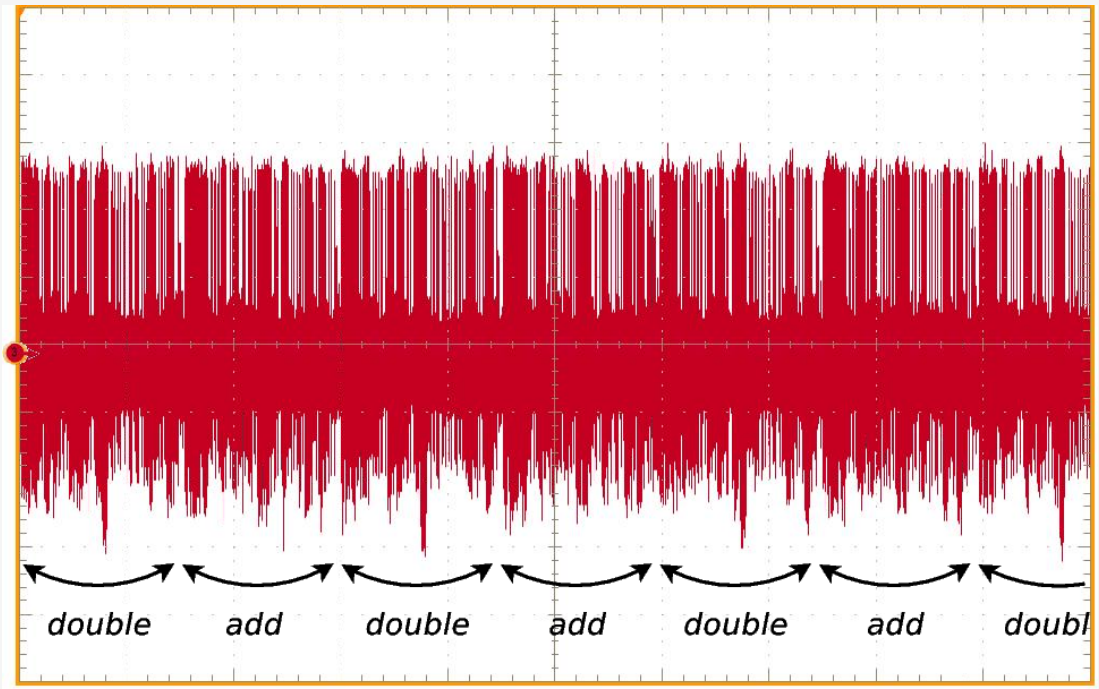

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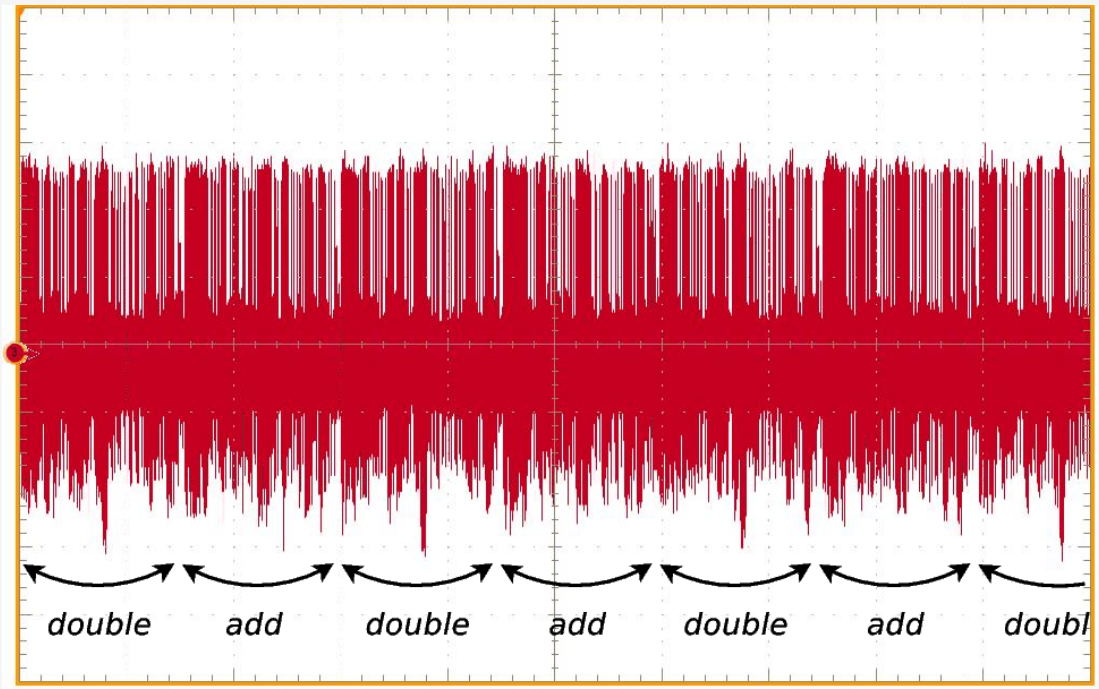

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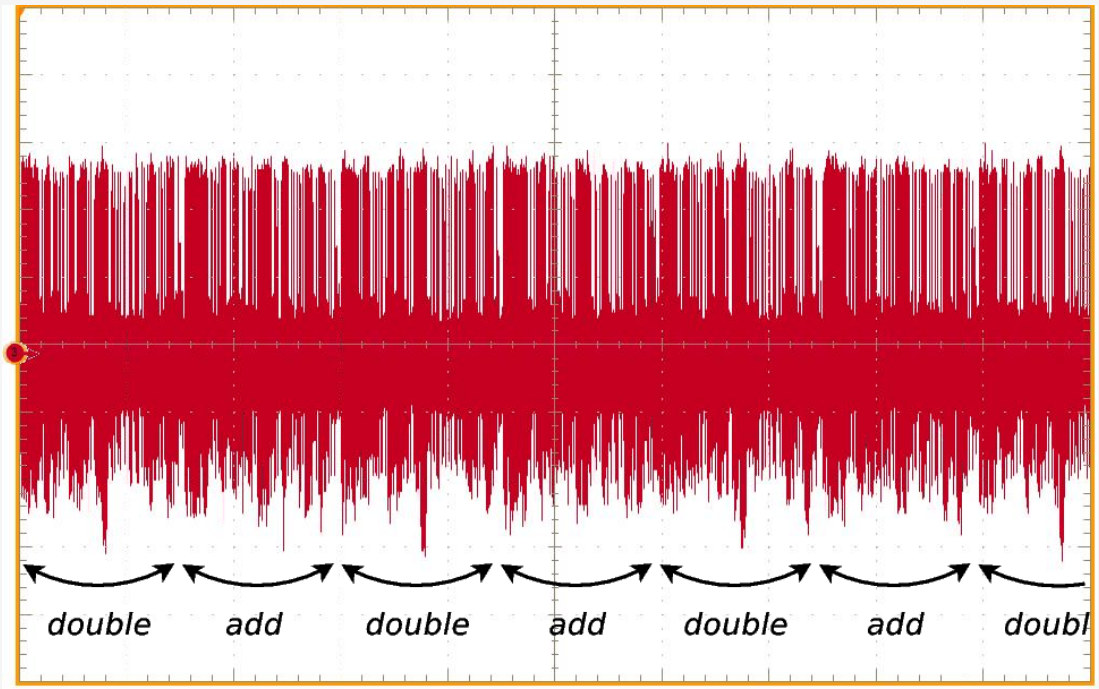

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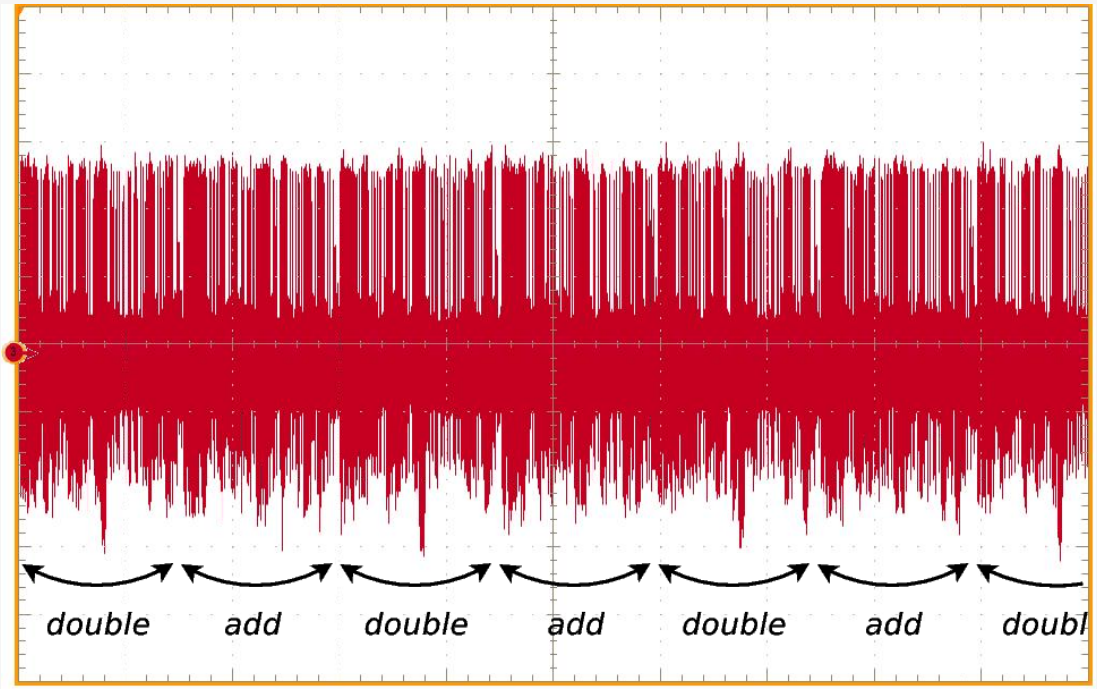


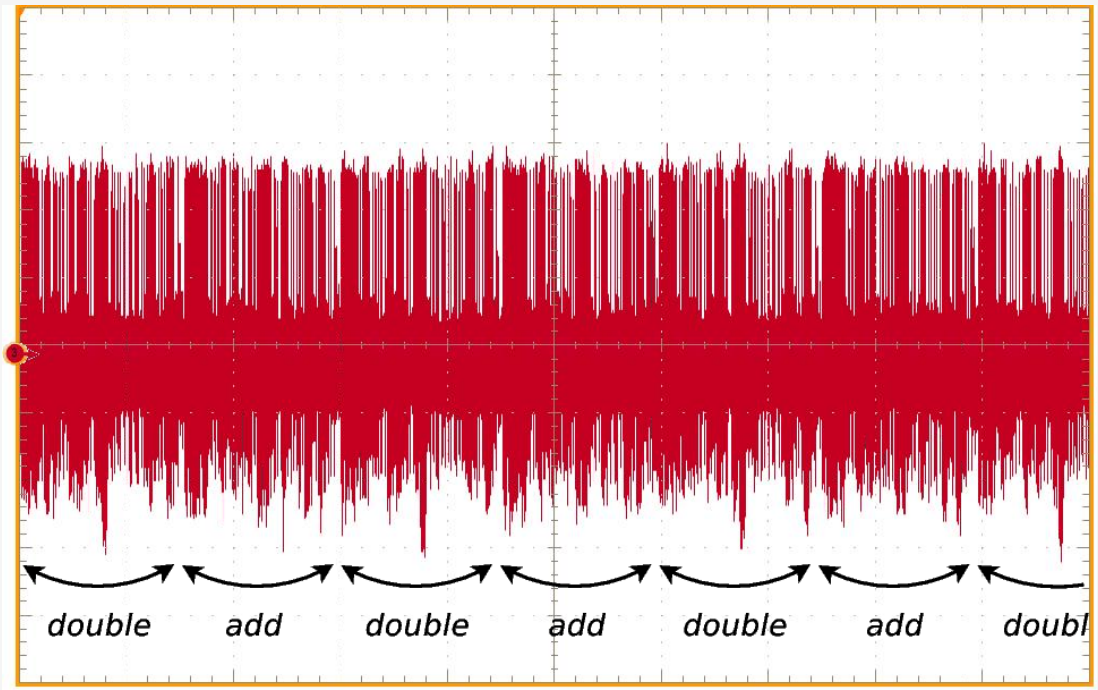
Input: $P \in \mathcal{E}(\mathbb{K}), d \in \mathbb{N}$
Output: $d \cdot P$

$R_0 \leftarrow \mathcal{O}$
$R_1 \leftarrow P$
for $i = \ell - 1$ **to** 0 **do**
  $R_{1-d_i} \leftarrow R_0 + R_1$
  $R_{d_i} \leftarrow 2R_{d_i}$
return R_0

Trick:

Y_1 and Y_2 computation can be avoided.

- Brier & Joye, PKC 2002
- Izu & Takagi, PKC 2002
- Fischer et al., ePrint 2002



Unified Formulas

A single formula for addition and doubling

Unified Formulas

A single formula for addition and doubling

- Homogeneous projective coordinates : 12M + 6S
- Edwards curves : 10M + 1S
- Twisted Edwards curves : 9M + 1S

Atomicity

Introduced in [Chevallier-Mames, Ciet & Joye, *Low-cost solutions for preventing simple side-channel analysis...*, ePrint 2003].

Idea : always repeat the same pattern of operations

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- S, M, S, S, S, M, S, S, M, S, M, ...

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$\rightarrow$ Cost

Atomicity for Elliptic Curves

Principle

Always repeat the same pattern :

Atomicity for Elliptic Curves

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Always repeat the same pattern :

- Multiplication
- Addition
- Negation
- Addition

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...

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- Multiplication
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...

No more squarings :-(

Atomicity for Elliptic Curves

Principle

Always repeat the same pattern :

- Multiplication
- Addition
- Negation
- Addition

- Multiplication
- Addition
- Negation
- Addition

...

No more squarings :-(
Many dummy additions/negations :-(

Longa Atomicity

Other patterns

In [Longa, *Accelerating the Scalar Multiplication on Elliptic Curve Cryptosystems over Prime Fields*, 2007] are proposed 2 new patterns :

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Longa Atomicity

Other patterns

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- Multiplication
- Negation
- Addition
- Multiplication
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- Squaring
- Negation
- Addition
- Multiplication
- Negation
- Addition
- Addition

Atomicity Improvement

Full paper: [Giraud & Verneuil, *Atomicity Improvement for Elliptic Curve Scalar Multiplication*, CARDIS 2010]

Two steps

- First define the largest atomic pattern possible
- Then remove as many possible dummy operations

Atomicity Improvement

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Two steps

- First define the largest atomic pattern possible
- Then remove as many possible dummy operations

Advantages

- Potentially applicable to every algorithm (no curve restriction)
- Prevents from the SPA at a lower cost than classical atomicity

Atomic Joye's Multiplication

Best pattern

	Add. 1	Add. 2	Dbl.
Sq.	$R_1 \leftarrow {Z_2}^2$	$R_1 \leftarrow {R_6}^2$	$R_1 \leftarrow {X_1}^2$
Add.	$\star$	$\star$	$R_2 \leftarrow Y_1 + Y_1$
Mult.	$R_2 \leftarrow Y_1 \cdot Z_2$	$R_4 \leftarrow R_5 \cdot R_1$	$Z_2 \leftarrow R_2 \cdot Z_1$
Add.	$\star$	$\star$	$R_4 \leftarrow R_1 + R_1$
Mult.	$R_5 \leftarrow Y_2 \cdot Z_1$	$R_5 \leftarrow R_1 \cdot R_6$	$R_3 \leftarrow R_2 \cdot Y_1$
Add.	$\star$	$\star$	$R_6 \leftarrow R_3 + R_3$
Mult.	$R_3 \leftarrow R_1 \cdot R_2$	$R_1 \leftarrow Z_1 \cdot R_6$	$R_2 \leftarrow R_6 \cdot R_3$
Add.	$\star$	$\star$	$R_1 \leftarrow R_4 + R_1$
Add.	$\star$	$\star$	$R_1 \leftarrow R_1 + W_1$
Sq.	$R_4 \leftarrow {Z_1}^2$	$R_6 \leftarrow {R_2}^2$	$R_3 \leftarrow {R_1}^2$
Mult.	$R_2 \leftarrow R_5 \cdot R_4$	$Z_3 \leftarrow R_1 \cdot Z_2$	$R_4 \leftarrow R_6 \cdot X_1$
Add.	$\star$	$R_1 \leftarrow R_4 + R_4$	$R_5 \leftarrow W_1 + W_1$
Sub.	$R_2 \leftarrow R_2 - R_3$	$R_6 \leftarrow R_6 - R_1$	$R_3 \leftarrow R_3 - R_4$
Mult.	$R_5 \leftarrow R_1 \cdot X_1$	$R_1 \leftarrow R_5 \cdot R_3$	$W_2 \leftarrow R_2 \cdot R_5$
Sub.	$\star$	$X_3 \leftarrow R_6 - R_5$	$X_2 \leftarrow R_3 - R_4$
Sub.	$\star$	$R_4 \leftarrow R_4 - X_3$	$R_6 \leftarrow R_4 - X_2$
Mult.	$R_6 \leftarrow X_2 \cdot R_4$	$R_3 \leftarrow R_4 \cdot R_2$	$R_4 \leftarrow R_6 \cdot R_1$
Sub.	$R_6 \leftarrow R_6 - R_5$	$Y_3 \leftarrow R_3 - R_1$	$Y_2 \leftarrow R_4 - R_2$

Outline

DPA/DEMA Protection

Classical countermeasures :

- Scalar blinding : $d' = d + r\#\mathcal{E}(\mathbb{K})$
- Point coordinates blinding : $(X : Y : Z) = (r^2X : r^3Y : rZ)$, $r \neq 0$
- Random curve isomorphism :
  $a' \leftarrow r^4 a$
  $b' \leftarrow r^6 b$
  $P' \leftarrow (r^2 X_P, r^3 Y_P, r Z_P)$
  $Q \leftarrow (x_{Q'}/r^2, y_{Q'}/r^3)$

Outline

Fault Protection

Classical countermeasures

- Redundancy, verification...
- Verify that $P, Q \in \mathcal{E}(\mathbb{K})$.

Fault Protection

Classical countermeasures

- Redundancy, verification...
- Verify that $P, Q \in \mathcal{E}(\mathbb{K})$.

Coherence tests

- Montgomery ladder invariant: $R_1 - R_0 = P$
- Right-to-left double-and-add-always

References

Thank you for your attention !

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