**Course Code : PHT 159 / PHT 154** **EZFY/RW – 22 / 1379**

# First / Second Semester B. Tech. ( CSE (AIML), CSE (CS) ) Examination

## INTRODUCTION TO QUANTUM COMPUTING

Time : 3 Hours ] [ Max. Marks : 60

**Instructions to Candidates :—**

(1) All questions are compulsory.
(2) Assume suitable data wherever necessary.
(3) Answer written with suitable steps, diagram will be given weightage.
(4) $\oplus$ – Symbol is for Tensor Product.
(5) **Q. 1** Solve any **Six**.

1\. (i) The accuracy in measurement of certain parameter of electron is related to uncertainty in measurement. Hence state uncertainty principle for position and momentum. 2(CO1)

(ii) Define terms Field, Bijective, Isomorphism and Abelian group. 2(CO2)

(iii) Eigen value and eigen vectors are the result of Hamilton operating on certain eigen function, what really happens when a matrix 'A' in $C^{n \times n}$ operates on vector 'V'. 2(CO2)

(iv) Why reversible gates are introduced in quantum computing ? 2(CO4)

(v) Explain quantum probabilistic systems. 2(CO3)

(vi) Stern-Gerlach experiment shows that electron in magnetic field will behave like spinning top. What it justifies with respect to wave function $\psi$ ? 2(CO3)

(vii) Two electron wave packet are represented by complex numbers as $C_1 = 4 - 3i$ and $C_2 = 1 - 2i$.

Prove that $|C_1 + C_2| \leq |C_1| + |C_2|$. 2(CO1)

(viii) What is Hadamard matrix ? 2(CO2)

(ix) Find the value of $i^{25}$ and represent $i^2$, $5i^2$ and $16i^5$ on complex plane. 2(CO1)

(x) Write the matrix corresponding to AND and NAND gates. 2(CO4)

2. (A) Certain electron accommodated in a one-dimensional box forms the standing wave with four maxima. The corresponding wave length is $\lambda$. Find the energy of the electron when it is $2^{nd}$ energy level. 4(CO1)

(B) Transition matrix form a Hadamard matrix when operates on vectors. Verify that following matrix is a Hadamard for the given vectors coefficients —

$$A = \begin{bmatrix} 2 & -\frac{3}{2} \\ -3 & \frac{5}{2} \end{bmatrix} \quad V_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \quad V_2 = \begin{bmatrix} 9 \\ -14 \end{bmatrix}$$

4(CO2)

(C) The Schrodinger equation in Hamiltonian form $H\psi = E\psi$, represent the eigen value equation. For a given Hamilton in matrix form 'H' as –

$$H = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} \text{ has eigen vectors } \psi_1 = \begin{bmatrix} -5 \\ 3 \\ 3 \end{bmatrix} \text{ and } \psi_2 = \begin{bmatrix} 1 \\ -9 \\ 6 \end{bmatrix}.$$

Find the eigen values for above eigen vectors and comment on the values obtained. 4(CO2)

3. (A) Inner product of the vectors compare the two different states of quantum system. Hence get the inner product of any two complex vector given by any $2 \times 1$ matrices. (Assume suitable matrices). Comment on the value which you obtain when the inner product is between the same vector. 4(CO3)

(B) In a probabilistic system, the adjacency matrix for it is :

$$M = \begin{pmatrix} 0 & \frac{1}{6} & \frac{5}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

Verify whether it is doubly stochastic or not ? If the dynamic 'M' operates on state —

$$\psi = \left[ \frac{1}{6}, \ \frac{1}{6}, \ 0 \right]^{T}$$

What new state you will obtain and what is its correlation with previous state ? 4(CO4)

(C) For the quantum system a unitary matrix U is given as :

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 1 \\ 0 & 0 & i \end{bmatrix}$$

Operates on state $[1\ 0\ 1]^T$ in two clicks, obtain the state of the system after every click and comment on your result. 4(CO4)

4. (A) A shooter is at '0', shooting through the two slits '1' and '2'. The probability of reaching the target '3, 4, 5, 6, 7' placed in front of slits is given by following matrix. Draw the weighted graphical representation of it.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1/3 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

4(CO3,4)

(B) How Stern - Gerlach experiment define the electron spin ? How it explains the logic of qubit ? 4(CO4)

(C) Find the commutator operator for –

$$\Omega_1 = \begin{pmatrix} 1 & -1-i \\ -1+i & 1 \end{pmatrix} \quad \text{and} \quad \Omega_2 = \begin{pmatrix} 0 & -1 \\ -1 & 2 \end{pmatrix}$$

4(CO5)

5. Write short notes on (any **Three**) :—

(I) Landauer's principle, control gate and reversible gate.

(II) Toffoli gate.

(III) Classical and Quantum Computers.

(IV) Hadamard gate.

(V) Young's Double Slit experiment.

(VI) Deutsch's Algorithm. 12(CO4,5)