Course Code : PHT 154/ PHT 159 **GHXW/MW – 22 / 1850**

First Semester B. Tech. (Artificial Intelligence and Machine Learning, Cyber Security and Information Technology) Examination

INTRODUCTION TO QUANTUM COMPUTING

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.
(2) Assume suitable data wherever necessary.
(3) Answer written with suitable steps, diagram will be given weightage.
(4) $\oplus$-Symbol is for Tensor Product.
(5) Q. 1. Solve any **Six**.

1. (i) "Nature love symmetry" which postulate this statement justified ? Give its significance with respect to particle mass. 2(CO1)

(ii) How one can establish equivalence between electron in wave and complex number ? 2(CO1)

(iii) What is linearly independent and linearly dependent set of vectors ? 2(CO2)

(iv) Give the significance of Tensor product. 2(CO2)

(v) What is assembling of quantum systems ? Explain with suitable example. 2(CO3)

(vi) Define the terms Observables, measuring and dynamics with respect to quantum system. 2(CO3)

(vii) Define 'Basis' and 'dimension'. 2(CO2)

(viii) How NOT gate operates on [0 > and |1> states ? 2(CO3)

(ix) How Hermitian matrix is different than unitary matrix ? Explain using example. 2(CO3)

(x) What is orthogonal and orthonormal basis ? 2(CO3)

2. (A) In a class room there are 60 students. All have same frequency and wavelength of their voice. They have started shouting in the class which leads to beat formation. Assuming only two sound waves interfering, obtain the velocity of group of waves superimposing on each other. 4(CO1)

(B) Define :

(i) Complex vector space.

(ii) Matrix multiplication relates to adjoint.

Verify the property by taking 2 x 2 complex matrices. 4(CO2)

(C) For following set of Matrices $\in$ V –
Calculate :

(1) Inner product, comment whether it degenerates or not ?

(2) Calculate Norm of V_1.

(3) Test it for Skew symmetric property.

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

4(CO2)

3. (A) Test whether given complex matrix is Symmetric, Hermitian, Unitary and can form self-adjoint operator —

$$A = \begin{bmatrix} 5 & 4 + 5i & 6 - 16i \\ 4 - 5i & 13 & 7 \\ 6 + 16i & 7 & -2.1 \end{bmatrix}$$

4(CO3)

(B) Given three pair basis – Check for orthogonality and orthonormality for them.

(i) $A = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (ii) $C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(iii) $E = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & 1 \end{bmatrix}$ $F = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix}$ 4(CO3)

(C) The best way to combine two quantum systems defined in two different vector space is to take their Tensor product. So, tensor product is fundamental building operation of quantum systems. Find the Tensor product of given vectors —

$$\begin{pmatrix} 3 + 2i & 5 - i \\ -6 - 3i & 9 + 3i \end{pmatrix} \otimes \begin{pmatrix} 1 & 23 + 4i \\ 0 & 1 \end{pmatrix}$$ 4(CO3)

4. (A) The assembling of the systems is also called as composite system. For such a composite system the tensor product of two system has matrix given below —

$M \otimes N =$

	0a	**0b**	**1a**	**1b**	**2a**	**2b**
0a	0	0	1/18	2/18	5/18	10/18
0b	0	0	2/18	1/18	10/18	5/18
1a	1/9	2/9	1/6	2/6	1/18	2/18
1b	2/9	1/9	2/6	1/6	2/18	1/18
2a	2/9	4/9	1/9	2/9	0	0
2b	4/9	2/9	2/9	1/9	0	0

Draw the weighted graph for 4^{th} column. 4(CO4)

(B) Write three spin matrices and calculate the commutators of the three spin matrices. Relate their results and comment. 4(CO4)

(C) Normalize the given ket and find the probability that the particle can be found in position x_5.

$|\psi> = [3 - 3i,\ 2 + 6i,\ 7 - 8i,\ 6.3 + 1.2i,\ 13i,\ 1 + 2i,\ 21.1]^T$ 4(CO4)

5. Write short notes on (any **Three**) :—

(I) Compare classical probabilistic and quantum systems.

(II) Deutsch's Algorithm.

(III) Quantum gates.

(IV) Obtain the matrix corresponding to following combinations of gates — NOT * AND * (NOT $\oplus$ NOT).

(V) Mathematics of quantum bit. 12(CO5)