

#### Monte Carlo of Molecular Systems

**MSSE Bootcamp** 

August 13, 2020

#### Monte Carlo Connection to Molecular Systems

According to **statistical mechanics** 

We can use MC to evaluate this integral!

$$\langle Q \rangle = \int_{V} Q(r^{N}) \rho(r^{N}) dr^{N}$$

- ${\it Q}$  quantity which depends on atomic coordinates  $(r^N)$
- (Q) average value of quantity Q (square brackets denote average)
- r<sup>N</sup> atomic coordinates with N dimensions
- $ho(r^N)$  probability density based on thermodynamic properties (beyond scope of this course)



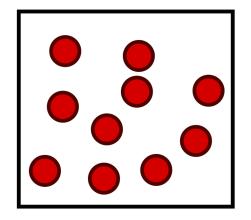


#### Monte Carlo Connection to Molecular Systems

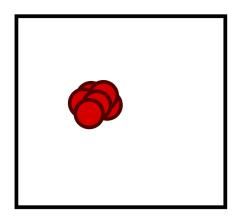
In order to evaluate this integral we have some special considerations

$$\langle Q \rangle = \int_{V} Q(r^{N}) \rho(r^{N}) dr^{N}$$

Because we have so many possible states, it is not effective to sample points with a uniform distribution.



Consider our 10 particles in a box.



This
configuration
(particles
stacked) is high
energy and not
likely to occur



### The Metropolis Monte Carlo Method

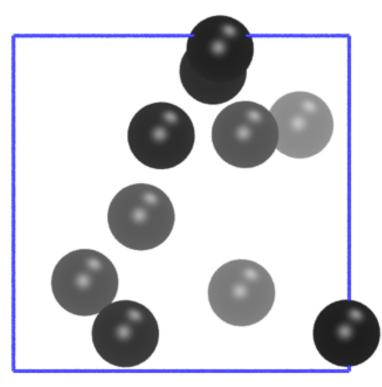
- No longer using a uniform distribution for coordinate generation.
- Instead, generate configurations with distribution  $\rho(r^N)$  the probability density based on thermodynamic properties.

Then, we can evaluate the integral as the average of the generated configurations:

$$\langle Q \rangle = \frac{1}{N} \sum_{i=1}^{N} Q(r_i^N)$$



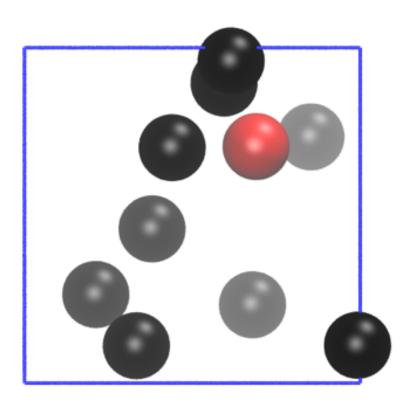




Generate an initial state *m* and calculate its energy.



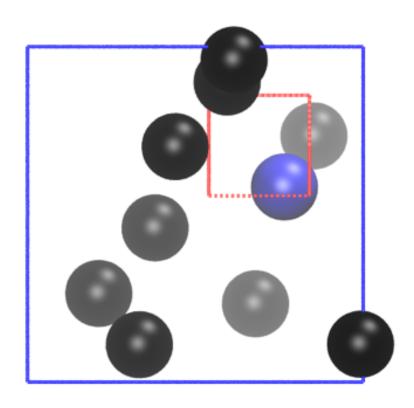




Choose an atom with uniform probability





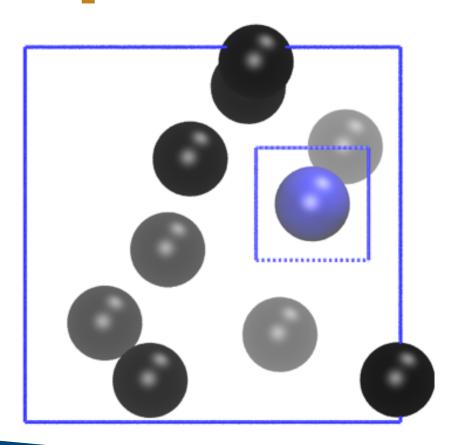


Attempt a random translation within a maximum distance.

Calculate the energy of the new state, n.







Accept or reject new state according to the Metropolis criterion





#### The Metropolis Criterion

Accept move based on the energy change resulting from moving the particle and system temperature.

$$P_{acc}(m \rightarrow n) = \min[1, e^{-\Delta U/T}]$$

This means we will always accept moves which result in a decrease in energy ( $-\Delta U$ ), and sometimes accept moves which are zero or positive.

In practice, we will generate a random number on the range zero to 1. If our calculated  $P_{acc}$  is greater than our generated number, we accept the configuration.

