Tracking the best Expert

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February 4, 2020

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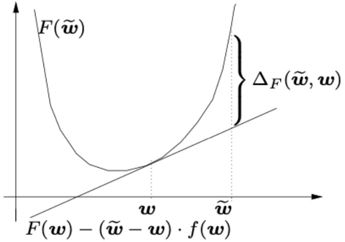
- There will be no mid-term exam.

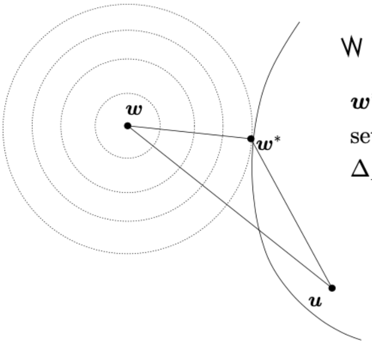
Bregman Divergences [Br,CL,Cs]

For any differentiable convex function F

$$\begin{aligned}\Delta_F(\tilde{\boldsymbol{w}}, \boldsymbol{w}) &= F(\tilde{\boldsymbol{w}}) - F(\boldsymbol{w}) - (\tilde{\boldsymbol{w}} - \boldsymbol{w}) \cdot \underbrace{\nabla_{\boldsymbol{w}} F(\boldsymbol{w})}_{f(\boldsymbol{w})} \\ &= F(\tilde{\boldsymbol{w}}) - \begin{array}{c}\text{supporting hyperplane} \\ \text{through } (\boldsymbol{w}, F(\boldsymbol{w}))\end{array}\end{aligned}$$



A Pythagorean Theorem [Br,Cs,A,HW]



$\boldsymbol{w}^*$ is projection of $\boldsymbol{w}$ onto convex set $\mathcal{W}$ w.r.t. Bregman divergence Δ_F:

$$\boldsymbol{w}^* = \operatorname*{argmin}_{\boldsymbol{u}\in\mathcal{W}} \Delta_F(\boldsymbol{u}, \boldsymbol{w})$$

Theorem:

$$\Delta_F(\boldsymbol{u}, w) \geq \Delta_F(\boldsymbol{u}, \boldsymbol{w}^*) + \Delta_F(\boldsymbol{w}^*, \boldsymbol{w})$$

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- Unnormalized RE is the Bregman divergence corresponding to the unnormalized entropy:

$$F(\mathbf{p}) = \sum_{i=1}^{n} p_i \log p_i - \sum_{i=1}^{n} p_i$$

Inequalities for Unnormalized Relative entropy

- No triangle inequality
  $\exists \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \;\; \mathrm{RE}(\mathbf{p}_1 \;\|\; \mathbf{p}_3) > \mathrm{RE}(\mathbf{p}_1 \;\|\; \mathbf{p}_2) + \mathrm{RE}(\mathbf{p}_2 \;\|\; \mathbf{p}_3)$

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- Generalized Pythagorean inequality For any closed convex set S and any point $\mathbf{p}_1 \notin S$, define the projection of $\mathbf{p}_1$ on S to be $\mathbf{p}_2 = \mathrm{argmin}_{\mathbf{u} \in S} \mathrm{RE}(\mathbf{p}_1 \ \| \ \mathbf{u})$, then:

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- Triangle inequality does not hold.
- Pythagoras inequality : For any closed convex set S and any point $\mathbf{v}_1 \notin S$, define the projection of $\mathbf{v}_1$ on S to be $\mathbf{v}_2 = \mathrm{argmin}_{\mathbf{u} \in S}\|\mathbf{v}_1 - \mathbf{u}\|^2$, then:
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Bregman divergence regularization

- Idea: Set $\mathbf{w}_{t+1}$ to be $\mathbf{u}$ that minimizes:

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- Efficient approximation Mirror Descent. Will be covered later.

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- The Projection Update After computing the unconstrained update, project the $\mathbf{w}_{t+1}$ onto a convex set.
- Does not allow the algorithm to over-commit to an extreme vector from which it is hard to recover.

Switching experts setup

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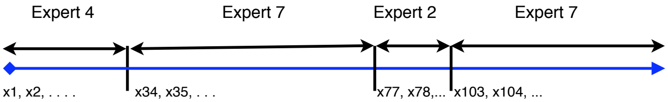

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- Requires maintaining $O\left(n^{k+1}\left(\frac{el}{k}\right)^k\right)$ weights.

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- There is an exponential weights algorithm with learning rate η that achieves (in the non-switching case) a bound

$$L_A \leq \min_i L_i + \frac{1}{\eta} \log n$$

- Then using the partition-expert algorithm for the switching-experts case we get a bound on the regret $\frac{1}{\eta}\left((k+1)\log n + k \log \frac{l}{k} + k\right)$

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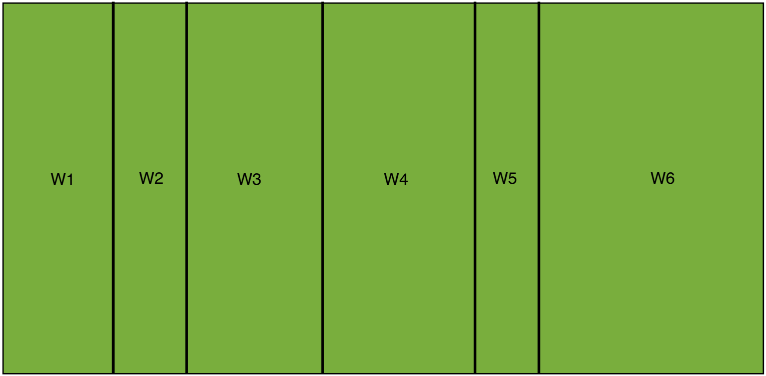
- Fixed-share:

$$\begin{aligned} pool &= \alpha \sum_{i=1}^{n} w^m_{t,i} \\ w^s_{t+1,i} &= (1-\alpha) w^m_{t,i} + \frac{1}{n-1} (pool - \alpha w^m_{t,i}) \end{aligned}$$

The fixed-share algorithm



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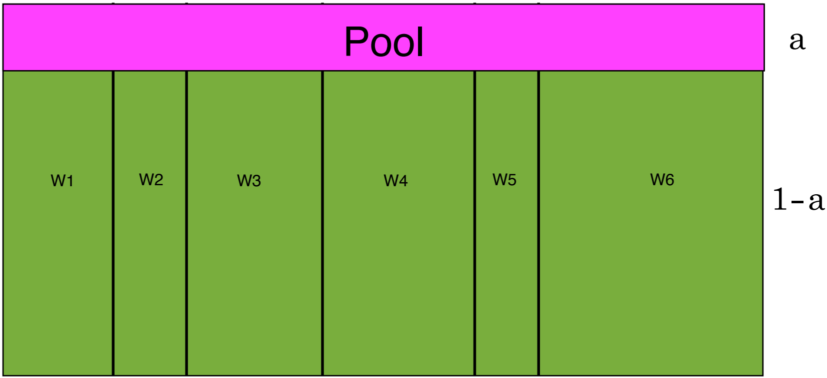

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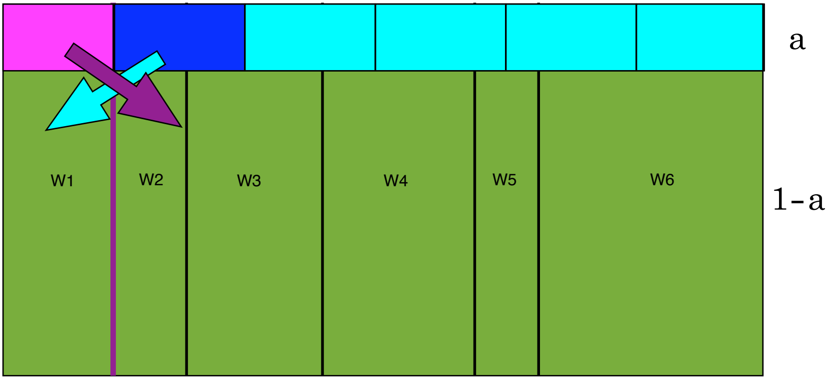

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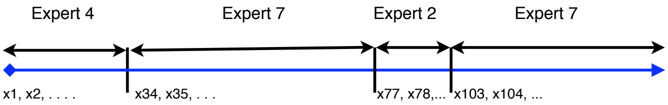
- The harder question is how to lower bound $\sum_{i=1}^{n} w^s_{l+1,i}$

Lower bounding the final total weight

- Fix some switching experts sequence:



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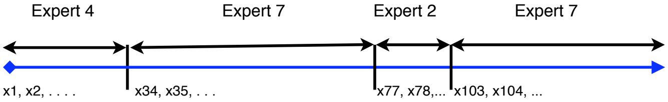

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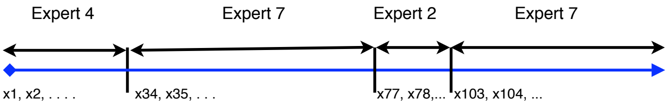

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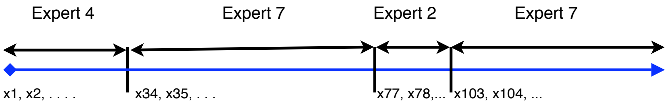

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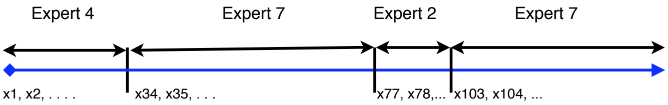

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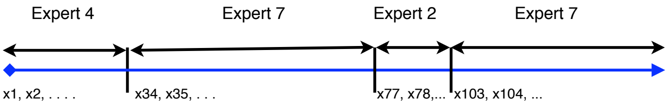

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  - $\frac{\alpha}{n-1}$ on iterations where a switch occurs.

Bound for arbitrary α

- Combining we lower bound the final weight of the last expert in the sequence

$$w^s_{l+1,e_k} \geq \frac{1}{n} e^{-\eta L_*} (1-\alpha)^{l-k-1} \left(\frac{\alpha}{n-1}\right)^k$$

Where L_* is the cumulative loss of the switching sequence of experts.

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- Combining the upper and lower bounds we get that for any sequence

$$L_A \leq L_* + \frac{1}{\eta}\left(\ln n + (l-k-1)\ln\frac{1}{1-\alpha} + k\left(\ln\frac{1}{\alpha} + \ln(n-1)\right)\right)$$

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Where

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- Not so for square loss!

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- In the fixed-share algorithm, the weight of a suboptimal expert never decreases below α/n.
- The algorithm does not concentrate only on the best expert, even if the last switch is in the distant past.
- The regret depends on the length of the sequence.

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- Requires that the loss be bounded.
- Works for square loss, but not for log loss!

Variable-share

$$
\begin{aligned}
pool &= \sum_{i=1}^{n} \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^m_{t,i} \\
w^s_{t+1,i} &= (1-\alpha)^{\ell_{t,i}} w^m_{t,i} + \frac{1}{n-1}\left(pool - \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^m_{t,i}\right)
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If $\ell_{t,i} = 0$, then expert i does not contribute to the pool.

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w^s_{t+1,i} &= (1-\alpha)^{\ell_{t,i}} w^m_{t,i} + \frac{1}{n-1}\left(pool - \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^m_{t,i}\right)
\end{aligned}
$$

If $\ell_{t,i} = 0$, then expert i does not contribute to the pool.
Expert can get fraction of the total weight arbitrarily close to 1.

Variable-share

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Expert can get fraction of the total weight arbitrarily close to 1.
Shares the weight quickly if $\ell_{t,i} > 0$

Bound for variable share

- $$\frac{1}{\eta}\ln n + \left(1 + \frac{1}{(1-\alpha)\eta}\right) L_* + k\left(1 + \frac{1}{\eta}\left(\ln n - 1 + \ln\frac{1}{\alpha} + \ln\frac{1}{1-\alpha}\right)\right)$$

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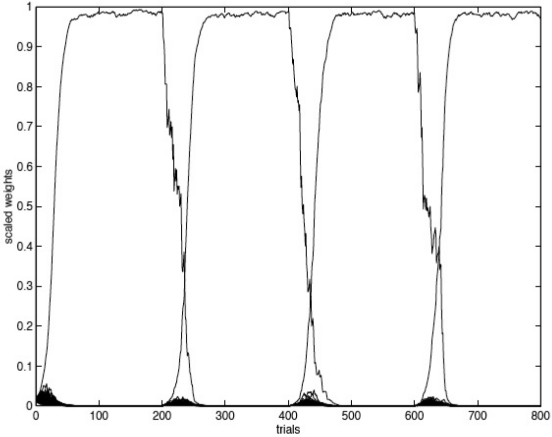
- α should be tuned so that it is (close to) $\frac{k}{2k+L_*}$

An experiment using variable share



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- Switching to Slides from Manfred Warmuth.