Tracking the best Expert

Yoav Freund

February 4, 2020

Based on “Tracking the best linear predictor” and “Tracking the best expert” by Herbster and Warmuth. Also, section 11.5 in Prediction learning and Games.

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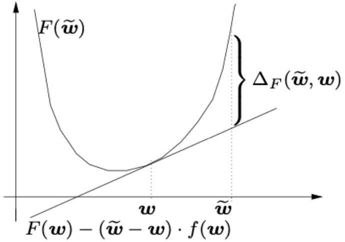
- There will be no mid-term exam.

Bregman Divergences [Br,CL,Cs]

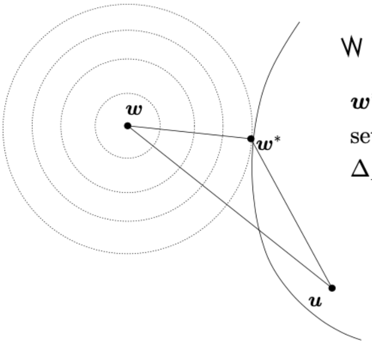
For any differentiable convex function F

$$\Delta_F(\tilde{\boldsymbol{w}}, \boldsymbol{w}) = F(\tilde{\boldsymbol{w}}) - F(\boldsymbol{w}) - (\tilde{\boldsymbol{w}} - \boldsymbol{w}) \cdot \underbrace{\nabla_{\boldsymbol{w}} F(\boldsymbol{w})}_{f(\boldsymbol{w})}$$

$$= F(\tilde{\boldsymbol{w}}) - \begin{array}{c}\text{supporting hyperplane} \\ \text{through } (\boldsymbol{w}, F(\boldsymbol{w}))\end{array}$$



A Pythagorean Theorem [Br,Cs,A,HW]



$\boldsymbol{w}^*$ is projection of $\boldsymbol{w}$ onto convex set $\mathcal{W}$ w.r.t. Bregman divergence Δ_F:

$$\boldsymbol{w}^* = \operatorname*{argmin}_{\boldsymbol{u} \in \mathcal{W}} \Delta_F(\boldsymbol{u}, \boldsymbol{w})$$

Theorem:

$$\Delta_F(\boldsymbol{u}, w) \geq \Delta_F(\boldsymbol{u}, \boldsymbol{w}^*) + \Delta_F(\boldsymbol{w}^*, \boldsymbol{w})$$

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- Unnormalized RE is the Bregman divergence corresponding to the unnormalized entropy:

$$F(\mathbf{p}) = \sum_{i=1}^{n} p_i \log p_i - \sum_{i=1}^{n} p_i$$

Inequalities for Unnormalized Relative entropy

- No triangle inequality
  $\exists \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3 \;\; \mathrm{RE}(\mathbf{p}_1 \;\|\; \mathbf{p}_3) > \mathrm{RE}(\mathbf{p}_1 \;\|\; \mathbf{p}_2) + \mathrm{RE}(\mathbf{p}_2 \;\|\; \mathbf{p}_3)$

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- Generalized Pythagorean inequality For any closed convex set S and any point $\mathbf{p}_1 \notin S$, define the projection of $\mathbf{p}_1$ on S to be $\mathbf{p}_2 = \mathrm{argmin}_{\mathbf{u} \in S} \mathrm{RE}(\mathbf{p}_1 \parallel \mathbf{u})$, then:

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- Pythagoras inequality : For any closed convex set S and any point $\mathbf{v}_1 \notin S$, define the projection of $\mathbf{v}_1$ on S to be $\mathbf{v}_2 = \text{argmin}_{\mathbf{u} \in S}\|\mathbf{v}_1 - \mathbf{u}\|^2$, then:
$$\forall \mathbf{v}_3 \in S; \quad \|\mathbf{v}_1 - \mathbf{v}_3\|^2 \geq \|\mathbf{v}_1 - \mathbf{v}_2\|^2 + \|\mathbf{v}_2 - \mathbf{v}_3\|^2$$

Bregman divergence regularization

- Idea: Set $\mathbf{w}_{t+1}$ to be $\mathbf{u}$ that minimizes:

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- Efficient approximation Mirror Descent. Will be covered later.

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- The Projection Update After computing the unconstrained update, project the $\mathbf{w}_{t+1}$ onto a convex set.
- Does not allow the algorithm to over-commit to an extreme vector from which it is hard to recover.

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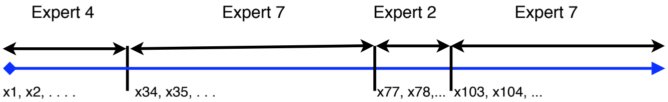

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- Requires maintaining $O\left(n^{k+1}\left(\frac{el}{k}\right)^k\right)$ weights.

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$$L_A \leq \min_i L_i + \frac{1}{\eta} \log n$$

- Then using the partition-expert algorithm for the switching-experts case we get a bound on the regret $\frac{1}{\eta}\left((k+1)\log n + k \log \frac{l}{k} + k\right)$

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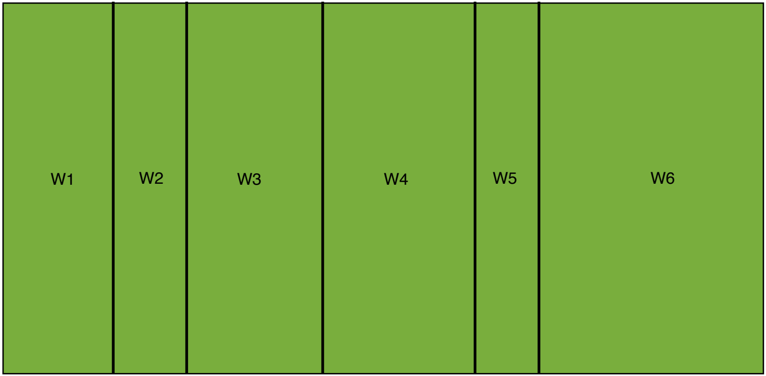
- Fixed-share:

$$\begin{aligned} pool &= \alpha \sum_{i=1}^{n} w^m_{t,i} \\ w^s_{t+1,i} &= (1-\alpha) w^m_{t,i} + \frac{1}{n-1} (pool - \alpha w^m_{t,i}) \end{aligned}$$

The fixed-share algorithm



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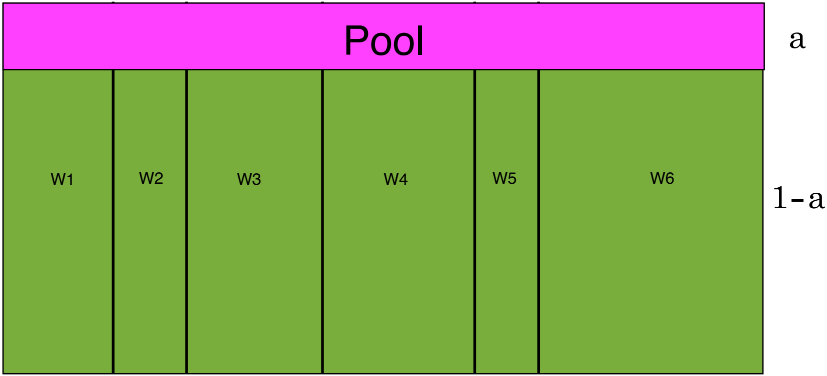

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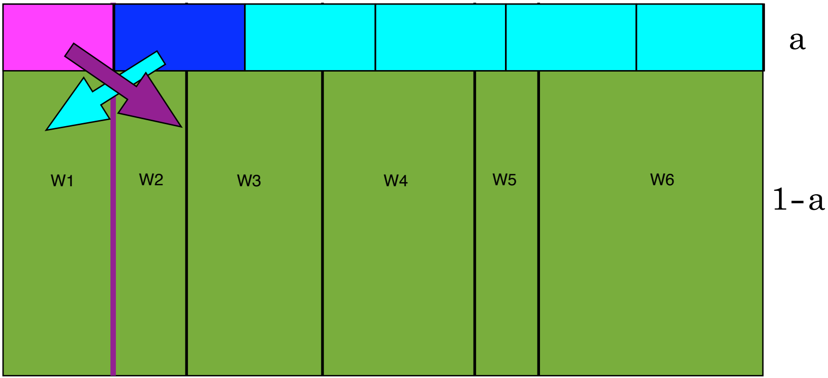

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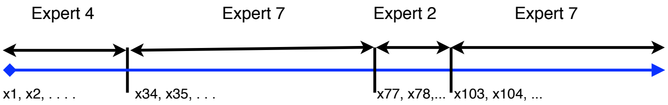
- The harder question is how to lower bound $\sum_{i=1}^{n} w^s_{l+1,i}$

Lower bounding the final total weight

- Fix some switching experts sequence:



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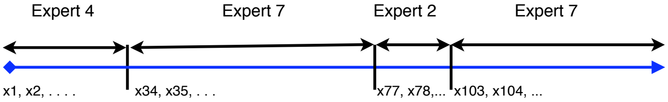

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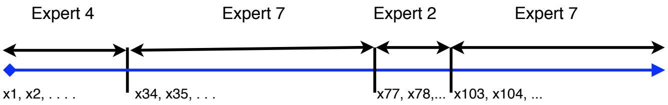

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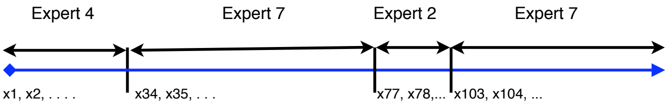

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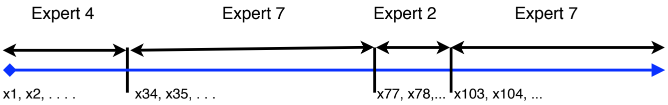

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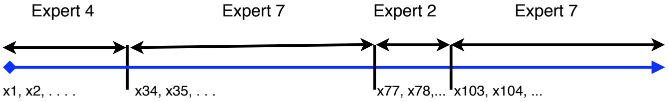

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  - $\frac{\alpha}{n-1}$ on iterations where a switch occurs.

Bound for arbitrary α

- Combining we lower bound the final weight of the last expert in the sequence

$$w^{s}_{l+1,e_k} \geq \frac{1}{n} e^{-\eta L_*} (1-\alpha)^{l-k-1} \left(\frac{\alpha}{n-1} \right)^{k}$$

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- Combining the upper and lower bounds we get that for any sequence

$$L_A \leq L_* + \frac{1}{\eta}\left(\ln n + (l-k-1)\ln\frac{1}{1-\alpha} + k\left(\ln\frac{1}{\alpha} + \ln(n-1)\right)\right)$$

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Where

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- Not so for square loss!

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- In the fixed-share algorithm, the weight of a suboptimal expert never decreases below α/n.
- The algorithm does not concentrate only on the best expert, even if the last switch is in the distant past.
- The regret depends on the length of the sequence.

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- Requires that the loss be bounded.
- Works for square loss, but not for log loss!

Variable-share

$$
\begin{aligned}
pool &= \sum_{i=1}^{n} \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^{m}_{t,i} \\
w^{s}_{t+1,i} &= (1-\alpha)^{\ell_{t,i}} w^{m}_{t,i} + \frac{1}{n-1}\left(pool - \left(1 - (1-\alpha)^{\ell_{t,i}}\right) w^{m}_{t,i}\right)
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If $\ell_{t,i} = 0$, then expert i does not contribute to the pool.

Variable-share

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If $\ell_{t,i} = 0$, then expert i does not contribute to the pool.
Expert can get fraction of the total weight arbitrarily close to 1.

Variable-share

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Shares the weight quickly if $\ell_{t,i} > 0$

Bound for variable share

- $$\frac{1}{\eta}\ln n + \left(1 + \frac{1}{(1-\alpha)\eta}\right)L_* + k\left(1 + \frac{1}{\eta}\left(\ln n - 1 + \ln\frac{1}{\alpha} + \ln\frac{1}{1-\alpha}\right)\right)$$

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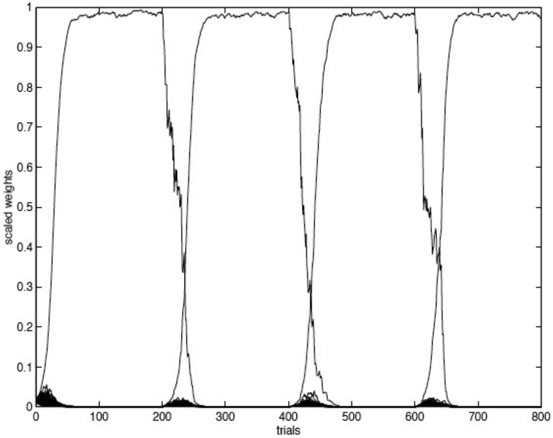
- α should be tuned so that it is (close to) $\frac{k}{2k+L_*}$

An experiment using variable share



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- Switching to Slides from Manfred Warmuth.