cs 584 f25: Assignment 0 - Review

Due by 9/1/2025 (0%)

Answer the following by computing and showing the steps involved

Part A: Vector Operations

Let:

$$p = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \quad q = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$$

Find:

1. $3p + 2q$
2. $\hat{p}$: a unit vector in the direction of p
3. $||p||$ and the angle of p relative to the positive y-axis
4. The direction cosines of p
5. The angle between p and q
6. $p \cdot q$ and $q \cdot p$
7. $p \cdot q$ using the angle between p and q
8. The scalar projection of q onto $\hat{p}$
9. A vector that is perpendicular to p
10. $p \times q$ and $q \times p$
11. A vector that is perpendicular to both p and q
12. The linear dependency between p, q, and r
13. $p^T q$ and pq^T

Part B: Matrix Operations

Let:

$$X = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 4 \\ 3 & 2 & -2 \end{bmatrix}, \quad Y = \begin{bmatrix} 4 & -1 & 2 \\ 3 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix}, \quad Z = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 4 & 5 \\ 3 & 1 & 2 \end{bmatrix}, \quad s = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$$

Find:

1. $X + 2Y$
2. XY and YX
3. $(XY)^T$ and $Y^T X^T$
4. $|X|$ and $|Z|$ (see question A-12)
5. The matrix (either X, Y, or Z) in which the row vectors form an orthogonal set

6. X^{-1} and Y^{-1} (see question B-5)
7. Z^{-1} (see question B-4)
8. The product Xs
9. The scalar projection of the rows of X onto the vector s (with s normalized)
10. The vector projection of the rows of X onto the vector s (with s normalized)
11. The linear combination of the columns of X using the elements of s
12. The solution t for the equation $Yt = s$
13. The solution t for the equation $Zt = s$ and the reason for it (see question B-7)

Part C: Eigenvalues and Eigenvectors

Let:

$$M = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}, \quad N = \begin{bmatrix} 5 & -3 \\ -3 & 6 \end{bmatrix}, \quad P = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$$

Find:

1. The eigenvalues and corresponding eigenvectors of M
2. The dot product between the eigenvectors of M
3. The dot product between the eigenvectors of N
4. The property of the eigenvectors of N and the reason for it (see question C-4)
5. The value of a trivial solution t to the equation $Pt = 0$
6. The value of two non-trivial solutions t to the equation $Pt = 0$
7. The only solution t to the equation $Mt = 0$ and the reason for having a single solution

Part D: Gradient Calculations (Do Not Submit in Python)

Let:

$$f(x) = 2x^2 - 1, \quad g(x) = 3x^2 + 4, \quad h(x, y) = x^2 + y^2 + xy$$

Find:

1. The first and second derivatives of $f(x)$ with respect to x: $f'(x)$ and $f''(x)$
2. The partial derivatives $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$
3. The gradient vector $\nabla h(x, y)$
4. The derivative $\frac{d}{dx} f(g(x))$ with and without using the chain rule for derivatives

Part E: Python implementation

Repeat questions A, B, C (Not D) using Python and prepare a Python notebook showing your computations.

Submission Instructions

1. **Step 1: Prepare your solution**
    - Prepare your solution in a PDF file (either typed and exported to PDF, or handwritten and scanned/photographed).
    - Prepare a Python notebook showing the Python solution.
    - Your name, student ID, course number, and semester must be clearly shown at the beginning of your report and the Python notebook.
    - Create a zip file containing all assignment components.
2. **Step 2: Upload your solution to Canvas**
    - Upload your solution to Canvas.
    - Your submission time/date will be determined based on the upload time.
    - There is no need to inform us if you need to use late days. Late days are deducted from your allocation automatically.