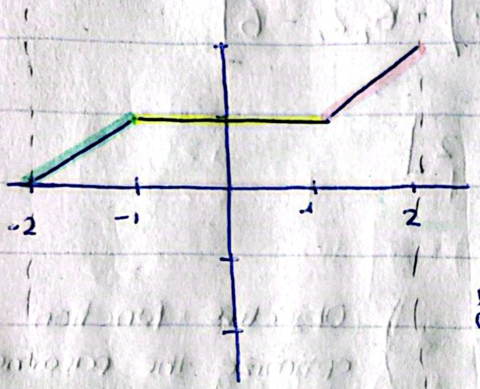
DEEP LEARNING - HW1

PROBLEM 1.a

$$f(x;\theta) = g(x+b_1) - g(-(x+b_2)) + b_3$$

$$\theta = [b_1, b_2, b_3] = [-1, 1, 1]$$



$$f(x;\theta) \begin{cases} x+2 & x \leq -1 \\ 1 & -1 \leq x \leq 1 \\ x & x \geq 1 \end{cases}$$

$$f(x;\theta) = g(x-1) - g(-x-1) + 1$$

PROBLEM 1b.

$\frac{\partial J_i}{\partial \theta}$

$$J_i(\theta) = (y^{(i)} - f(x^{(i)};\theta))^2$$

$$\frac{\partial J_i(\theta)}{\partial \theta} = 2 \cdot (f - y) \cdot \frac{\partial f}{\partial \theta}$$

$$\begin{cases} \frac{\partial f}{\partial \theta_1} = 1 & \text{if } x + b_1 > 0 \rightarrow x_1 > 1 \\ \frac{\partial f}{\theta_2} = 1 & \text{if } -(x+b_2) > 0 \rightarrow x < -1 \\ \frac{\partial f}{\theta_3} = 1 & \text{allways} \end{cases}$$

$(x = 2, y = 5)$
- $\frac{\partial f}{\theta_1} \rightarrow 2 \cdot (2-5) = -6$
- $\partial f / \theta_2 \rightarrow 0$
- $\frac{\partial f}{\theta_3} \rightarrow 2 \cdot (2-5) = -6$

$(x = -2, y = -1)$
- 0
- $2 \cdot (0 - (-1)) = 2$
- $2 \cdot (0 - (-1)) = 2$

$(x=0, y=2) \begin{cases} \to 0 \\ \to 0 \\ \to 2\cdot(1-2) = -2 \end{cases}$

$$J(\theta) = \frac{1}{3}\sum J_i(\theta) = \frac{1}{3}\cdot\left[-6, 2, -6\right]$$

$$\frac{\partial J}{\partial \theta} = \left[-2, 2/3, -2\right]$$

Problem 1c

$\theta_{initial} = [-1, 1, 1]$

Objective function to optimize the parameters

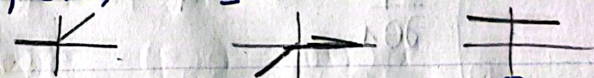
$$\frac{\partial J(\theta)}{\partial \theta} = [-2, 2/3, -2]$$

learning process with a step size η

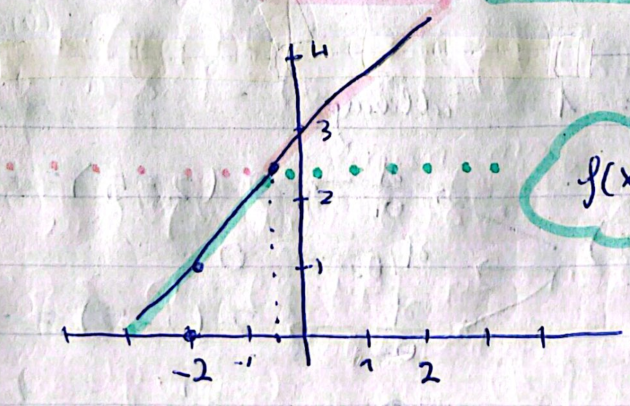
$$\theta_{opt} = \theta_{ini} - \eta \nabla J(\theta)$$

$$\theta_{opt} = [-1, 1, 1] - 3/4 \cdot [-2, 2/3, -2]$$

$$\theta_{opt} = [0.5, 0.5, 2.5]$$



$$f(x;\theta) = g\left(x+\frac{1}{2}\right) - g\left(-\left(x+\frac{1}{2}\right)\right) + \frac{5}{2}$$

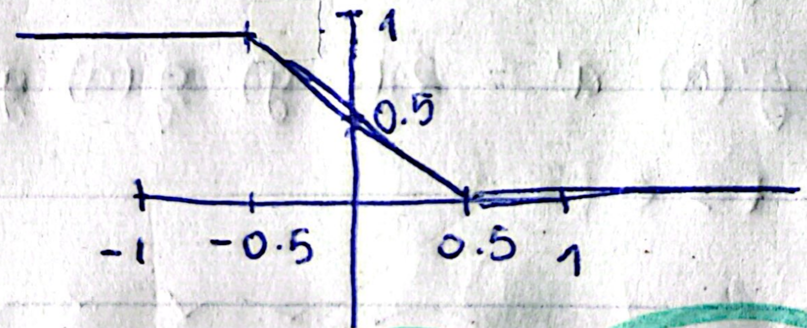


$f(x;\theta) = \{ \; x+3$

PROBLEM 2a

$$f(x;\theta) = g(w_2 \cdot g(w_1 x + b_1) + b_2)$$

$$\theta = [b_1, w_1, b_2, w_2] = [0.5, 1, 1, -1]$$



$$f(x;\theta) = g(-1 \cdot g(x+0{,}5)+1)$$

$x = -1 \rightarrow f(-1) = 1$

$x = 0 \rightarrow f(0) = 0{,}5$

$x = 1 \rightarrow f(1) = 0$

$$f(x) = \begin{cases} 1 & x < -1/2 \\ -x & -1/2 < x < 1/2 \\ 0 & x > 1/2 \end{cases}$$

$$a>0 \rightarrow -1 \cdot g(x+0{,}5)+1>0$$
$$1>g(x+0{,}5)$$
$$b>0 \rightarrow w_1 x + b_1 > 0$$
$$x+0{,}5>0$$

PROBLEM 2B

$$f(x;\theta) = g(\overbrace{w_2 \cdot g(\underbrace{w_1 \cdot x + b_1}_{b}) + b_2}^{a})$$

$$J_i(\theta) = (y_i - f(x;\theta))^2$$

$$\frac{\partial J}{\partial b_1}(\theta) = 2\cdot(f-y)\cdot f'(x)$$
$$= 2\cdot(f-y)\cdot g'(a)\cdot a' = 2\cdot(f-y)\cdot g'(a)\cdot w_2 \cdot g'(b)\cdot \overset{1}{b'}$$

$$\frac{dg}{dt}(t) = 1 \quad \text{if } t>0$$

$$\frac{\partial J}{\partial b_1}(\theta) = 2\cdot(f-y)\cdot 1 \cdot w_2 \cdot 1 \cdot 1 \quad \text{if } \begin{matrix} a>0 \\ b>0 \end{matrix}$$
$$= 2\cdot\left[g(w_2\cdot g(w_1 x + b_1)+b_2) - y\right]\cdot w_2 \quad \begin{matrix} a>0 \\ b>0 \end{matrix}$$
$$= 2\cdot\left[g(-g(x+0{,}5)+1) - y\right]\cdot(-1)$$

$$\frac{\partial J}{\partial w_1}(\theta) = 2\cdot(f-y)\cdot f'(x) = 2\cdot(f-y)\cdot g'(a)\cdot a' =$$
$$= 2\cdot(f-y)\cdot g'(a)\cdot w_2 \cdot g'(b)\cdot x$$

$$\frac{dg}{dt}(t) = 1 \quad \text{if } t>0$$

$$\frac{\partial J}{\partial w_1}(\theta) = 2\cdot(f-y)\cdot 1 \cdot w_2 \cdot 1 \cdot x \quad \text{if } \begin{matrix} a>0 \\ b>0 \end{matrix}$$
$$= 2\cdot\left[g(-g(x+0{,}5)+1-y\right]\cdot(-1)\cdot x$$

$$\frac{\partial J}{\partial b_2}(\theta) = 2\cdot(f-y)\cdot f'(x) = 2\cdot(f-y)\cdot g'(a)\cdot a'$$
$$= 2\cdot(f-y)\cdot g'(a)\cdot 1$$

$$\frac{\partial J(\theta)}{\partial b_2} = 2\cdot(f-y) \quad \text{if } a>0 \rightarrow 2\cdot\left[g(-g(x+0{,}5)+1)-y\right]$$

$$\frac{\partial J}{\partial w_2}(\theta) = 2\cdot(f-y)\cdot f'(x) = 2\,(f-y)\cdot \overset{1}{g'(a)}\cdot g(b)$$

$$\frac{\partial J}{\partial w_2} = 2\cdot\left[g(-g(x+0{,}5)+1)-y\right]\cdot 1 \cdot g(x+0{,}5)$$
$$\text{if } a>0$$

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