Demographic Matrix Population Models

SSA 200

Applications to SSAs

- Models are designed to output useful metrics on future resiliency and redundancy
    - Future abundance, extinction probability, population growth rate
- Models allow us to predict future condition of the populations and characterize uncertainties

Lecture outline

- Simple model construction
  - Review of data types/analysis
  - From conceptual to quantitative
- Incorporate environmental covariates and density dependence
- Model environmental and parametric uncertainty

Constructing the model

1. Choose state variables (age, stage)
   - Dependent on data and species
2. Use demographic data to estimate vital rates for each state
   - Fecundity, survival probability, recruitment probability
3. Use state-specific vital rate estimates to create matrix model

State variables

- Age classes (Leslie Matrix)
  - Equal time intervals & all individuals advance at next time
    - Short lived species with age-specific data
- Stage classes (Lefkovitch Matrix)
  - Unequal time intervals
  - Population divided by developmental stage or size
    - Difficult to age individuals but can get length, height, etc.
    - Juvenile, subadult
    - Seeds, dormancy, small plants, large plants

Constructing the model

1. Choose state variables (age, size, stage)
   - Dependent on data and species
2. Use demographic data to estimate vital rates for each state
   - Fecundity, survival probability, recruitment probability
3. Use state-specific vital rate estimates to create matrix model

Demographic data

- Depends on ecology/life history of species

Data types
Number of offspring per female
Nest/den success or failure
Number of young returning next year
Radio telemetry
Individual mark-recapture/resight
Annual plot census (plants)
Growth rate

Demographic vital rate
Fecundity
Breeding success probability
Recruitment probability
Survival probability (seasonal or annual)
Transition probability (annual)

alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

Estimate vital rates

- Fecundity
  - Number of offspring/female -> Poisson GLM
  - Successful breeding (yes/no) -> Binomial GLM
- Annual survival/mortality
  - Radio telemetry -> known-fate models
  - Individual CMR -> Cormack-Jolly-Seber (CJS) models
  - Proportion of marked plants alive next year

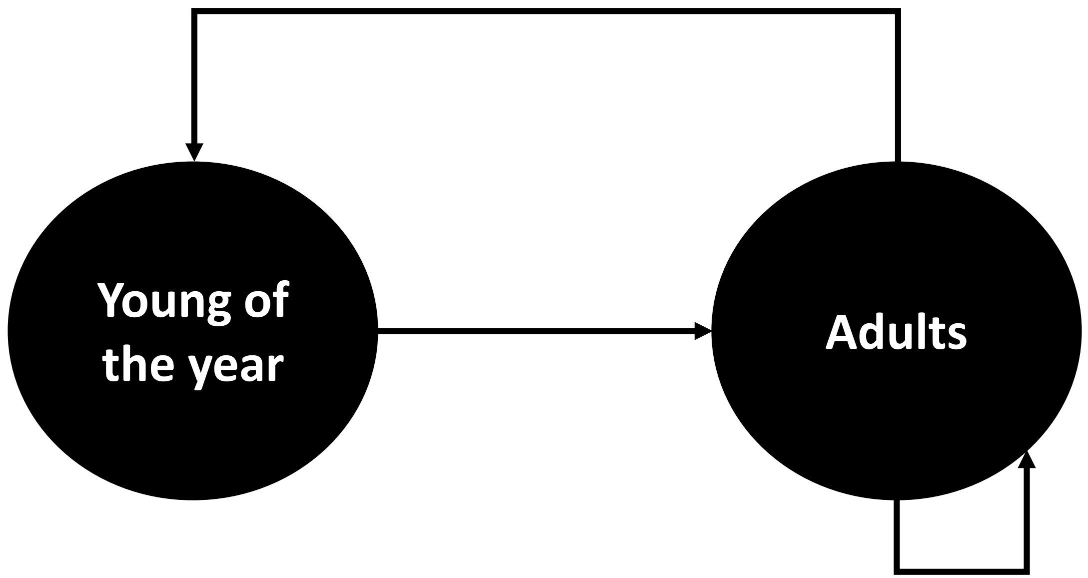
- OR – use values reported in literature/expert opinion

Constructing the model

1. Choose state variables (age, size, stage)
    - Dependent on data and species
2. Use demographic data to estimate vital rates for each state
    - Fecundity, survival probability, recruitment probability
3. Use state variables and state-specific vital rate estimates to create a conceptual model and matrix model

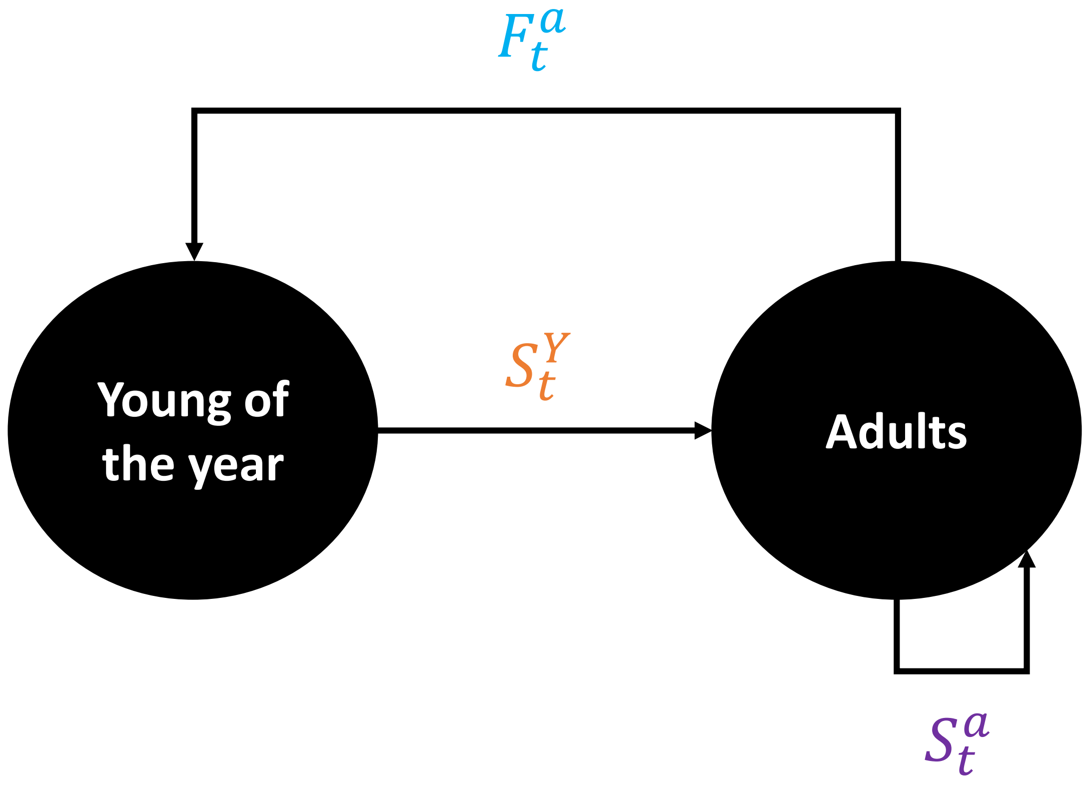
Conceptual model

- Species with two stage classes
  - Survival
    - Adults – CMR data
    - Young of year – in literature
  - Fecundity
    - Number of offspring/ adult female



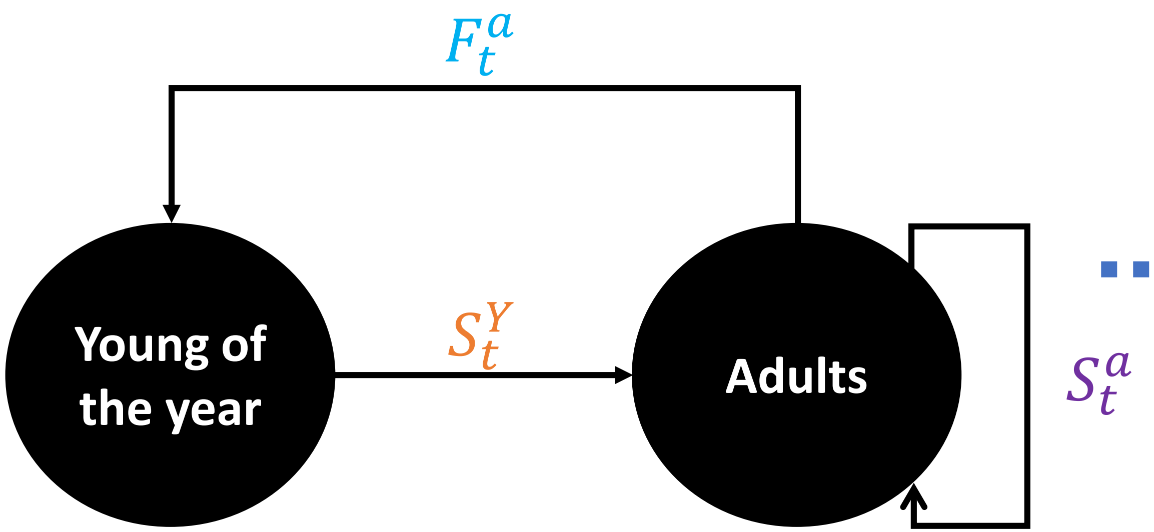
Conceptual model

- Species with two stage classes
  - Survival
    - Adults – CMR data
    - Young of year – estimate in literature
  - Fecundity
    - Number of offspring/ adult female



Create matrix model

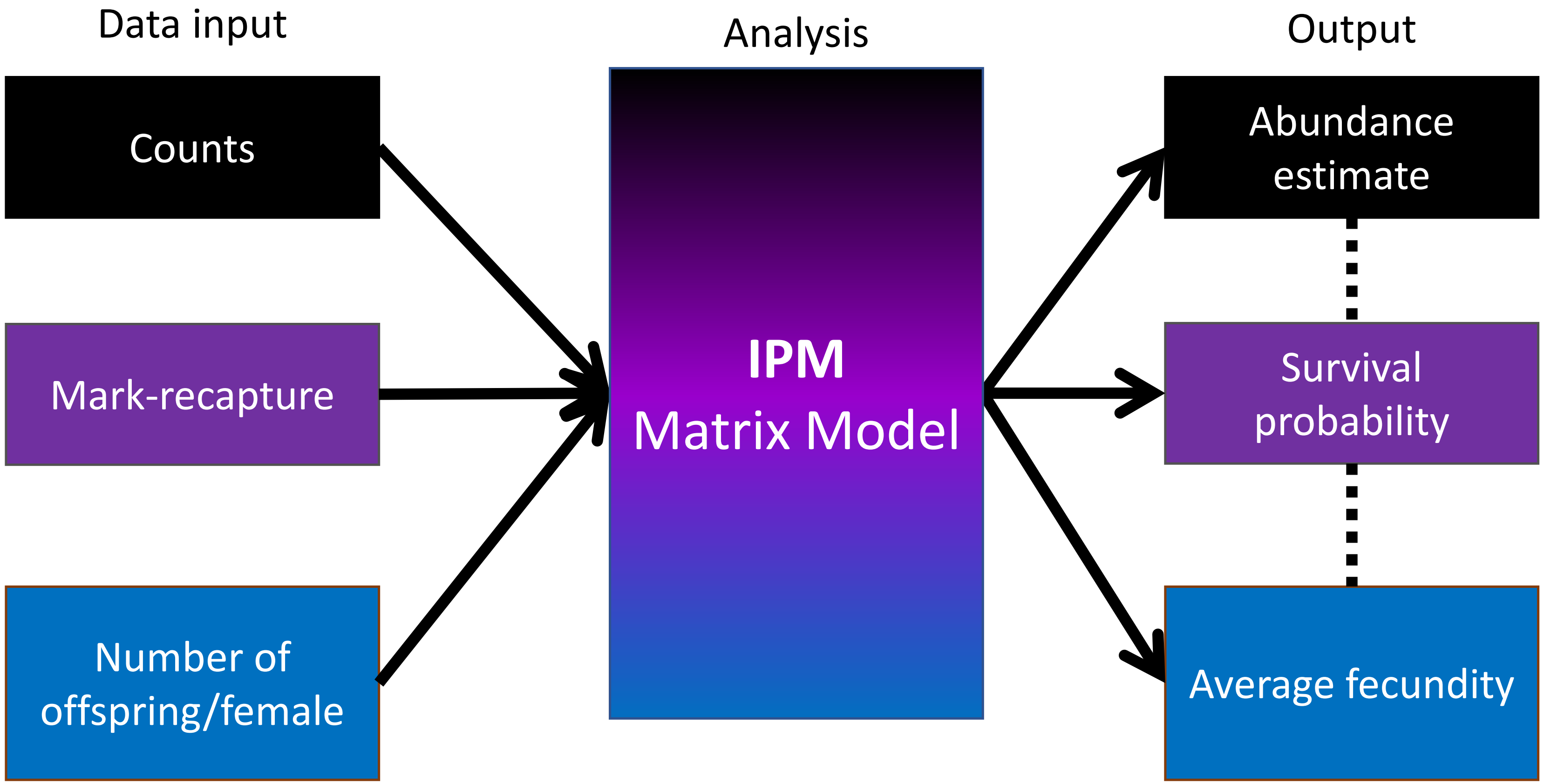
- Matrix models are used to present, analyze, and project population dynamics



$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{a} \end{bmatrix} = \begin{bmatrix} 0 & F_t^a \\ S_t^Y & S_t^a \end{bmatrix} \times \begin{bmatrix} N_t^Y \\ N_t^a \end{bmatrix}$$

Integrated population models

$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{a} \end{bmatrix} = \begin{bmatrix} 0 & F_t^a \\ S_t^Y & S_t^a \end{bmatrix} \times \begin{bmatrix} N_t^Y \\ N_t^a \end{bmatrix}$$



- Combine demographic data with counts
- Use the model to make projections

Matrix projections

$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{a} \end{bmatrix} = \begin{bmatrix} 0 & F_t^a \\ S_t^Y & S_t^a \end{bmatrix} \times \begin{bmatrix} N_t^Y \\ N_t^a \end{bmatrix}$$

$$N_{t+1} = A * N_t \longleftrightarrow N_{t+1} = \lambda * N_t$$

Matrix projection outputs

- Abundance over time
- Population growth rate
  - Lambda (λ)
    - λ = 1.0 stationary
    - λ = 1.10 increasing 10% per year
    - λ = 0.90 decreasing 10% per year
- Extinction and/or quasi-extinction risk
- Sensitivity and elasticity

Population Resiliency

Sensitivity and elasticity

$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{A} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \times \begin{bmatrix} N_{t}^{Y} \\ N_{t}^{A} \end{bmatrix}$$

$$Sensitivity = \frac{\delta\lambda}{\delta a_{2,2}}$$

$$Elasticity = \frac{a_{2,2}}{\lambda}\frac{\delta\lambda}{\delta a_{2,2}}$$

Sensitivity is the rate of change in population growth (λ) with respect to a change in any element of the matrix.

Elasticity analysis estimates the effect of a **proportional** change in the demographic rates on population growth (λ).

How to estimate sensitivity and elasticity

- **Program R – Package ‘PopBio’**

Population matrix → $\begin{bmatrix} 0 & F_t^A \\ S_t^Y & S_t^A \end{bmatrix}$ → $\begin{bmatrix} 0 & 1.2 \\ 0.3 & 0.8 \end{bmatrix}$

Elasticity matrix → $\begin{bmatrix} 0 & 0.222 \\ 0.222 & 0.554 \end{bmatrix}$

Simple future condition assessments

- Using sensitivity and/or elasticity output
  - Results indicate population growth is most sensitive to **adult survival**
    - Conceptual modeling and lit review suggest that adult survival is negatively affected by drought frequency
    - Drought frequency will increase over next 50 years
  - What can we expect given this information?
    - Adult survival will likely decrease
    - Population growth will likely decrease
    - If climate predictions are accurate, future resiliency will decrease

alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

Forms of uncertainty

- Partial controllability
- Observational uncertainty
- Environmental variation
- Ecological uncertainty
- Demographic stochasticity

Incorporating uncertainty

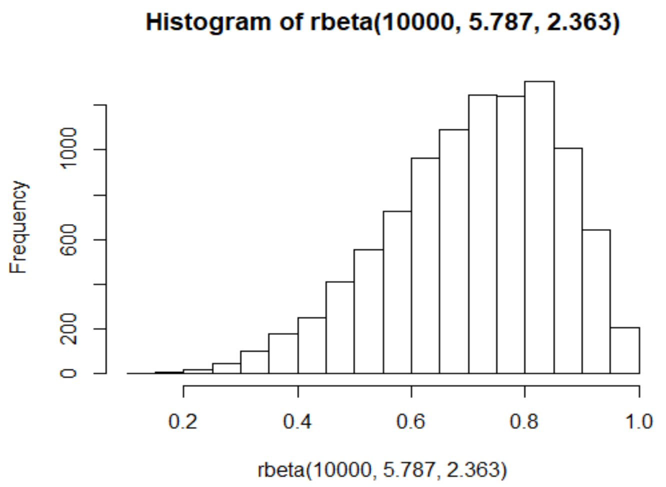
- Use statistical distributions and functional relationships
  - Environmental variation/stochasticity
  - Demographic stochasticity
  - Ecological/structural uncertainty
    - Density dependence
  - Parametric uncertainty

Environmental stochasticity

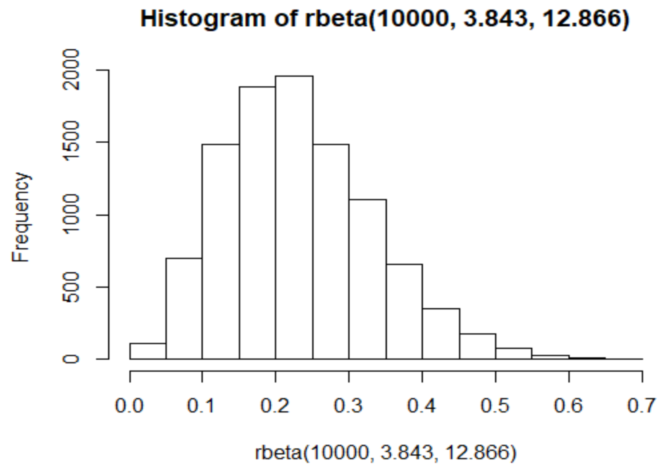
- Survival parameters typically drawn from a beta distribution
  - Continuous but restricted between 0 and 1
  - Very flexible
- Fecundity parameters have two typical methods
  - Log-normal, bounded by 0 and infinity
  - Poisson distribution summed over all the individuals in the population

Survival rate distribution

- S = 0.71, S.D. = 0.15 →
  - Beta1 =5.787 Beta2 =2.363 →

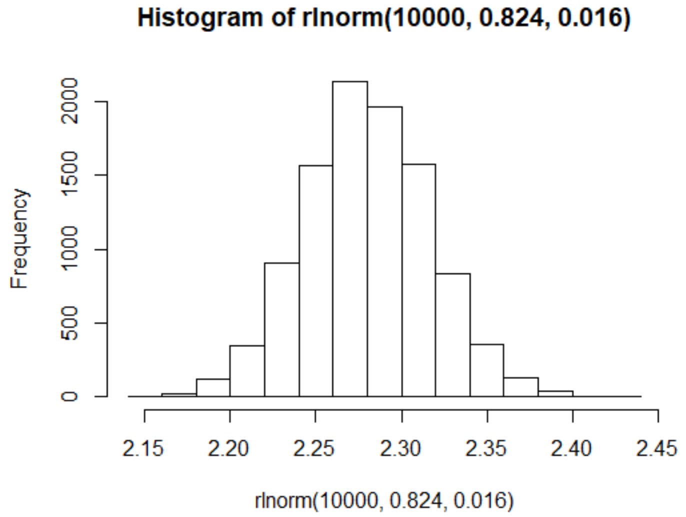


- S = 0.23, S.D. = 0.1 →
  - Beta1 =3.843 Beta2 =12.866 →

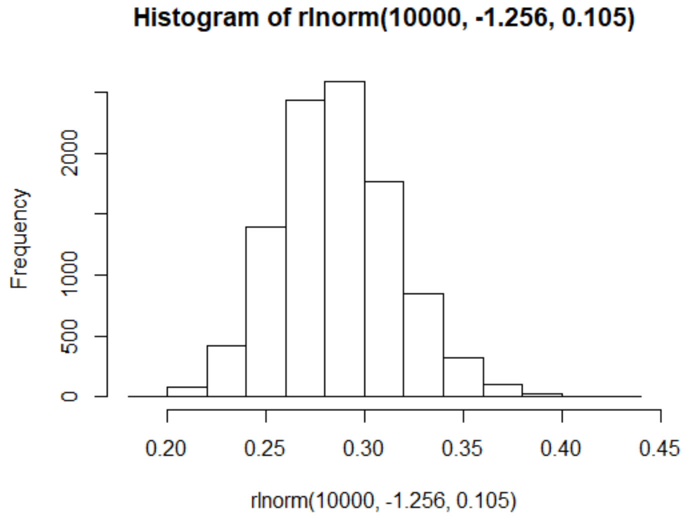


Fecundity distribution

- F = 2.3, S.D. = 0.3 →
  - s1= 0.824, s2 = 0.016 →



- F = 0.3, S.D. = 0.1 →
  - s1= -1.256, s2 = 0.105 →



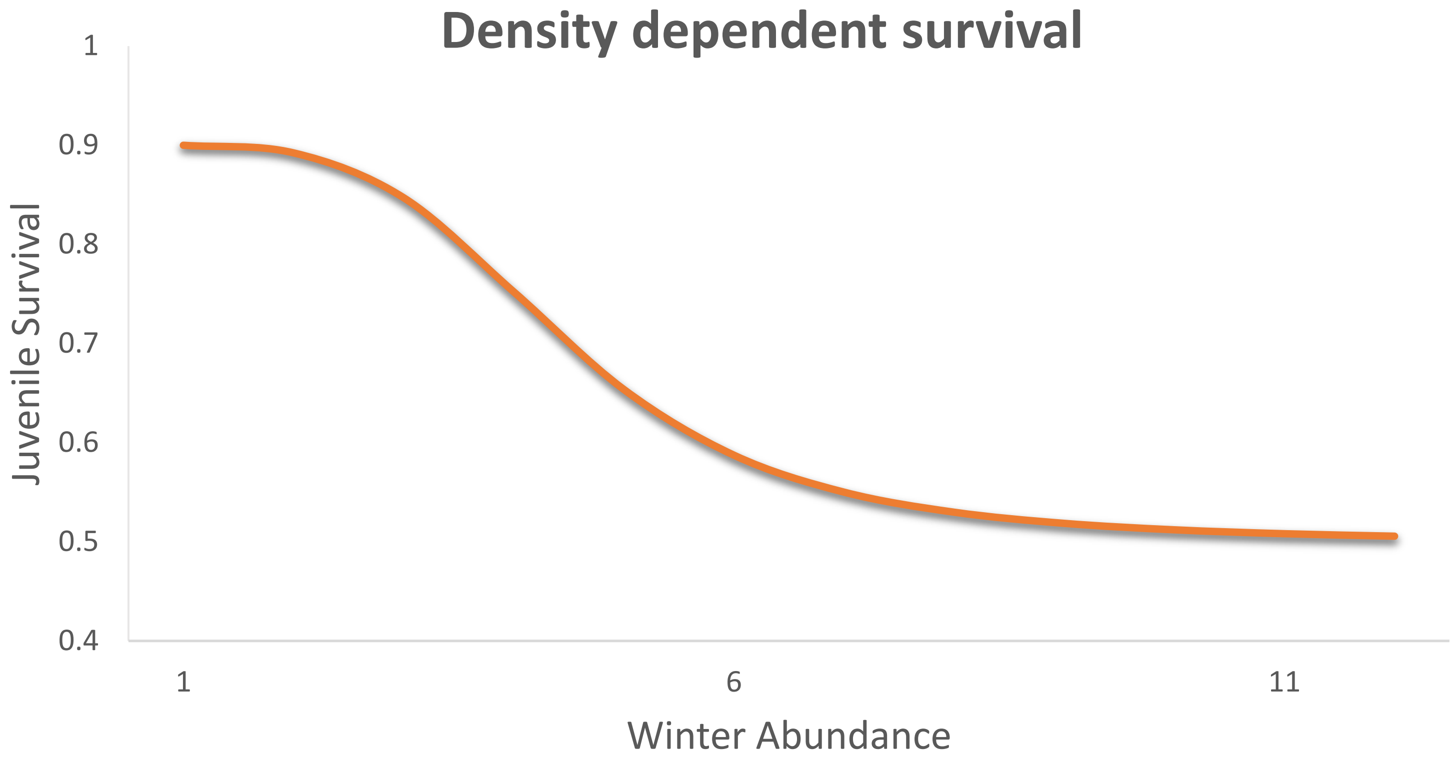
Demographic stochasticity

- Animals live or die as whole animals - not as fractions
  - Without demographic stochasticity mean survival probability = 0.8
    - 6 individuals * 0.8 = 4.8 individuals at the next time step

  - With demographic stochasticity
    - Model survival using a Binomial distribution
      - Computer picks 6 random numbers: 0.32, 0.89, 0.81, 0.11, 0.94, 0.70
      - 3 out of the 6 individuals die because their random picks were less than the mean of 0.8

Ecological or structural uncertainty

- Density dependence
  - Model parameters are a function of population density



Modeling density dependence

Threshold Density dependence

- If the population exceeds a ceiling threshold, fecundity is equal to zero
  - if (n[i,j] > ncrit) F[i,j] = 0

- As the population approaches some ceiling threshold, fecundity gets smaller and smaller
  - F[i,j]= F[i,j]*(1-n[i,j]/ncrit)

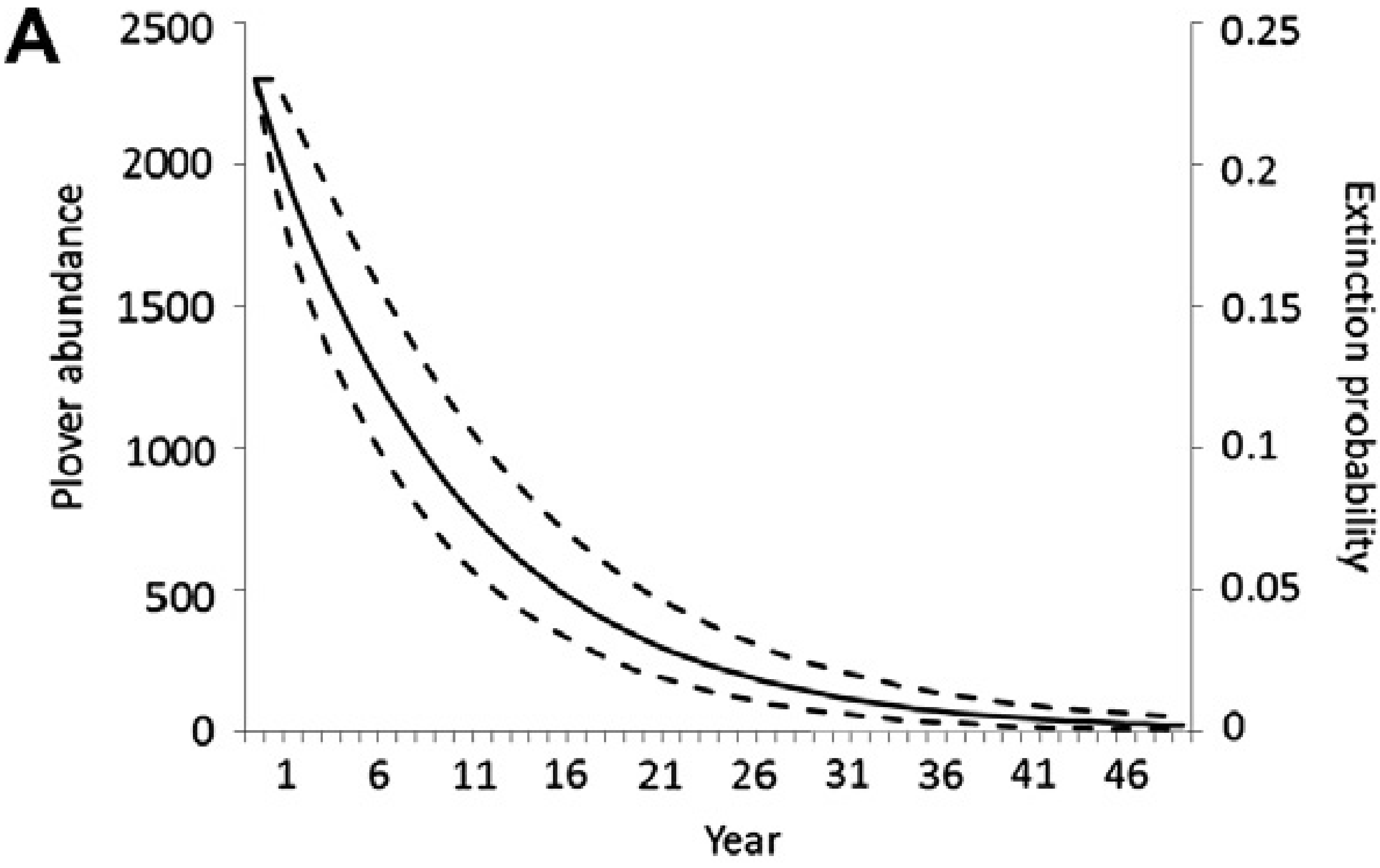
Parametric uncertainty

- Parameters values are not precisely known
    - Variance or standard deviation estimates for parameters estimated over years conflate environmental variation with sampling variance
        - Sampling variance is the result of only using a specific number of individuals or locations to study a phenomenon
        - Happens with every wildlife study because we can’t study every individual in every location

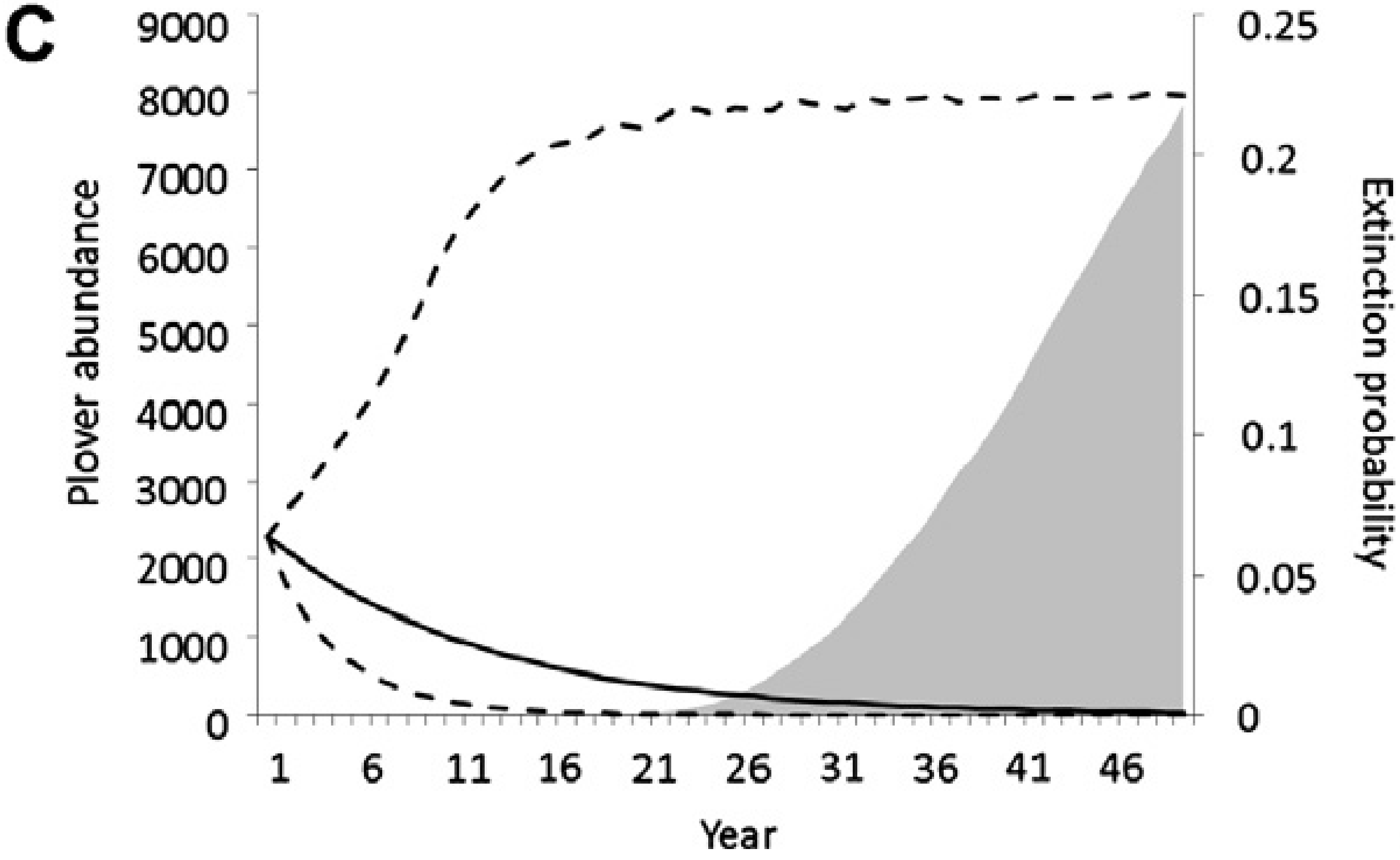
Great, so what do we do about it?

- Most models will discard sampling variance
- Not always a good idea!
    - Pretending that we know system parameters with precision
        - Less variation in the model predictions
        - Could affect assessment of management choices

Projection without sampling variance

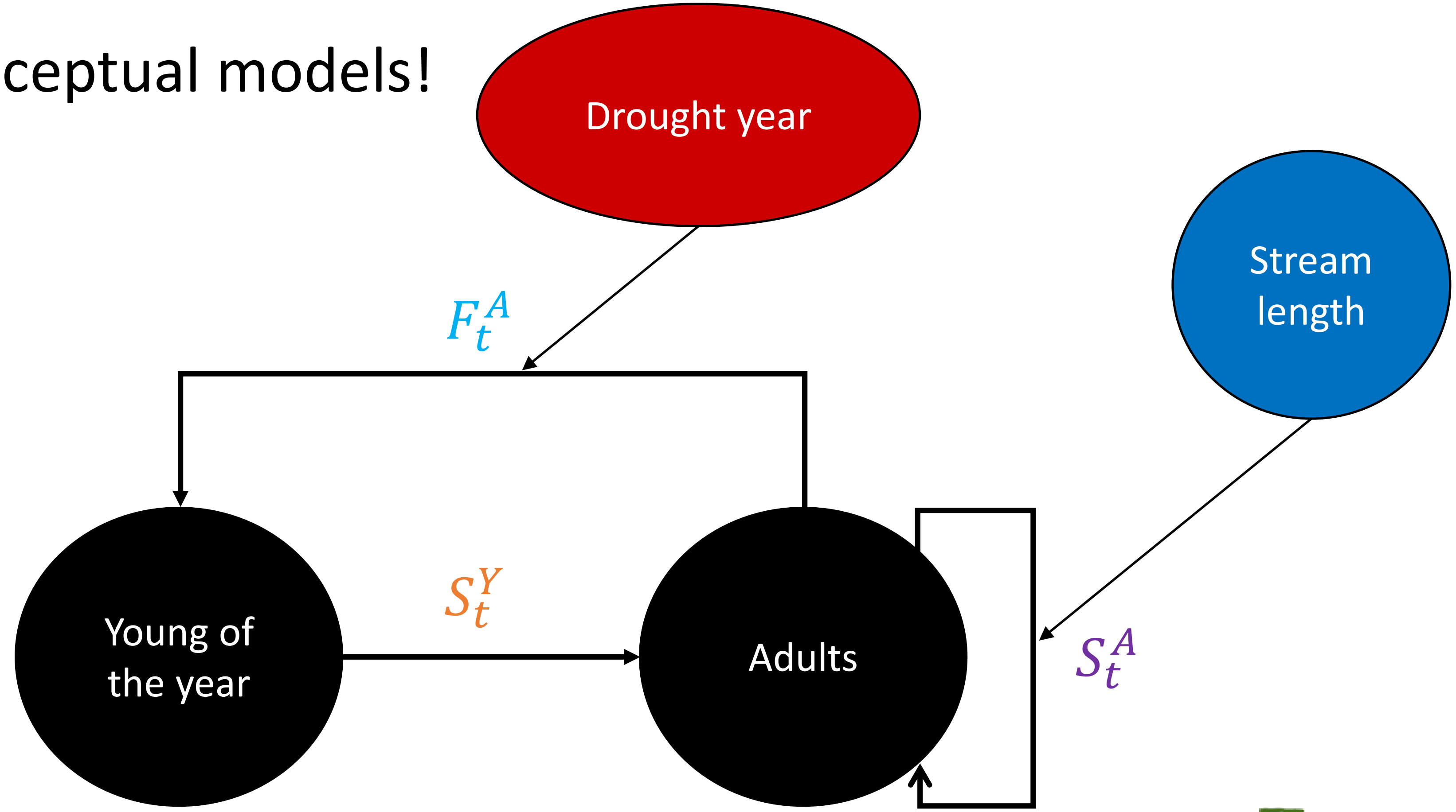


Projection with sampling variance



Modeling environmental effects

- Back to conceptual models!

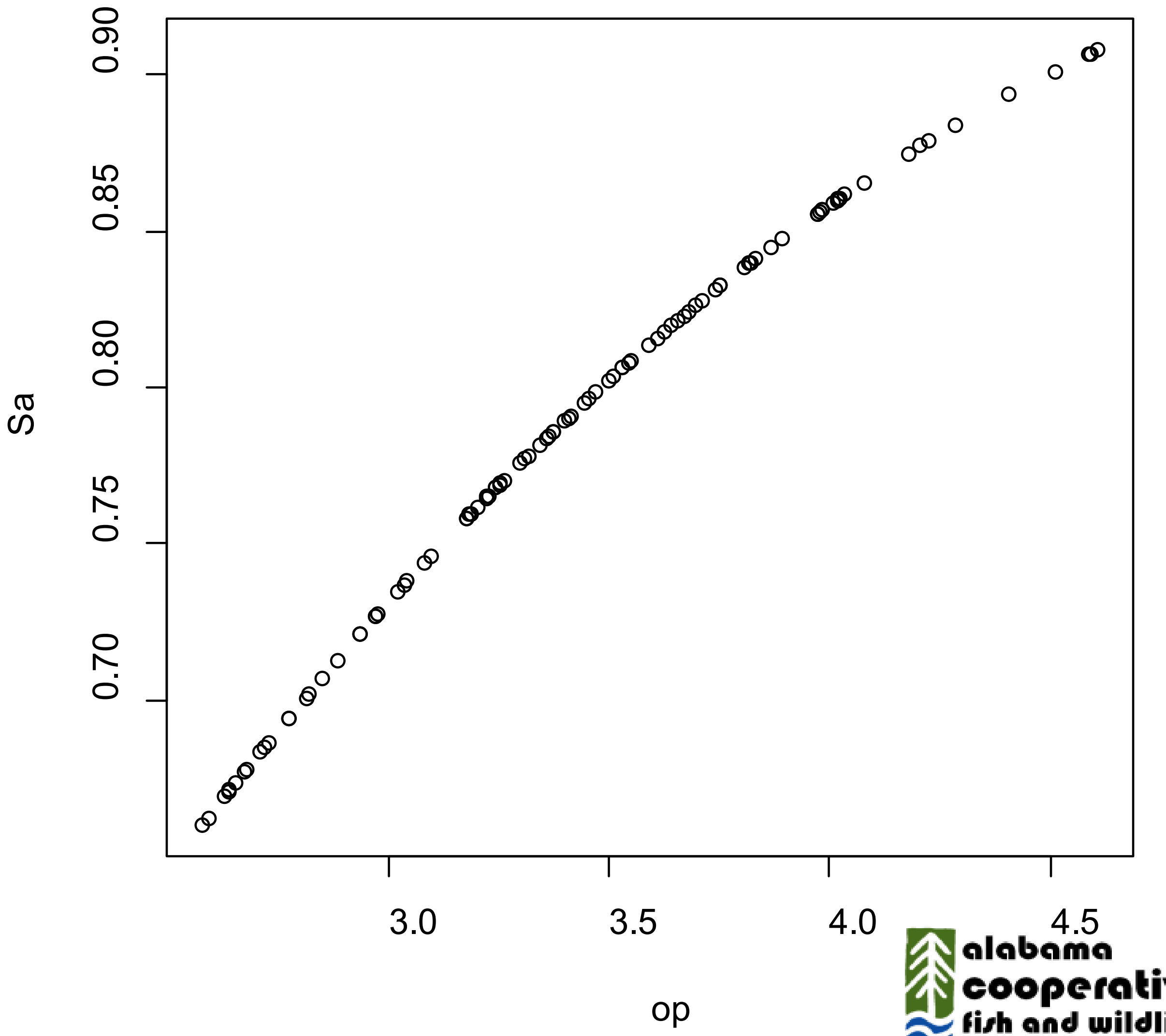


Incorporating environmental covariates

- Conditionally linked events
  - Use **IF→THEN** statements to link a demographic parameter to some other randomized event
    - E.g., if a Bernoulli trial for drought returns a 1, then mean fecundity is 1.1 offspring per female, but if it returns a 0 then mean fecundity is 2.3 offspring per female

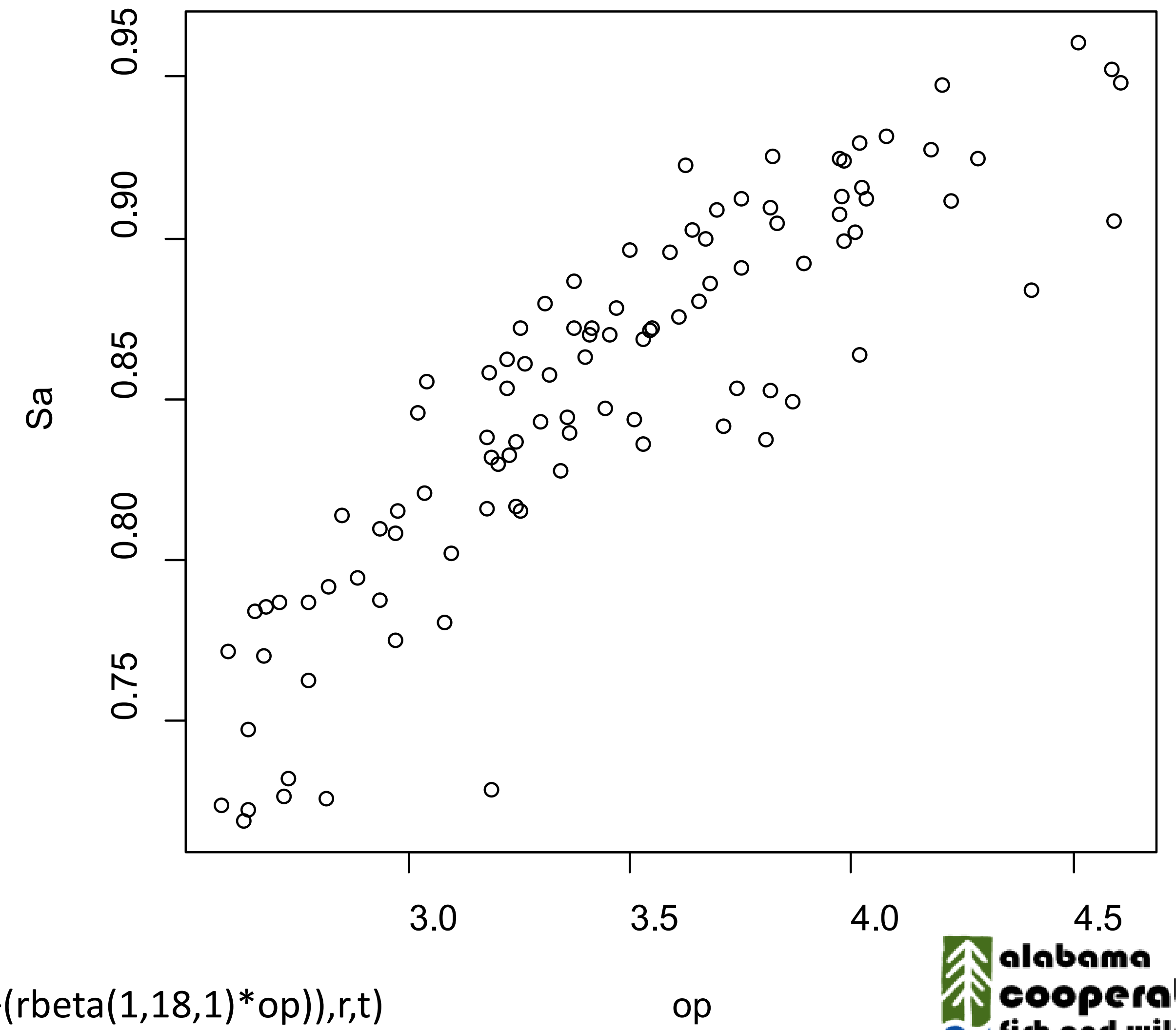
Environmental covariates

- Adult survival (Sa) can be a function of some other environmental parameter/variable
  - (“op” for other parameter, e.g., stream length)



Environmental covariates

- Adult survival (Sa) can be a function of “op” but it has variability



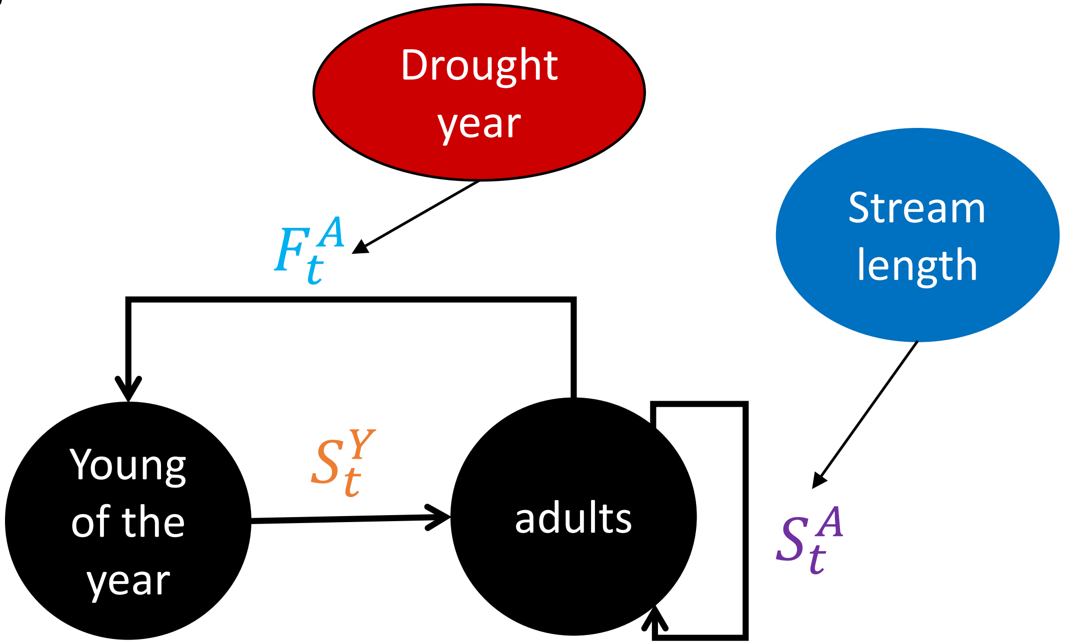
Sa=matrix(plogis(-rnorm(1,1.4,.1)+(rbeta(1,18,1)*op)),r,t)

alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

Inputting scenarios

- Use the conceptual model and sensitivity analysis to guide scenarios
  - i.e., what ecological factors affect the most sensitive parameters?
- Design scenarios to explore the expected range of future variation in important covariates



Using this structure to build GLMs

- Generate lots of output values (abundance, P(extinction), etc.) with lots of corresponding input values
- Use a multi-variate GLM to assess the importance of each variable of interest:
    - P$(extinction) \sim b_1(Initial\ N) + b_2(drought\ freq) + b_3(MaxPop)$ …
    - This is a binomial GLM

- What factors most effect the output metric of interest?

alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

Sonoran desert tortoise example

- MDR = mean drought rate
- NAI = Initial Number of adults
- MaxPop = habitat based maximum population size
  - You could input different values of MDR, NAI or MaxPop to predict the corresponding P(Qe100), i.e., input alternative future scenarios

- $P(Qe100) = -5.602 + (18.42\ x\ MDR) - (5.363e-6x\ NAI) - (1.797e-6x\ MaxPop)$

Review

- What metrics are available for assessing population resiliency using a demographic matrix model?
- What sources of uncertainty may make it difficult to predict population dynamics?
- How can we incorporate uncertainty in our population projections?

Questions?