Demographic Matrix Population Models

SSA 200

alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

Applications to SSAs

- Models are designed to output useful metrics on future resiliency and redundancy
  - Future abundance, extinction probability, population growth rate
- Models allow us to predict future condition of the populations and characterize uncertainties

Lecture outline

- Simple model construction
  - Review of data types/analysis
  - From conceptual to quantitative
- Incorporate environmental covariates and density dependence
- Model environmental stochasticity, demographic stochasticity, and parametric uncertainty

Constructing the model

1. Choose state variables (age, stage)
    - Dependent on data and species
2. Use demographic data to estimate vital rates for each state
    - Fecundity, survival probability, recruitment probability
3. Use state-specific vital rate estimates to create matrix model

State variables

- Age classes (Leslie Matrix)
    - Equal time intervals & all individuals advance at next time
        - Short lived species with age-specific data
- Stage classes (Lefkovitch Matrix)
    - Unequal time intervals
    - Population divided by developmental stage or size
        - Difficult to age individuals but can get length, height, etc.
        - Juvenile, subadult
        - Seeds, dormancy, small plants, large plants

Constructing the model

1. Choose state variables (age, size, stage)
   - Dependent on data and species
2. Use demographic data to estimate vital rates for each state
   - Fecundity, survival probability, recruitment probability
3. Use state-specific vital rate estimates to create matrix model

Demographic data

- Depends on ecology/life history of species

Data types
Number of offspring per female
Nest/den success or failure
Number of young returning next year
Radio telemetry
Individual mark-recapture/resight
Annual plot census (plants)
Growth rate

Demographic vital rate
Fecundity
Breeding success probability
Recruitment probability
Survival probability (seasonal or annual)
Transition probability (annual)

Estimate vital rates

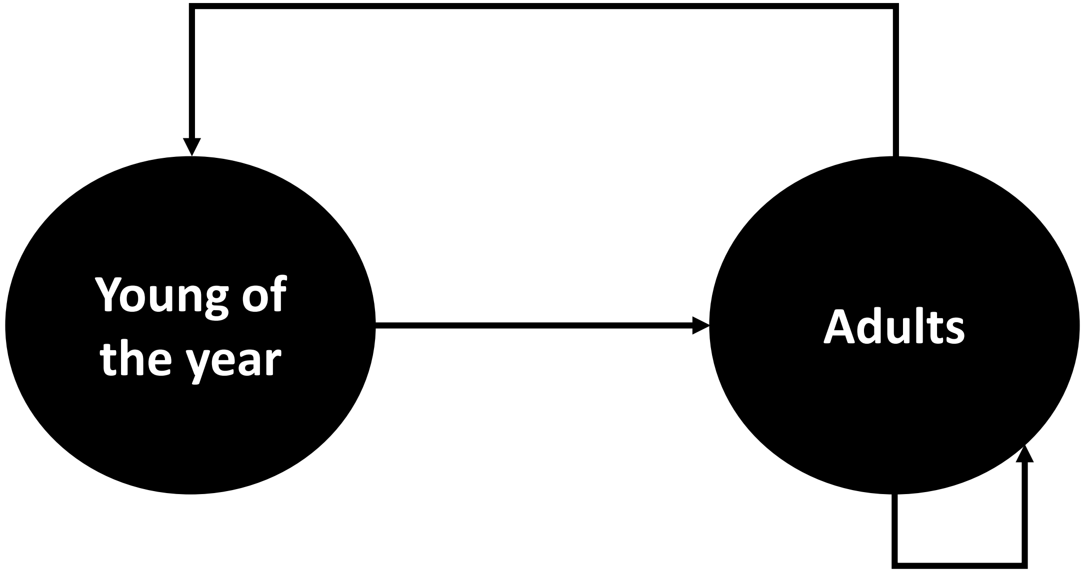
- Fecundity
    - Number of offspring/female -> Poisson GLM
    - Successful breeding (yes/no) -> Binomial GLM
- Annual survival/mortality
    - Radio telemetry -> known-fate models
    - Individual CMR -> Cormack-Jolly-Seber (CJS) models
    - Proportion of marked plants alive next year
- OR – use values reported in literature/expert opinion

Constructing the model

1. Choose state variables (age, size, stage)
   - Dependent on data and species
2. Use demographic data to estimate vital rates for each state
   - Fecundity, survival probability, recruitment probability
3. Use state variables and state-specific vital rate estimates to create a conceptual model and matrix model

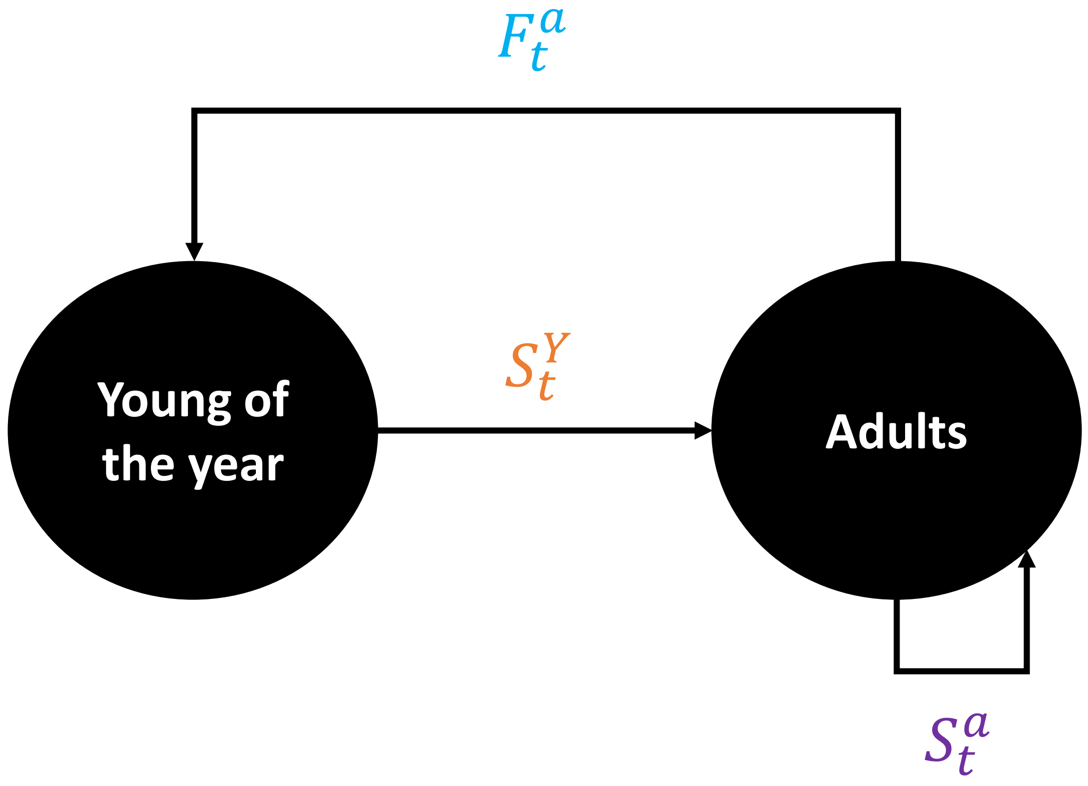
Conceptual model

- Species with two stage classes
  - Survival
    - Adults – CMR data
    - Young of year – estimate in literature
  - Fecundity
    - Number of offspring/ adult female



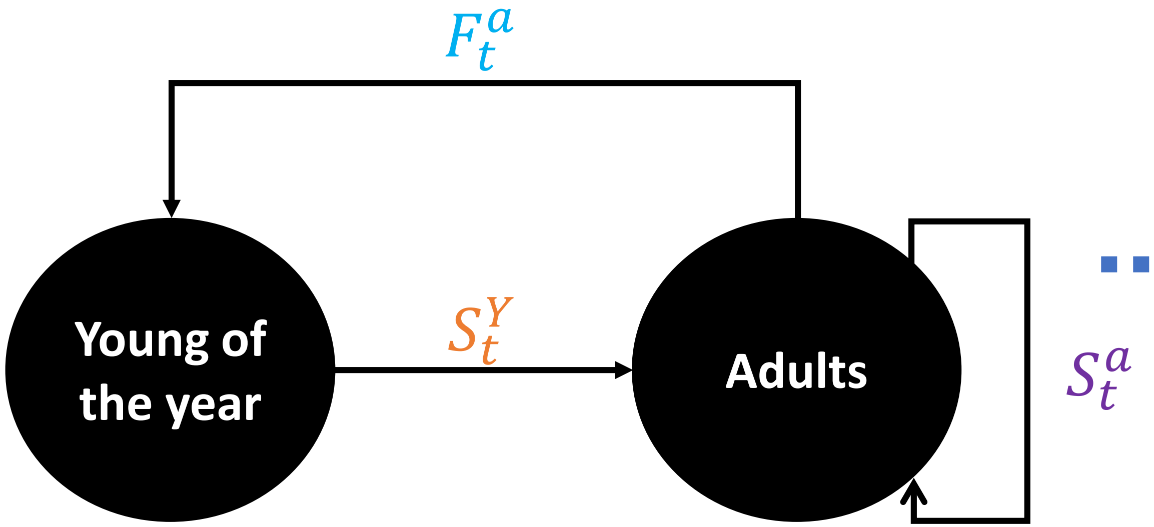
Conceptual model

- Species with two stage classes
  - Survival
    - Adults – CMR data
    - Young of year – estimate in literature
  - Fecundity
    - Number of offspring/ adult female



Create matrix model

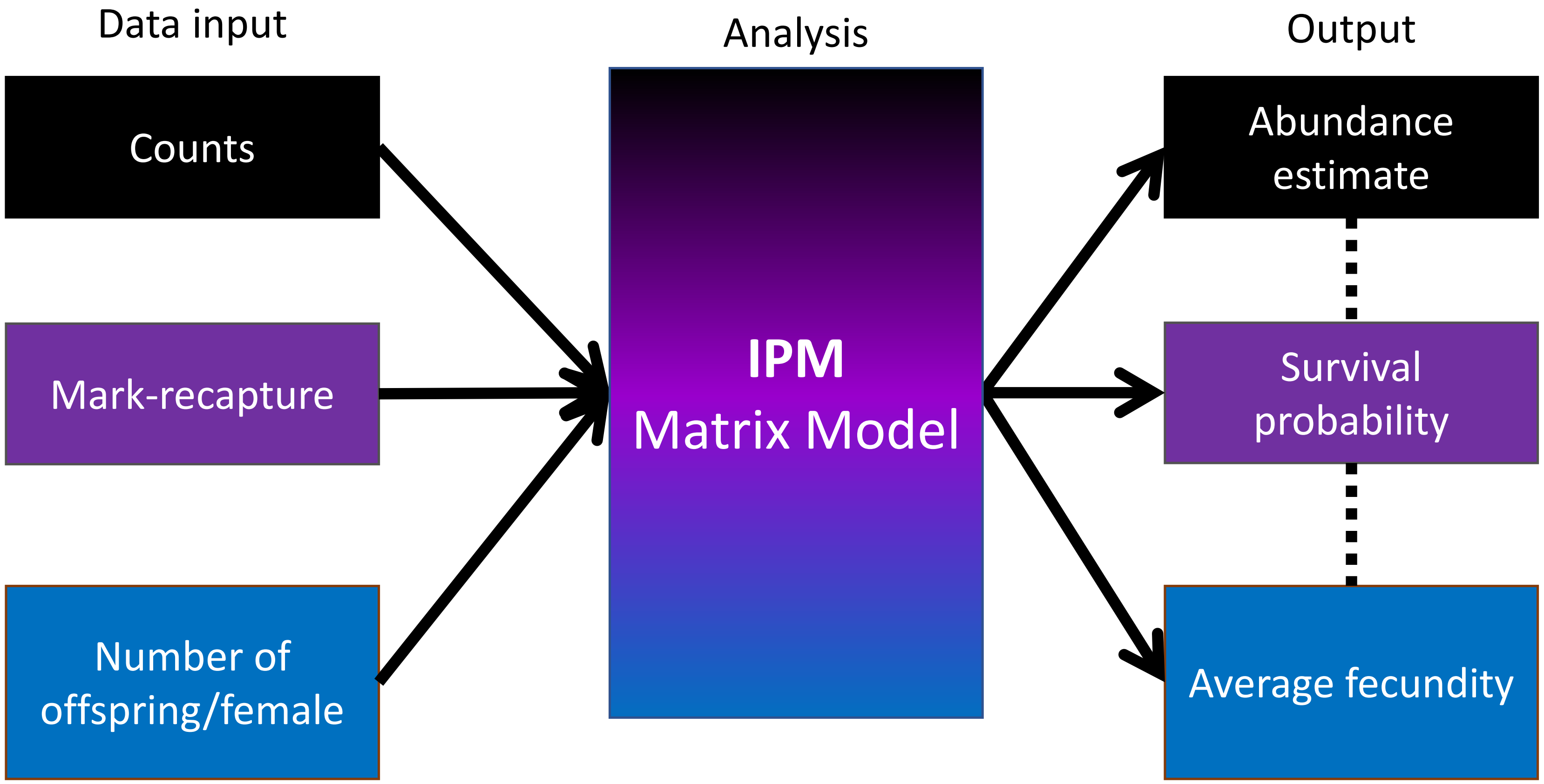
- Matrix models are used to present, analyze, and project population dynamics



$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{a} \end{bmatrix} = \begin{bmatrix} 0 & F_t^a \\ S_t^Y & S_t^a \end{bmatrix} \times \begin{bmatrix} N_t^Y \\ N_t^a \end{bmatrix}$$

Integrated population models

$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{a} \end{bmatrix} = \begin{bmatrix} 0 & F_t^a \\ S_t^Y & S_t^a \end{bmatrix} \times \begin{bmatrix} N_t^Y \\ N_t^a \end{bmatrix}$$



- Combine demographic data with counts
- Use the model to make projections

Matrix projections

$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{a} \end{bmatrix} = \begin{bmatrix} 0 & F_t^a \\ S_t^Y & S_t^a \end{bmatrix} \times \begin{bmatrix} N_t^Y \\ N_t^a \end{bmatrix}$$

$$N_{t+1} = \mathrm{A} * N_t \longleftrightarrow N_{t+1} = \lambda * N_t$$

Matrix projection outputs

- Abundance over time
- Population growth rate
  - Lambda (λ)
    - λ = 1.0 stationary
    - λ = 1.10 increasing 10% per year
    - λ = 0.90 decreasing 10% per year
- Extinction and/or quasi-extinction risk
- Sensitivity and elasticity

Population Resiliency

alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

Sensitivity and elasticity

$$\begin{bmatrix} N_{t+1}^{Y} \\ N_{t+1}^{A} \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \times \begin{bmatrix} N_{t}^{Y} \\ N_{t}^{A} \end{bmatrix}$$

$$Sensitivity = \frac{\delta\lambda}{\delta a_{2,2}}$$

$$Elasticity = \frac{a_{2,2}}{\lambda}\frac{\delta\lambda}{\delta a_{2,2}}$$

Sensitivity is the rate of change in population growth (λ) with respect to a change in any element of the matrix.

Elasticity analysis estimates the effect of a **proportional** change in the demographic rates on population growth (λ).

How to estimate sensitivity and elasticity

- **Program R – Package ‘PopBio’**

Population matrix → $\begin{bmatrix} 0 & F_t^A \\ S_t^Y & S_t^A \end{bmatrix}$ → $\begin{bmatrix} 0 & 1.2 \\ 0.3 & 0.8 \end{bmatrix}$

Elasticity matrix → $\begin{bmatrix} 0 & 0.222 \\ 0.222 & 0.554 \end{bmatrix}$

Simple future condition assessments

- Using sensitivity and/or elasticity output
  - Results indicate population growth is most sensitive to **adult survival**
    - Conceptual modeling and lit review suggest that adult survival is negatively affected by drought frequency
    - Drought frequency will increase over next 50 years
  - What can we expect given this information?
    - Adult survival will likely decrease
    - Population growth will likely decrease
    - If climate predictions are accurate, future resiliency will decrease

Forms of uncertainty

- Partial controllability
- Observational uncertainty
- Environmental variation
- Ecological uncertainty
- Demographic stochasticity

Incorporating uncertainty

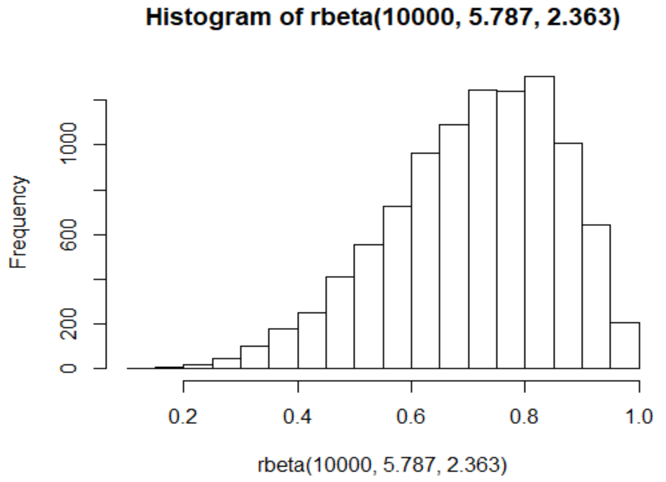
- Use statistical distributions and functional relationships
  - Environmental variation/stochasticity
  - Demographic stochasticity
  - Ecological/structural uncertainty
    - Density dependence
  - Parametric uncertainty

Environmental stochasticity

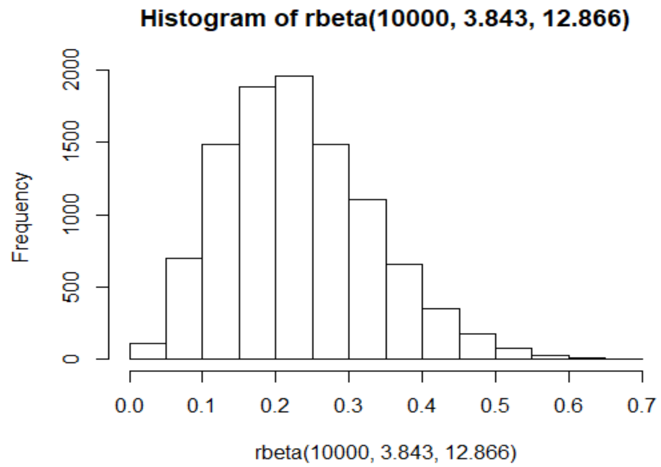
- Survival parameters typically drawn from a beta distribution
    - Continuous but restricted between 0 and 1
    - Very flexible
- Fecundity parameters have two typical methods
    - Log-normal, bounded by 0 and infinity
    - Poisson distribution summed over all the individuals in the population

Survival rate distribution

- S = 0.71, S.D. = 0.15 →
  - Beta1 =5.787 Beta2 =2.363 →



- S = 0.23, S.D. = 0.1 →
  - Beta1 =3.843 Beta2 =12.866 →

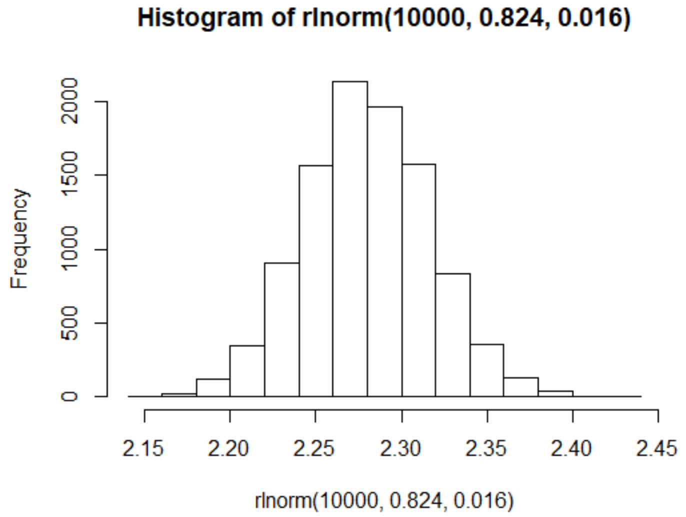


alabama cooperative fish and wildlife research unit

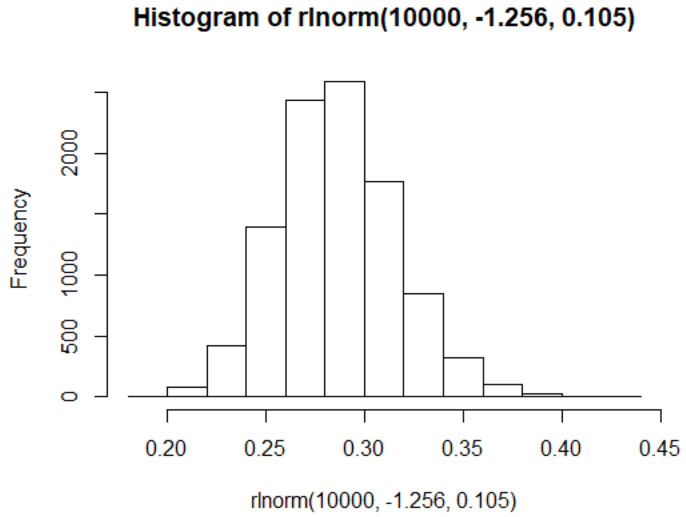
U.S. FISH & WILDLIFE SERVICE

Fecundity distribution

- F = 2.3, S.D. = 0.3 →
  - s1= 0.824, s2 = 0.016 →



- F = 0.3, S.D. = 0.1 →
  - s1= -1.256, s2 = 0.105 →



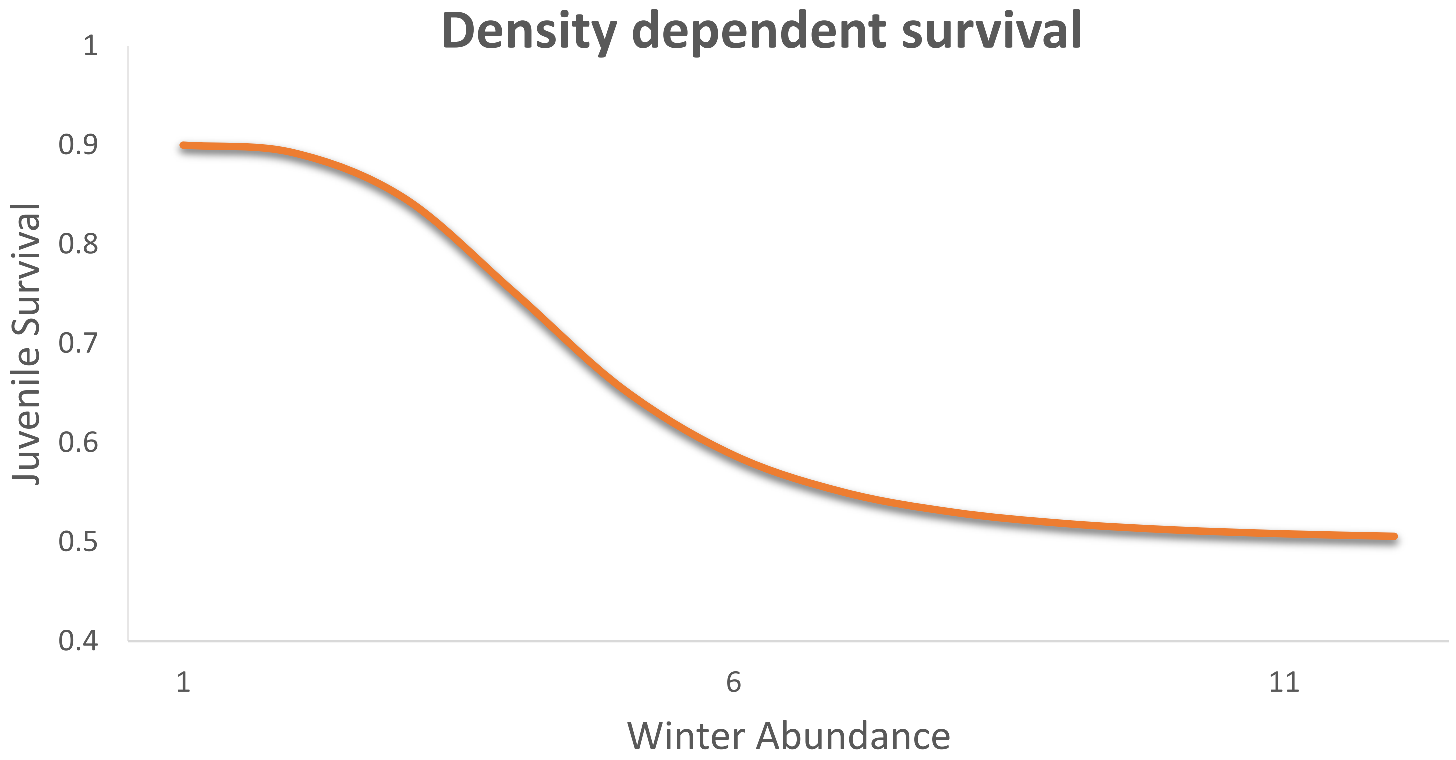
Demographic stochasticity

- Animals live or die as whole animals - not as fractions
  - Without demographic stochasticity mean survival probability = 0.8
    - 6 individuals * 0.8 = 4.8 individuals at the next time step
  - With demographic stochasticity
    - Model survival using a Binomial distribution
      - Computer picks 6 random numbers: 0.32, 0.89, 0.81, 0.11, 0.94, 0.70
      - 3 out of the 6 individuals die because their random picks were less than the mean of 0.8

alabama cooperative fish and wildlife research unit

Ecological or structural uncertainty

- Density dependence
  - Model parameters are a function of population density



Modeling density dependence

Threshold Density dependence

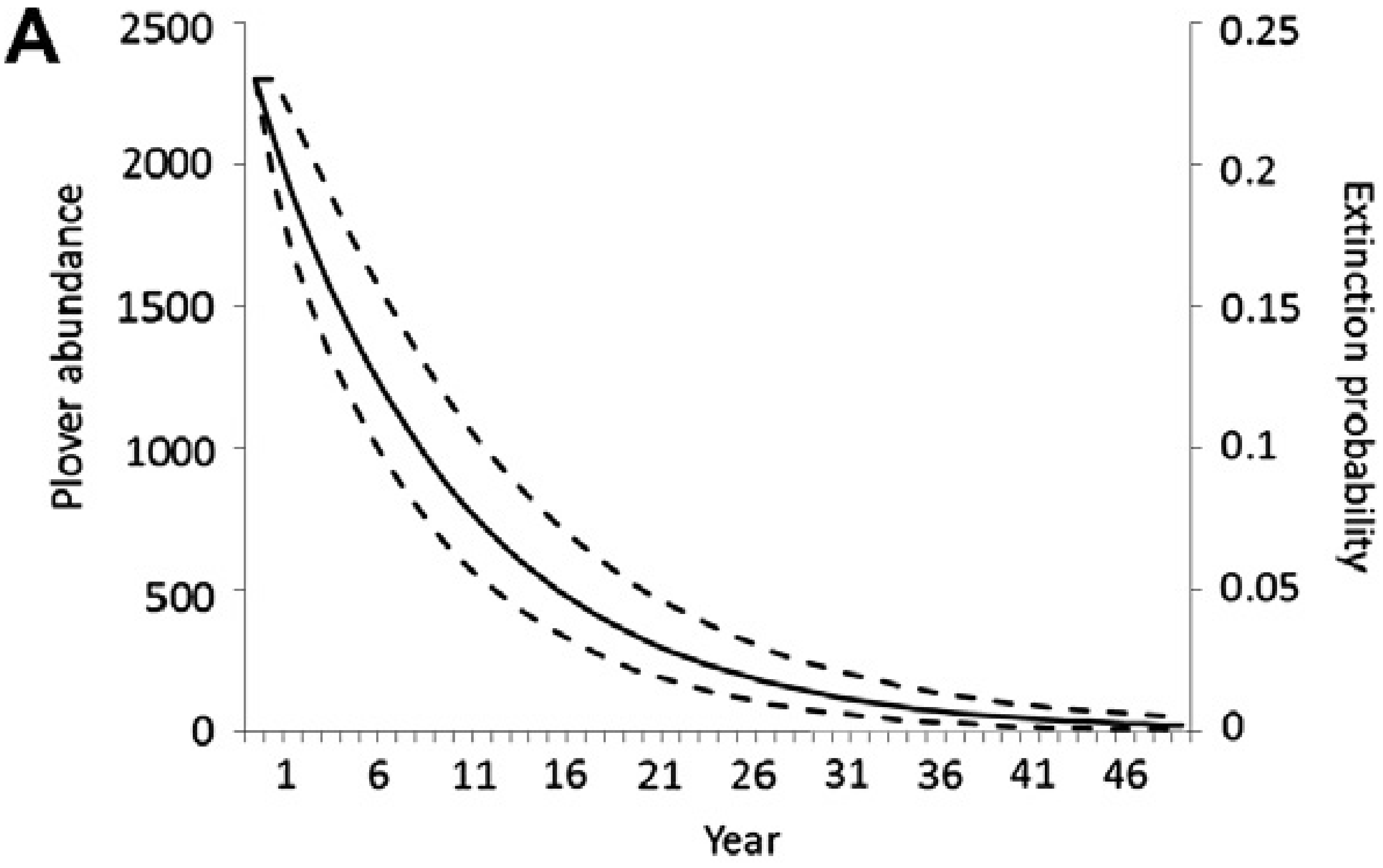
- If the population exceeds a ceiling threshold, fecundity is equal to zero
  - if (n[i,j] > ncrit) F[i,j] = 0

- As the population approaches some ceiling threshold, fecundity gets smaller and smaller
  - F[i,j]= F[i,j]*(1-n[i,j]/ncrit)

Parametric uncertainty

- Parameters values are not precisely known
    - Variance or standard deviation estimates for parameters estimated over years conflate environmental variation with sampling variance
        - Sampling variance is the result of only using a specific number of individuals or locations to study a phenomenon
        - Happens with every wildlife study because we can’t study every individual in every location

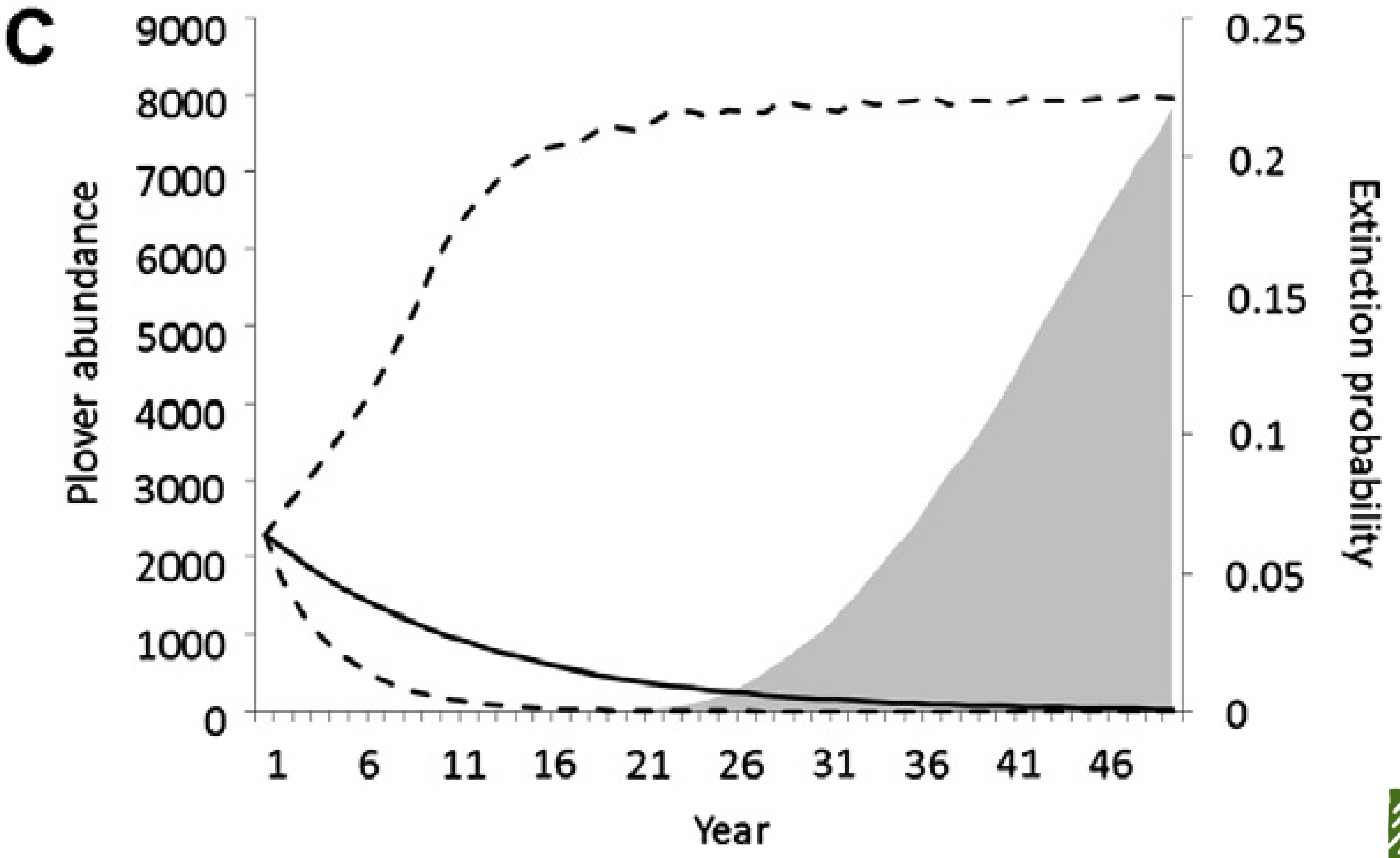
Projection without sampling variance



Great, so what do we do about it?

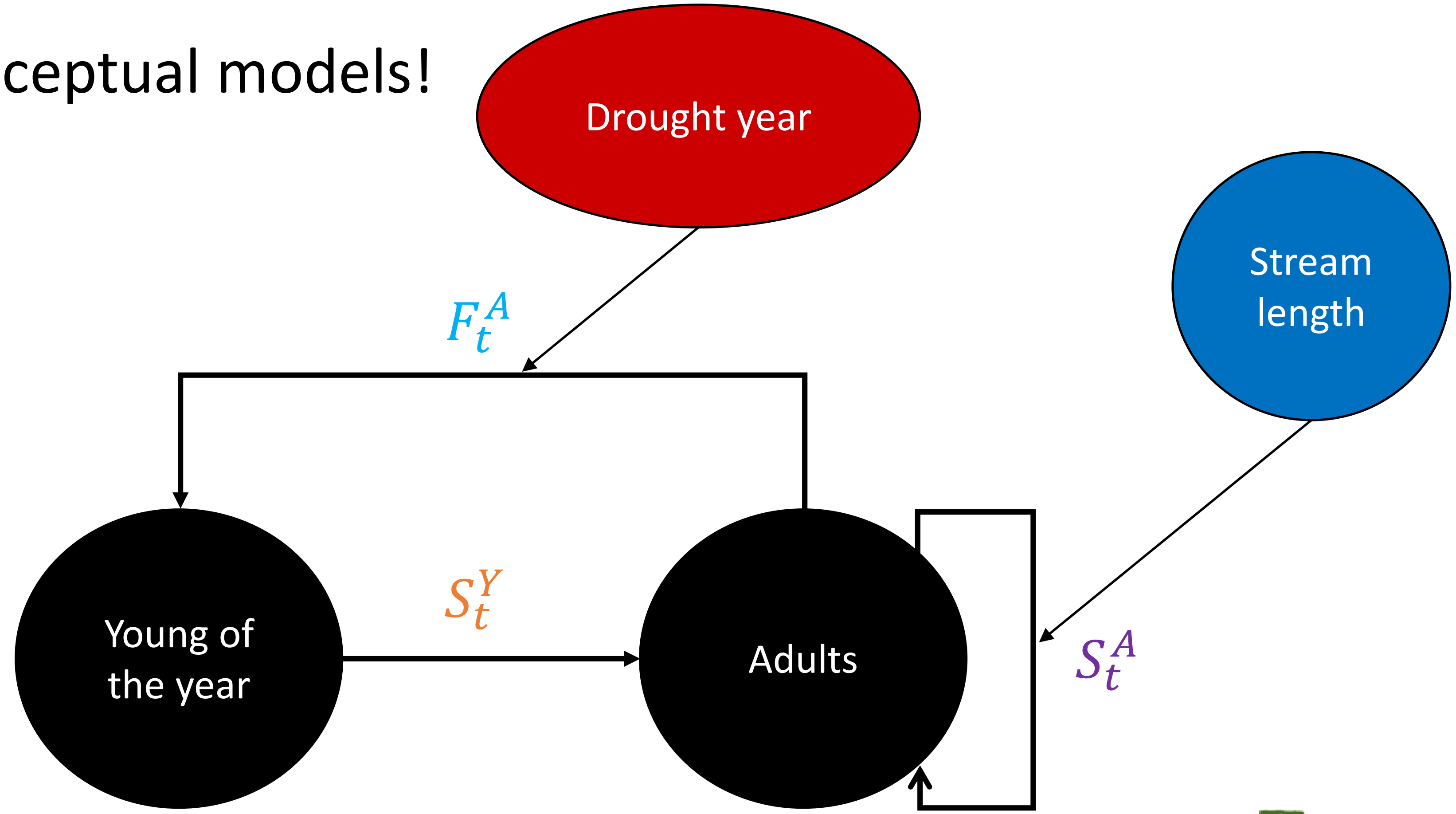
- Modeled by selecting a random value for the mean and variance from a distribution that represents potential parameter values
- The sampled mean and variance are used to generate a new distribution for the parameter
- The vital rate or population growth rate is then randomly selected from the new distribution

Projection with sampling variance



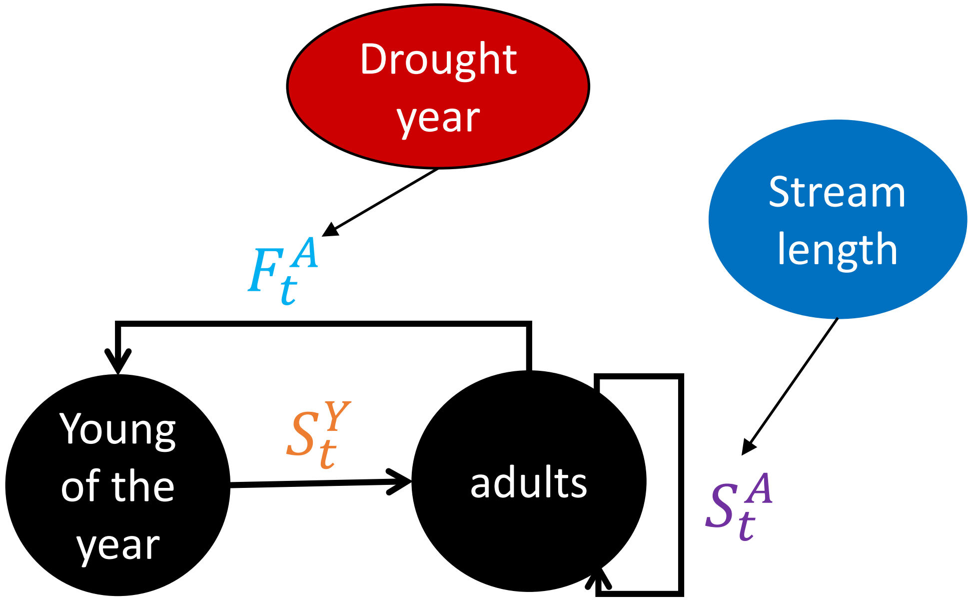
Modeling environmental effects

- Back to conceptual models!



Incorporating environmental covariates

- Conditionally linked events
  - Use **IF→THEN** statements to link a demographic parameter to some other randomized event
    - E.g., if a Bernoulli trial for drought returns a 1, then mean fecundity is 1.1 offspring per female, but if it returns a 0 then mean fecundity is 2.3 offspring per female

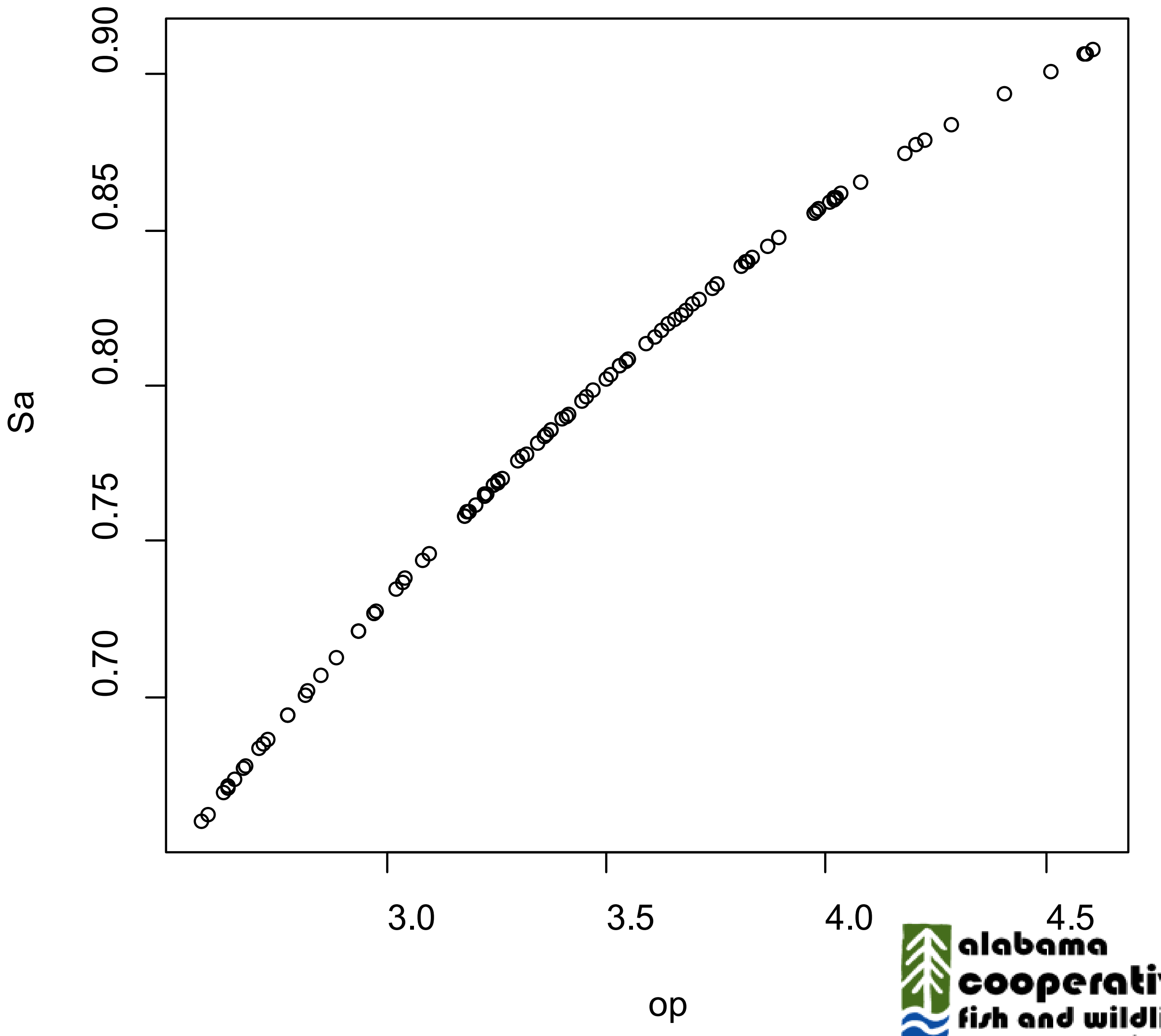


alabama cooperative fish and wildlife research unit

U.S. FISH & WILDLIFE SERVICE

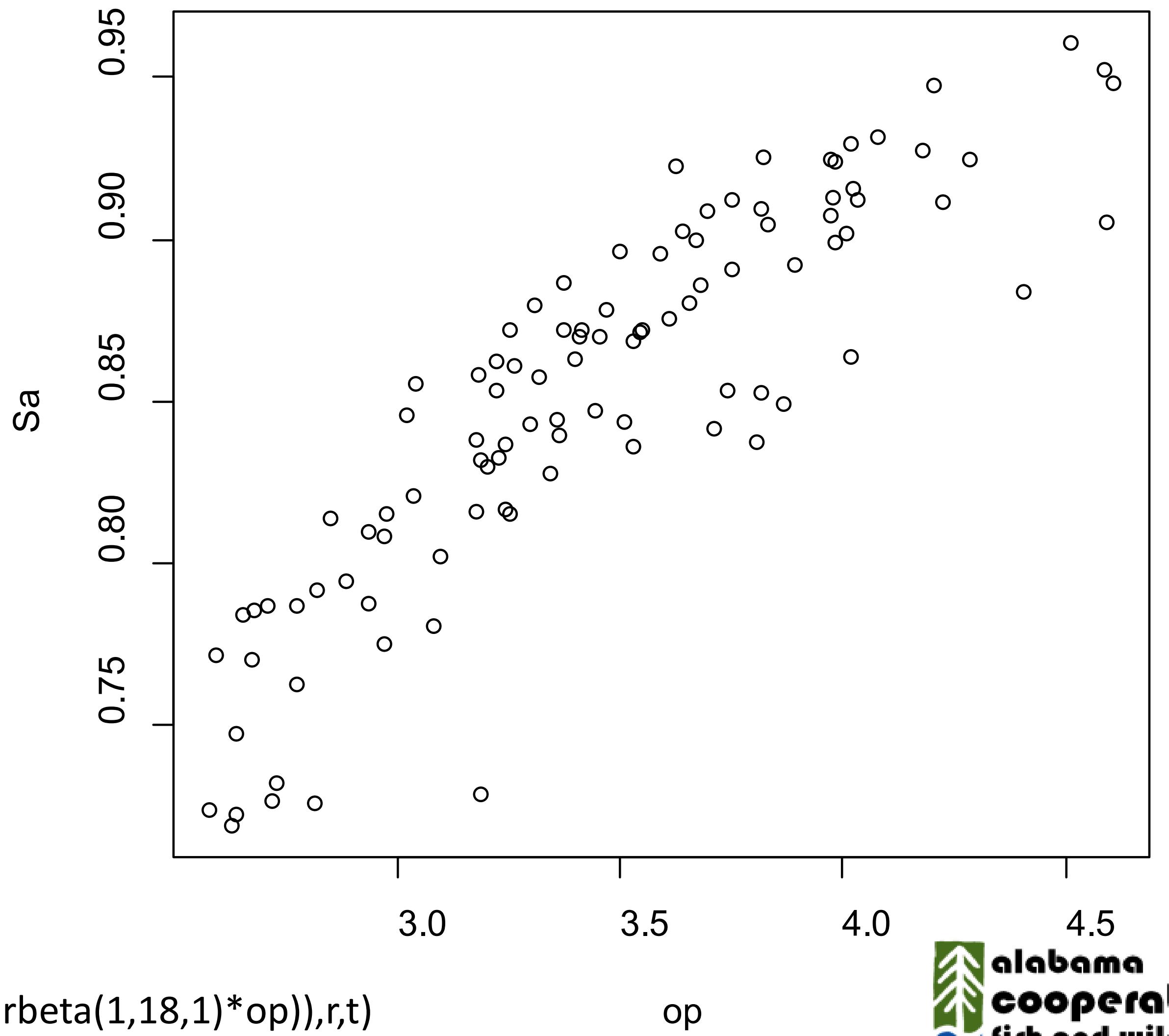
Environmental covariates

- Adult survival (Sa) can be a function of some other environmental parameter/variable
  - (“op” for other parameter, e.g., stream length)



Environmental covariates

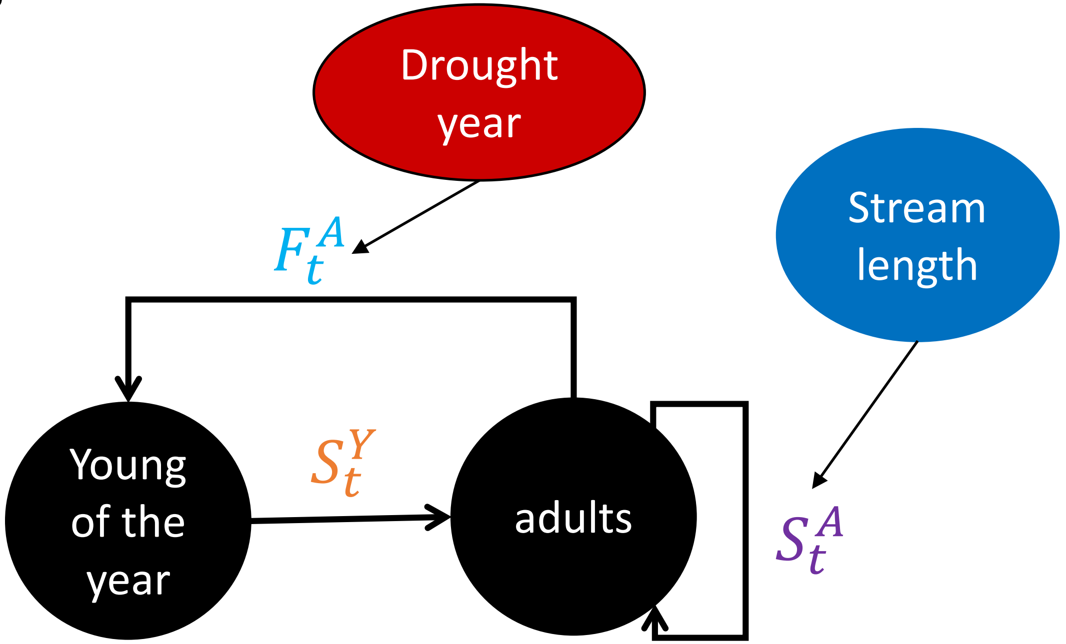
- Adult survival (Sa) can be a function of “op” but it has variability



Sa=matrix(plogis(-rnorm(1,1.4,.1)+(rbeta(1,18,1)*op)),r,t)

Inputting scenarios

- Use the conceptual model and sensitivity analysis to guide scenarios
  - i.e., what ecological factors affect the most sensitive parameters?
- Design scenarios to explore the expected range of future variation in important covariates



Using this structure to build GLMs

- Generate lots of output values (abundance, P(extinction), etc.) with lots of corresponding input values
- Use a multi-variate GLM to assess the importance of each variable of interest:
  - $\mathrm{P}(extinction) \sim b_1(Initial\ N) + b_2(drought\ freq) + b_3(MaxPop) \ldots$
    - This is a binomial GLM

- Determine which factors most effect the output metric of interest

Sonoran desert tortoise example

- MDR = mean drought rate
- NAI = Initial Number of adults
- MaxPop = habitat based maximum population size
    - You could input different values of MDR, NAI or MaxPop to predict the corresponding P(Qe100), i.e., input alternative future scenarios

- $P(Qe100) = -5.602 + (18.42\ x\ MDR) - (5.363e - 6x\ NAI) - (1.797e - 6x\ MaxPop)$

Review

- What metrics are available for assessing population resiliency using a demographic matrix model?
- What sources of uncertainty may make it difficult to predict population dynamics?
- How can we incorporate uncertainty in our population projections?

Questions?