COMPUTER SCIENCE & ENGINEERING
TEXAS A&M UNIVERSITY

Application of Physics-informed Neural Networks to a Linear Elasticity problem.

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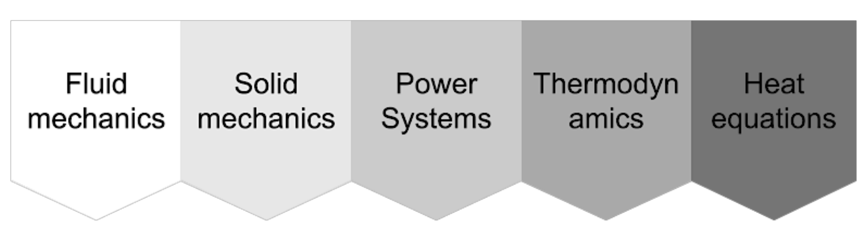
Introduction

Physics-informed neural networks (PINN for short) are neural networks that can perform either data driven discovery of parameters or solution of given problems which have partial differential equations. They do this by utilizing the given constraints, such as boundary/initial conditions, governing equations, and body forces to restrict the output to physically viable solutions.

Overall, PINN is a very valuable tool and can be applied to a plethora of scientific fields, as shown below.



Problem Set-up

The problems are set up with the following governing equations:

$$\sigma_{ij,j} + f_i = 0$$

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij},$$

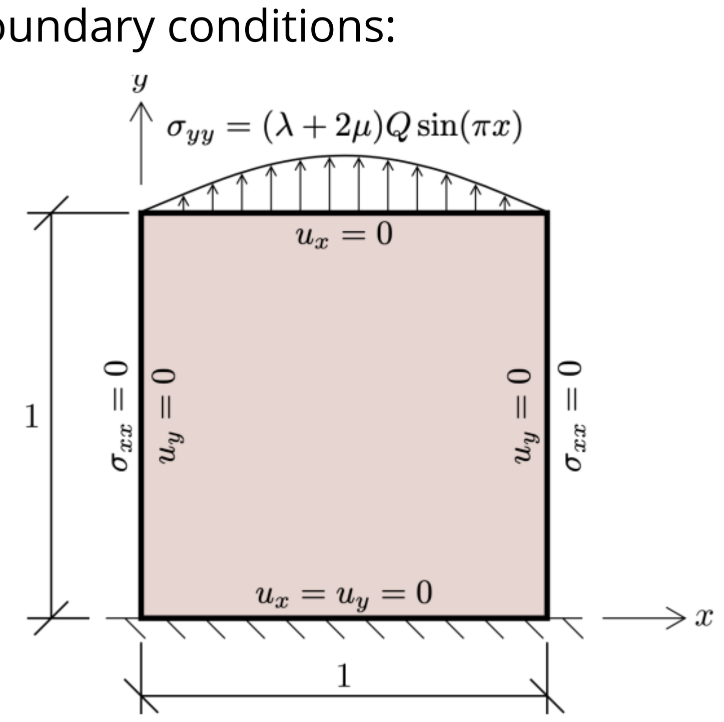
$$\boldsymbol{\varepsilon_{ij}} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{u_{i,j}} + \boldsymbol{u_{j,i}}).$$

Body forces:

$$f_x = \lambda[4\pi^2\cos(2\pi x)\sin(\pi y) - \pi\cos(\pi x)\,Qy^3] + \mu[9\pi^2\cos(2\pi x)\sin(\pi y) - \pi\cos(\pi x)\,Qy^3]$$

$$f_y = \lambda[-3\sin(\pi x)\,Qy^2 + 2\pi^2\sin(2\pi x)\cos(\pi y) + \mu[-6\sin(\pi x)\,Qy^2 + 2\pi^2\sin(2\pi x)\cos(\pi y) + (\pi^2\sin(\pi x)\,Qy^4) \div 4].]$$

And boundary conditions:



Results & Process

Process/Architecture:

The problems are forward and inverse problems, respectively. The parameters being identified are lambda and mu in the inverse problem, and the forward problem is solving the equations for the displacement (which can be differentiated to find the stress).

The architecture of the networks are as follows:

- Written in Pytorch.
- Width & Depth: 4 layers of 30 neurons each.
- Activation function: hyperbolic tangent.
- Changing point (proportion of Adam optimizer to LBFGS): 0.5
- Data used is synthetically generated with NumPy (exact solution is used for inverse problem for target data)

Results:

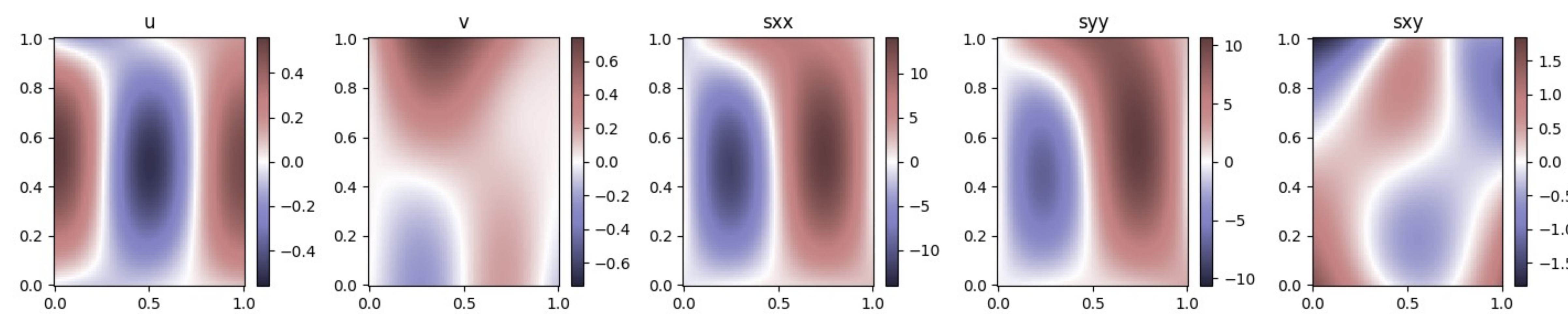


Figure 1: Results from inverse problem model; u and v represent displacement in the x and y directions, respectively. Sxx, syy, and sxy each represent the Cauchy stress tensor in terms of the different dimensions.

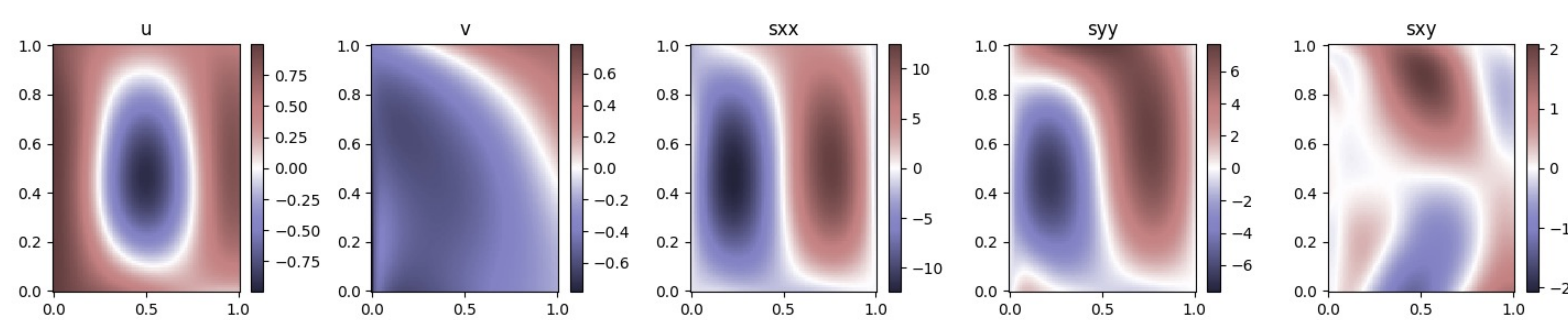
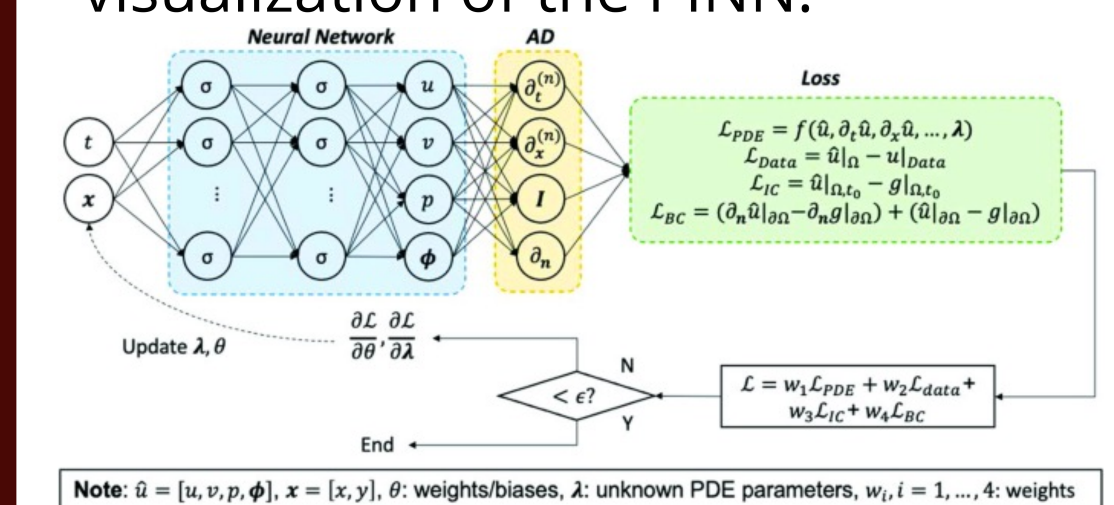


Figure 2: Results from forward problem model; u and v represent displacement in the x and y directions, respectively. Sxx, syy, and sxy each represent the Cauchy stress tensor in terms of the different dimensions.

As shown above, the plain PINN utilized was able to adequately capture the displacement, u and v (symbolically written as U_x and U_y), and the Cauchy stress tensor, sxx, syy, and sxy (written symbolically as σ_{xx}, σ_{yy}, and σ_{xy}). The graphs represent the solution for these terms given x and y coordinates. Figure 1 finds said solutions by discovering the values for lambda and mu (1 and 0.5 respectively) with data and the given solution for the displacements, and Figure 2 finds said solutions by learning from the data and boundary conditions.

Discussion

As the results show, plain PINN is moderately successful for applications in linear elasticity and can properly identify parameters as well as solve the forward problem with minimal changes between the two methodologies. Below is a visualization of the PINN.



Despite the success PINN has had in these various applications, there remains a large amount of research to be done. Some areas which are lacking in research are error estimation in PINN, certainty levels in PINN, and many others.

References

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