Professor David Meyer
Anna Preger, A15587676
Math 111A

The Ephemerality of Eternal Triangles

Introduction

Love is a difficult concept to define and has fascinated humans for centuries. Even mathematicians have demonstrated interest in the dynamics of love, presenting multiple models to interpret these human connections. As Hannah Fry expresses in her book The Mathematics of Love, "I wouldn't be qualified to describe the intangible thrill, all-consuming passion, or world-ending despair that love can bring." [1] We will not be trying to define or model love in this project, but rather to understand the possible dynamics of romantic relationships through a mathematical lens. By doing so, we are combining a popular interest with a powerful tool. We hope this paper will help make Mathematics more tangible and exciting to the general audience. To make the project more interesting, we have decided to model a love triangle – also known as an eternal triangle – where two suitors are interested in the same girl. We are curious about how appropriate it is to call this type of relationship an "eternal" triangle. **Furthermore, this model could be of interest in social psychology as a novel tool for measuring relationships.** It will be interesting to visualize these interactions across time, and maybe predict the potential outcomes of a love triangle.

Model Description

In 1988, Steven Strogatz published the article "Love Affairs and Differential Equations," in which he proposed a simple model for the dynamics of love affairs, modeling emotions by means of Ordinary Differential Equations. [2] Years later, he brought the concept back in his

book “Nonlinear Dynamics and Chaos,” where he introduced four different romantic styles: the eager beaver, the narcissistic nerd, the cautious lover, and the hermit. [3,4] We will be using this classification with some slight changes to characterize the three lovers in our model. Our characters will be an eager beaver, a narcissistic nerd, and a hermit, since most models found in the literature explore the relationship between cautious lovers.

In our paper we classify individuals as synergic or non-synergic, securely attached or insecurely attached. These characteristics, when combined two by two, yield the four different romantic styles previously mentioned. We say that an eager beaver is synergic and securely attached, a narcissistic nerd is synergic and insecurely attached, and a hermit is non-synergic and insecurely attached.

Synergy in this paper is the name given to the enhancement of certain humans’ reactions when they feel love. For example, certain people become more physically attracted to someone the more they feel for them. We consider synergy to be responsible for that increase in the reaction to a partner’s appeal. It does not only grow as the appeal of partner *y* grows, but also as the feelings of a person *x* grow. However, in this paper we will not apply synergy as an enhancer of appeal, as it has been done previously. Here, to be more in accordance with the definition of a synergic individual, our synergy will enhance the function of return in response to the partner’s feelings as well. This will mean that a synergic person’s own feelings encourage them to respond in an increased rate to their partner’s feelings. One could say that synergic individuals are encouraged by their own emotions. The more they feel the more fuel they add to those feelings. To account for the synergy, we will simply introduce two factors of (1+S(x)) in the equation, multiplying both R(y) and I(α_y). When an individual is non-synergic, S(x) = 0 and the return function is never enhanced.

In terms of attachment style, a secure individual is said to be encouraged by their partner's feelings, and therefore they will present a **return** function R that matches them. This means that negative feelings are responded by negative values, whereas positive feelings are responded by positive values. Look at Figure 2 to visualize the behavior of the return function of a secure individual. An insecure individual, on the other hand, presents this same behavior up to a certain point, a threshold where the partner's feelings become so high that they overwhelm person x, causing a negative reaction, modeled by a negative return value. Take a look at Figure 1 to visualize this behavior.

We will also be assuming that an individual's response to their partner's appeal will be an increasing function. We will call this response the **instinct** $I(\alpha_y)$, and the higher the partner's appeal, the more attracted an individual is to them. This function will therefore be modeled by f2 – in other words, we assume that every individual presents a secure response to their partner's appeal. We define appeal as a constant reflecting a partner's physical, financial, cultural, educational, intellectual properties. It is also circumstantial, and we assume that every partner's appeal is different in the eyes of the individual whose feelings are being measured. The values of the instinct function start off negative as a response to unappealing partners, which gives rise to a certain aversion. They are positive for positive appeal, indicating that the more appealing an individual is the better will be their partner's response, namely instinct.

Notice that three basic processes are assumed to take part in love dynamics: synergy, return, and instinct. So, we start with the following differential equation:

$$\frac{dx}{dt} = R(y) + I(\alpha_y)$$

Synergy is not in the equation above, which indicates that it is the rate of change in feelings of a non-synergic individual. We modify it to account for synergy in synergic individuals, yielding the following:

$$\frac{dx}{dt} = \big(1 + S(x)\big) * R(y) + \big(1 + S(x)\big) * I(\alpha_y)$$

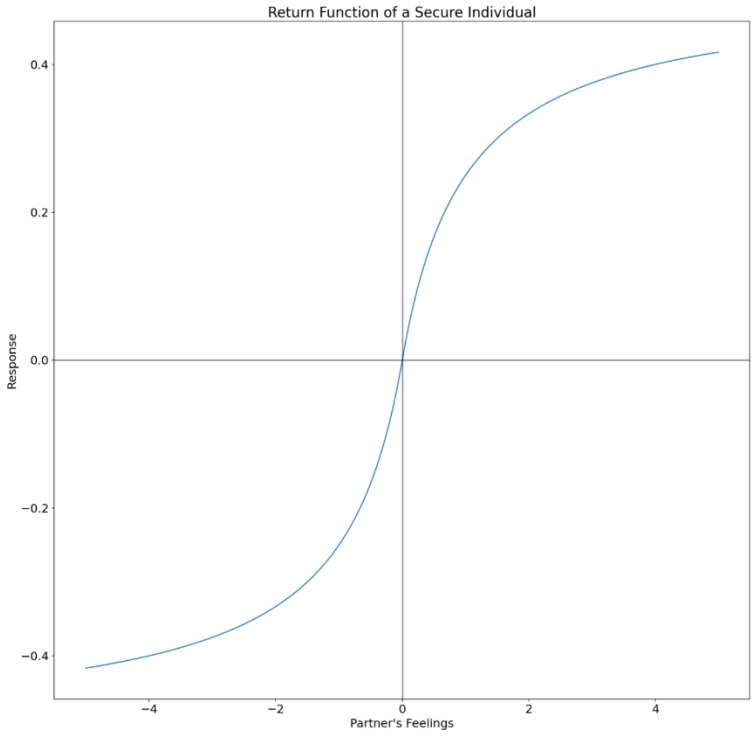

Figure 1

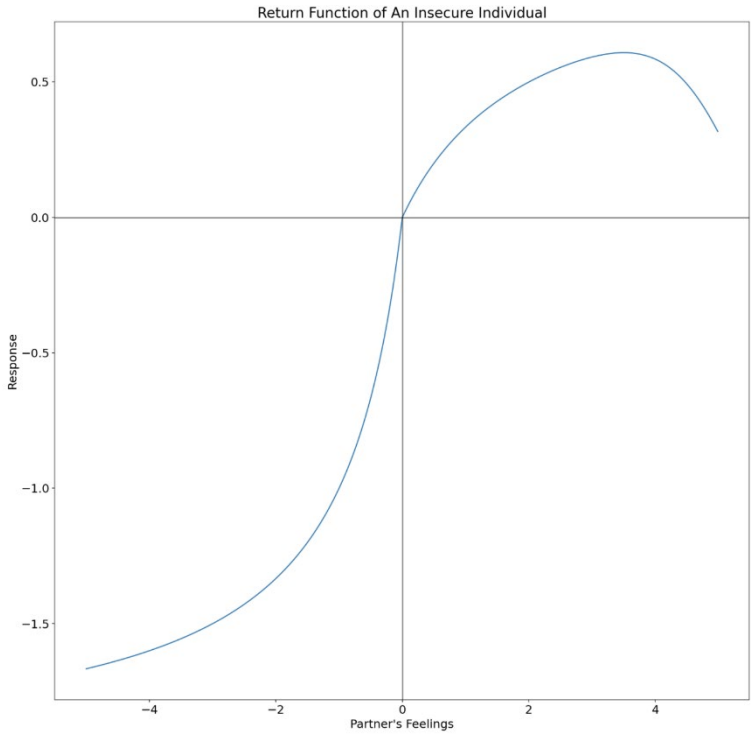

Figure 2

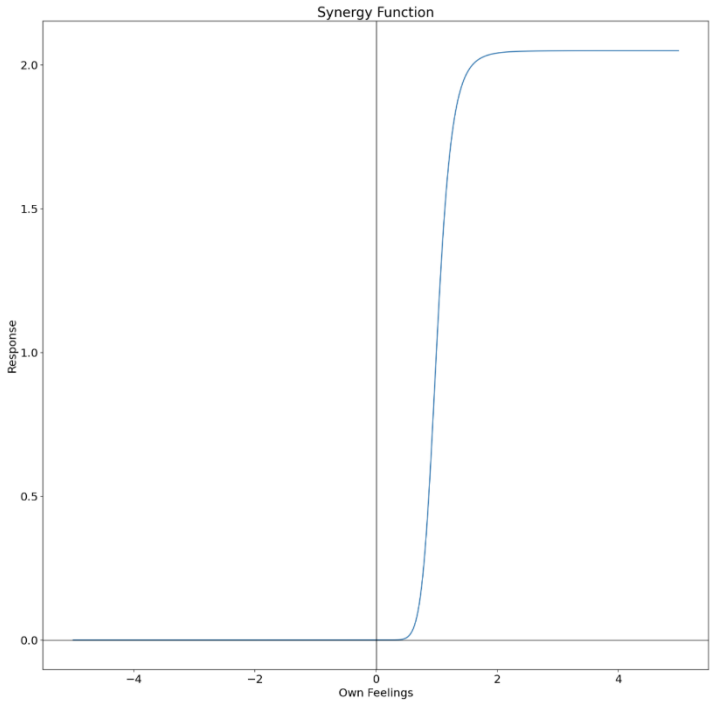

Figure 3

These are the equations corresponding to the functions used in our model:

$$R = f_1 = \begin{cases} \dfrac{2x}{(1-x)}, & x \leq 0 \\ \left(\dfrac{x}{2+x}\right)\left[\dfrac{(10^6 - x^8)}{(10^6 + x^8)}\right], & x > 0 \end{cases}$$

$$R = I = f_2 = \begin{cases} \dfrac{f_- x}{(x-1)}, & x \leq 0 \\ \dfrac{f_+ x}{(x+1)}, & x > 0 \end{cases}$$

$$S = \begin{cases} 0\,, & x \leq 0 \\ \dfrac{s\,x^8}{(x^8+1)}, & x > 0 \end{cases}$$

Where f_1 and f_2 are the response functions of insecure and non-secure individuals, respectively. Although we attempted to write our own functions to model love, the functions above – obtained and modified from Rinaldi's paper (except for f_1, which I deemed more appropriate than the equations proposed in literature) – work together in a satisfactory way [14]. Had we used other functions, different dynamics would have been observed. This suggests that this model might not be very reliable, since functions of love are obviously not set in stone. This model can still be used as a powerful tool for the observation of dynamic processes.

To fit a love triangle, we let:

- MB(t) = Mark's love / hate for Bridget at time t
- DB(t) = Daniel's love / hate for Bridget at time t
- BM(t) = Bridget's love / hate for Mark at time t
- BD(t) = Bridget's love / hate for Daniel at time t

The resulting equations measuring the rate of change of these feelings over time (~per day) are as follows:

$$\frac{dMB}{dt} = \big(1 + S(MB)\big) * f_2(BM) + \big(1 + S(MB)\big) * f_2(\alpha_{BM})$$

$$\frac{dDB}{dt} = f_1(BD) + f_2(\alpha_{BD})$$

$$\frac{dBM}{dt} = \big(1 + S(BM)\big) * f_1(MB) + \big(1 + S(MB)\big) * f_2(\alpha_M) - \beta BD$$

$$\frac{dBD}{dt} = \big(1 + S(BD)\big) * f_1(DB) + \big(1 + S(MB)\big) * f_2(\alpha_D) - \beta BM$$

Where *S* stands for the synergy function, the first $f_{1,2}$ is the return function R (f_1 for insecure and f_2 for secure individuals), and the second f_2 is the instinct function I. The parameter α_i is person i's appeal to their partner, and β is a coefficient that accounts for the impact of Bridget's feelings for one partner on her feelings for the other.

As stated before, the models found in the literature mostly examine the love dynamics between two cautious partners, and do not account for synergy. This idea was introduced once by Rinaldi in 1997 and we will be approaching our model similarly [14]. However, in Rinaldi's model synergy was only applied to the instinct function of response to appeal. Here, we also apply it to our return function to account for an individual's self-regard. Furthermore, previous models include a process called "oblivion," a forgetting coefficient that allows feelings to decay exponentially at the loss of a partner. However, we will not account for this in our model. It could be argued that feelings do not decrease once we lose a partner – they are simply compartmentalized and left untouched. They might turn into nostalgia or longing, but they are not assumed to be eradicated or even to lose intensity.

Summarizing the functions present in our model, we have:

- R_x – the return function corresponding to the reaction to partner's feelings. If they have an insecure attachment style, $R_x=f_1$. If the attachment style is secure, $R_x = f_2$
- I_x - a source of interest and describes the reaction of individual x to the partner's appeal "reflecting physical, financial, educational, intellectual, cultural properties".
- S_x – the synergy function enhances the reaction of synergic individuals, namely their return function Rx.

Let us set some initial conditions for the problem. First, we need to understand the scenario we are trying to model. Mark and Bridget are friends who have known each other for a few months. Daniel and Bridget have just met and exchanged numbers. Mark is smart, good-looking and from the same cultural background as Bridget, which appeals to her. They are both in college, whereas Daniel is older and has a job. He is also smart and good-looking, and his age and independence make him slightly more appealing to Bridget. Similarly, Bridget is also smart and good looking, but because she is still a college student she is not as appealing to Daniel as she is to Mark.

To account for Bridget and Mark's friendship and for the fact that Daniel and Bridget have recently met, we let BM(0) = 0.5, MB(0) = 0.5, DB(0) = 0, BD(0) = 0. Furthermore, taking into consideration what has been stated above, we assume that Bridget's appeal to Mark is equal to 1.2, Mark's appeal to Bridget is equal to 1, Daniel's appeal to Bridget is equal to 1.2, and Bridget's appeal to Daniel is equal to 1. We set $\beta = 0.2$.

These initial conditions give rise to the following observations, over the course of 12 months:

Figure 4

The first two graphs are pieces of the phase portraits representing Mark and Bridget's feelings, and Daniel and Bridget's feelings. Notice that as Mark's feelings grow, Bridget's do as well, until Mark's feelings start to overwhelm Bridget and her feelings decay. In Daniel and Bridget's graph as Daniel's feelings grow (y axis), Bridget's feelings also grow, until his feelings are too intense, and her feelings start to decay.

We have built a widget using python that allows us to test different values for the characters' initial feelings towards each other, as well as their appeal and the coefficient β. This allows us to change the initial conditions, and if, for example, we increase Bridget's initial feelings, we observe a change of behavior in the graph. In this scenario Bridget and Mark end up both loving each other, whereas Bridget and Daniel seem to hate each other, as shown in Figure 5.

Figure 5

We have also produced phase portraits that give us a clearer idea of the different possible outcomes, with $\beta = 0.5$ for a clearer visualization of its impact (although our convention says

that $\beta = 0.2$). The results can be found in Figure 6 and Figure 7. We are obviously not able to produce a flow field that accounts for the dynamic interaction with the third character of the triangle, so we have kept this value fixed and plotted it twice, with $DB_1 = 0$, $DB_2 = 4$. The same procedure was repeated for the second couple (Figures 8, 9).

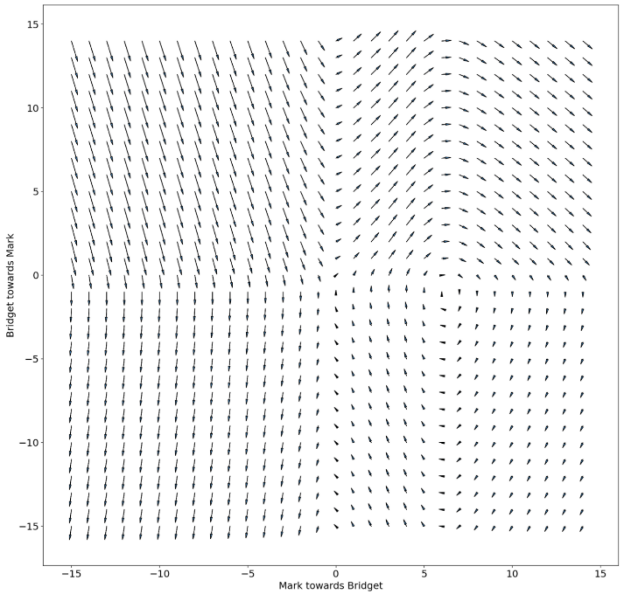

Figure 6

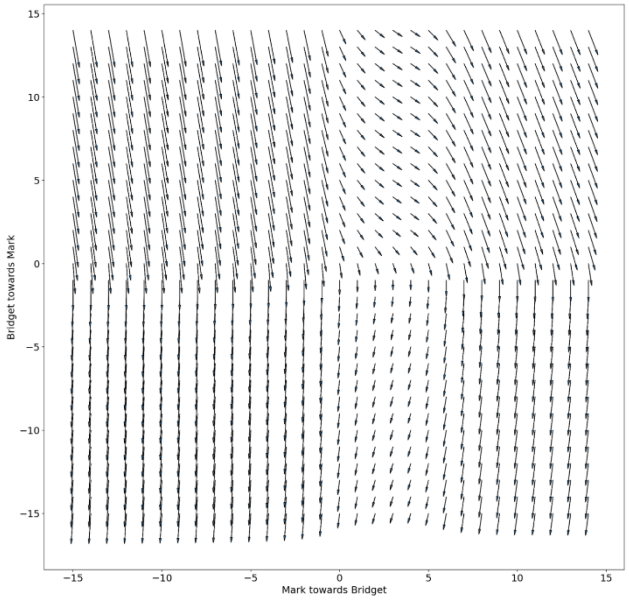

Figure 7

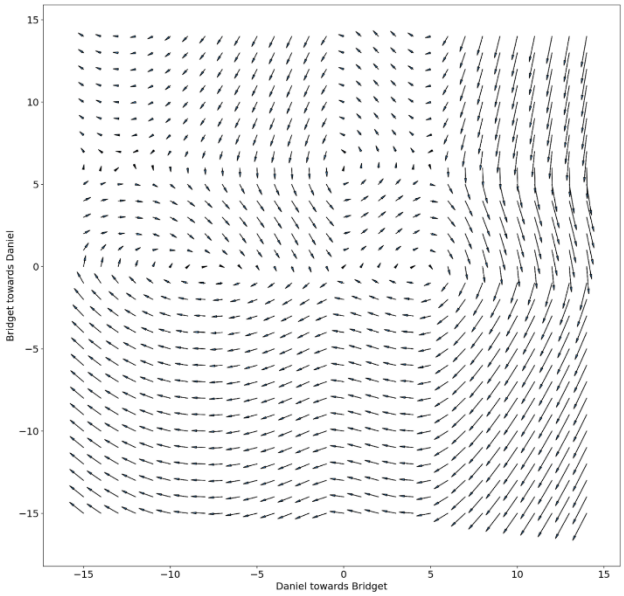

Figure 8

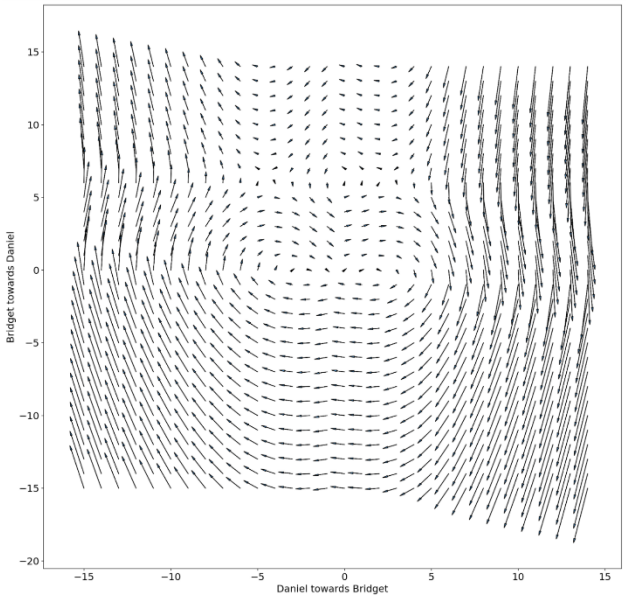

Figure 9

Both flow fields show cyclic dynamics in the interactions between lovers. More specifically, our couples seemed to easily get stuck into love and hate cycles. The magnitude of the vectors in both flow fields increases as we introduce Beta, indicating that changes in the feelings occur faster. In Mark and Bridget's flow field, higher feelings for another partner seem to doom their relationship, and we do not observe the cyclic dynamics from when there was no other partner involved. In Daniel and Bridget's flow field the dynamics seem to only have been enhanced, but not altered. The impact of the third person's feelings will not be as great as observed in the figures above since we let $\beta = 0.2$.

To visualize the impact of the feelings for Daniel on Bridget's feelings for Mark and on the feelings for Mark on Bridget's feelings for Daniel, we plot a phase portrait with different conditions, yielding 8 different trials. The results are as follows:

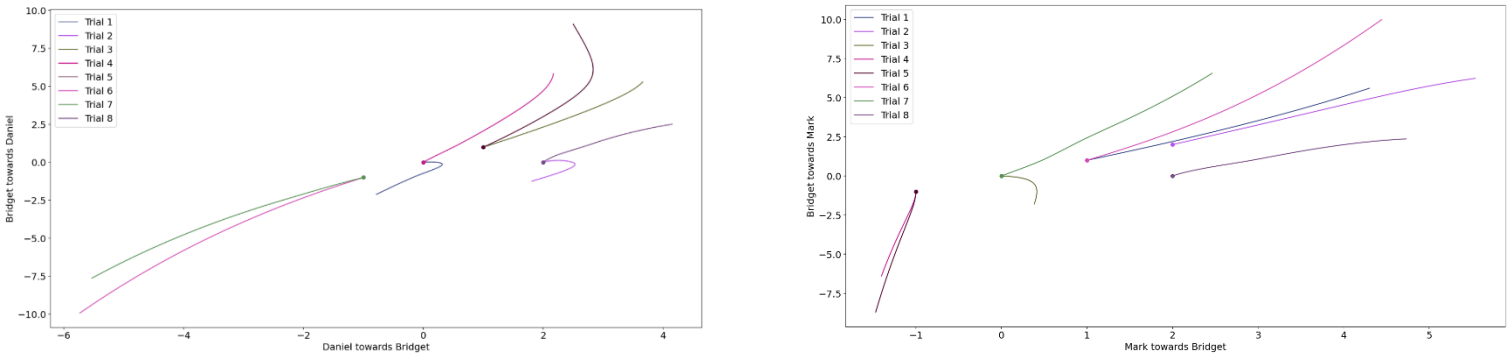

Figure 10

The dips and curves observed in the phase portrait above are not observed when $\beta = 0$. This confirms the behavior of β shown in the flow fields. Positive feelings increase at a slower pace and negative feelings decrease faster. The behavior of change is also affected, as ever positive increasing feelings became negative with the introduction of β. It is possible to check the python code and notebook present in this paper for a better notion of what the values used in the phase portraits were.

Conclusion

This model presents a lot of qualities, but also a lot of weaknesses. It was a challenging assignment that culminated in the production of a code that could be extended to other works. We were able to produce a graph of feelings over time, phase portraits with several different initial conditions, and flow charts. The main issue with this model is the attempt of writing functions that model a human being's response to its surroundings. It is a brute simplification of real life, and may scholars used equations that would be favorable to the derivation of interesting and peculiar conclusions, such as cyclic dynamics. A change in the equations for the return functions from the ones found in literature produced results extremely different. Furthermore, small alterations in the values passed into the functions also resulted in different outcomes than expected, which may not reflect reality. Finally, it can be tricky to quantify love and to find an appropriate range for feelings, which leads authors once again to choose values that would work to their favor.

Nevertheless, the answer to our initial question has been found: it might not be so appropriate to denominate a love triangle as "eternal." Independently from initial conditions, one result was consistent in every trial: at least one of the characters developed a decay of feelings for another character over a period of 12 months. From every plot made with the widget the results are clear: either no one will end up with their feelings reciprocated or one couple will, but all three characters do not have increasing positive feelings at the same time. Eternal Triangles do not seem to last.

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