FDS

TV

1) SVD. $\boxed{A = \text{Original matrix}}$

Step1:- A^TA [Either square or rectangle matrix]

Step 2:- Find out eigen values.

$$|A^TA - \lambda I| = 0.$$

$\boxed{U - \text{Use original matrix}}$

$\boxed{\text{Eigen vectors in descending order.}}$

Step 3:-

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}_{3\times3}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}_{2\times2}$$ For 2x2 matrix

$$\sigma_i = \sqrt{\lambda_i}$$

$\boxed{\text{Arrange } \lambda \text{ in descending order.}}$

$\sigma_1 = \sqrt{\lambda_1},$

Step 4:- Eigen vector calculation.

$$A x = \lambda x \Rightarrow [\underbrace{A}_{(A^TA)} - \lambda I] x = 0.$$

Step 5:-

Normalised Vector

$$v_1^T = [1 \; 2 \; 1] \Rightarrow \frac{1}{\sqrt{1^2+2^2+1^2}}, \frac{2}{\sqrt{1^2+2^2+1^2}}$$

$$v_2^T = [\quad]$$

$$v_3^T = [\quad]$$

Normalised vectors.

$$V = [v_1 \; v_2 \; v_3]$$

Step 6:- $\boxed{U_i = \frac{AV_i}{\sigma_i}}$ $U_1 = \frac{AV_1}{\sigma_1}$, $U_2 = \frac{AV_2}{\sigma_2}$, $U_3 = \frac{AV_3}{\sigma_3}$.

Step 7:- ε , Σ , σ

$$\boxed{A = U \Sigma V^T}$$

2) LDA. $x_1 = (1,2), (3,4), (5,6) \longrightarrow S_1$

$x_2 = (2,1), (3,2) (4,5)$. $\longrightarrow S_2$

Step 1:- μ_1 , μ_2

$$\mu_1 = \frac{1+3+5}{3}, \quad \mu_2 = \frac{2+\cancel{4}+6}{3}$$

μ_1 , μ_2

Step 2: Element-wise subtraction.

$$x_1 - \mu_1$$

Step 3: Covariance matrix calculation. $[-2 \cdot 3$

$$[x_1][x_1]^T / \text{No. of elements.}$$

Step 4: $S_W \Rightarrow S_1 + S_2$

Step 5: $S_W^{-1} \Rightarrow \frac{1}{ab-cd} (\operatorname{adj} A)$

$$\Rightarrow \frac{1}{|A|} (\operatorname{adj} A)$$

Step 6:

$$S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

Step 7:

$$v = S_W^{-1} (\mu_1 - \mu_2) \rightarrow (\mu_{y_1} + \mu_{y_2})$$

$$\hookrightarrow (\mu_{x_1} + \mu_{x_2})$$

3) PCA – Only 1 u_1, u_2

$x_T = (1,2), (2,4) (3,2$

Step 1, Step 2, Step 3 same for PCA.

Step 4: Eigen values

$$|S - \lambda I| = 0.$$

Step 5: Eigen Vectors

$$[A - \lambda I] X = 0.$$

$X \neq x_1, x_2, x_3$ (3×3) $\quad X = x_1, x_2$ (2×2)

Example $x_1 = x_2 = k$

$x_1 = 2x_2.$

Step 6:- Eigen Vectors are called principle components of A.

Shortcuts

#1) Adjoint A 2×2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A_{3\times3} \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 \\ -1 & 2 & 1 & -1 & 2 \\ 1 & 2 & -1 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 \end{bmatrix}$$

→ first two numbers

first two rows

Moving row wise we should write in column wise.

Skipping 1st row & column

$$\begin{bmatrix} (1-4) & (2-2) & \\ (-2-2) & (1-1) & \\ (4+1) & (-2-2) & \end{bmatrix}$$

$\boxed{\text{Instead of calculating co-factor matrix}}$

5) Characteristic Equation

$$\lambda^3 - [\text{trace}(A)]\lambda^2 + \frac{1}{2}\left[[tr(A)]^2 - (tr(A^2))\right]\lambda - \det A.$$

tr(A) - Sum of elements in diagonal of a matrix.

6) Rank of matrix.

det A $\neq 0$ $\boxed{\text{Rank}=3}$ [for 3x3 matrix]

det A $= 0$, Take minor

det $A_{2\times2} \neq 0$ $\boxed{\text{Rank}=2}$

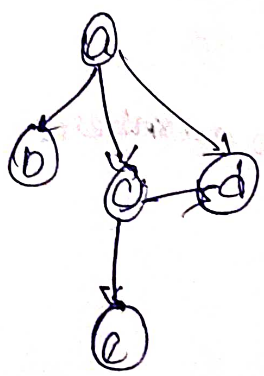
det $A_{2\times2} = 0$ $\boxed{\text{Rank}=1}$

Mod2	Mod3
SVD	Eigen Val
LDA	Eigen Vectors
PCA	Rank.
	Characteristic Equation

Mod3 (Few more topics)

7) Adjacency matrix

Based on dimension.



$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & & & & \\ b & & 0 & & & \\ c & & & 0 & & \\ d & & & & 0 & \\ e & & & & & 0 \end{array}$$

Transition matrix - No. of out degree from each node.

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ b & & & & & \\ c & & & & & \\ d & & & & & \end{array} \Rightarrow \frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

3 → Total no. of out degrees from node A.

If data is given.

push-ups pull-ups

Run Rowing

Summation of probability = 0 or 1

8) Markov chain problem - probability.
(Transition matrix, Transition Problem)

9) Inner Products

10) Polygons.

11) BFS

12) DFS

13) Metropolis algo - Theory.

14) Decision Tree.

Mod 4

15) Newton Problem.

$$F_0 \Rightarrow P = \frac{F'(x)}{F''(x)}$$

$$x_{k+1} \Rightarrow x_k - \frac{F'(x)}{F''(x)}$$

$$\boxed{x_1 = x_0 - \frac{F'(x_0)}{F''(x_0)}}$$

$$\boxed{x_2 = x_1 - \frac{F'(x_1)}{F''(x_1)}}$$

For 10 marks upto 3 iterations

For 6 to 7 marks upto 2 iterations

16) Analytical Method.

$$D = F_{xx}(a,b)\, F_{yy}(a,b) - [F_{x,y}(a,b)]^2$$

$$\frac{\partial^2 f}{\partial x^2}(a,b)\, \frac{\partial^2 f}{\partial y^2}(a,b) - \left[\frac{\partial^2 f}{\partial x \partial y}(a,b)\right]^2$$

$D > 0$, $F_{xx}(a,b) > 0$ — Local minima.

a,b

$D > 0$ $F_{xx}(a,b) < 0$ — Local maxima.

$D < 0$ (a,b) — Saddle points.

17) Steepest Ascent Method — Directional

18) $$\nabla f(x) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

All eigen +ve — +ve definite

All eigen -ve — -ve definite

One +ve, One -ve — Indefinite.

Module 5

Programming Question:

Mod 6

Exploratory data analytics - Theory

Module 6

Data Phase I - Theory.

Visualization of various charts:-

Inputs given, Need to write data representation.

Node - link diagram
Heat Map.
} Types of qns.

Module 7

K-Means clustering - 1 qn. -*- R Code & Numerical.

based on euclidean distance (Either own data or based on qn)

[Explain detailed steps] - in answer.

ML based techniques - R Codes important.

KNN NaiveBayes

Plots - R Codes.

- Bar
- Scatter
- Line
- Box etc.

* Help command

Mean & Weight

If ranges are given, taken an assumed mean and calculate the values.

~~Banker~~ Inverse of matrix – $A^{-1} = \frac{adj A}{adj}$

Theory

Mod 1

Mod 2 →

Mod 6 – First half.

Mod 5 – Second half

Gradient descent

4–5 pages – theory qn.

Theory qp pattern

$10 \times 10 = 100$.