Seasonal adjustment of economic data
The X-11 core of X-13

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Seasonal adjustment, 30 January 2023

Unobserved components model

Non-technical set up (I/II)

Data : Trend-cycle Seasonal Calendar Irregular

Unobserved components model

Non-technical set up (I/II)

Data : Trend-cycle Seasonal ~~Calendar~~ Irregular

Unobserved components model
Non-technical set up (I/II)

Data : Trend-cycle Seasonal Irregular

Assumption $\rightsquigarrow$ Linearised data

- No calender
- No outlier
- Data transformed

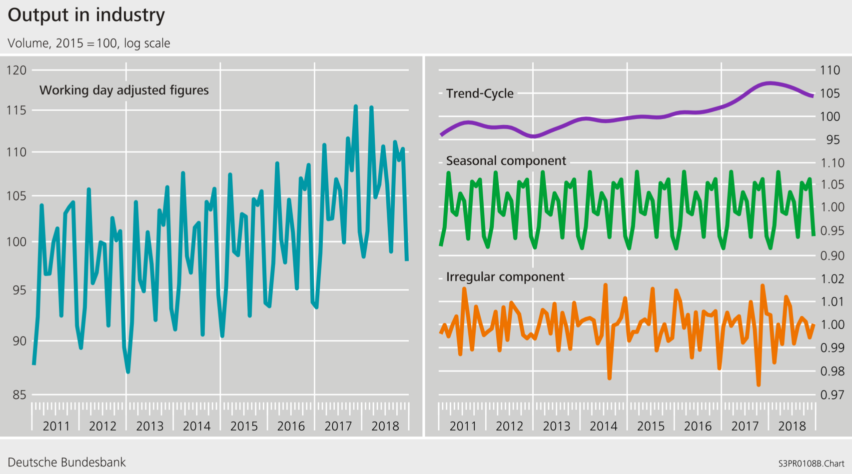
Unobserved components model
Non-technical set up (I/II)

Data	:	Trend-cycle	Seasonal	Irregular
		↓	↓	↓
		long-term growth path	recurring movements	remaining movements

Assumption ⇝ Linearised data

- No calender
- No outlier
- Data transformed

Unobserved components model
Non-technical set up (II/II)



Unobserved components model

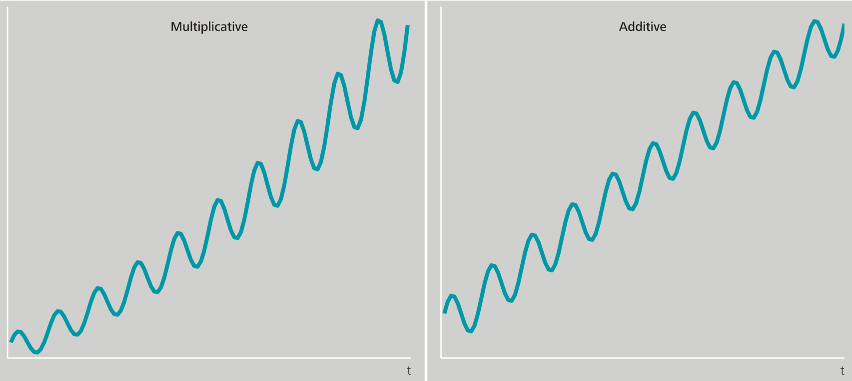
Technical set up (I/IV)

$$
\begin{array}{ccccccc}
\text{Data} & : & \text{Trend-cycle} & & \text{Seasonal} & & \text{Irregular} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
y_t & = & t_t & & s_t & & i_t
\end{array}
$$

Unobserved components model
Technical set up (II/IV)

Decomposition of observed time series into unobservable components

Trend-cyclical and seasonal component



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Unobserved components model
Technical set up (III/IV)

Data	:	Trend-cycle		Seasonal		Irregular
$\downarrow$		$\downarrow$		$\downarrow$		$\downarrow$
y_t	$=$	t_t	$*$	s_t	$*$	i_t

Seasonal effect...

- Proportional to trend-cycle $\rightsquigarrow$ multiplicative decomposition
- Seasonally adjusted series:

$$\frac{y_t}{s_t} = t_t * i_t$$

Unobserved components model
Technical set up (IV/IV)

Data	:	Trend-cycle		Seasonal		Irregular
$\downarrow$		$\downarrow$		$\downarrow$		$\downarrow$
y_t	$=$	t_t	$+$	s_t	$+$	i_t

Seasonal effect...

- Independent of level of trend-cycle $\rightsquigarrow$ additive decomposition
- Seasonally adjusted series :

$$y_t - s_t = t_t + i_t$$

X-11 family
X-11 and major innovations

X-11: Seasonal adjustment based on linear filters

- Simple moving averages
- Henderson filter
- Periodic specific moving averages

X-11-ARIMA: Series extension via ARIMA forecast

X-12-ARIMA: RegARIMA for outliers and calendar

X-13-ARIMA-SEATS: Adaption of TRAMO and inclusion of SEATS

Estimation strategy I/II

$$\begin{array}{ccccccc} \text{Data} & : & \text{Trend-cycle} & & \text{Seasonal} & & \text{Irregular} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ y_t & = & t_t & * & s_t & * & i_t \end{array}$$

1. Estimate trend $\rightsquigarrow \widehat{t_t}$

Estimation strategy I/II

$$\begin{array}{ccccccc} \text{Data} & : & \text{Trend-cycle} & & \text{Seasonal} & & \text{Irregular} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ y_t & = & t_t & * & s_t & * & i_t \end{array}$$

1. Estimate trend $\rightsquigarrow \widehat{t_t}$
2. Remove trend from series $\rightsquigarrow \frac{y_t}{\widehat{t_t}} = \widehat{s_t * i_t}$

Estimation strategy I/II

$$\begin{array}{ccccccc} \text{Data} & : & \text{Trend-cycle} & & \text{Seasonal} & & \text{Irregular} \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ y_t & = & t_t & * & s_t & * & i_t \end{array}$$

1. Estimate trend $\rightsquigarrow \widehat{t_t}$
2. Remove trend from series $\rightsquigarrow \frac{y_t}{\widehat{t_t}} = \widehat{s_t * i_t}$
3. Estimate season $\rightsquigarrow \widehat{s_t}$

Estimation strategy I/II

$$
\begin{array}{ccccccc}
\text{Data} & : & \text{Trend-cycle} & & \text{Seasonal} & & \text{Irregular} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
y_t & = & t_t & * & s_t & * & i_t
\end{array}
$$

1. Estimate trend $\rightsquigarrow \widehat{t_t}$
2. Remove trend from series $\rightsquigarrow \frac{y_t}{\widehat{t_t}} = \widehat{s_t * i_t}$
3. Estimate season $\rightsquigarrow \widehat{s_t}$
4. Remove season from series $\rightsquigarrow \frac{y_t}{\widehat{s_t}} = \widehat{t_t * i_t}$

Estimation strategy (II/II)

Basic principle of the X-11 seasonal adjustment algorithm in JDemetra+

Workflow diagram

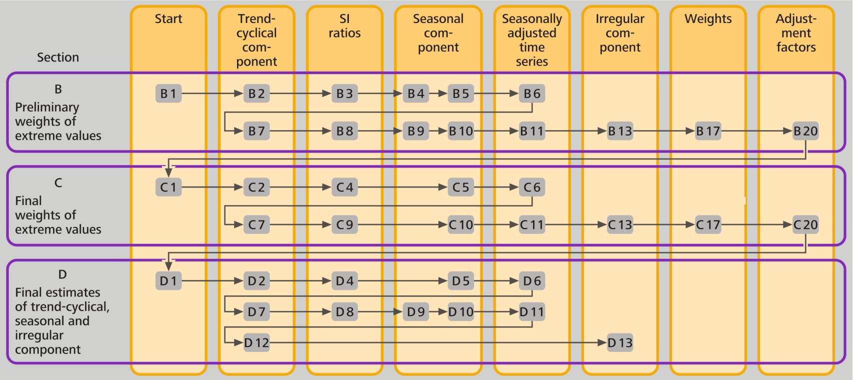

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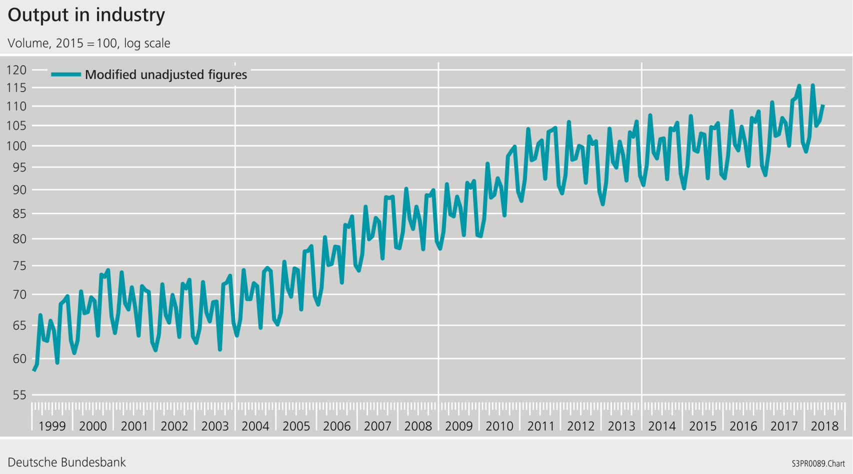
Table D 1
Input series



Digression I
Trend filter calculation

Weighted moving average $\rightsquigarrow$ MA$_{2x12}$ filter

	$t-6$	$t-5$	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$
weight of MA$_{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	
		$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
weight of MA$_{2x12}$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{24}$

$$\boxed{\hat{t}_t = \frac{1}{24}\,y_{t-6} + \frac{1}{12}\,y_{t-5} + \cdots + \frac{1}{12}\,y_t + \cdots + \frac{1}{12}\,y_{t+5} + \frac{1}{24}\,y_{t+6}}$$

Table D 2 (I/II)

Step 1: estimation of provisional trend-cyclical component

Preliminary trend-cycle

$$\hat{t}_t = \frac{1}{24}\,y_{t-6} + \frac{1}{12}\,y_{t-5} + \cdots + \frac{1}{12}\,y_t + \cdots + \frac{1}{12}\,y_{t+5} + \frac{1}{24}\,y_{t+6}$$

Interpretation

- Modified unadjusted figures $\rightsquigarrow$ Smoothed by centred moving average over 13 months

Table D 2 (II/II)
Step 1: estimation of provisional trend-cyclical component

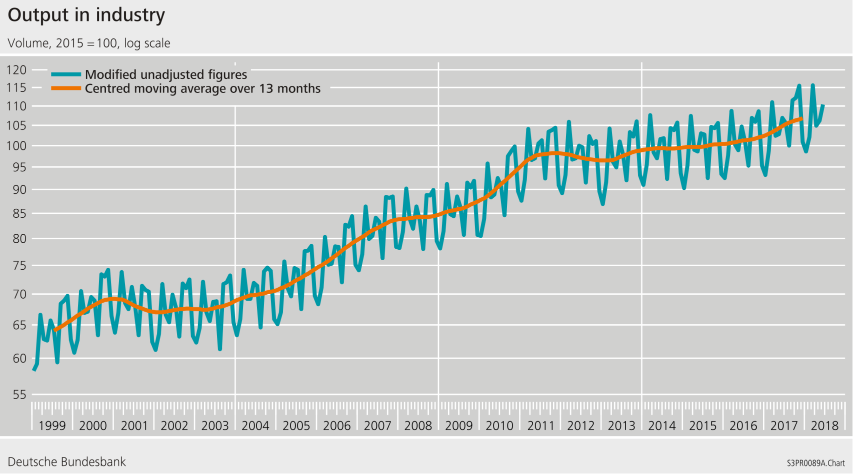


Table D 4 (I/II)
Step 2: determination of seasonal-irregular component

Preliminary seasonal-irregular

$$\boxed{(\widehat{si})_t = \frac{y_t}{\hat{t}_t}}$$

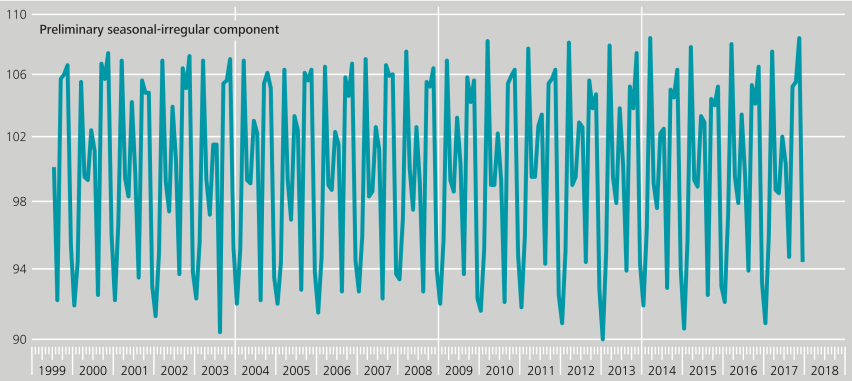
Interpretation

- Modified unadjusted figures $\rightsquigarrow$ Removal of preliminary trend-cycle

Table D 4 (II/II)
Step 2: determination of seasonal-irregular component

Output in industry

Volume, 2015 = 100, log scale



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Digression II
Seasonal filter calculation

Weighted moving average $\rightsquigarrow$ MA$_{3x3}$ filter

	$t-24$	$t-12$	t	$t+12$	$t+24$
weight of MA$_3$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		
		$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
			$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
weight of MA$_{3x3}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

$$\boxed{\hat{s}_t = \frac{1}{9}\,(\widehat{si})_{t-24} + \frac{2}{9}\,(\widehat{si})_{t-12} + \frac{3}{9}\,(\widehat{si})_t + \frac{2}{9}\,(\widehat{si})_{t+12} + \frac{1}{9}\,(\widehat{si})_{t+24}}$$

Table D 5 (I/III)
Step 3: estimation of seasonal component

Preliminary seasonal

$$\hat{s}_t = \frac{1}{9}\,(\widehat{si})_{t-24} + \frac{2}{9}\,(\widehat{si})_{t-12} + \frac{3}{9}\,(\widehat{si})_{t} + \frac{2}{9}\,(\widehat{si})_{t+12} + \frac{1}{9}\,(\widehat{si})_{t+24}$$

Interpretation

- Preliminary seasonal-irregular $\rightsquigarrow$ Smoothed by 3×3 seasonal filter within each month

Missing values

- Imputation $\rightsquigarrow$ "Copy-paste"

Table D 5 (II/III)
Step 3: estimation of seasonal component

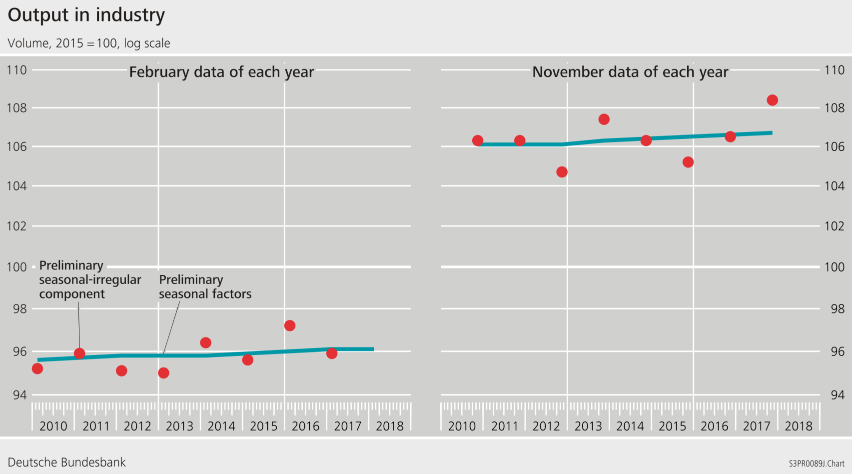


Table D 5 (III/III)
Step 3: estimation of seasonal component

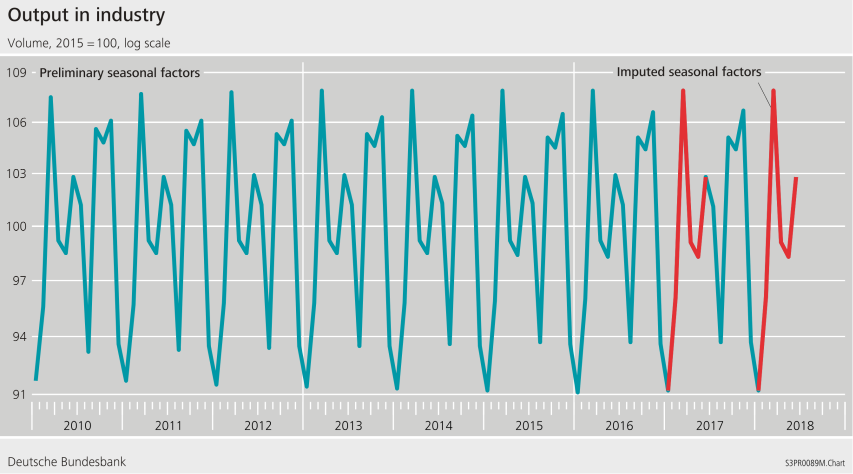


Table D 6 (I/II)

Step 4: determination of seasonally adjusted series

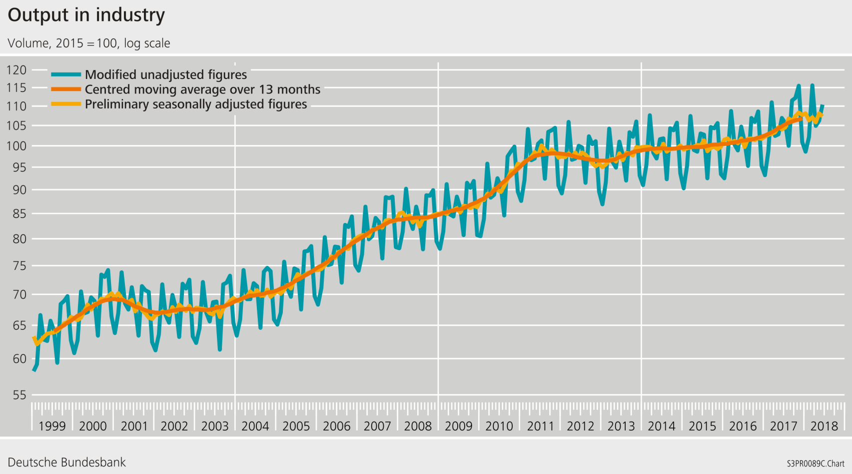
Preliminary seasonally adjusted figures

$$\boxed{\hat{y}_t^{(sa)} = \frac{y_t}{\hat{s}_t}}$$

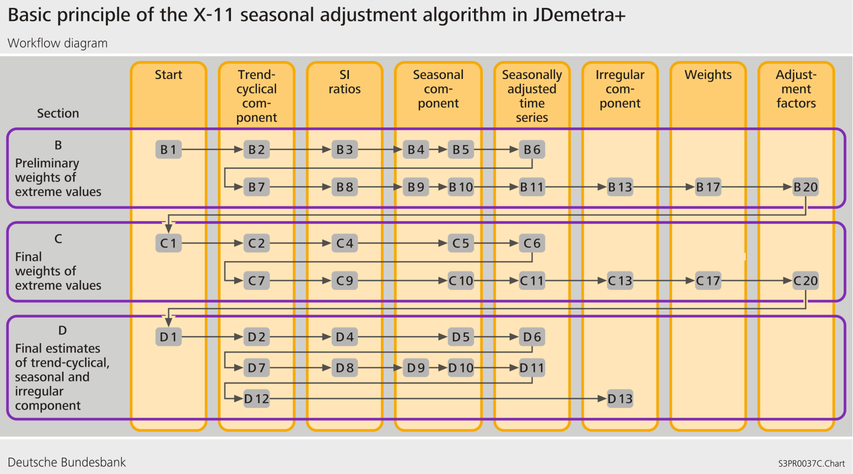
Interpretation

– Modified unadjusted figures $\rightsquigarrow$ Removal of preliminary seasonal

Table D 6 (II/II)
Step 4: determination of seasonally adjusted series



Recap: Estimation strategy



Next iteration

Final estimates

- Preliminary estimates $\rightsquigarrow$ Refinement

Linear filters

- Extended options $\rightsquigarrow$ Henderson & seasonal filters
- User customisation $\rightsquigarrow$ Month-specific seasonal filters, down-weighing of extreme SI ratios

Table D 7 (I/III)

Another preliminary trend-cycle

$$\hat{t}_t = \sum_k \text{weight}_k^{(m)} \cdot \hat{y}_{t+k}^{(sa)}$$

Interpretation

- Preliminary seasonally adjusted figures $\rightsquigarrow$ Smoothed by Henderson filter over m months

Preservation

- Polynomial trend $\rightsquigarrow$ Order 3

Table D 7 (II/III)

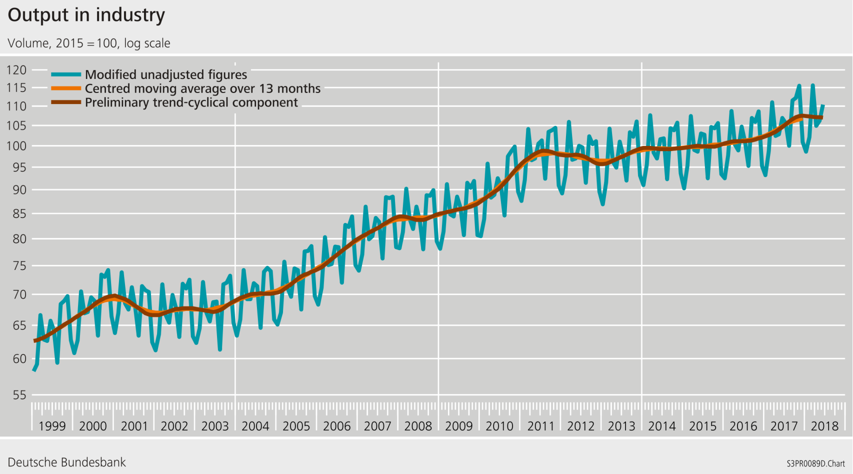
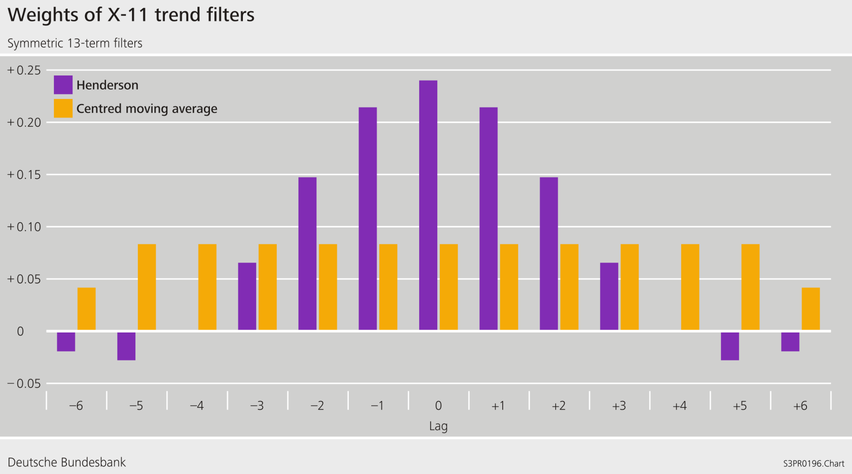


Table D 7 (III/III)



Henderson filters (I/II)

Automatic choice: I/C ratio

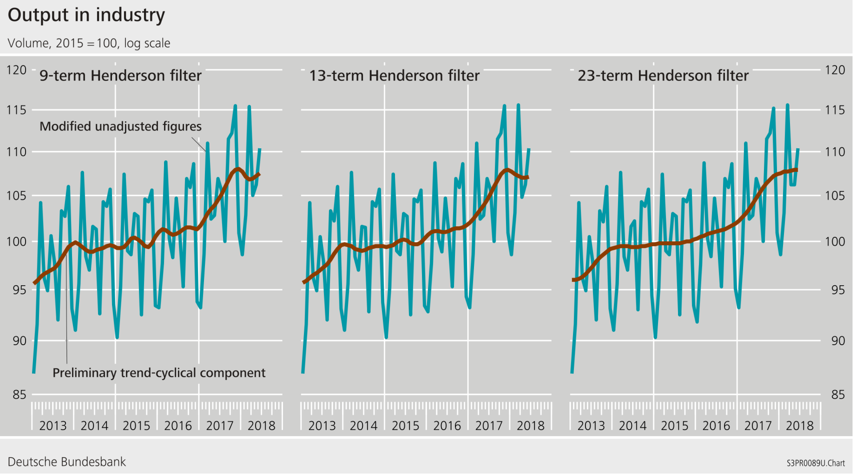
$$\text{I/C} = \frac{\sum_t \left|\tilde{i}_t/\tilde{i}_{t-1} - 1\right|}{\sum_t \left|\tilde{t}_t/\tilde{t}_{t-1} - 1\right|}, \quad \text{with} \quad \begin{array}{lcl} \tilde{i}_t & = & \text{temporary irregular} \\ \tilde{t}_t & = & \text{temporary trend-cycle} \end{array}$$

	Decision rule		
I/C	$[0, 1)$	$[1, 3.5)$	$[3.5, \infty)$
Henderson filter (m)	9-term	13-term	23-term

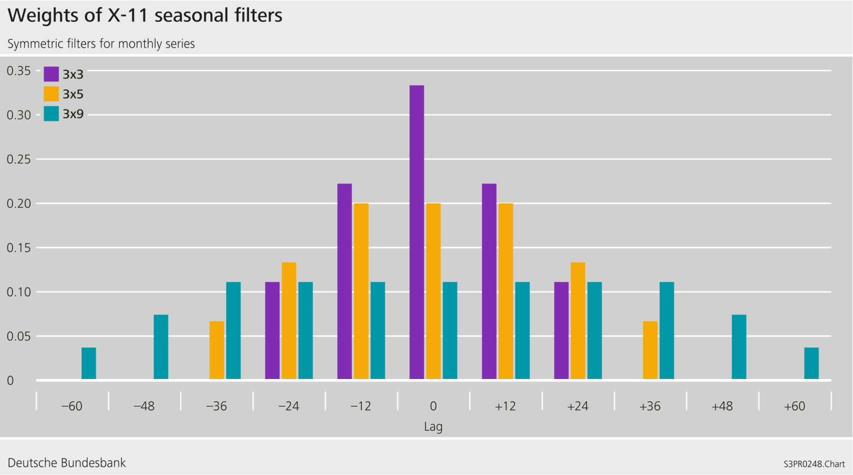
Interpretation

- I/C large $\rightsquigarrow$ Dominance of irregular (i.e. long filter)
- I/C small $\rightsquigarrow$ Dominance of trend-cycle (i.e. short filter)

Henderson filters (II/II)
User choice



Tables D 8 & D 10
Seasonal filters: common options



Seasonal filters (I/II)
Automatic choice: I/S ratio

$$\text{I/S} = \frac{\sum_t \left|\tilde{i}_t/\tilde{i}_{t-12} - 1\right|}{\sum_t \left|\tilde{s}_t/\tilde{s}_{t-12} - 1\right|}, \quad \text{with} \quad \begin{array}{lcl} \tilde{i}_t & = & \text{temporary irregular} \\ \tilde{s}_t & = & \text{temporary seasonal} \end{array}$$

	Decision rule				
I/S	$[0, 2.5)$	$[2.5, 3.5]$	$(3.5, 5.5)$	$[5.5, 6.5]$	$(6.5, \infty)$
Seasonal filter	3×3	*	3×5	*	3×9

* $\rightsquigarrow$ Maximum of five I/S recalculations under omission of the respective last year, application of the 3×5 seasonal filter if still no decision is found.

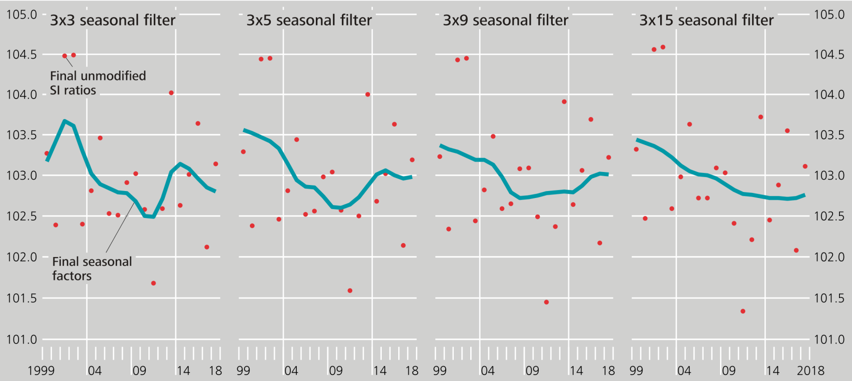
Interpretation

- I/S large $\rightsquigarrow$ Dominance of irregular (i.e. long filter)
- I/S small $\rightsquigarrow$ Dominance of seasonal (i.e. short filter)

Seasonal filters (II/II)
User choice

Output in industry

Volume, 2015 = 100, log scale, June data of each year



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General problems

Symmetric filters

- Finite data $\rightsquigarrow$ Endpoint problematic
- Remedy
  - Asymmetric non-centred variants (Henderson & seasonal filters)
  - ARIMA forecast

Linear filters

- Extreme data $\rightsquigarrow$ High sensitivity
- Remedy $\rightsquigarrow$ Down-weighting, replacement (SI ratios)

Extreme values
Identification

Extreme SI ratio

– Irregular $\rightsquigarrow$ Large deviation from expectation

Benchmark $\rightsquigarrow$ Moving standard deviation (σ_t)

– Homoskedastic $\rightsquigarrow$ Calculation for all periods together

– Heteroskedastic $\rightsquigarrow$ Division of periods into different groups or periodic specific calculation

$$\Downarrow$$

Cochran test

Final weights for irregular

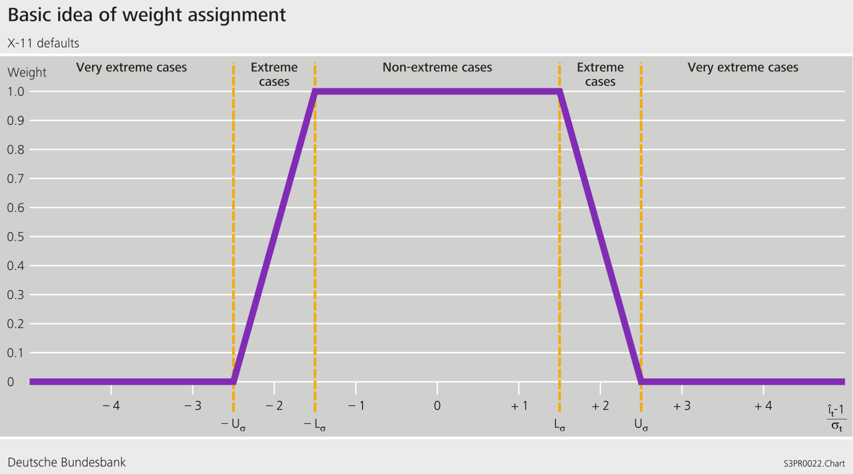
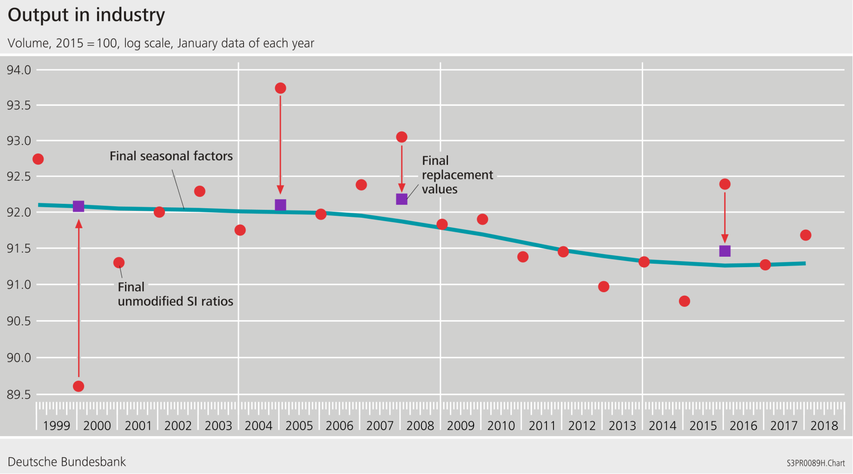
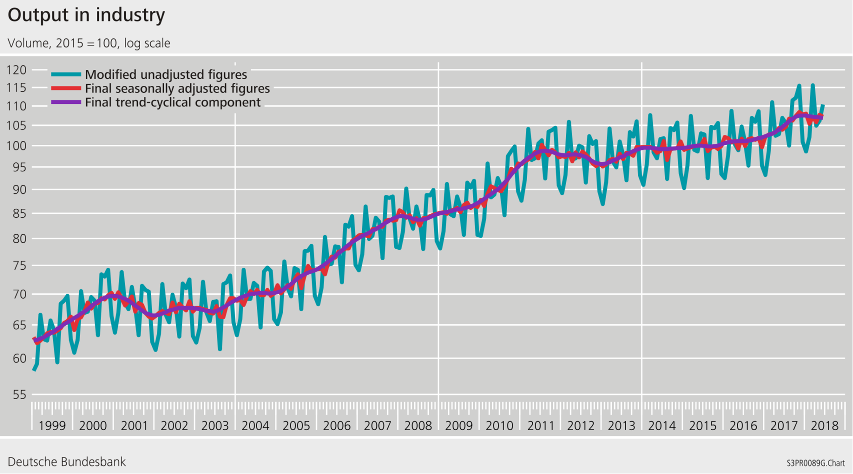


Table D 9 & D 10
Final replacement values



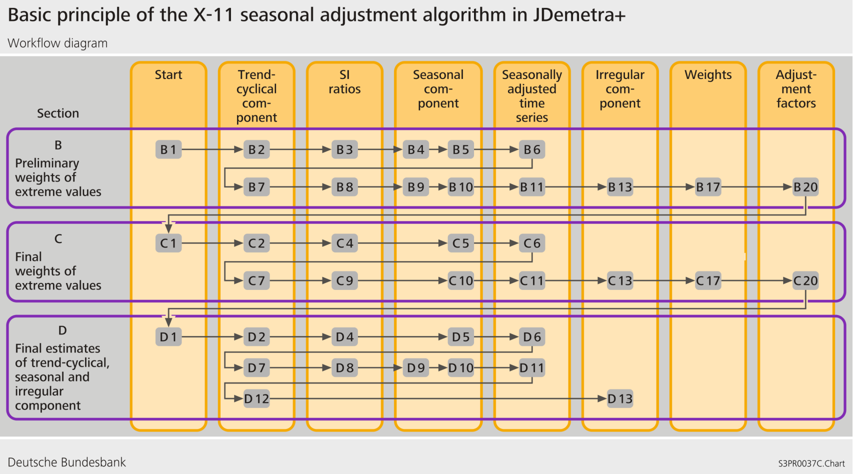
Tables D 11 & D 12
Final results



Automatic Choices
Depoutot & Planas (1998)

Θ_1	Θ_{12} 0.95	0.8	0.7	0.6	0.5	0.4	0.2	0.0
0.95	S3x15 H(23)	S3x15 H(23)	S3x9 H(23)	S3x5 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)
0.8	S3x15 H(23)	S3x15 H(23)	S3x9 H(23)	S3x5 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)
0.7	S3x15 H(23)	S3x15 H(23)	S3x9 H(23)	S3x5 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)
0.6	S3x15 H(17)	S3x15 H(17)	S3x9 H(17)	S3x5 H(17)	S3x5 H(23)	S3x3 H(23)	S3x3 H(23)	S3x3 H(23)
0.5	S3x15 H(13)	S3x15 H(13)	S3x9 H(13)	S3x5 H(13)	S3x5 H(13)	S3x3 H(17)	S3x3 H(23)	S3x3 H(23)
0.4	S3x15 H(13)	S3x15 H(13)	S3x9 H(13)	S3x5 H(13)	S3x5 H(13)	S3x3 H(13)	S3x3 H(17)	S3x3 H(23)
0.2	S3x15 H(9)	S3x15 H(9)	S3x9 H(9)	S3x5 H(9)	S3x3 H(9)	S3x3 H(9)	S3x3 H(13)	S3x3 H(23)
0.0	S3x15 H(9)	S3x15 H(9)	S3x9 H(9)	S3x5 H(9)	S3x3 H(9)	S3x3 H(9)	S3x3 H(9)	S3x3 H(23)

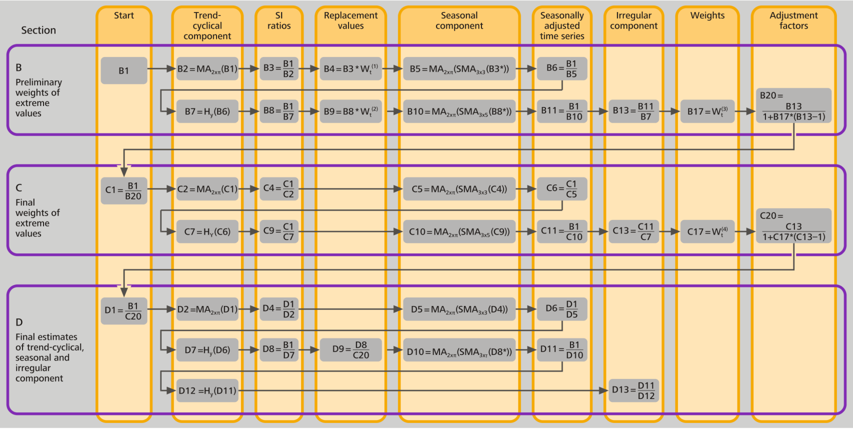
Recap: Estimation strategy



Recap: Estimation strategy

The automatic X-11 routine in JDemetra+ for a multiplicative time series model

Workflow diagram



$H_y(.)$: Henderson moving average of length y, with y determined based on I/C ratio or specified by user. $MA_{2x\pi}$: Moving average of length $2 \times \pi$, with π = frequency of the series. $SMA_{3x\tau}$: Moving average, applied on a period-by-period basis, of length $3\,x\,\tau$ with τ set determined based on I/S ratio or spezified by user. In the latter case the chosen SMA is *always used*. $W_t^{(i)}$: Weights obtained based on moving standard deviation of irregular component.

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S3IN0530.Chart

Summary

- X-11 is the workhorse for estimating the seasonal effects
- X-11 uses moving averages

References

- Depoutot, R. und Planas, C. (1998) Comparing seasonal adjustment and trend extraction filters with application to a model-based selection of X11 linear filters. Eurostat Working Papers.
- Eurostat (2015), ESS Guidelines on Seasonal Adjustment, ISSN 2315-0815.