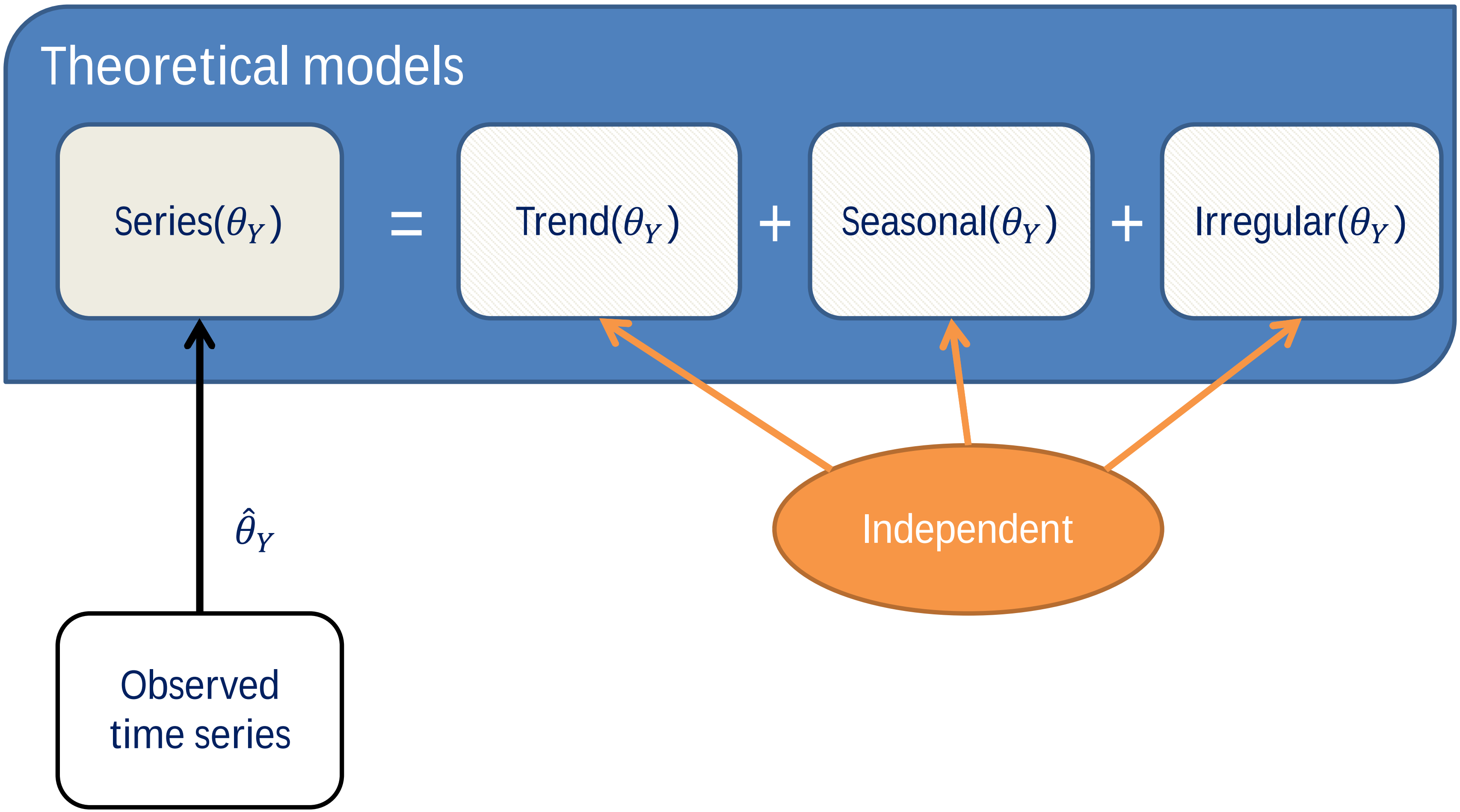
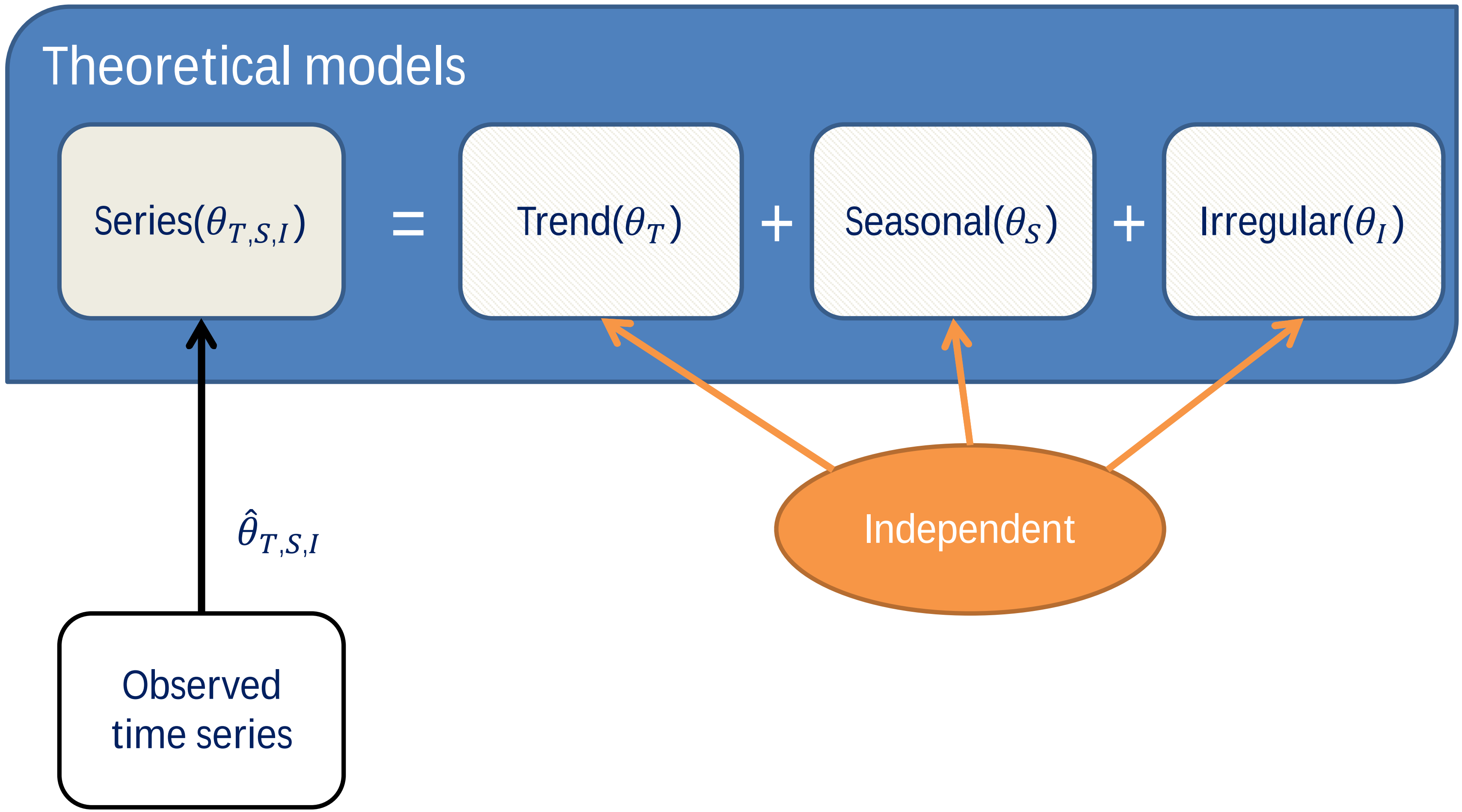
Model-Based Decomposition

ESTP training

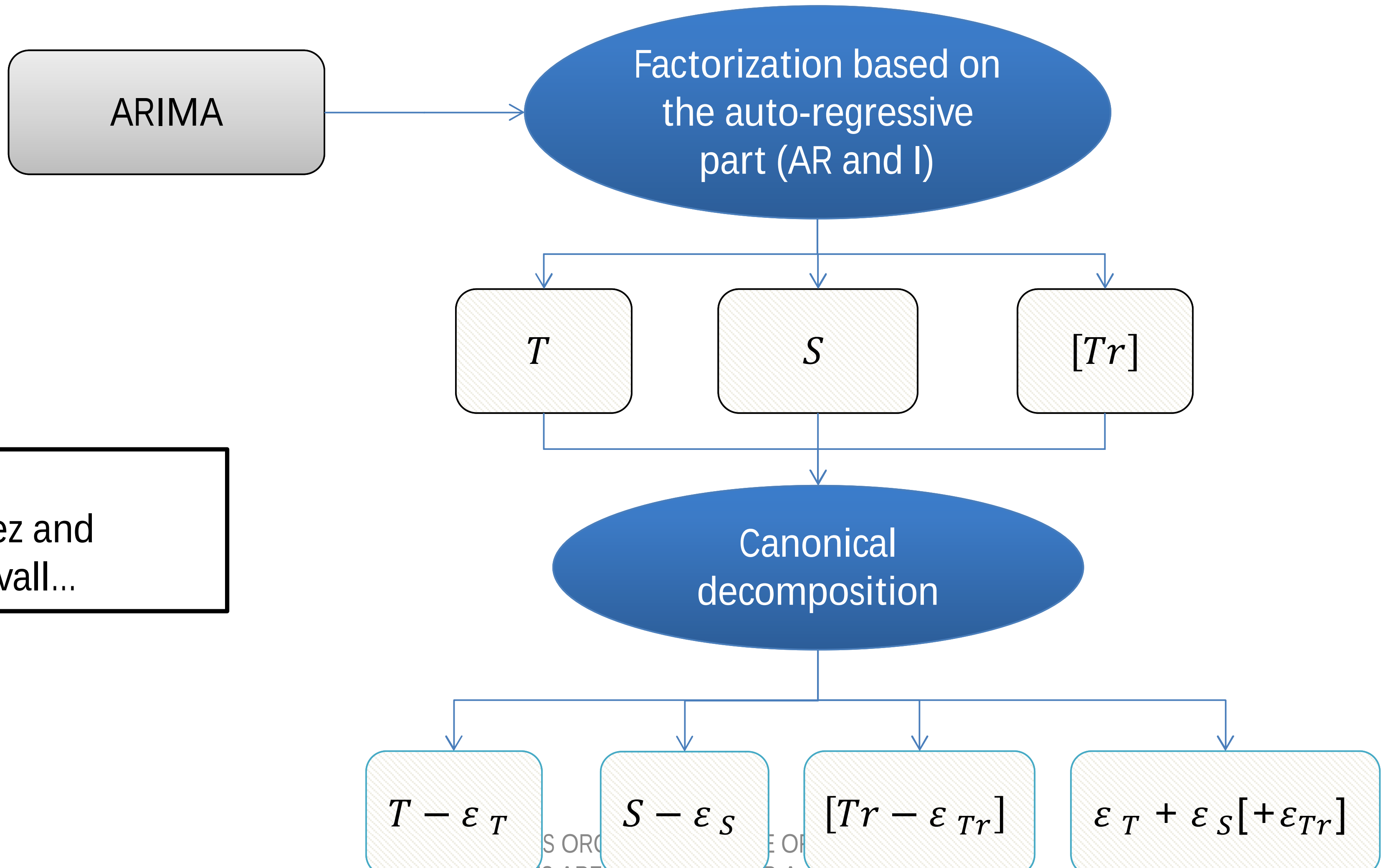
Unobserved components model (AMB)



Unobserved components model (STS)



SEATS decomposition

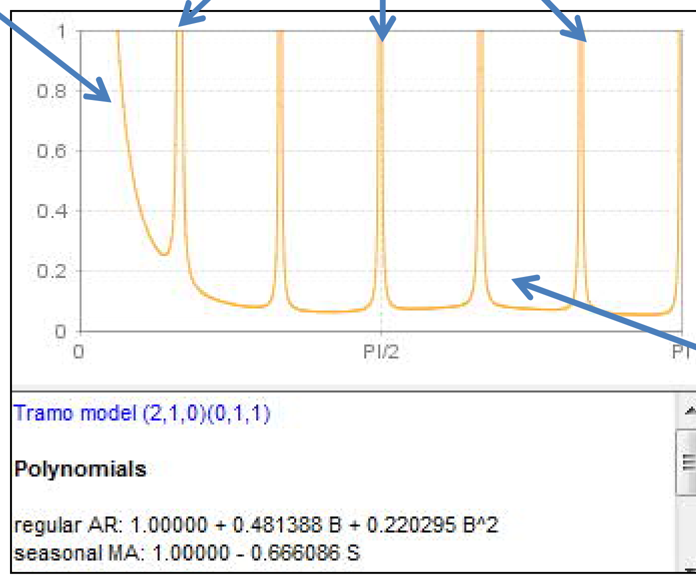


See:
Gomez and Maravall…

(Pseudo-)spectrum *≡ another way of representing a stochastic model*

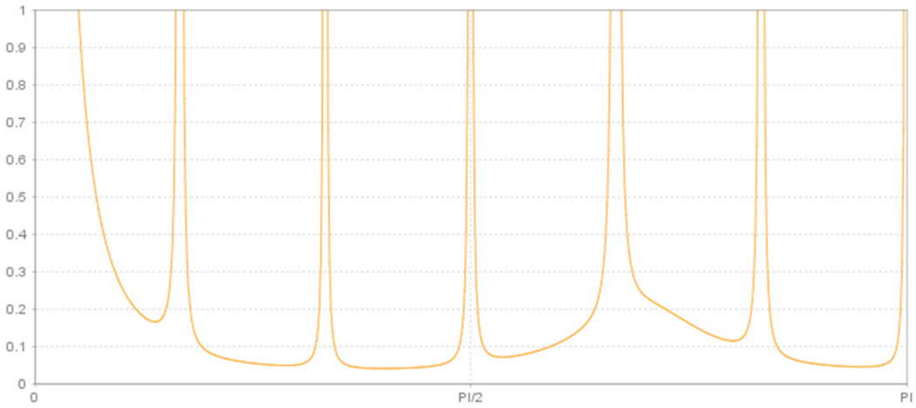
Low-frequencies (trend)

Seasonal-frequencies



TD frequency

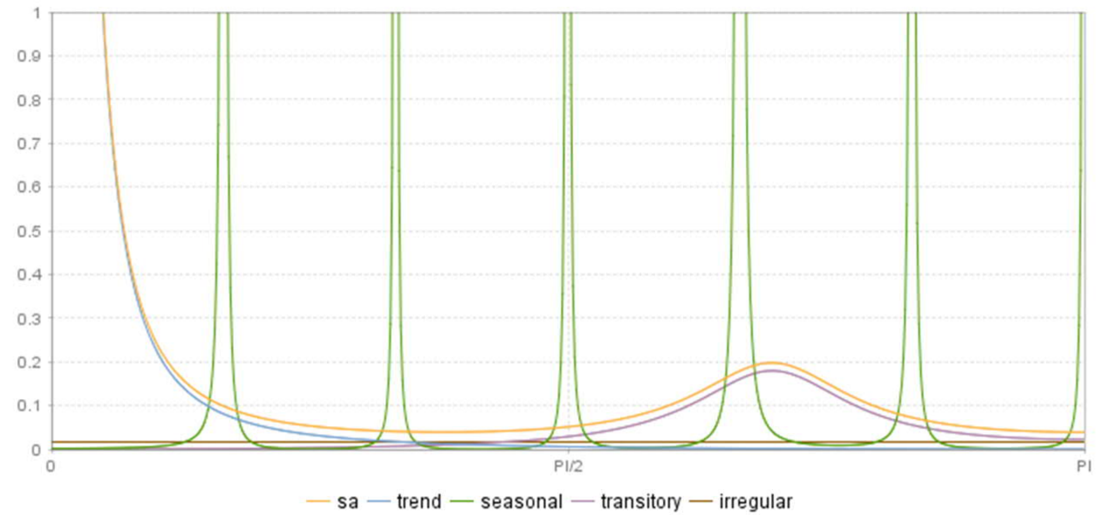
Unobserved components model. Spectral analysis

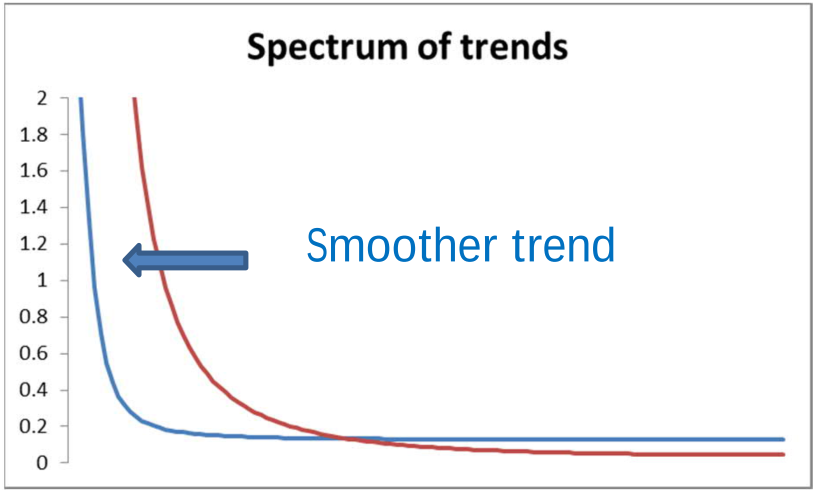


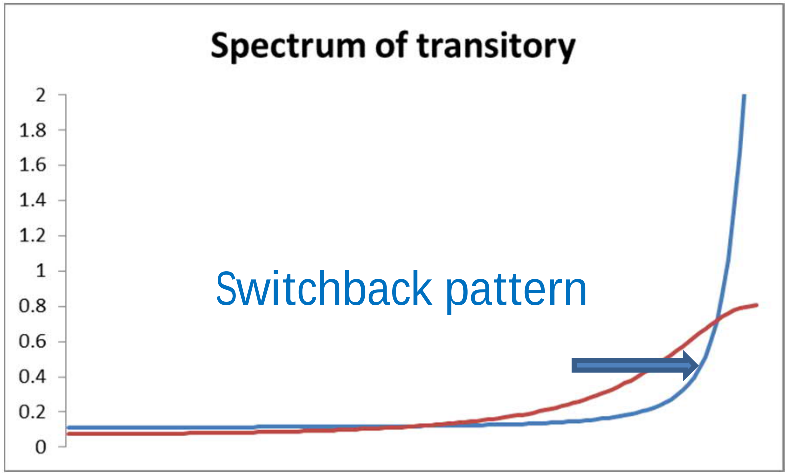
$$S_Y = S_T + S_S + S_{Tr} + S_I$$

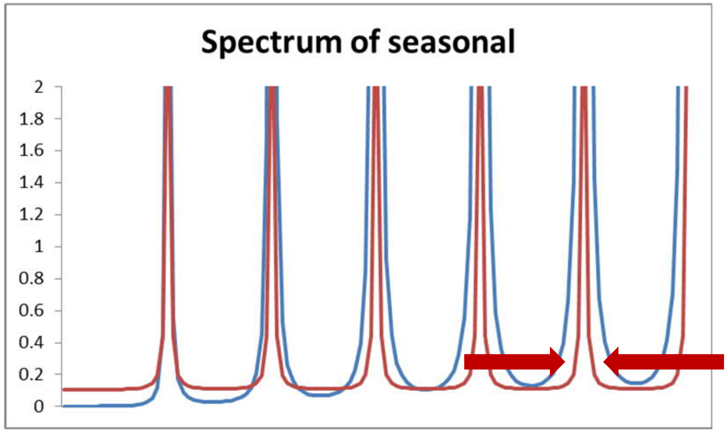
$$S_{SA} = S_T + S_{Tr} + S_I = S_Y - S_S$$

Independent components !



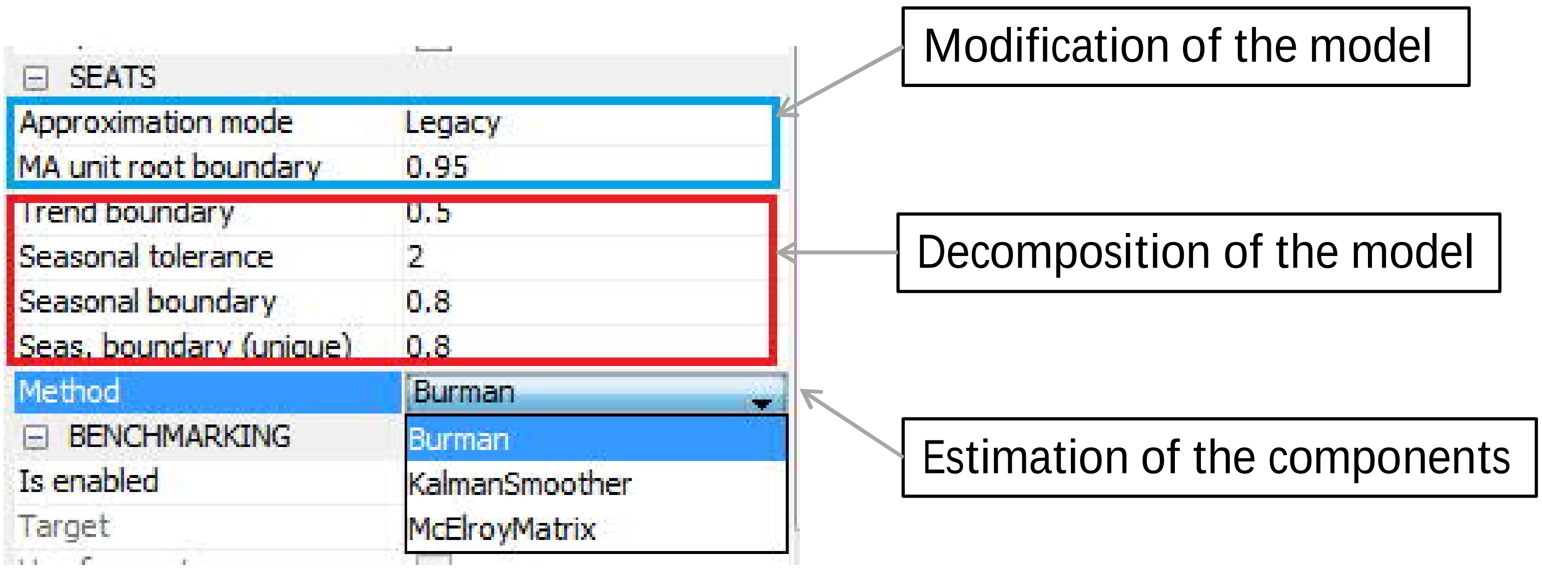






More stable seasonal

SEATS in JD+


SEATS
Approximation mode
Legacy
MA unit root boundary
0.95
Trend boundary
0.5
Seasonal tolerance
2
Seasonal boundary
0.8
Seas. boundary (unique)
0.8
Method
Burman
BENCHMARKING
Is enabled
Target
Burman
KalmanSmoother
McElroyMatrix
Modification of the model
Decomposition of the model
Estimation of the components

SEATS decomposition (I)

- Factorization of the AR polynomial

$$\varphi(B)\Delta(B) = \prod(1 - \alpha_i B)$$

  - Trend-cycle
    - α_i real,
      - $\alpha_i \geq k$
    - α_i complex,
      - $|\alpha_i| \geq k, \arg(\alpha_i) \leq c$

$\left(c = \frac{\pi}{s}\right) \sim cycle\ length \geq two\ years$

TrendCycleSelector.java

```java
public boolean accept(final Complex root) {
    Complex iroot = root.inv();
    if (root.getIm() == 0) {
        return iroot.getRe() >= m_bound;
    } else {
        if (iroot.abs() >= m_bound) {
            double arg = Math.abs(iroot.arg());
            if (arg <= m_lfreq) {
                return true;
            }
        }
        return false;
    }
}
```

– Seasonal

- α_i real,
  - $\alpha_i < -l$
- α_i complex,
  - $|\arg(\alpha_i) - f_s| \leq e$

f_s *seasonal frequency*

SeasonalSelector.java

```java
public boolean accept(final Complex root) {
    if (Math.abs(root.getIm()) < 1e-6) {
        if (1/root.getRe() < -m_k)
            return true;
        else
            return false;
    }

    double pi = 2 * Math.PI / m_freq;
    double arg = Math.abs(root.arg());
    double eps=m_epsphi/180*Math.PI;
    for (int i = 1; i <= m_freq / 2; ++i) {
        if (Math.abs(pi * i - arg) <= eps)
            return true;
    }
    return false;
```

– Transitory (I)

- All other roots

Trend boundary	*k*
Seasonal tolerance (degree)	*e*
Seasonal boundary	*l*
Seas.boundary (unique)	*l (no seasonal part)*

Seats decomposition (II)

- Impact of the parameters
  - k
    - Small: possible « noisy » trend
    - k ≈ 1: more stable trend
  - e
    - Large (>5): possible short term cycle in the seasonal (for instance, stochastic TD)→erratic seasonal
  - l
    - Small (<.8): higher risk of erratic seasonal

- General consideration: threshold effects are unavoidable (only in case of AR polynomials)

Non decomposable models

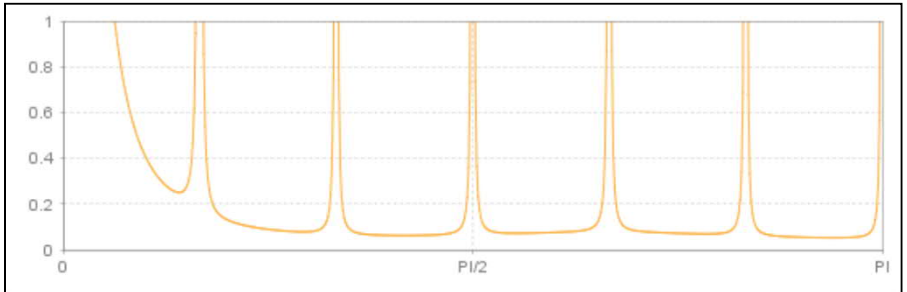
- Some models are not decomposable (often due to complex models)
  - Best solution: change yourself the model in Tramo
  - Otherwise:
    - Legacy: Seats search for another SARIMA model, as similar as possible to the original model
    - Noisy: Seats add noise in the initial model (→ I = Tr[= 0])

Estimation of the components

- 3 solutions, strictly equivalent (except for SD)
  - Burman algorithm (WK filters): legacy solution, fastest
  - Kalman smoother: more stable, exact SD
  - [Matrix computation]
- Exception:
  - Burman and Matrix computations are unstable if quasi-unit roots in MA →Fix MA unit roots boundary
  - No such problem with the Kalman smoother

- Unit roots in MA →fixed seasonal or linear trend (?)

Example



Tramo model (2,1,0)(0,1,1)

Polynomials

regular AR: 1.00000 + 0.481388 B + 0.220295 B^2
seasonal MA: 1.00000 - 0.666086 S

Regular AR inverse roots

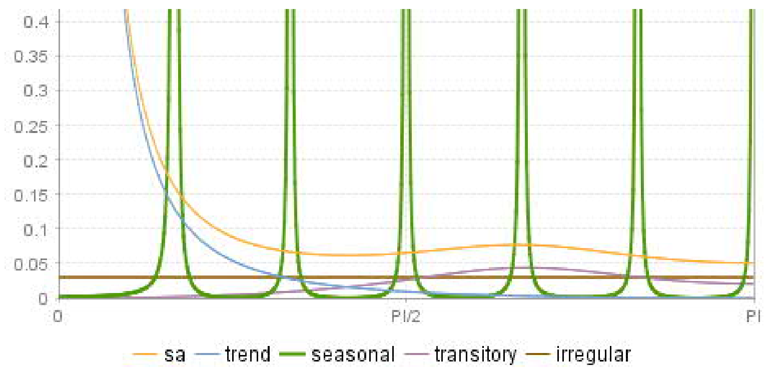
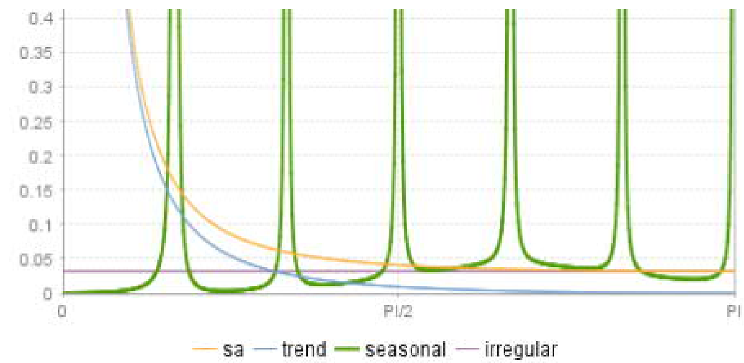
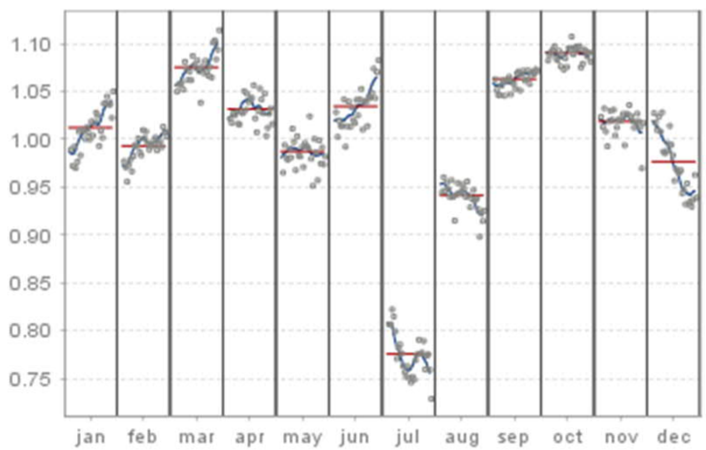
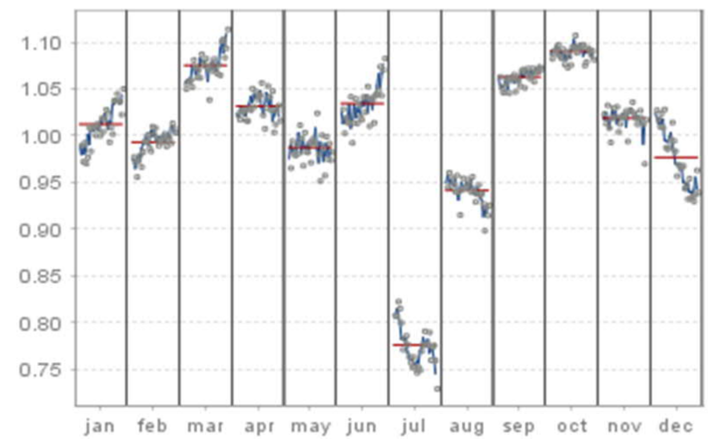
argument=2.1093, modulus=0.4694 (seasonal frequency)
argument=-2.1093, modulus=0.4694 (seasonal frequency)

II. Epsphi=2

trend
D: 1.00000 - 2.00000 B + B^2
MA: 1.00000 + 0.0332784 B - 0.966722 B^2
Innovation variance: **0.06036**
seasonal
AR: 1.00000 + 0.481388 B + 0.220295 B^2
D: 1.00000 + B + B^2 + B^3 + B^4 + B^5 + B^6 + B^7 + B^8 + B^9 + B^10 + B^11
MA: 1.00000 + 0.845739 B + 0.508682 B^2 + 0.350842 B^3 + 0.159239 B^4 - 0.0382435 B^5 - 0.222372 B^6 - 0.376268 B^7 - 0.475568 B^8 - 0.506799 B^9 - 0.397686 B^10 - 0.171174 B^11 - 0.463080 B^12 - 0.213313 B^13
Innovation variance: **0.14606**
irregular
Innovation variance: **0.19978**

I. Epsphi=.01

trend
D: 1.00000 - 2.00000 B + B^2
MA: 1.00000 + 0.0332784 B - 0.966722 B^2
Innovation variance: **0.06036**
seasonal
D: 1.00000 + B + B^2 + B^3 + B^4 + B^5 + B^6 + B^7 + B^8 + B^9 + B^10 + B^11
MA: 1.00000 + 1.19980 B + 1.15311 B^2 + 1.30832 B^3 + 1.11157 B^4 + 0.864344 B^5 + 0.654553 B
0.409564 B^7 + 0.243908 B^8 - 0.0639808 B^9 - 0.204553 B^10 - 0.381637 B^11
Innovation variance: **0.03297**
transitory
AR: 1.00000 + 0.481388 B + 0.220295 B^2
MA: 1.00000 - 0.555652 B - 0.444348 B^2
Innovation variance: **0.05784**
irregular
Innovation variance: **0.18760**

Epsphi=.01	Epsphi=2
	
	

SEATS analysis

- What matters?
  - Understanding the differences between the “theoretical components” and their “estimators”
    - For instance: “dips” in the spectrum of the estimator
  - Understanding the properties of the estimators
    - Model of Irregular ≠ white noise, negative ac(1) in many SA estimators, …
  - Understanding PsiE-weights

$$y_t = \sum_{i \le t} \psi_i \varepsilon_i \Rightarrow \hat{s}_t = \nu(B, F) y_t = \nu(B, F) \sum_{i \le t} \psi_i \varepsilon_i$$

$$\Rightarrow \hat{s}_t = \sum_{i \le t} \xi_{s,i}^{-} \varepsilon_i + \sum_{i > t} \xi_{s,i}^{+} \varepsilon_i$$

$$\Rightarrow \hat{s}_{t|T} = \sum_{i \le t} \xi_{s,i}^{-} \varepsilon_i + \sum_{t < i \le T} \xi_{s,i}^{+} \varepsilon_i$$

$$\Rightarrow \hat{r}_{t|T} = \sum_{t > T} \xi_{s,i}^{+} \varepsilon_i$$

Final remarks

- What matters?
  - Impact of the model on
    - SA/S "smoothness" ⇒ Be very careful with stationary AR roots
    - Revisions
  - Use PsiE-weights to understand/anticipate revisions
  - Model-based diagnostics
    - Variance estimators<> variance estimates
      - Should not happen if the original model is well defined
      - Be careful with non decomposable models / fixed models / "bad" models
  - To go further, see:
    - "SEASONAL ADJUSTMENT AND SIGNAL EXTRACTION IN ECONOMIC TIME SERIES", by R. Gomez and A. Maravall