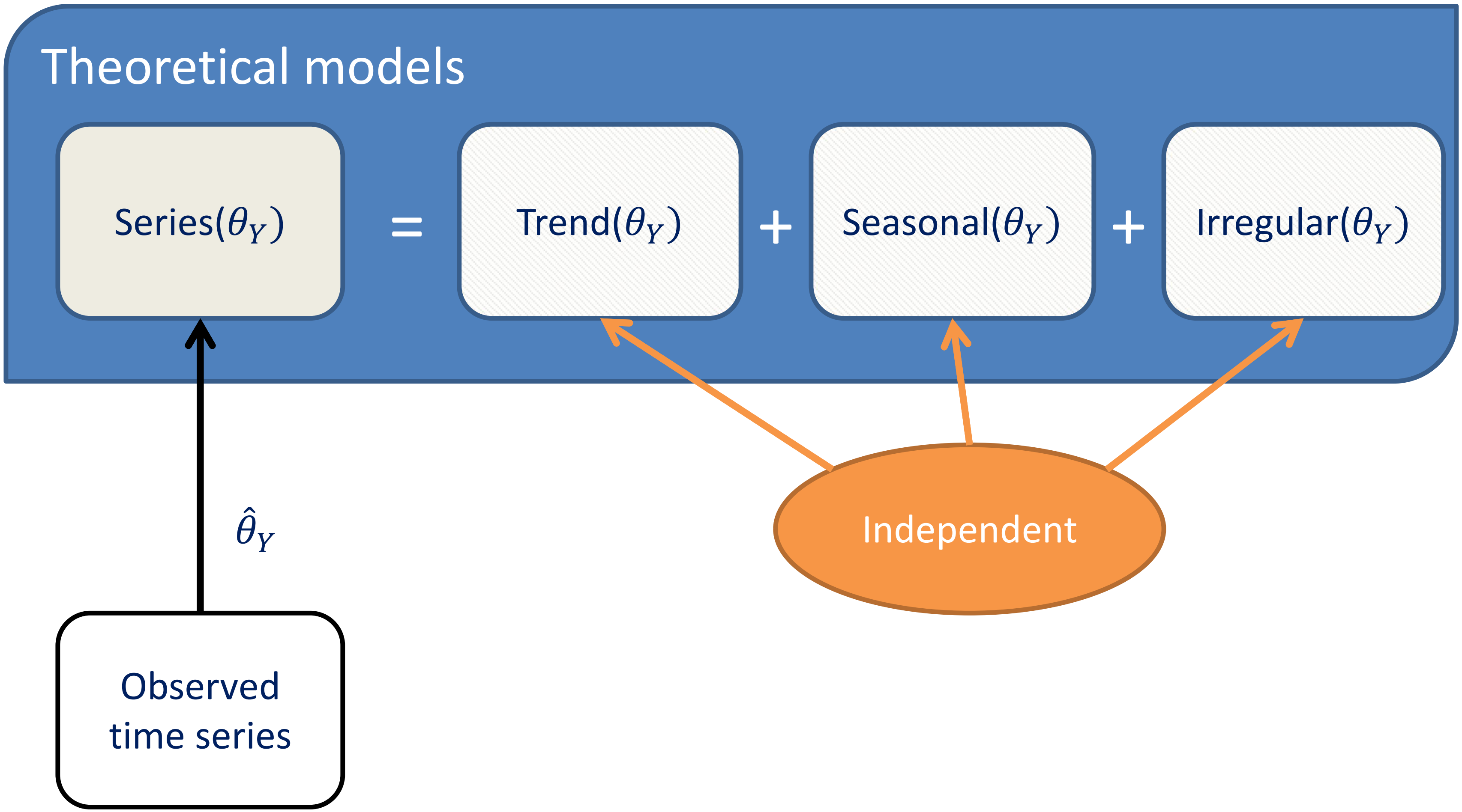
European Commission

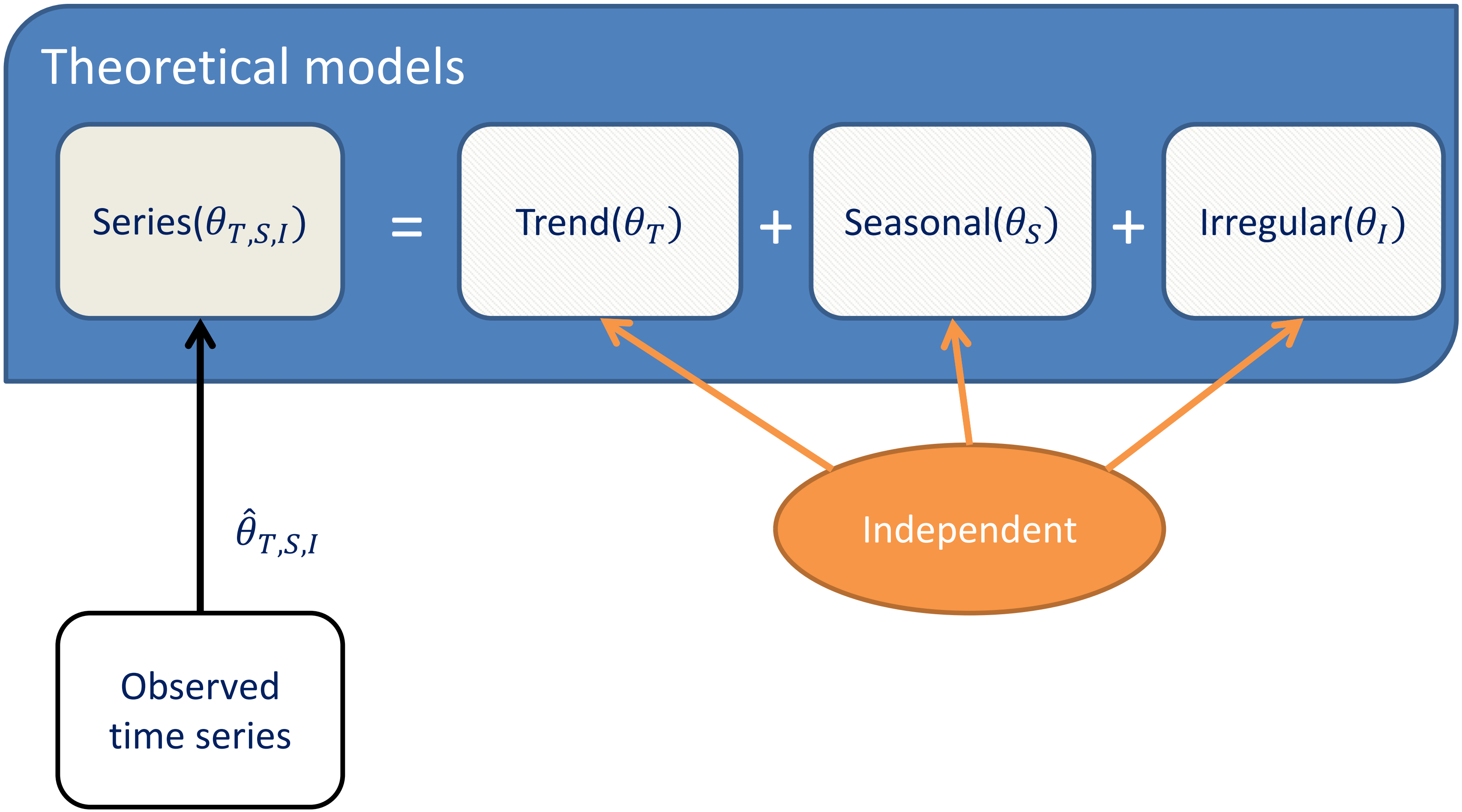
Model-Based Decomposition

ESTP training

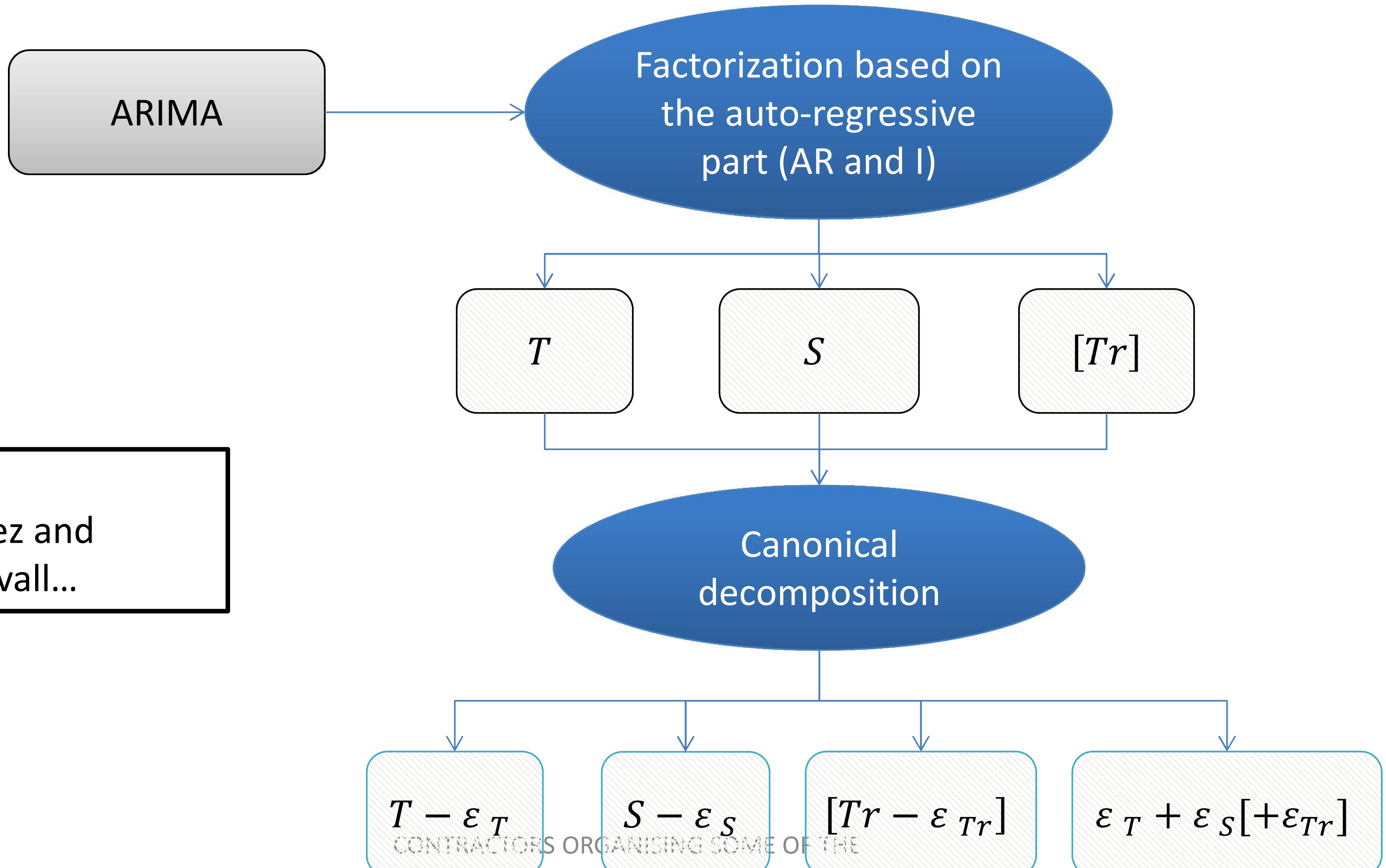
Unobserved components model (AMB)



Unobserved components model (STS)



SEATS decomposition



See:
Gomez and Maravall…

Arima model (definition)

- Backward operator
  - $B^k x_t = x_{t-k}$
  - $(1 - B^k)x_t = x_t - x_{t-k}$
- Auto-regressive model
  - $(1 + \varphi_1 B + \cdots + \varphi_p B^p)x_t = \Phi(B)x_t = \varepsilon_t$
  - $x_t = \varepsilon_t - \varphi_1 x_{t-1} - \cdots - \varphi_p x_{t-p}$ (≠R, X13)
- Moving average model
  - $x_t = \Theta(B)x_t = (1 + \theta_1 B + \cdots + \theta_q B^q)\varepsilon_t$
  - $x_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$

Arima models (definitions)

- ARMA (p, q)
  - $\Phi(B)x_t = \Theta(B)\varepsilon_t$
- SARIMA$(p, d, q)(bp, bd, bq)_s$
  - $(1-B)^d(1-B^s)^{bd}\Phi(B)\Phi_s(B^s)x_t = \Theta(B)\Theta_s(B^s)\varepsilon_t$

- Airline model
  - $(1-B)(1-B^s)x_t = (1+\theta B)(1+\theta_s B^s)\,\varepsilon_t$

  - Interpretation of $x_t = x_{t-12} + \theta_s\varepsilon_{t-12} + \varepsilon_t$: *same as previous year, partially corrected for the "committed error"*

Arima model properties

- Autocovariance function(ACF): only defined for stationary models
  - $x_t = \sum_{k=0}^{\infty} \psi_{t-k} \varepsilon_{t-k}$ ψ *generated by* $\frac{\Theta(B)}{\Phi(B)}$
  - $acgf = \frac{\Theta(B)}{\Phi(B)} \frac{\Theta(F)}{\Phi(F)} = \Psi(B)\, \Psi(F)$
- (Pseudo-)spectrum = Fourier's transform of the ACF (with extension to non-stationary models)
  - Counterpart of the periodogram
  - Contribution of each frequency to the variance of the series

Wiener-Kolmogorov filters

$$\hat{s}_t = k_s \frac{\Psi_s(B)\Psi_s(F)}{\Psi(B)\Psi(F)} y_t = k_s \frac{\Theta_s(B)\Phi_n(B)\Theta_s(F)\Phi_n(F)}{\Theta(B)\Theta(F)} y_t$$

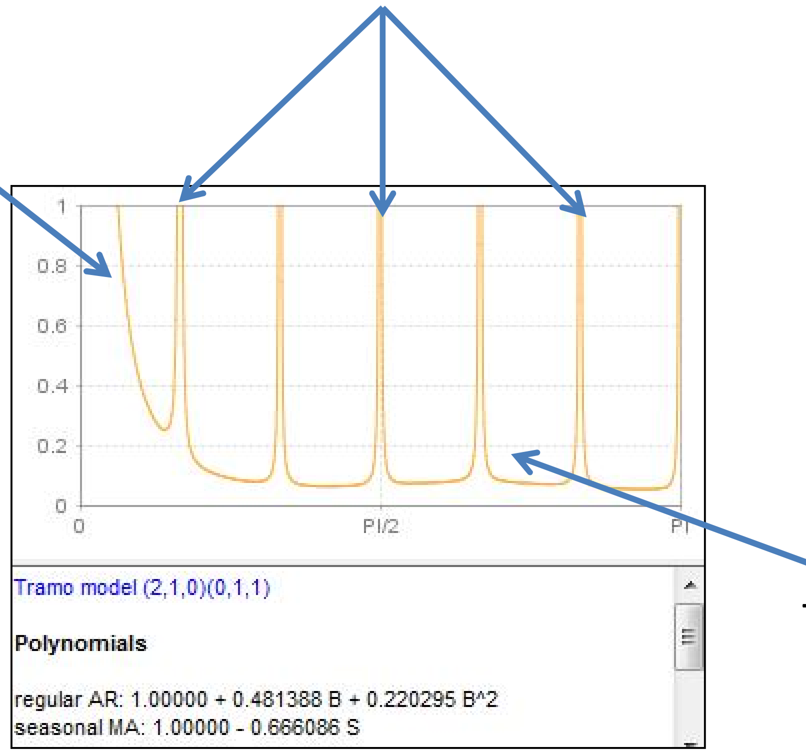
$$\hat{s}_t = k_s \frac{\Theta_s(B)\Phi_n(B)\Theta_s(F)\Phi_n(F)\Theta_s(B)}{\Theta(B)\Theta(F)\Phi(B)} \varepsilon_t$$

$$\hat{s}_t = k_s \frac{\Theta_s(B)\Phi_n(F)\Theta_s(F)}{\Phi_s(B)\Theta(F)} \varepsilon_t \quad (\psi_e\, weights)$$

(Pseudo-)spectrum *≡ another way of representing a stochastic model*

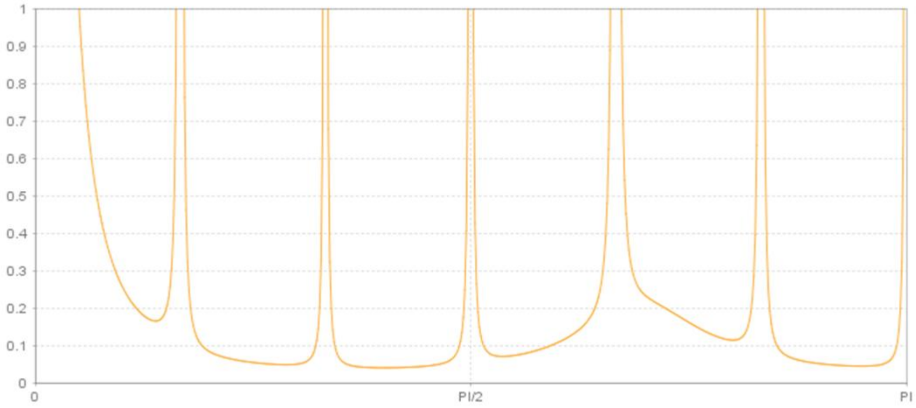
Low-frequencies (trend)

Seasonal-frequencies



TD frequency

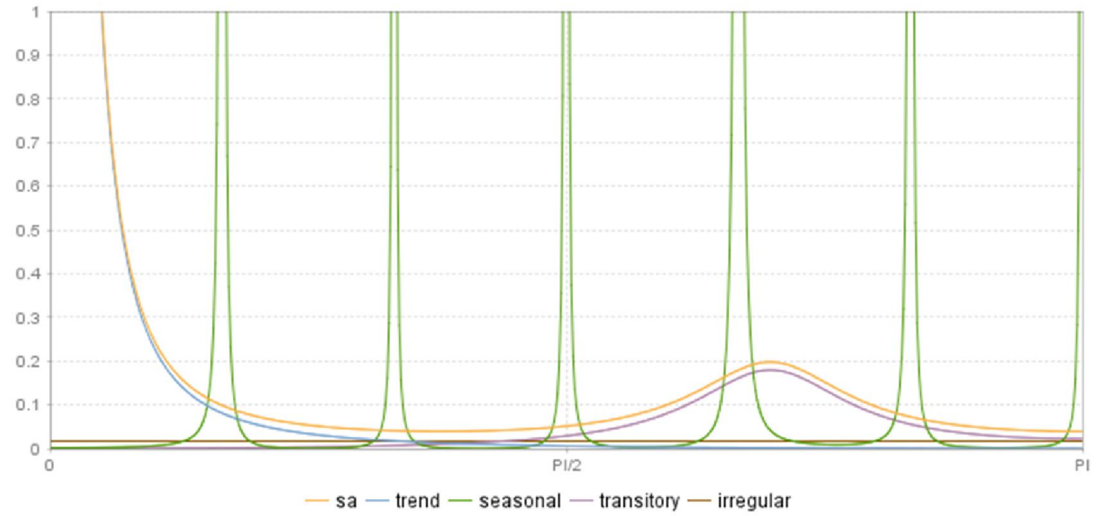
Unobserved components model. Spectral analysis

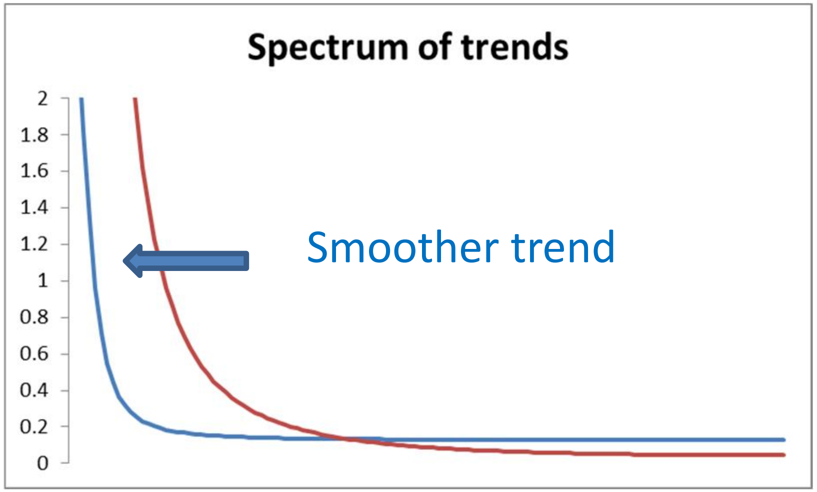


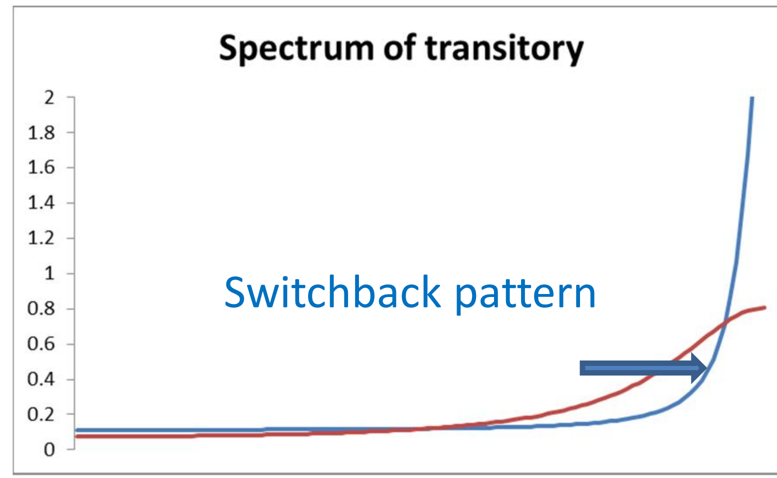
$$S_Y = S_T + S_S + S_{Tr} + S_I$$

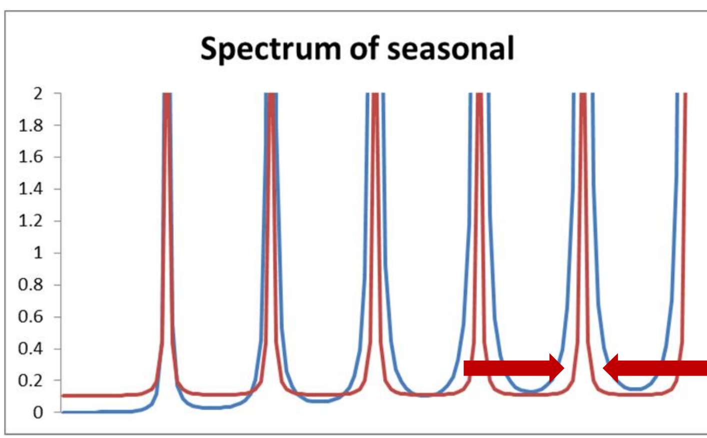
$$S_{SA} = S_T + S_{Tr} + S_I = S_Y - S_S$$

Independent components !



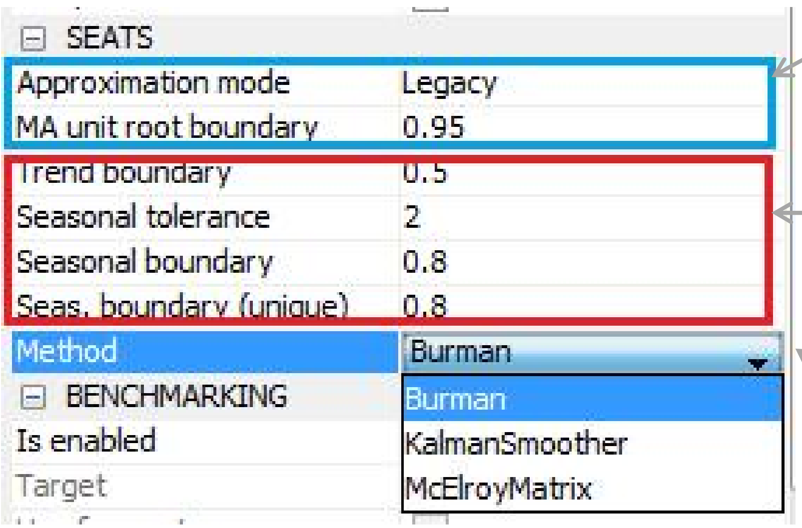






More stable seasonal

SEATS in JD+



Modification of the model

Decomposition of the model

Estimation of the components

SEATS decomposition (I)

- Factorization of the AR polynomial

$$\varphi(B)\Delta(B) = \prod (1 - \alpha_i B)$$

  - Trend-cycle
    - α_i real,
      - $\alpha_i \geq k$
    - α_i complex,
      - $|\alpha_i| \geq k, \arg(\alpha_i) \leq c$

$\left(c = \frac{\pi}{s}\right) \sim cycle\ length \geq two\ years$

TrendCycleSelector.java

```
public boolean accept(final Complex root) {
    Complex iroot = root.inv();
    if (root.getIm() == 0) {
        return iroot.getRe() >= m_bound;
    } else {
        if (iroot.abs() >= m_bound) {
            double arg = Math.abs(iroot.arg());
            if (arg <= m_lfreq) {
                return true;
            }
        }
        return false;
    }
}
```

– Seasonal

- α_i real,
  - $\alpha_i < -l$
- α_i complex,
  - $|\arg(\alpha_i) - f_s| \leq e$

f_s *seasonal frequency*

SeasonalSelector.java

```java
public boolean accept(final Complex root) {
    if (Math.abs(root.getIm()) < 1e-6) {
        if (1/root.getRe() < -m_k)
            return true;
        else
            return false;
    }

    double pi = 2 * Math.PI / m_freq;
    double arg = Math.abs(root.arg());
    double eps=m_epsphi/180*Math.PI;
    for (int i = 1; i <= m_freq / 2; ++i) {
        if (Math.abs(pi * i - arg) <= eps)
            return true;
    }
    return false;
```

– Transitory (I)

- All other roots

Trend boundary	*k*
Seasonal tolerance (degree)	*e*
Seasonal boundary	*l*
Seas.boundary (unique)	*l (no seasonal part)*

Seats decomposition (II)

- Impact of the parameters
  - k
    - Small: possible « noisy » trend
    - k ≈ 1: more stable trend
  - e
    - Large (>5): possible short term cycle in the seasonal (for instance, stochastic TD)→erratic seasonal
  - l
    - Small (<.8): higher risk of erratic seasonal

- General consideration: threshold effects are unavoidable (only in case of AR polynomials)

Non decomposable models

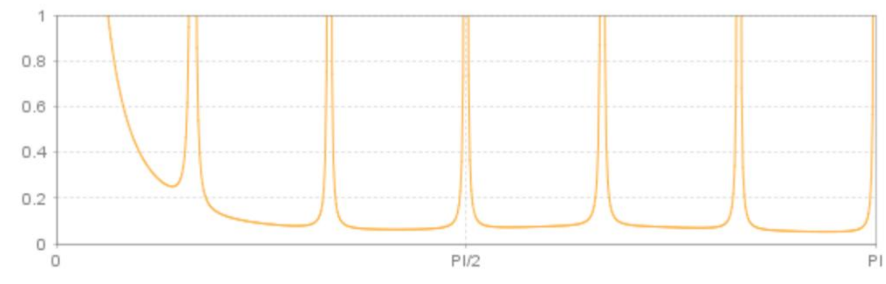
- Some models are not decomposable (often due to complex models)
  - Best solution: change yourself the model in Tramo
  - Otherwise:
    - Legacy: Seats search for another SARIMA model, as similar as possible to the original model
    - Noisy: Seats add noise in the initial model (→ I = Tr[= 0])

Estimation of the components

- 3 solutions, strictly equivalent (except for SD)
  - Burman algorithm (WK filters): legacy solution, fastest
  - Kalman smoother: more stable, exact SD
  - [Matrix computation]
- Exception:
  - Burman and Matrix computations are unstable if quasi-unit roots in MA →Fix MA unit roots boundary
  - No such problem with the Kalman smoother

- Unit roots in MA →fixed seasonal or linear trend (?)

Example



Tramo model (2,1,0)(0,1,1)

Polynomials

regular AR: 1.00000 + 0.481388 B + 0.220295 B^2
seasonal MA: 1.00000 - 0.666086 S

Regular AR inverse roots

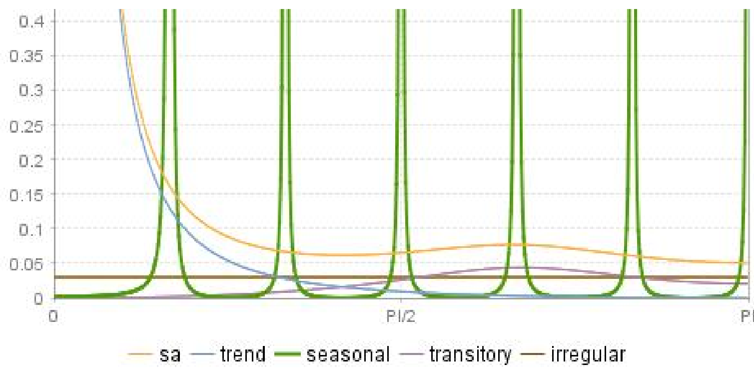
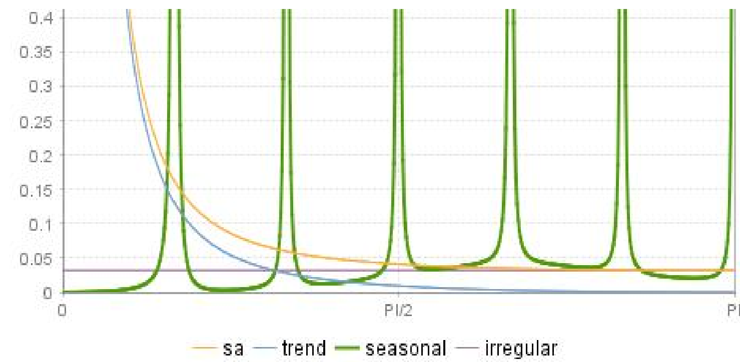
argument=2.1093, modulus=0.4694 (seasonal frequency)
argument=-2.1093, modulus=0.4694 (seasonal frequency)

II. Epsphi=2

trend
D: 1.00000 - 2.00000 B + B^2
MA: 1.00000 + 0.0332784 B - 0.966722 B^2
Innovation variance: **0.06036**
seasonal
AR: 1.00000 + 0.481388 B + 0.220295 B^2
D: 1.00000 + B + B^2 + B^3 + B^4 + B^5 + B^6 + B^7 + B^8 + B^9 + B^10 + B^11
MA: 1.00000 + 0.845739 B + 0.508682 B^2 + 0.350842 B^3 + 0.159239 B^4 - 0.0382435 B^5 - 0.222372 B^6 - 0.376268 B^7 - 0.475568 B^8 - 0.506799 B^9 - 0.397686 B^10 - 0.171174 B^11 - 0.463080 B^12 - 0.213313 B^13
Innovation variance: **0.14606**
irregular
Innovation variance: **0.19978**

I. Epsphi=.01

trend
D: 1.00000 - 2.00000 B + B^2
MA: 1.00000 + 0.0332784 B - 0.966722 B^2
Innovation variance: **0.06036**
seasonal
D: 1.00000 + B + B^2 + B^3 + B^4 + B^5 + B^6 + B^7 + B^8 + B^9 + B^10 + B^11
MA: 1.00000 + 1.19980 B + 1.15311 B^2 + 1.30832 B^3 + 1.11157 B^4 + 0.864344 B^5 + 0.654553 B
0.409564 B^7 + 0.243908 B^8 - 0.0639808 B^9 - 0.204553 B^10 - 0.381637 B^11
Innovation variance: **0.03297**
transitory
AR: 1.00000 + 0.481388 B + 0.220295 B^2
MA: 1.00000 - 0.555652 B - 0.444348 B^2
Innovation variance: **0.05784**
irregular
Innovation variance: **0.18760**

<table>
<tr><th>Epsphi=.01</th><th>Epsphi=2</th></tr>
<tr><td>
</td><td>
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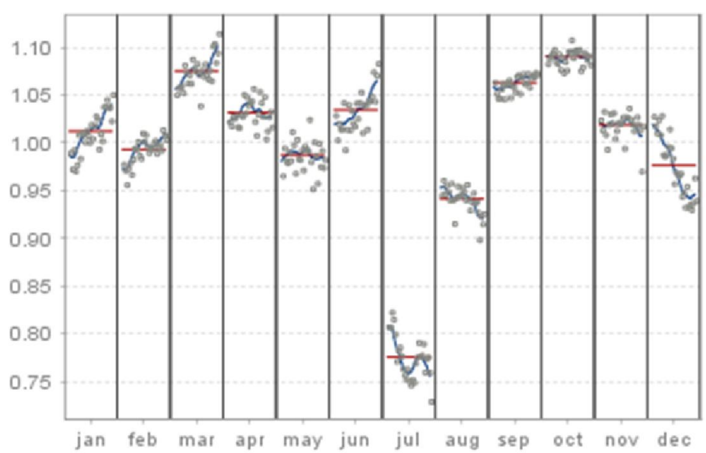
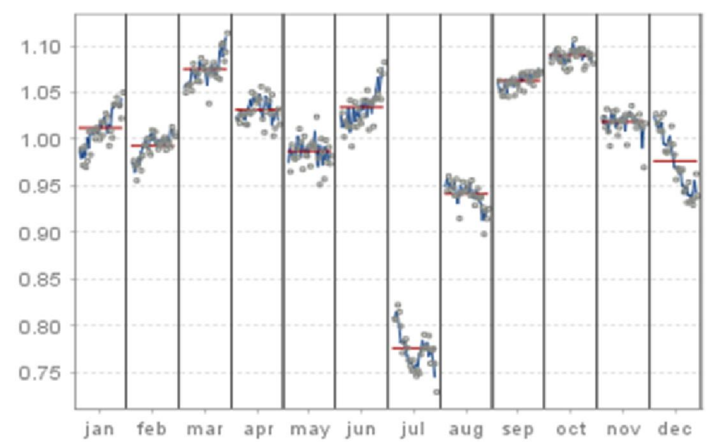
<tr><td>
</td><td>
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SEATS analysis

- What matters?
  - Understanding the differences between the “theoretical components” and their “estimators”
    - For instance: “dips” in the spectrum of the estimator
  - Understanding the properties of the estimators
    - Model of Irregular ≠ white noise, negative ac(1) in many SA estimators, …
  - Understanding PsiE-weights

$$y_t = \sum_{i\le t} \psi_i \varepsilon_i \Rightarrow \hat{s}_t = \nu(B,F) y_t = \nu(B,F) \sum_{i\le t} \psi_i \varepsilon_i$$

$$\Rightarrow \hat{s}_t = \sum_{i\le t} \xi_{s,i}^{-} \varepsilon_i + \sum_{i>t} \xi_{s,i}^{+} \varepsilon_i$$

$$\Rightarrow \hat{s}_{t|T} = \sum_{i\le t} \xi_{s,i}^{-} \varepsilon_i + \sum_{t<i\le T} \xi_{s,i}^{+} \varepsilon_i$$

$$\Rightarrow \hat{r}_{t|T} = \sum_{t>T} \xi_{s,i}^{+} \varepsilon_i$$

Final remarks

- What matters?
  - Impact of the model on
    - SA/S "smoothness" ⇒ Be very careful with stationary AR roots
    - Revisions
  - Use PsiE-weights to understand/anticipate revisions
  - Model-based diagnostics
    - Variance estimators<> variance estimates
      - Should not happen if the original model is well defined
      - Be careful with non decomposable models / fixed models / "bad" models
  - To go further, see:
    - "SEASONAL ADJUSTMENT AND SIGNAL EXTRACTION IN ECONOMIC TIME SERIES", by R. Gomez and A. Maravall