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Andreev Reflection and Proximity effect

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The Andreev Reflection is the key mechanism for the superconducting proximity effect. It provides phase correlations in a system of non-interacting electrons at mesoscopic scales, i.e. over distances much larger than the microscopic lengths : Fermi wavelength and elastic electron mean free path. This field of research has attracted an increasing interest in the recent years in part because of the tremendous development of nanofabrication technologies, and also because of the richness of the involved quantum effects. In this paper we review some recently achieved advances. We also discuss new open questions, in particular non-equilibrium effects and proximity effect in systems with ferromagnetic elements.

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1. Introduction

The proximity effect is the occurrence of superconducting-like properties in non-superconducting materials placed in electrical contact with a superconductor (S). It was understood in the sixties that superconducting correlation could extend over a large length scale in a normal metal (N), even in the absence of attractive electron-electron interactions.[1–3] It is instructive to see that many of the basic phenomena in particular regarding tunneling spectroscopy were understood although the mesoscopic language was not used. However the technology did not allow yet to really manufacture complex normal-superconducting circuits in a controllable way at this scale. Most experiments were carried out on thin film layers and some key experiments were not accessible. The two main difficulties - patterning at submicron scale and control of S-N interfaces - have been overcome recently when state-of-the-art nanofabrication technologies became available in research laboratories. In the last ten years a large variety of normal-superconductor hybrid structures including phase-sensitive devices has been studied, leading to a clarification of the basic mechanism as well as a good understanding of many non-intuitive observations. This understanding strongly benefited from the progresses in mesoscopic physics which called the attention to phase-coherent phenomena in disordered systems.[4–8]

It is worth noticing that the important length and energy scales in the proximity effect are the same as those governing quantum transport or thermodynamic effects in mesoscopic systems. The Thouless energy or "correlation energy" E_c defined as $\hbar/\tau_D$ in a disordered metal is of central importance since it characterizes the time $\tau_D = L^2/D$ required by a single electron to feel the sample boundary. Here $\hbar$ is the Planck constant, L is the sample length and D is the diffusion coefficient. As will be seen, this energy scale plays in some sense the role of the energy gap in a non-interacting metal in proximity contact with a superconductor. The fact that the actual energy gap is determined not by a pairing interaction as in a BCS superconductor but by the diffusion of single quasiparticles in the normal metal is a simple result of the remote interaction felt by the electronic state when entering the device.

The role of the Andreev reflection[9] is central to the proximity effect since it provides the elementary mechanism for converting single electron states from a normal metal to Cooper pairs in the superconducting condensate. As will be illustrated in this review, the actual proximity effect is the result of an interplay between Andreev reflection at the N-S interface and long-range coherence in the normal metal.

This paper is organized as follows. We first recall the main properties of Andreev reflection. We then illustrate some consequences on the conductance of the N-S junction and of the normal metal itself. Both the sub-gap conductance of a tunnel junction and the re-entrance effect are now well understood at least in the case of non-interacting metals. In the subsequent paragraphs we focus on recently addressed situations which are now open topics including non-equilibrium phenomena, thermopower, shot noise, density of states and proximity effects in ferromagnetic metals.

2. The Andreev Reflection

The Andreev Reflection provides a conversion of the dissipative electrical current in the normal metal into a dissipationless supercurrent. The signatures of the Andreev Reflection are :

1. A two-electron process : Because of the existence of an energy gap at

the Fermi energy in the density of states of the superconductor, the transfer of single quasiparticle states with an energy ϵ below the gap Δ is forbidden. This situation holds when one restricts oneself to the first-order process as it is the case for an opaque tunnel barrier. Indeed the Giaver tunneling experiments demonstrated that the tunnel conductance of a N-I-S junction directly probes the density of states of a superconductor.[10] However, another type of transfer is possible when second order processes are allowed. An incoming electron can be transferred into the superconductor if a second electron is also transferred through the interface thus forming a Cooper pair into the superconductor. In terms of single excitations, this process is equivalent to the reflection of a hole.

The consequences of Andreev reflection on the current voltage characteristics of a S-N junction were studied in detail in the so-called BTK theory.[11] The barrier strength was characterized by a simple parameter Z ranging from 0 for a perfect metallic contact to ∞ for a low transparency tunnel barrier. With this definition, the transparency reads $t = 1/(1+Z^2)$. The Andreev process is significant when the transparency of the barrier is high. For a perfect contact ($Z=0$) the sub-gap conductance was found to be twice the normal state conductance thus demonstrating the double charge transfer.

2. Retro-reflection : This curious feature was noticed by Andreev in his original paper on thermal properties of the intermediate state of superconductors. It was observed in particular by Benistant et al.[12] in an elegant experiment with a pure silver single crystal showing that all three components of the velocity changed sign upon reflection on a superconducting interface. The Andreev reflection is a perfect retro-reflection only for electrons incident at the Fermi energy.[13] When the energy is above the Fermi energy, the incident electron $(E_F+\epsilon, k_F+\delta k/2)$ and the reflected hole $(E_F-\epsilon, -k_F+\delta k/2)$ have different wavelengths in the normal metal. The wavevector mismatch is linear in energy : $\delta k = 2\epsilon/\hbar v_F$. In a purely balistic system this results into resonance effects.[14,15]

3. Coherence properties : The most important property for the proximity effect is the phase coherence of the process. The reflected hole carries information both on the phase of the electron state and on the macroscopic phase Φ of the superconductor. Let us assume that the pair potential is fixed and given by $\Delta e^{i\Phi}$. For a state with energy ϵ above the Fermi energy the phase change can be written as : $\delta\phi = \Phi + \arccos(\epsilon/\Delta)$. One can see that the Andreev reflection of

a state at the Fermi energy, $\epsilon = 0$, is accompanied by a phase shift of $\pi/2$. The influence of this phase shift on the resistance of a N-S junction and the difference between Andreev reflection on a superconducting interface and optical reflection on a phase conjugated mirror was recently discussed by C. Beenakker.[16] The $\pi/2$ phase shift is at the origin of the finite resistance of a diffusive N-S junction at zero temperature.

In a diffusive metal, the phase shift leads to a loss of interferences beyond an energy-dependent coherence length given by $L_\epsilon = \sqrt{\hbar D/\epsilon}$. This mesoscopic length characterizes how far the two electrons from a Cooper pair leaking from the superconductor will diffuse in phase in the normal metal. It appears naturally in the propagation equation for the pair amplitude (Usadel equation). The ultimate cut-off[17] is the single electron phase memory length L_ϕ as for the weak localization effect.

4. Role of impurities : As far as phase-breaking events can be ignored the presence of impurities does not suppress the quantum interference effects. On the contrary, the diffusion on impurities provides a mechanism to re-direct the trajectories to the interface, therefore enhancing the tranfer at the interface. This is the coherent multiple scattering effect whose importance was emphasized by van Wees et al. in Ref. [18]. The superposition of multiple coherent transfers through the interface in presence of disorder is at the origin of the so-called reflectionless tunneling.[19] Namely, the superposition of many second order processes add up to give a first order process of much larger amplitude. In other words, the presence of disorder (in practice confinement) in the normal metal results in a strong enhancement of the conductance of the junction. This enhancement is suppressed by phase-breaking effects such as inelastic processes or external magnetic field. Because of the very special relationship between trajectories of electrons and holes, it is possible that some of the phase-breaking processes which are operant in weak localization or Aharonov-Bohm effects where trajectories are well separated might have a smaller dephasing effect here.[20]

5. Andreev reflection vs Cooper pair transfer : The Andreev reflection of an electron (or a hole) is equivalent to the transfer of single Cooper pairs in (or out) of the superconducting condensate. The proximity effect is due to the presence of Cooper pairs leaking into the normal metal. The way the pair density builds up in the normal metal is strongly influenced by the presence of impurities, tunnel barrier or boundaries. In the theory of non-equilibrium superconductivity[21]

the presence of Cooper pairs in the normal metal is described by the anomalous Green function $F^R(x,\epsilon) = -i\sin\Theta(x,\epsilon)$, conveniently expressed through a proximity angle $\Theta(x,\epsilon)$ which is a complex function of both the position x and the energy ϵ.

6. Role of spin : In the classical Andreev reflection between a pure metal and a BCS superconductor the spin degree of freedom can be ignored : The condensate is formed of Cooper pairs with opposite spins and the spin up and spin down bands of electrons in the N-metal are identical. The picture is the following : An incident electron with spin up (down) is tranferred into the superconductor together with a second electron with spin down (up) to form a Cooper pair. The reflected hole has a spin up (down) since it is associated with a missing down (up) electron. As we will see later, the situation is strongly altered in ferromagnets where the energy bands are spin-dependent.

3. The subgap conductance of a N-S interface

The Andreev reflection describes the elementary microscopic processes that occur at an ideal N-S boundary. The description of proximity effect requires additional ingredients that are not included in the simple models of the interface.[11] The zero bias anomaly observed in a semiconductor-superconductor junction by Kastalsky et al.[22] could only be understood by taking into account the multiple coherent scattering[18] on the impurities which brings back the electrons and holes on the interface with the result of increasing the effective conductance. This non-local coherent effect is generic to the proximity effect. The effect of disorder is dramatic. Hekking and Nazarov[23] predicted that the tunnel conductance G_T should be enhanced by a factor proportional to G_T/G_N, G_N being the metallic conductance of the N metal in the vicinity of the junction. The scattering matrix theory[18,19,24,25] which extends the Landauer-Buttiker approach of mesoscopic transport to include the Andreev reflection at a superconducting interface provides a good description of the effect of multiple scattering. This effect was implicitly taken into account in the older quasiclassical theory based upon the Larkin-Ovchinnikov theory of non-equilibrium superconductivity.[21,26–28] In many practical situations (disordered metals) the latter approach leads to a simple circuit equation for the energy-dependent proximity angle $\Theta(\epsilon, x)$.[29,30]

The phase-sensitivity was demonstrated by Pothier et al.[31] in a loop shaped N-S metallic circuit having two N-S junctions in parallel. As in Ref. [22], the subgap conductance showed a strong enhancement of the Andreev conductance near zero bias. Interestingly, the conductance was periodic with respect to the magnetic flux in the superconducting loop. This experiment clearly illustrates the existence of an interference effect between the two N-S junctions and therefore demonstrates the sensitivity of the Andreev current to the superconducting phase. Further recent theoretical works show that this phase sensitivity can provide useful informations on the quantum fluctuation of the phase of the superconducting island in a superconducting transistor.[32,33]

4. The spectral conductance and the re-entrance effect

As discussed above, the conductance of a N-S junction is enhanced by the coherent multiple scattering due to the disorder in the metal. The proximity effect can also be directly observed on the metallic conductance itself if the conductance of the metal is much smaller than the barrier conductance or if the design is such that the N-S interface is not in the measured circuit. A serie of experiments on Aharonov-Bohm loop circuits[34,35] and Andreev interferometers[35,36] have elucidated the length and temperature dependences of the phase sensitive contribution to the conductance.

It is well established now that the metallic conductance of a normal metal N in proximity with a superconductor S exhibits a non-monotonic temperature and voltage dependence with a maximum at kT or $eV \approx E_c$ (or equivalently $\sqrt{\hbar D/k_B T}$ or $\sqrt{\hbar D/eV} \approx L$). Here L is the length of the normal metal and $E_c = \hbar D/L^2$ is the Thouless energy. This so-called re-entrance effect first recognized in Ref. [[37]] was also observed in a variety of N-S systems including doped semiconductors[38] and two dimensional electron gases connected to a superconductor.[39] In the latter case, the position of the maximum could be controlled in-situ by changing the diffusion coefficient by an external gate voltage.[40]

The physics of the re-entrance effect now is well understood[28,41–43] in the case of non interacting electrons. The point is that one can define a energy-dependent spectral conductance[37] for the electron transport through the structure. The measured conductance is the convolution of this spectral conductance with the electron energy distribution function. Interestingly, in both N-S and S-N-S devices the temperature $kT = E_c$ is a crossover temperature below which the conductance diverges in the S-N-S junction (Josephson short circuit) or returns to the normal state resistance in the N-S junction (re-entrance effect).

At temperatures $kT > E_c$ larger than the Thouless energy, the magnetoconductance oscillations amplitude decays slowly with temperature with a $1/T$ power law.[34] This behaviour is in clear contrast with the exponential decay of the Josephson current over L_T in a S-N-S junction at high temperature. The physical meaning of this long-range effect is that even at relatively high temperatures, electrons at the Fermi level form pairs which remain coherent over the whole sample length. The relative weight of this population is about E_c/k_BT, this factor gives the appropriate order of magnitude for the long-range correction to the metallic conductance.[44]

5. The Thermopower

The first recognized consequence of Andreev reflection is the absence of energy transfer through the interface. Actually the initial motivation of the original Andreev work[9] was aimed at a quantitative understanding of thermal conductivity experiments in a type I superconductor that is made of alternate layers of normal and superconducting domains. However, to-date, most experiments on mesoscopic N-S devices have focused on electrical transport. Only recently the attention was called to other properties such as thermopower and thermal conductivity.[45]

In a homogeneous ordinary metal the thermopower is very small because the electrical conductivity is energy independent. This is not true for Kondo alloys where the magnetic scattering time is strongly dependent on the electron energy. Thus, alloys such as AuFe provide useful materials for low temperature thermocouple sensors. In a N-S device it is now understood that the spectral conductance is strongly peaked at a characteristic energy given by the Thouless energy. This is the origin of the re-entrance effect discussed above. Accordingly the thermopower is expected to be larger than in a plain normal metal.

Recently J. Eom et al.[46] succeeded in the measurements of the thermopower of mesoscopic Andreev interferometers. The experiment consists in imposing a thermal gradient in a Au wire in contact with a micron size loop shaped superconducting circuit. Various sample designs were realized. The thermopower was obtained from the voltage drop between the reservoirs. It oscillates as a function of the magnetic field with a fundamental period corresponding to one flux quantum in the superconducting loop. This experiment demonstrates the phase-dependence of the thermopower in a mesoscopic Andreev interferometer.

6. Noise experiments

Shot noise is a consequence of current fluctuations in electrical transport due to the discrete nature of the carriers. It has been recognized to yield information on the statistics of the charge carrier and on the correlation of their transmission. This makes noise experiments an unique tool in the field of mesoscopic quantum transport.[47] The shot noise in N-S and S-N-S junctions has been investigated recently. Using a calibrated dc-SQUID set-up Jehl et al.[48] have observed the doubling of current shot noise in a low impedance NbAl structure, in agreement with the theoretical predictions of de Jong and Beenakker.[49] This enhancement originates from the fact that the current is carried in the superconductor by Cooper pairs in units of 2e.

In the case of a S-N-S junction biased above the critical current, multiple Andreev reflections (MAR) take place as recognized by the subharmonic gap structures in the I-V characteristic. According to this mechanism, an electron is Andreev-reflected at one side of the junction as a hole which, after diffusion to the other interface is Andreev-reflected as an electron and then continue the cycle. At each cycle a Cooper pair is transferred and an energy eV is gained. The process ends when the quasiparticle reaches the gap energy and therefore escape into the superconductor. A possible charge transfer mechanism at low voltage ($eV \ll 2\Delta$) is the coherent transfer of multiple charge quanta $q* = 2\Delta/V$. In such a case a strongly enhanced shot noise is expected at low voltage.[50] Serious indications of this enhanced shot noise mechanism have been obtained recently by Dieleman et al.[51] and Hoss et al..[52] More experiments are needed to check the nature of the effect.

7. The density of states

The density of states is a meaningful quantity which has been first investigated in the early days of proximity superconductivity.[53] The samples were made of a tunnel barrier in contact of the N-side of a N-S bilayer with a highly transparent interface. The recent development of new fabrication techniques enabled a new kind of experiment where the density of states can be probed at distinct locations.[54] The density of states shows a depression at the Fermi energy with a characteristic energy of order of the Thouless energy. The experiments have been successfully compared to calculations from the Usadel equations[55] provided a noticeably large spin-flip rate is included. When the normal metal is a "closed system" disconnected from any electron reservoirs a true energy gap is expected provided that the system is disordered or chaotic.[56] For example in a S-N-S junction where N is a small island, a mini-gap is expected with a strong dependence on the phase difference between the two superconducting electrodes.[43,57] In the ballistic case, special trajectories lead to low energy sub-gap states which may depend on the dimensionality[15] or on the shape of the N-metal.[56] New techniques such as local spectroscopy with a low-temperature Scanning Tunneling Microscope are welcome for getting more insight into the spatial and phase dependence of the density of states in N-S structures.

8. The Josephson effect in SNS structures

Let us consider an ideal S-N-S structure which consists of a normal metal N in-between two superconductors S1 and S2 with respective phases $+\Phi/2$ and $-\Phi/2$. Because of the confinement induced by the superconducting gap, the electron energy levels in the normal metal consist of phase dependent Andreev bound-states which, as predicted long ago by Kulik,[58] can carry a finite supercurrent. The total supercurrent is given by a summation over contribution of the current carrying states which all depend upon the phase difference Φ between the two superconductors. For a system in thermal equilibrium, the occupation probabilities of each state is given by the Fermi-Dirac distribution function. When the phase difference is zero, for each bound state there is another degenerate bound state (time-reversed state) that carries the same current in the opposite direction and therefore the total current is zero.

When the phase difference in a S-N-S junction is non zero, the energies of these states are different giving rise to a finite supercurrent at low temperature. At high temperature, since the thermal population of states with opposite current is almost the same the supercurrent is suppressed. The suppression occurs when $k_B T$ is of order of the spacing between states with opposite currents. This picture remains valid in diffusive systems. In this case this spacing is of order of the Thouless energy. Measurements of the Josephson critical current in S-N-S long junctions[59] revealed that the characteristic energy eI_c/G_N at low temperature is of the order of the Thouless energy of the sample.[60] This shows again that the Thouless energy is the relevant energy scale in a diffusive metal in proximity with a superconductor. As the sample L is varied, there is a crossover between the superconducting gap for short junctions ($L < \xi_s$) and the Thouless energy for long junctions ($L > \xi_s$).[61]

9. Effect of non-equilibrium electron distributions

According to the above description of the Josephson effect, one sees that the suppression of the critical current at high temperature results from a compensation of spectral currents with large amplitudes and alternating signs. This exact compensation only occurs for a true thermal equilibrium distribution function. A small deviation from thermal equilibrium of the electron distribution in the normal metal should result in a large enhancement of the critical current.

A non-equilibrium distribution can be induced into the metal by injection of hot electrons in a four terminal device.[62] This steady state method has been used recently to demonstrate the operation of a mesoscopic S-N-S transistor[63] where the supercurrent of the S-N-S junction could be commanded by a control current. These ideas have been implemented experimentally to perform energy spectroscopy of the supercurrent in a S-N-S Josephson junction.[64] At the lowest temperature and for an optimized design of the control reservoirs the sign of the supercurrent could even be reversed, resulting in a π-junction.[65] An enhancement of the supercurrent under non-equilibrium injection was demonstrated recently in a three terminal planar device made of aluminum on GaAs. This non-equilibrium induced supercurrent was found to exceed the equilibrium supercurrent at high temperature.[66]

A non-equilibium distribution can also be obtained dynamically. Let us consider for simplicity a voltage-biased S-N-S junction. According to the Josephson equation the phase difference $\Phi = 2eVt/\hbar$ becomes time-dependent, leading to a fast oscillation of the Andreev levels. Several theoretical predictions have been made recently for this regime. F. Zhou et al.[67] have calculated the conductance enhancement due to the dynamical state in the case of a long ($L > L_T$) junction for which the equilibrium critical current is suppressed. Argaman[68] has considered the non-equilibrium Josephson effect in the same limit. In a recent experiments, K. W. Lehnert et al.[69] have observed half-integer Shapiro steps in a Nb-InAs-Nb junction which persist to high temperatures in the absence of a critical current. The observed power law of their amplitude, which is reminiscent of that of the proximity-induced AB oscillations[34] in a N-S interferometer, was attributed to the combined time-dependent contributions of both the distribution function and the spectral current. Current-voltage measurements in a mesoscopic diffusive metallic S-N-S junction[70] also revealed strong deviations from the resistively-shunted junction model which are due to this non-equilibrium dynamical effect.

10. The Meissner effect

Like a conventional superconductor, a proximity superconductor expulses the magnetic flux : it is the Meissner effect.[1] A. Mota et al. measured a variety of coaxial structures made of a superconducting core (Nb) surrounded by a normal metal envelope (Cu, Ag, Au, ...).[71] These quasi-millimetric samples are in the mesoscopic regime because they are very pure, so that the coherence length are of the order of the sample size. The price to pay is the use of ultra low temperatures (about $100\mu K$) for reaching the interesting regime $L \approx L_T$.

At intermediate temperatures, the observed diamagnetic behaviour is well described by the quasiclassical theory.[72] A puzzling paramagnetic reentrance effect[71] has been demonstrated to appear at very low temperature when the coherence lengthes are of the order of the diameter of the structure. These results motivated a number of theoretical studies. C. Bruder and Y. Imry propose an interpretation in terms of a whispering gallery for electrons which will not "see" the N-S interface,[73] whereas Fauchère et al. propose the existence of a repulsive interaction in the normal metal reponsible for paramagnetic instability at the N-S interface.[74] This topic is still an open question.

11. Ferromagnetic metal-Superconductor systems

As opposed to noble normal metals, itinerant ferromagnetic metals represent an example of a system with strong electron-electron interaction leading to an order state of the electron spins. The Andreev reflection picture is strongly modified because the incoming electron and the Andreev reflected hole occupy opposite spin bands.[75] An immediate consequence is the total suppression of Andreev reflection in a fully spin-polarized metal.[76] This effect has been used recently to measure the degree of spin polarization (P) of various ferromagnetic metals using direct conductance measurements through a superconducting point contact.[77] The spin polarization is obtained directly from the differential conductance using an adaptation of the BTK theory to include spin polarization. With increasing spin polarization the subgap conductance drops from about twice the normal state conductance for non-polarized metals to a small value in the highly polarized metals. Materials with spin polarization up to 90% could be characterized this way. Similar observations were made using nanofabricated contacts between a superconductor and ferromagnetic nanoparticles.[78] This technique gives an alternative method with higher energy resolution than photo-emission and less material constraints as compared to single particle tunneling experiments. Besides this non-perturbative effect, the injection of a spin polarized current[79] was shown to suppress of pairing in a superconductor.

The possible existence of long-range proximity effects in a ferromagnet has been addressed in several puzzling experiments. According to the naive expectation, the strong Fermi wavevector mismatch between spin up and spin down energy bands should lead to a very short effective coherence length inside the ferromagnet. Using the simplified view of a Stoner model with an exchange energy U much larger than the superconducting gap energy, the coherence length is of order of $L_U = \sqrt{\hbar D/2\pi U}$ in a single domain ferromagnet with an electron diffusion coefficient D. For transistion metal such as cobalt, this length is of order of a few nanometers. This short range effect has been investigated intensively in ferromagnetic-superconductor multilayers.[80–82] However, several recent experiments[83–85] have shown strong resistance drops in mesoscopic ferromagnetic structures that cannot be understood by the present models. A re-entrance effect similar to that observed in non-interacting metals (see above) was even observed in Ref. [[85]]. A characteristic length scale of 200 nm was inferred which is compatible with the absence of Aharonov-Bohm oscillations mentioned in this work.

Interesting theoretical predictions have been reported recently. Leadbeater et al.[86] have investigated the subgap conductance in ferromagnetic mesoscopic structures in the case where the exchange energy is of order of the superconducting energy gap. Fazio et al.[87] have computed the local density of states on both sides of the interface. Zhou et al.[88] predicted long-range proximity as due to the penetration of the triplet part of the superconducting condensate wave function in the ferromagnetic metal. This is made possible in the mesoscopic regime in presence of spin-orbit interaction in the superconductor. Several authors have recently stressed an important consequence of the spin polarization which actually leads to an opposite effect.[89,90] Since the total spin of a Cooper pair in the superconductor is zero, there can be no spin current across the interface. Because of the spin polarization of the current in the ferromagnet there must be a spin accumulation near the interface and therefore an enhanced interface resistance.

Obviously these systems call for further investigation in order to understand the origin of the unexpected observations. An extensive characterization of these hybrid structures at the mesoscopic scale is now highly desirable : magnetic domain structure, density of states in F and S, presence of Josephson current in SFS structures, etc.. Also the determination of the significant characteristic lengths (phase memory length, spin memory length, spin orbit diffusion length, ..) needs substantial experimental efforts.

12. Summary and perspectives

The recent years have shown the subtle role of the Andreev reflection in the phenomena arising in structures made of a normal metal in contact with a superconductor. It has been understood that the Thouless energy is the characteristic energy and that the induced correlations in the normal metal decay only as $1/T$. Many aspects including the case of the ferromagnetic metals and the non-equilibrium effects remain unsettled.

Andreev reflection is also a instrument for studying new physical effects like in atomic contacts where it has been possible to measure the transmission of individual channels.[91] Also at the atomic scale, the superconducting proximity effect recently observed in single-walled nanotubes[92] raises interesting questions such as the role of electron-electron interactions in one-dimensional systems. In the future, the consequences of Andreev reflection will likely play a significant role in new areas of mesoscopic physics such as the physics of Charge Density Wave (CDW) systems. These systems are known to develop a macroscopic phase coherent condensed state. Recent experiments on patterned niobium based[93] CDW crystals or blue bronze thin films device[94] have shown novel mesoscopic phenomena.

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