SPECIALIZATION PROJECT IN THEORETICAL PHYSICS

Supercurrent transport by Andreev Bound States in external field

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Abstract

Here you give a summary of your your work and your results. This is like a management summary and should be written in a clear and easy language, without many difficult terms and without abbreviations. Everything you present here must be treated in more detail in the main report. You should not give any references to the report in the summary – just explain what you have done and what you have found out. The Summary and Conclusions should be no more than two pages.

You may assume that you have got three minutes to present to the Rector of NTNU what you have done and what you have found out as part of your thesis. (He is an intelligent person, but does not know much about your field of expertise.)

Acknowledgement

I would like to thank the following persons for their great help during ...

If the project has been carried out in cooperation with an external partner (e.g., a company), you should acknowledge the contribution and give thanks to the involved persons.

You should also acknowledge the contributions made by your supervisor(s).

O.N.

(Your initials)

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Abbreviations

FTA Fault tree analysis

MTTF Mean time to failure

RAMS Reliability, availability, maintainability, and safety

Chapter 1

Introduction

Chapter 2

Superconductivity

Two fundamental properties associated to superconductivity are 1) zero electrical resistance giving rise to *supercurrents*, for temperatures below some critical temperature T_c and 2) complete expulsion of magnetic field below T_c, known as the Meissner effect [1, 2]. The theory behind these properties was presented by Bardeen, Cooper and Schrieffer in 1957 and is known as the BCS-theory [3].

2.1 The Meissner effect

Meissner and Ochsenfeld discovered in 1933 [1] that applied magnetic field, H, below some critical limit H_c, would be expelled in the superconductor for temperatures below T_c, resulting in zero field inside the superconductor, $B = \mu_0(H+M) = 0$, so that $M = -H$. The superconductor is thus a perfect diamagnet with susceptibility

$$\chi = \frac{dM}{dH} = -1. \tag{2.1}$$

This is called the Meissner effect and is a consequence of induced screening supercurrents at the surface of the superconductor. No current can exist only on the surface of a material as this would imply a finite current in a layer of zero thickness requiring infinite density of free charge. Consequently, the screening current must exist at some finite distance, λ_L, into the superconductor and thus letting the external magnetic field penetrate to a depth λ_L. This penetration

depth will depend on the density of superconducting carriers (Cooper pairs) and is a result from the London equations [4] and Ampere's law.

The Meissner effect breaks down as the external field is increased to above the critical limit H_c. Depending on the material we will then get full (in type I superconductors) or partial (in type II superconductors) penetration of magnetic flux and the superconductor will go from the superconducting state into the normal or mixed state, respectively.

2.2 BCS theory

The BCS theory is based on the appearance of so called *Cooper pairs* which conventionally are formed by a phonon-mediated attractive interaction between two electrons overwinning the Coulomb repulsion [3]. The Cooper pairs are bosonic...

2.2.1 The BCS Hamiltonian

The Hamiltonian of the system will consist of two parts, describing the non-interacting and interacting electrons, respectively. A given state is defined by the momentum $\mathbf{k}$ and spin σ. In the second quantization formalism the annihilation- and creation operators, $c_{\mathbf{k},\sigma}$ and $c^\dagger_{\mathbf{k},\sigma}$, will destroy and create an electron in the corresponding state, respectively. The number operator $n_{\mathbf{k},\sigma} = c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$ counts the number of electrons in the state. The non-interacting part of the Hamiltonian will simply be the energy of each state, $\epsilon_{\mathbf{k}} = \hbar^2 k^2/2m$, times the number operator and summed over all states. This will thus be the first term in Hamiltonian (2.2). The interacting part of the Hamiltonian will describe a scattering process where two electrons into the states ($\mathbf{k}$, σ) and ($\mathbf{k}'$,σ') are scattered to the states ($\mathbf{k}+\mathbf{q}$, σ) and ($\mathbf{k}'-\mathbf{q}$, σ'), i.e. ($\mathbf{k}$, σ) and ($\mathbf{k}+\mathbf{q}$, σ) are destroyed by the annihilation operators while ($\mathbf{k}+\mathbf{q}$, σ) and ($\mathbf{k}'-\mathbf{q}$, σ') are created by the creation operators. We must also include a matrix element $V_{\mathbf{k},\mathbf{k}'}$ including both the attractive phonon-mediated interaction and the repulsive Coulomb interaction, between the electrons. The second term in the Hamiltonian (2.2) describe this interaction. The total Hamiltonian in-

cluding both the non-interacting and the interacting term is thus given as

$$H = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\mathbf{k}',\mathbf{q},\sigma,\sigma'} V_{\mathbf{k},\mathbf{k}'}(\mathbf{q},\omega) \mathbf{c}^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma} \mathbf{c}^{\dagger}_{\mathbf{k}'-\mathbf{q},\sigma'} \mathbf{c}_{\mathbf{k},\sigma} \mathbf{c}_{\mathbf{k}',\sigma'}. \tag{2.2}$$

We define $\varepsilon_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}} - \mu$ as the energy above the Fermi surface. We have used the chemical potential, μ, in the place of the Fermi energy, ϵ_F, as these two quantities are essentially the same in all relevant cases. The attractive interaction will only be valid in a small energy range, ω, above the Fermi-surface, and for electrons on opposite sides of the Fermi-surface. We may therefore let $\mathbf{k}' = -\mathbf{k}$. Due to the Pauli principle we will in most cases find the electrons in the Cooper pairs in opposite spin states, so we will also let $\sigma' = -\sigma$. By now changing the dummy indices, the Hamiltonian takes the form

$$H - \mu N = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow} c_{\mathbf{k}',\uparrow} c_{-\mathbf{k}',\downarrow}, \tag{2.3}$$

where N is the number of electrons. Henceforth we will write H in place of $H - \mu N$.

2.2.2 Mean field Approximation

We will use mean field approximation to simplify the Hamiltonian and assume the fluctuations around the expectation values to be small such that we can write

$$c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} = \langle c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} \rangle + c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} - \langle c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} \rangle \equiv \langle c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} \rangle + \delta_{\mathbf{k}}, \tag{2.4}$$

and only keep δ to the first order. By also defining the *gap parameter* as follows

$$\Delta_{\mathbf{k}'} = \sum_{\mathbf{k}} V_{\mathbf{k},\mathbf{k}'} \langle c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} \rangle, \tag{2.5}$$

the Hamiltonian will simplify to

$$\begin{aligned} H &= \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma} + \sum_{\mathbf{k}} \left[\Delta^*_{\mathbf{k}} c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} - \Delta_{\mathbf{k}} \langle c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} \rangle \right] \\ &= -\sum_{\mathbf{k}} \Delta_{\mathbf{k}} \langle c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} \rangle + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \left[c^\dagger_{\mathbf{k},\uparrow} c_{\mathbf{k},\uparrow} + c^\dagger_{-\mathbf{k},\downarrow} c_{-\mathbf{k},\downarrow} \right] + \sum_{\mathbf{k}} \left[\Delta^*_{\mathbf{k}} c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} \right] \\ &= \sum_{\mathbf{k}} \left[\varepsilon_{\mathbf{k}} - \Delta_{\mathbf{k}} \langle c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} \rangle \right] + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} \left[c^\dagger_{\mathbf{k},\uparrow} c_{\mathbf{k},\uparrow} - c_{-\mathbf{k},\downarrow} c^\dagger_{-\mathbf{k},\downarrow} \right] + \sum_{\mathbf{k}} \left[\Delta^*_{\mathbf{k}} c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} \right] \\ &= E_0 + \sum_{\mathbf{k}} \begin{pmatrix} c^\dagger_{\mathbf{k},\uparrow} & c_{-\mathbf{k},\downarrow} \end{pmatrix} \begin{pmatrix} \varepsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & -\varepsilon_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k},\uparrow} \\ c^\dagger_{-\mathbf{k},\downarrow} \end{pmatrix} \equiv E_0 + \sum_{\mathbf{k}} \varphi'^\dagger_{\mathbf{k}} H'_{\mathbf{k}} \varphi'_{\mathbf{k}} \end{aligned} \tag{2.6}$$

where we have used the standard commutation relations for fermions (A.1c) and defined

$$E_0 \equiv \sum_{\mathbf{k}} \left[\varepsilon_{\mathbf{k}} - \Delta_{\mathbf{k}} \langle c^\dagger_{\mathbf{k},\uparrow} c^\dagger_{-\mathbf{k},\downarrow} \rangle \right], \quad H'_{\mathbf{k}} = \begin{pmatrix} \varepsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & -\varepsilon_{\mathbf{k}} \end{pmatrix} \quad \text{and} \quad \varphi'_{\mathbf{k}} \equiv \begin{pmatrix} c_{\mathbf{k},\uparrow} \\ c^\dagger_{-\mathbf{k},\downarrow} \end{pmatrix}.$$

The Hamiltonian (2.6) can be diagonalized by inserting $U_{\mathbf{k}} U^\dagger_{\mathbf{k}} = I$, where U is a unitary matrix:

$$U_{\mathbf{k}} = \begin{pmatrix} u_{\mathbf{k}} & -v^*_{\mathbf{k}} \\ v_{\mathbf{k}} & u^*_{\mathbf{k}} \end{pmatrix}, \qquad U^\dagger_{\mathbf{k}} = \begin{pmatrix} u^*_{\mathbf{k}} & v^*_{\mathbf{k}} \\ -v_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \tag{2.7}$$

and $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ satisfy the relation

$$|u_{\mathbf{k}}|^2 + |v_{\mathbf{k}}|^2 = 1. \tag{2.8}$$

This will be satisfied if we write $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ on the form

$$u_{\mathbf{k}} = e^{i\alpha} \cos\theta_{\mathbf{k}}, \qquad v_{\mathbf{k}} = e^{i\beta} \sin\theta_{\mathbf{k}}. \tag{2.9}$$

Our Hamiltonian will now be on the form

$$H = E_0 + \sum_{\mathbf{k}} \varphi^\dagger_{\mathbf{k}} H_{\mathbf{k}} \varphi_{\mathbf{k}} \tag{2.10}$$

with $H_{\mathbf{k}} = U_{\mathbf{k}}^{\dagger} H'_{\mathbf{k}} U_{\mathbf{k}}$ and $\varphi_{\mathbf{k}} \equiv U_{\mathbf{k}}^{\dagger} \varphi'_{\mathbf{k}}$, i.e.

$$\varphi_{\mathbf{k}} \equiv \begin{pmatrix} \gamma_{\mathbf{k},\uparrow} \\ \gamma^{\dagger}_{-\mathbf{k},\downarrow} \end{pmatrix} = \begin{pmatrix} u^*_{\mathbf{k}} & v^*_{\mathbf{k}} \\ -v_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k},\uparrow} \\ c^{\dagger}_{-\mathbf{k},\downarrow} \end{pmatrix}. \tag{2.11}$$

The new fermionic operators $\gamma_{\mathbf{k},\uparrow}$ and $\gamma^{\dagger}_{-\mathbf{k},\downarrow}$ are describing excitations of so called *quasiparticles*.

2.2.3 Diagonalization of the BCS Hamiltonian

We need to find what values of $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ that will satisfy the relation (2.8) and diagonalize $H_{\mathbf{k}}$:

$$\begin{aligned} H_{\mathbf{k}} = U_{\mathbf{k}}^{\dagger} H'_{\mathbf{k}} U_{\mathbf{k}} &= \begin{pmatrix} u^*_{\mathbf{k}} & v^*_{\mathbf{k}} \\ -v_{\mathbf{k}} & u_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} \varepsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} & -\varepsilon_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} u_{\mathbf{k}} & -v^*_{\mathbf{k}} \\ v_{\mathbf{k}} & u^*_{\mathbf{k}} \end{pmatrix} \\ &= \begin{pmatrix} \varepsilon_{\mathbf{k}} \left(|u_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^2 \right) + \Delta_{\mathbf{k}} u^*_{\mathbf{k}} v_{\mathbf{k}} + \Delta^*_{\mathbf{k}} u_{\mathbf{k}} v^*_{\mathbf{k}} & \Delta_{\mathbf{k}} u^{*2}_{\mathbf{k}} - \Delta^*_{\mathbf{k}} v^{*2}_{\mathbf{k}} - 2\varepsilon_{\mathbf{k}} u^*_{\mathbf{k}} v^*_{\mathbf{k}} \\ \Delta^*_{\mathbf{k}} u^2_{\mathbf{k}} - \Delta_{\mathbf{k}} v^2_{\mathbf{k}} - 2\varepsilon_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}} & -\left[\varepsilon_{\mathbf{k}} (|u_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^2) + \Delta_{\mathbf{k}} u^*_{\mathbf{k}} v_{\mathbf{k}} + \Delta^*_{\mathbf{k}} u_{\mathbf{k}} v^*_{\mathbf{k}} \right] \end{pmatrix}. \end{aligned} \tag{2.12}$$

For the off-diagonal elements to be zero we must have $\Delta^*_{\mathbf{k}} u^2_{\mathbf{k}} - \Delta_{\mathbf{k}} v^2_{\mathbf{k}} - 2\varepsilon_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}} = 0$. We write $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ on the form given in equation (2.9) and write $\Delta_{\mathbf{k}} = |\Delta_{\mathbf{k}}| e^{i\varphi}$. This yields

$$\begin{aligned} 0 &= \Delta^*_{\mathbf{k}} u^2_{\mathbf{k}} - \Delta_{\mathbf{k}} v^2_{\mathbf{k}} - 2\varepsilon_{\mathbf{k}} u_{\mathbf{k}} v_{\mathbf{k}} \\ &= |\Delta_{\mathbf{k}}| e^{i(\alpha+\beta)} \cos^2\theta \left(e^{i(\alpha-\beta-\varphi)} - e^{-i(\alpha-\beta-\varphi)} \tan^2\theta_{\mathbf{k}} - 2\frac{\varepsilon_{\mathbf{k}}}{|\Delta_{\mathbf{k}}|} \tan\theta_{\mathbf{k}} \right), \end{aligned}$$

which gives

$$\alpha - \beta = \varphi \qquad \text{and} \qquad \tan\theta_{\mathbf{k}} = -\frac{\varepsilon_{\mathbf{k}}}{|\Delta_{\mathbf{k}}|} \pm \sqrt{\frac{\varepsilon^2_{\mathbf{k}}}{|\Delta_{\mathbf{k}}|^2} + 1}. \tag{2.13}$$

$u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ will thus be satisfied by

$$\begin{aligned} |u_{\mathbf{k}}|^2 &= \cos^2\theta = \frac{1}{2}\left(1 \pm \frac{\varepsilon^{\pm}_{\mathbf{k}}}{\sqrt{\varepsilon^{\pm 2}_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}} \right) \\ |v_{\mathbf{k}}|^2 &= \sin^2\theta = \frac{1}{2}\left(1 \mp \frac{\varepsilon^{\pm}_{\mathbf{k}}}{\sqrt{\varepsilon^{\pm 2}_{\mathbf{k}} + |\Delta_{\mathbf{k}}|^2}} \right). \end{aligned} \tag{2.14}$$

We let $\varepsilon_{\mathbf{k}}^{+} > 0$ and $\varepsilon_{\mathbf{k}}^{-} < 0$ and notice that we get $|u_{\mathbf{k}}| = 1$ and $|v_{\mathbf{k}}| = 0$ when $\Delta_{\mathbf{k}} = 0$, i.e. when there is no attraction between the electrons and thus in the limit of the *normal* state, according to equation (2.5). We calculate the diagonal terms of $H_{\mathbf{k}}$ (2.12) and find

$$H_{\mathbf{k}} = \begin{pmatrix} E_{\mathbf{k}} & 0 \\ 0 & -E_{\mathbf{k}} \end{pmatrix} \tag{2.15}$$

where we have defined

$$E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2}. \tag{2.16}$$

as the quasiparticle excitation energy. It is now clear why $\Delta_{\mathbf{k}}$ is refered to as the *gap-parameter* as it gives a gap in the excitation spectrum of the quasiparticles $\varphi_{\mathbf{k}}$. Moreover, we get

$$k^{\pm} = k_F \sqrt{1 + \frac{\varepsilon_{\mathbf{k}}^{\pm}}{\mu}} = k_F \sqrt{1 \pm \frac{\sqrt{E_{\mathbf{k}}^2 - |\Delta_{\mathbf{k}}|^2}}{\mu}} \tag{2.17}$$

where $\mu = \hbar^2 k_F / 2m$ and $\varepsilon_{\mathbf{k}}^{\pm} = \pm\sqrt{E_{\mathbf{k}} - |\Delta_{\mathbf{k}}|^2}$ is obtained from equation (2.16). We notice how we get a fourfold degeneracy of relevant states, $(k^+, k^-, -k^+, -k^-)$, for each $E_{\mathbf{k}}$. From equation (2.14) we see that the quasiparticle excitation $\gamma_{\mathbf{k},\uparrow}^{\dagger}$ from equation (2.11) will be electronlike, since we have $u_{\mathbf{k}} \to 1$ and $v_{\mathbf{k}} \to 0$ as $\Delta \to 0$ and $c_{\mathbf{k},\uparrow}^{\dagger}$ creates an electron while $c_{-\mathbf{k},\downarrow}$ destroys an electron, leaving a hole. Similarly, $\gamma_{\mathbf{k},\uparrow}$ will be holelike. Moreover, from equation (2.17) we see that $\pm k^+$ ($\pm k^-$) corresponds to energy above(below) the Fermi surface and thus $\pm k^+$ ($\pm k^-$) are electron(hole)-like excitations.

For convenience we introduce a new variable, η, defined in the following way

$$\eta = \begin{cases} \arccos\left(\frac{E_{\mathbf{k}}}{|\Delta_{\mathbf{k}}|}\right), & \text{if } E_{\mathbf{k}} < |\Delta_{\mathbf{k}}| \\ i\,\text{arccosh}\left(\frac{E_{\mathbf{k}}}{|\Delta_{\mathbf{k}}|}\right), & \text{if } E_{\mathbf{k}} > |\Delta_{\mathbf{k}}|\,. \end{cases} \tag{2.18}$$

Then we can write

$$\frac{|u_{\mathbf{k}}|}{|v_{\mathbf{k}}|} = e^{i\eta}. \tag{2.19}$$

2.2.4 Bogoliubov-de Gennes Equations

In the description above we assumed the Hamiltonian to be position-invariant so that the wave functions could be considered as simple plane waves, $\sim \exp(i\mathbf{k}\cdot\mathbf{r})$. We took the potential $V(\boldsymbol{r})$ and the vector potential, $\boldsymbol{A}$, to be zero and the simply replaced the Hamiltonian for a single particle system,

$$h(\mathbf{r}) = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2 - \mu(\mathbf{r}) + V(\mathbf{r}), \tag{2.20}$$

with $\varepsilon_{\mathbf{k}} = \hbar^2 k^2/2m - \mu$. For systems where we can not do this simplification we introduce field operators:

$$\psi(\mathbf{r},t) \equiv \sum_{\mathbf{k}} U(\mathbf{r},t)\varphi_{\mathbf{k}}, \qquad \psi^\dagger(\mathbf{r},t) \equiv \sum_{\mathbf{k}} \varphi^\dagger_{\mathbf{k}} U^\dagger(\mathbf{r},t) \tag{2.21}$$

and rewrite the Hamiltonian in equation (2.6) as

$$H = E_0 + \int d^3r\, \psi^\dagger(\mathbf{r},t)\begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix}\psi(\mathbf{r},t) \equiv E_0 + \int d^3r\, \psi^\dagger(\mathbf{r},t) H(\mathbf{r})\psi(\mathbf{r},t). \tag{2.22}$$

Again the Hamiltonian may be diagonalized by setting $U^\dagger(\mathbf{r},t)H(\mathbf{r})U(\mathbf{r},t) = H_{\mathbf{k}}$, or equally $H(\mathbf{r})U(\mathbf{r},t) = U(\mathbf{r},t)H_{\mathbf{k}}$, where $H_{\mathbf{k}}$ is on the form given in equation (2.15). By separating these equations for each eigenvalue in $H_{\mathbf{k}}$ we get the *Bogoliubov de Gennes equations* (BdG equations) [5]:

$$\begin{aligned} \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix}\begin{pmatrix} u(\mathbf{r},t) \\ v(\mathbf{r},t) \end{pmatrix} &= E_{\mathbf{k}}\begin{pmatrix} u(\mathbf{r},t) \\ v(\mathbf{r},t) \end{pmatrix}, \\ \begin{pmatrix} -h(\mathbf{r}) & -\Delta^*(\mathbf{r}) \\ -\Delta(\mathbf{r}) & h(\mathbf{r}) \end{pmatrix}\begin{pmatrix} -v(\mathbf{r},t) \\ u(\mathbf{r},t) \end{pmatrix} &= E_{\mathbf{k}}\begin{pmatrix} -v(\mathbf{r},t) \\ u(\mathbf{r},t) \end{pmatrix}. \end{aligned} \tag{2.23}$$

From equation (2.11) we have $\gamma^\dagger_{\mathbf{k},\uparrow} = u(\mathbf{r},t)c^\dagger_{\mathbf{k},\uparrow} + v(\mathbf{r},t)c_{-\mathbf{k},\downarrow}$. By writing $\gamma^\dagger_{\mathbf{k},\sigma} = u_\sigma(\mathbf{r},t)c^\dagger_{\mathbf{k},\sigma} + v_{-\sigma}(\mathbf{r},t)c_{-\mathbf{k},-\sigma}$ and $\gamma_{\mathbf{k},\sigma} = u_\sigma(\mathbf{r},t)c^\dagger_{-\mathbf{k},-\sigma} + v_{-\sigma}(\mathbf{r},t)c_{\mathbf{k},\sigma}$, we can represent $\gamma^\dagger_{\mathbf{k},\sigma}$ and $\gamma_{\mathbf{k},\sigma}$ by the vectors $\Psi_{e,\sigma}$ and

$\Psi_{h,\sigma}$, respectively, where Ψ is a vector of the form $\left(u_\uparrow, u_\downarrow, v_\uparrow, v_\downarrow\right)^T$. We get:

$$\begin{aligned}
\gamma^\dagger_{\mathbf{k},\uparrow} \to \Psi_{e,\uparrow} &= \begin{pmatrix} u(\mathbf{r},t) \\ 0 \\ 0 \\ v(\mathbf{r},t) \end{pmatrix}, \qquad \gamma^\dagger_{-\mathbf{k},\downarrow} \to \Psi_{e,\downarrow} = \begin{pmatrix} 0 \\ u(\mathbf{r},t) \\ -v(\mathbf{r},t) \\ 0 \end{pmatrix}, \\
\gamma_{\mathbf{k},\uparrow} \to \Psi_{h,\uparrow} &= \begin{pmatrix} v^*(\mathbf{r},t) \\ 0 \\ 0 \\ u^*(\mathbf{r},t) \end{pmatrix}, \qquad \gamma_{-\mathbf{k},\downarrow} \to \Psi_{h,\downarrow} = \begin{pmatrix} 0 \\ -v^*(\mathbf{r},t) \\ u^*(\mathbf{r},t) \\ 0 \end{pmatrix}.
\end{aligned} \tag{2.24}$$

We define the 2×2-matrices $\hat{H}(\mathbf{r}) \equiv \hat{\sigma}_0 h(\mathbf{r})$ and $\hat{\Delta}(\mathbf{r}) \equiv i\hat{\sigma}_2\Delta(\mathbf{r})$ where $\hat{\sigma}_0$ is the identity matrix and $\hat{\sigma}_i$ with $(i = 1,2,3)$ are the Pauli matrices, see equation (A.2) in the appendix. Moreover, we define $\vec{u}(\mathbf{r},t) \equiv \begin{pmatrix} u_\uparrow(\mathbf{r},t) & u_\downarrow(\mathbf{r},t) \end{pmatrix}^T$ and $\vec{v}(\mathbf{r},t) \equiv \begin{pmatrix} v_\uparrow(\mathbf{r},t) & v_\downarrow(\mathbf{r},t) \end{pmatrix}^T$. The BdG-equations (2.23) can then be written more compact:

$$\begin{pmatrix} \hat{H}(\mathbf{r}) & \hat{\Delta}(\mathbf{r}) \\ \hat{\Delta}^\dagger(\mathbf{r}) & -\hat{H}(\mathbf{r}) \end{pmatrix} \begin{pmatrix} \vec{u}(\mathbf{r},t) \\ \vec{v}(\mathbf{r},t) \end{pmatrix} = E_{\mathbf{k}} \begin{pmatrix} \vec{u}(\mathbf{r},t) \\ \vec{v}(\mathbf{r},t) \end{pmatrix}. \tag{2.25}$$

2.3 Andreev reflection

When an electron with momentum, $\mathbf{k}^+ = k_x^+\hat{x} + k_y^+\hat{y} + k_z^+\hat{z}$, and spin, σ, in the normal metal is propagating towards the interface between the normal metal and a superconductor, it will be scattered with certain probabilities of transmission and reflection. We choose the coordinate system such that the intersection is placed in the yz-plane, see figure !!!REFFIG!!!. There are two possible ways the electron could be transmitted and reflected. The electron may be transmitted into the superconductor as an electron-like quasiparticle such that the energy of the transmitted quasiparticle is on the *same* side of the Fermi surface, i.e. with momentum $\mathbf{q}^+ = q_x^+\hat{x} + q_y^+\hat{y} + q_z^+\hat{z}$ and spin σ, or as a hole-like quasiparticle by crossing the Fermi surface, i.e. with momentum $-\mathbf{q}^- = -q_x^-\hat{x} - q_y^-\hat{y} - q_z^-\hat{z}$ and spin σ. We have negative sign since the wave direction of a hole is opposite of the direction of its wave vector, as explained in section 2.2.3 !!!OBS PASS PÅ AT

DETTE ER FORKLART RETT STED!!!. The electron may be reflected, either in the normal way, i.e. as an electron with momentum, $\mathbf{k}_r^+ = -k_x^+\hat{x} + k_y^+\hat{y} + k_z^+\hat{z}$, and the same spin, σ, or by *Andreev reflection* [6]. In Andreev reflection the incoming electron goes into the superconductor and form a Cooper pair with an electron of opposite spin, leaving a reflected hole with momentum $\mathbf{k}^-$ and spin $-\sigma$. We will here ignore the spin degeneracy and express the wave vectors as $\psi(\mathbf{r}) = \begin{pmatrix} u_\mathbf{k}(\mathbf{r}) & v_\mathbf{k}(\mathbf{r}) \end{pmatrix}^T$. In the simplest case we consider plane waves, i.e. energies, $E_\mathbf{k}$, as given in equation (2.16) with corresponding wave numbers, $\mathbf{k}^\pm$ and wave vectors of the form [7]

$$\psi_{k^+}(\mathbf{r}) = \begin{pmatrix} u_0 e^{i\alpha} \\ v_0 e^{i\beta} \end{pmatrix} e^{i\mathbf{k}^+\mathbf{r}} \quad \text{and} \quad \psi_{k^-}(\mathbf{r}) = \begin{pmatrix} v_0 e^{-i\beta} \\ u_0 e^{-i\alpha} \end{pmatrix} e^{i\mathbf{k}^-\mathbf{r}}. \tag{2.26}$$

The incoming, reflected and transmitted wave vectors will in this notation take the form

$$\begin{aligned} \psi_i(\mathbf{r}) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i\mathbf{k}^+\mathbf{r}} \\ \psi_r(\mathbf{r}) &= r_{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{i\mathbf{k}_r^+\mathbf{r}} + r_{eh} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\mathbf{k}^-\mathbf{r}} \\ \psi_t(\mathbf{r}) &= t_{ee} \begin{pmatrix} u_0 e^{i\alpha} \\ v_0 e^{i\beta} \end{pmatrix} e^{i\mathbf{q}^+\mathbf{r}} + t_{eh} \begin{pmatrix} v_0 e^{-i\beta} \\ u_0 e^{-i\alpha} \end{pmatrix} e^{-i\mathbf{q}^-\mathbf{r}}, \end{aligned} \tag{2.27}$$

where r_{ee}, r_{eh}, t_{ee}, and t_{eh} represent the probabilities of normal reflection, Andreev reflection, electron-like transmission and hole-like transmission with crossing of the Fermi surface, respectively. We notice how the normal reflection will give opposite value of the x-component of the wave vector while the others remain the same, like one expects in a normal reflection. In the Andreev reflection, however, we have retro reflection and the hole will thus move along the same path as the incoming electron. See figure !!!!FIGREF!!!!.

In equation (2.16) we found that only energies above the energy gap, $|\Delta_\mathbf{k}|$, are allowed for the quasiparticles. Consequently, when $E_\mathbf{k} < |\Delta_\mathbf{k}|$ the amplitudes t_{ee} and e_{eh} will be zero and only reflection (either normal or Andreev reflection) is allowed. States with such energies in SNS-junctions would thus be trapped in the normal metal and the states reflected via Andreev reflec-

tion is referred to as Andreev Bound States (ABS).

2.4 Josephson current

A Josephson junction is a device consisting of two superconductors that is brought into contact via a *weak link*, in which the *critical current* is much lower. The critical current is the maximum supercurrent that can exist in the superconductor and is related to the density of Cooper pairs. Josephson effect describes two important phenomena of supercurrents in a Josephson junction [8]. Firstly Josephson predicted that supercurrents would flow through the Josephson junction even without any applied voltage. Secondly if the junction was driven by an external current exceeding the critical current, electromagnetic waves would be radiated. We will here focus on the first phenomena.

There are several ways to construct a weak link, and we will in this project consider the *SNS-junction*, i.e. a junction consisting of two superconductors, separated by a normal metal.

The Josephson current is in general dependent on the full quasiparticle spectrum, but in the short junction regime the continuous spectrum ($E > \Delta_0$) does not contribute to the current. blblbabla

The number operator, N, of the Cooper pairs in the superconductor, and the superconducting phase φ are canonical conjugate variables. Hence

$$\dot{N} = -\frac{1}{\hbar}\frac{\partial H}{\partial \varphi} \qquad \dot{\varphi} = \frac{1}{\hbar}\frac{\partial H}{\partial N}. \tag{2.28}$$

The tunnelig current from a superconductor S_1 with number of particles N_1 to a superconductor S_2 with number of particles N_2 through a weak link will be given as

$$I = q\dot{N}_1 = -q\dot{N}_2 \tag{2.29}$$

where $q = -2e$ is the charge of a Cooper pair. Using equation (2.28) in this expression gives

$$I = \frac{2e}{\hbar}\frac{\partial H}{\partial \varphi_1} = -\frac{2e}{\hbar}\frac{\partial H}{\partial \varphi_2}. \tag{2.30}$$

The phase difference is defined $\Delta\varphi = \varphi_1 - \varphi_2$, and as only the phase difference, not the individual phases, has physical meaning, we let $\partial\varphi_1 \to \partial\Delta\varphi$ and $\partial\varphi_2 \to -\partial\Delta\varphi$. Hence

$$I = \frac{2e}{\hbar}\frac{\partial H}{\partial(\Delta\varphi)}. \tag{2.31}$$

Taking the expectation value of this gives

$$I = \frac{2e}{\hbar}\frac{\partial F}{\partial(\Delta\varphi)} \tag{2.32}$$

where F is the free energy, since

$$\frac{\partial F}{\partial(\Delta\varphi)} = -\frac{1}{\beta}\frac{1}{Z}\frac{\partial Z}{\partial(\Delta\varphi)} = -\frac{1}{\beta Z}\mathrm{Tr}\left[-\beta\frac{\partial H}{\partial(\Delta\varphi)}e^{-\beta H}\right] = \frac{1}{Z}\mathrm{Tr}\left[\frac{\partial H}{\partial(\Delta\varphi)}e^{-\beta H}\right] = \left\langle\frac{\partial H}{\partial(\Delta\varphi)}\right\rangle \tag{2.33}$$

with Z as the partition function:

$$Z = e^{-\beta F} = \mathrm{Tr}\left[e^{-\beta H}\right]. \tag{2.34}$$

This shows how the current is *phase-driven*. blablabla

2.5 Free Energy

The diagonal Hamiltonian in equation (2.10) is on the form as a free fermion gas:

$$\begin{aligned} H &= E_0 + \sum_{\mathbf{k}}\begin{pmatrix}\gamma^\dagger_{\mathbf{k},\uparrow} & \gamma_{-\mathbf{k},\downarrow}\end{pmatrix}\begin{pmatrix}E_{\mathbf{k}} & 0\\ 0 & -E_{\mathbf{k}}\end{pmatrix}\begin{pmatrix}\gamma_{\mathbf{k},\uparrow}\\ \gamma^\dagger_{-\mathbf{k},\downarrow}\end{pmatrix} \\ &= E_0 + \sum_{\mathbf{k}}\left[E_{\mathbf{k}}\gamma^\dagger_{\mathbf{k},\uparrow}\gamma_{\mathbf{k},\uparrow} - E_{\mathbf{k}}\left(1-\gamma^\dagger_{-\mathbf{k},\downarrow}\gamma_{-\mathbf{k},\downarrow}\right)\right] \\ &= E_0 + \sum_{\mathbf{k}} E_{\mathbf{k}}\left(N_\uparrow + N_\downarrow - 1\right) \end{aligned} \tag{2.35}$$

and the partition function of the system will be

$$\begin{aligned} Z &= \sum e^{-\beta H} = e^{-\beta E_0} \sum_{N_\uparrow, N_\downarrow} e^{-\beta \sum_k E_{\mathbf{k}}(N_\uparrow + N_\downarrow - 1)} = e^{-\beta E_0} \prod_{\mathbf{k}} e^{\beta E_{\mathbf{k}}} \sum_{N_\uparrow} e^{-\beta E_{\mathbf{k}} N_\uparrow} \sum_{N_\downarrow} e^{-\beta E_{\mathbf{k}} N_\downarrow} \\ &= e^{-\beta E_0} \prod_{\mathbf{k}} e^{\beta E_{\mathbf{k}}} \left(1 + e^{-\beta E_{\mathbf{k}}}\right)^2 = e^{-\beta E_0} \prod_{\mathbf{k}} \left(2\cosh\left(\frac{\beta E_{\mathbf{k}}}{2}\right)\right)^2 \end{aligned} \tag{2.36}$$

This expression may now be used to find the free energy of the system:

$$F = -\frac{1}{\beta}\ln(Z) = E_0 - 2k_B T \sum_{\mathbf{k}} \ln\left[2\cosh\left(\frac{E_{\mathbf{k}}}{2k_B T}\right)\right]. \tag{2.37}$$

Chapter 3

Physical System

We consider a two-dimensional SNS-junction in the xy-plane and parallel to the x-axis, with $x = -L/2$ and $x = L/2$ at interface between the normal metal and the leftmost (L) and rightmost (R) superconductor, respectively, see Fig. !!!REFFIG!!!. We use the position-space representation $\Psi(x,y) = \begin{pmatrix}\vec{u}(x,y) & \vec{v}(x,y)\end{pmatrix}^T$ as used in equation (2.25). We consider s-wave superconductors such that the gap parameter, $\Delta(x)$, is constant in each superconductor, with equal magnitude, Δ_0, but allow for different phases, ϕ_L and ϕ_R. Necessarily, the gap parameter is zero in the normal metal. The overall gap parameter is

$$\Delta(x) = \Delta_0 \left(e^{i\phi_L}\Theta(-L/2 - x) + e^{i\phi_R}\Theta(x - L/2) \right), \tag{3.1}$$

where $\Theta(x)$ is the Heaviside step function. The Hamiltonian will be on the form given in equation (2.20). We allow for different chemical potential, μ_S and μ_N, but we assume the effective mass to be equal in the superconductor and the normal metal, i.e. $m_S = m_N \equiv m$. Moreover, we let $V(x)$ be a delta-potential barrier at the interfaces and allow for different strength, i.e. $V(x) = V_L\delta(x + L/2) + V_R\delta(x - L/2)$. The overall Hamiltonian is then

$$h(x,y) = h_S(x,y)\big(\Theta(-L/2 - x) + \Theta(x - L/2)\big) + h_N(x,y)\Theta(x + L/2)\Theta(L/2 - x) + V_L\delta(x) + V_R\delta(x - L) \tag{3.2}$$

where $h_{S/N}(x,y) = \frac{1}{2m}\left(-i\hbar\nabla - q\mathbf{A}(x,y)\right)^2 - \mu_{S/N}$ and $\mathbf{A}$ is the vector potential allowing for an external magnetic field.

We will look at three different situations. First we will consider the system when there is no external field and no barriers at the intersections. Secondly, we will include the barriers but keep the external field to zero. Finally, we will include an external field, but have transparent barriers. Our goal is to find the ABS-energies in each situation and use this to find the Josephson current.

Chapter 4

Andreev Bound State energies in SNS-junction

In this chapter we will find the ABS energies in the four situations.

4.1 ABS energies without barriers or applied field

We will first consider the system with no applied field ($\mathbf{A} = 0$) and no barriers ($V_L = V_R = 0$).

4.1.1 Andreev reflection amplitude

The probability amplitudes from chapter section 2.3 may be found by using the boundary conditions in the interface between the normal metal and the super conductor. With no barriers the boundary conditions yields

$$\begin{aligned} \psi_i(0,y) + \psi_r(0,y) &= \psi_t(0,y) \\ \frac{\partial}{\partial x}\psi_i(0,y) + \frac{\partial}{\partial x}\psi_r(0,y) &= \frac{\partial}{\partial x}\psi_t(0,y). \end{aligned} \tag{4.1}$$

Solving these equations give the amplitudes

$$\begin{aligned} r_{eh} &= \frac{2e^{-i\varphi}}{\frac{u_0}{v_0}\frac{k_x^- + q^-}{k_x^+ + k_x^-}\left(1 + \frac{k_x^+}{q_x^+}\right) + \frac{v_0}{u_0}\frac{k_x^+ - q_x^-}{k_x^+ + k_x^-}\left(1 - \frac{k_x^-}{k_x^+}\right)} \\ r_{ee} &= \left(\frac{u_0}{v_0}\frac{k_x^+ + q^-}{k_x^+ + k_x^-} + \frac{v_0}{u_0}\frac{k_x^+ - q^-}{k_x^+ + k_x^-}\right)e^{i\varphi} r_{eh} \\ t_{ee} &= \frac{1}{v_0}\frac{k_x^- + q_x^-}{k_x^+ + k_x^-}e^{-i\beta} r_{eh} \\ t_{eh} &= \frac{1}{u_0}\frac{k_x^+ - q_x^-}{k_x^+ + k_x^-}e^{i\alpha} r_{eh}, \end{aligned} \tag{4.2}$$

where $\varphi = \alpha - \beta$ is the phase of the gap parameter as shown in section 2.2.3. Since we are considering Andreev Bound States we are especially interested in the r_{eh}-amplitude, and see that it's expression may be simplified if we use $k_x^\pm \approx q_x^\pm$:

$$r_{eh} = \frac{v_0}{u_0}e^{-i\varphi} \equiv e^{-i\eta}e^{-i\varphi}, \tag{4.3}$$

where η is as defined in equation (2.18). Similarly the amplitude of an incoming hole which is Andreev reflected as an electron will be $r_{he} = r_{eh}^*$:

$$r_{he} = \frac{v_0}{u_0}e^{i\varphi} = e^{-i\eta}e^{i\varphi}. \tag{4.4}$$

Consequently, the Andreev reflection of an electron (hole) give a phase shift of $-\eta \mp \varphi$.

4.1.2 Bohr-Sommerfeld quantization

For the Andreev Bound states we have $E_\mathbf{k} < \Delta_0$ which yields $\eta = \arccos(E_\mathbf{k}/\Delta_0)$,according to equation(2.18). The Bohr-Sommerfeld quantization condition require the total phase obtained by the state in a whole cycle to be a multiple of 2π. An electron starting at the left interface traveling towards the right interface with an angle θ, see fig. !!!REF!!!!, would gain a phase of $L(k_x^+ + k_y^+ \tan\theta)$ before it is Andreev reflected at the interface with the amplitude r_{eh} and thus is gaining a phase of $\eta - \varphi$. The state would then continue as a hole traveling back along the same trajectory, and thus obtaining a phase of $-L(k_x^- + k_y^- \tan\theta)$. Hence, the total phase in the

quantization condition gives

$$\begin{aligned} 2\pi n = \oint d\phi &= \int_L^R \pm\boldsymbol{k}^{\pm}\cdot d\boldsymbol{l} + \phi^R_{(eh)(he)} + \int_R^L \pm\boldsymbol{k}^{\mp}\cdot d\boldsymbol{l} + \phi^L_{(he)(eh)} \\ &= L(k_x^+ - k_x^-) + L\tan\theta(k_y^+ - k_y^-) - 2\eta \pm \Delta\varphi \end{aligned} \tag{4.5}$$

where $\phi^{R/L}_{(eh)(he)} = -\eta \mp \varphi_{R/L}$ is the phase from Andreev reflection of a electron (hole) and we have defined the phase difference $\Delta\varphi \equiv \varphi_L - \varphi_R$. Again we let $k_x^+ \approx k_x^-$ and $k_y^+ \approx k_y^-$ such that the two first terms vanish and we simply get

$$2\pi n = -2\eta \pm \Delta\varphi. \tag{4.6}$$

4.1.3 ABS energy

We can then find the Andreev energy levels:

$$E_{\mathbf{k}} = \Delta_0 \cos\eta = \Delta_0 \cos\left(\frac{\Delta\varphi}{2}\right). \tag{4.7}$$

4.2 ABS energies with barriers

We will in this section allow for barriers. In this case the Andreev reflection amplitude will be more complicated so that we can not express it as a phase shift like we did above. We will instead find the energies by setting up the wave-functions in each region, insert the wave-functions in the boundary conditions and solve the system. The Hamiltonian of the system will be as given in equation (3.2), but with no external field so that we can set $\boldsymbol{A}$ to zero. Hence, $h_{S/N}(x,y) = -\hbar^2\nabla^2/2m - \mu_{S/N}$.

4.2.1 Boundary conditions

Charge conservation yields continuous wave-functions at the interfaces:

$$\begin{aligned}\Psi_L(-L/2,y) &= \Psi_N(-L/2,y) \equiv \Psi(-L/2,y),\\ \Psi_R(L/2,y) &= \Psi_N(L/2,y) \equiv \Psi(L/2,y).\end{aligned} \tag{4.8}$$

We find the boundary conditions for the derivatives by integrating the BdG-equations (2.25):

$$\begin{aligned}
0 &= \lim_{\epsilon\to 0}\int_{-L/2-\epsilon}^{-L/2+\epsilon} E_{\mathbf{k}}\Psi(x,y)dx = \lim_{\epsilon\to 0}\int_{-L/2-\epsilon}^{-L/2+\epsilon}\begin{pmatrix}\hat{H}(x,y) & \hat{\Delta}(x)\\ \hat{\Delta}^\dagger(x) & -\hat{H}(x,y)\end{pmatrix}\Psi(x,y)dx\\
&= \lim_{\epsilon\to 0}\int_{-L/2-\epsilon}^{-L/2^-}\begin{pmatrix}h_S(x,y)\hat{\sigma}_0 & i\Delta(x)\hat{\sigma}_2\\ -i\Delta^*(x)\hat{\sigma}_2 & -h_S(x,y)\hat{\sigma}_0\end{pmatrix}\Psi_L(x,y)dx + \begin{pmatrix}V_L\hat{\sigma}_0 & 0\\ 0 & -V_L\hat{\sigma}_0\end{pmatrix}\Psi(-L/2,y)\\
&\quad + \lim_{\epsilon\to 0}\int_{-L/2^+}^{-L/2+\epsilon}\begin{pmatrix}h_N(x)\hat{\sigma}_0 & 0\\ 0 & -h_S(x)\hat{\sigma}_0\end{pmatrix}\Psi_N(x,y)dx\\
&= \begin{pmatrix}\hat{\sigma}_0 & 0\\ 0 & -\hat{\sigma}_0\end{pmatrix}\left(V_L\Psi(-L/2,y) - \frac{\hbar^2}{2m}\lim_{\epsilon\to 0}\left(\int_{-L/2-\epsilon}^{-L/2^-}\frac{\partial^2}{\partial x^2}\Psi_L(x,y)dx + \int_{-L/2^+}^{-L/2+\epsilon}\frac{\partial^2}{\partial x^2}\Psi_N(x,y)dx\right)\right)\\
&= \begin{pmatrix}\hat{\sigma}_0 & 0\\ 0 & -\hat{\sigma}_0\end{pmatrix}\left(V_L\Psi(-L/2,y) - \frac{\hbar^2}{2m}\left(\frac{\partial\Psi_N}{\partial x}\Big|_{x=-L/2} - \frac{\partial\Psi_L}{\partial x}\Big|_{x=-L/2}\right)\right),
\end{aligned}$$

and the boundary condition at the left interface for the derivatives is thus

$$\frac{\partial\Psi_N}{\partial x}\Big|_{x=-L/2} - \frac{\partial\Psi_L}{\partial x}\Big|_{x=-L/2} = \frac{2m}{\hbar^2}V_L\Psi(-L/2,y). \tag{4.9}$$

Similarly, we get

$$\frac{\partial\Psi_R}{\partial x}\Big|_{x=L/2} - \frac{\partial\Psi_N}{\partial x}\Big|_{x=L/2} = \frac{2m}{\hbar^2}V_R\Psi(L/2,y). \tag{4.10}$$

as boundary condition at the right interface.

4.2.2 Wave functions in the superconduction region

The solution of the BdG-equations (2.25) that satisfy the boundary conditions will be on the form

$$\Psi_{\mathbf{k}^\pm}(\boldsymbol{r}) = \begin{pmatrix} \vec{u}_{\mathbf{k}} \\ \vec{v}_{\mathbf{k}} \end{pmatrix} e^{i\boldsymbol{k}^\pm \cdot \boldsymbol{r}}, \tag{4.11}$$

Hence, the time-independent Schrödinger equation yields

$$h(\boldsymbol{r})\Psi_{\mathbf{k}^\pm}(\boldsymbol{r}) = \left(\frac{\hbar^2 k^{\pm 2}}{2m} - \mu\right)\Psi_{\mathbf{k}^\pm}(\boldsymbol{r}) \equiv \varepsilon_k^\pm \Psi_{\mathbf{k}^\pm}(\boldsymbol{r}), \tag{4.12}$$

where $h(\boldsymbol{r})$ is either h_S or h_N, depending on the region. For the eigenvalue problem in the BdG-equations (2.25) we must calculate

$$\begin{aligned} 0 &= \det \begin{pmatrix} \left(\varepsilon_{\mathbf{k}}^\pm - E_{\mathbf{k}}\right)\hat{\sigma}_0 & i\Delta\hat{\sigma}_2 \\ -i\Delta^*\hat{\sigma}_2 & \left(-\varepsilon_{\mathbf{k}}^\pm - E_{\mathbf{k}}\right)\hat{\sigma}_0 \end{pmatrix} \\ &= \left(\varepsilon_{\mathbf{k}}^\pm - E_{\mathbf{k}}\right)^2 \left(\varepsilon_{\mathbf{k}}^\pm + E_{\mathbf{k}} + \frac{\Delta_0^2}{\varepsilon_{\mathbf{k}}^\pm - E_{\mathbf{k}}}\right)^2. \end{aligned} \tag{4.13}$$

As $E_{\mathbf{k}} = \varepsilon_{\mathbf{k}}^\pm$ would give zero in the denominator when Δ_0 is non-zero, the only solution to the above equation (4.13) is

$$E_{\mathbf{k}}^2 = \varepsilon_{\mathbf{k}}^{\pm 2} + \Delta_0^2 \tag{4.14}$$

which agrees with the energies obtained in equation (2.16). $k^\pm$ and $\varepsilon_{\mathbf{k}}^\pm$ will be as in equation (2.17). We will here only consider positive energies, $E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^{\pm 2} + \Delta_0^2}$. For $\varepsilon_{\mathbf{k}}^+ = +\sqrt{E_{\mathbf{k}}^2 - \Delta_0^2} = i\Delta_0 \sin\eta$ we get the (non-normalized) wave-functions describing electronlike quasiparticles:

$$\Psi_{e,\uparrow}^\pm(x,y) = \begin{pmatrix} e^{i(\eta+\varphi)} \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{i(\pm k_x^+ x + k_y^+ y)} \quad \text{and} \quad \Psi_{e,\downarrow}^\pm(x,y) = \begin{pmatrix} 0 \\ e^{i(\eta+\varphi)} \\ -1 \\ 0 \end{pmatrix} e^{i(\pm k_x^+ x + k_y^+ y)}, \tag{4.15}$$

Where $\Psi^{+}_{e,\sigma}$ are right-going waves, while $\Psi^{-}_{e,\sigma}$ are left-going waves. Similarly, for $\varepsilon^{-}_{k} = -\sqrt{E_k^2 - \Delta_0^2} = -i\Delta_0 \sin\eta$ we get the wave-functions describing holelike quasiparticles:

$$\Psi^{\pm}_{h,\uparrow}(x,y) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ e^{i(\eta-\varphi)} \end{pmatrix} e^{i(\pm k_x^- x + k_y^- y)} \quad \text{and} \quad \Psi^{\pm}_{h,\downarrow}(x,y) = \begin{pmatrix} 0 \\ -1 \\ e^{i(\eta-\varphi)} \\ 0 \end{pmatrix} e^{i(\pm k_x^- x + k_y^- y)}, \tag{4.16}$$

where $\Psi^{+}_{h,\sigma}$ are left-going waves, while $\Psi^{-}_{h,\sigma}$ are right-going waves. The direction of the waves is determined from the group velocity:

$$\boldsymbol{v}_g = \frac{1}{\hbar}\frac{\partial E_{\mathbf{k}}}{\partial \boldsymbol{k}} = \frac{\varepsilon^{\pm}_{\mathbf{k}}}{E_{\mathbf{k}}}\frac{\hbar \boldsymbol{k}^{\pm}}{m}. \tag{4.17}$$

We allow for different phases, φ_L and φ_R, in each region. As we will only consider Andreev bound states, i.e. with energies $E_{\mathbf{k}} < \Delta_0$, the wave vectors (2.17) will get imaginary parts and must vanish in the superconductors. Consequently there will be no incoming wave-functions from the superconductor into the normal metal with such energies and we need only to consider the outgoing wave-functions in the superconducting regions. We let $k_y^+ \approx k_y^- \equiv k_y$ and the total wave functions in the left (L) and right (R) region will thus be

$$\begin{aligned} \Psi_L(x - L/2, y) &= \Psi_L(x - L/2) e^{ik_y y} \\ \Psi_R(x + L/2, y) &= \Psi_R(x + L/2) e^{ik_y y} \end{aligned} \tag{4.18}$$

with $\Psi_{L/R}(x \mp L/2)$ defined as

$$\Psi_L(x-L/2) = a_1 \begin{pmatrix} e^{i(\eta+\varphi_L)} \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{-ik_x^+ x} + a_2 \begin{pmatrix} 0 \\ e^{i(\eta+\varphi_L)} \\ -1 \\ 0 \end{pmatrix} e^{-ik_x^+ x} + a_3 \begin{pmatrix} 1 \\ 0 \\ 0 \\ e^{i(\eta-\varphi_L)} \end{pmatrix} e^{ik_x^- x} + a_4 \begin{pmatrix} 0 \\ -1 \\ e^{i(\eta-\varphi_L)} \\ 0 \end{pmatrix} e^{ik_x^- x}$$

$$\Psi_R(x+L/2) = b_1 \begin{pmatrix} e^{i(\eta+\varphi_R)} \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{ik_x^+ x} + b_2 \begin{pmatrix} 0 \\ e^{i(\eta+\varphi_R)} \\ -1 \\ 0 \end{pmatrix} e^{ik_x^+ x} + b_3 \begin{pmatrix} 1 \\ 0 \\ 0 \\ e^{i(\eta-\varphi_R)} \end{pmatrix} e^{-ik_x^- x} + b_4 \begin{pmatrix} 0 \\ -1 \\ e^{i(\eta-\varphi_R)} \\ 0 \end{pmatrix} e^{-ik_x^- x}. \tag{4.19}$$

We have here absorbed a phase factor $\exp(\pm ik_x L/2)$ in the coefficients in order to simplify the boundary equations.

4.2.3 Normal region

In the normal region the gap parameter, $\Delta(x)$, is zero, and so $u_0 = 1$ and $v_0 = 0$ and the eigenvalues are E_k such that $\varepsilon_k^{\pm} = \pm E_k$. The corresponding eigenvectors are

$$\Psi_{e,\uparrow}(\boldsymbol{r}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{i\boldsymbol{k}^+\cdot\boldsymbol{r}}, \quad \Psi_{e,\downarrow}(\boldsymbol{r}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{i\boldsymbol{k}^+\cdot\boldsymbol{r}}, \quad \Psi_{h,\uparrow}(\boldsymbol{r}) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{i\boldsymbol{k}^-\cdot\boldsymbol{r}}, \quad \Psi_{h,\downarrow}(\boldsymbol{r}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{i\boldsymbol{k}^-\cdot\boldsymbol{r}}. \tag{4.20}$$

We must here allow both right- and leftgoing waves and the total wave function in the normal region becomes

$$\Psi_N(x-L/2, y) = \Psi_N(x-L/2)e^{ik_y y} \tag{4.21}$$

with

$$\begin{aligned}\Psi_N(x-L/2) = c_1\begin{pmatrix}1\\0\\0\\0\end{pmatrix}e^{ik^+x} + c_2\begin{pmatrix}1\\0\\0\\0\end{pmatrix}e^{-ik^+x} + c_3\begin{pmatrix}0\\1\\0\\0\end{pmatrix}e^{ik^+x} + c_4\begin{pmatrix}0\\1\\0\\0\end{pmatrix}e^{-ik^+x}\\ + c_5\begin{pmatrix}0\\0\\1\\0\end{pmatrix}e^{ik^-x} + c_6\begin{pmatrix}0\\0\\1\\0\end{pmatrix}e^{-ik^-x} + c_7\begin{pmatrix}0\\0\\0\\1\end{pmatrix}e^{ik^-x} + c_8\begin{pmatrix}0\\0\\0\\1\end{pmatrix}e^{-ik^-x}.\end{aligned} \tag{4.22}$$

4.2.4 ABS energy

We use the boundary conditions found in section 4.2.1:

$$\begin{aligned}&\Psi_L(-L/2) - \Psi_N(-L/2) = 0\\ &\Psi_R(L/2) - \Psi_N(L/2) = 0\\ &\frac{\partial\Psi_N(x)}{\partial x}\Big|_{x=-L/2} - \frac{\partial\Psi_L(x)}{\partial x}\Big|_{x=-L/2} - Z_L k_x \Psi_L(-L/2) = 0\\ &\frac{\partial\Psi_R(x)}{\partial x}\Big|_{x=L/2} - \frac{\partial\Psi_N(x)}{\partial x}\Big|_{x=L/2} - Z_R k_x \Psi_R(L/2) = 0\end{aligned} \tag{4.23}$$

and insert the equations in a homogeneous matrix equation of the form $\mathrm{M}\left(a_1\cdots a_4\ b_1\cdots b_4\ c_1\cdots c_8\right)^T = 0$ where M is a 16×16-matrix, see appendix !!!APPENDIX!!!. We have let $k_x^+ \approx k_x^- \equiv k_x$ and defined the barrier strengths

$$Z_L = \frac{2mV_L}{\hbar^2 k_x^2} \quad \text{and} \quad Z_R = \frac{2mV_R}{\hbar^2 k_x^2}. \tag{4.24}$$

The determinant of M is found to be

$$\det(\mathrm{M}) = \left(8e^{i\eta}\right)^4\left[\sin^2\frac{\Delta\varphi}{2} - (1+\zeta)\sin^2\eta\right]^2 \tag{4.25}$$

which must be zero in order for the equations to have non-trivial solutions, and thus the energy is

$$E = \Delta_0 \cos\eta = \Delta_0 \sqrt{\frac{\cos^2\frac{\Delta\varphi}{2} + \zeta}{\zeta + 1}}. \tag{4.26}$$

In these equations we have let $\Delta\varphi = \varphi_L - \varphi_R$ be the phase difference between the left and right semiconductor. We have also introduced ζ which measures the effect of the barriers and is given as

$$\zeta = Z^2 + z^2 \sin(k_F L)\left[Z\cos(k_F L) + \left(\frac{z^2}{4} - 1\right)\sin(k_F L)\right] \tag{4.27}$$

where Z and z are defined as

$$Z = \frac{Z_L + Z_R}{2} \quad \text{and} \quad z = \sqrt{Z_L Z_R}. \tag{4.28}$$

In the limit with no barrier, i.e. $\zeta = 0$ we see that equation (4.26) yields (4.7).

4.3 ABS energies with applied field

We will now apply a magnetic field, $\vec{B}$, to the junction, and let the barriers be transparent. This imply modifying the Hamiltonian to

$$h_N(\boldsymbol{r}) = \frac{1}{2m_N}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}(\boldsymbol{r})\right)^2 + q\varphi - \mu_N \tag{4.29}$$

with $\boldsymbol{A}(\boldsymbol{r})$ as the vector potential and φ as the scalar potential.

Our strategy now is the same as in section 4.1. We want to express the problem in the phase of the wave function and use the quantization condition to find the energy.

4.3.1 Guage transformation

As we must have Gauge invariance we may do the transformation

$$\begin{aligned} \boldsymbol{A}' &= \boldsymbol{A} + \nabla f \\ \varphi' &= \varphi - \frac{\partial f}{\partial t} \end{aligned} \tag{4.30}$$

where f is any function of position and time. Doing such transformation imply a transformation in the wavefunction Ψ as well. Considering the time-dependent Schrödinger equation yields

$$\begin{aligned} i\hbar\frac{\partial \Psi}{\partial t} &= \left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}\right)^2 + q\varphi - \mu_N\right]\Psi \\ &= \left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}' + q\nabla f\right)^2 + q\varphi' + q\frac{\partial f}{\partial t} - \mu_N\right]\Psi \end{aligned} \tag{4.31}$$

which gives

$$i\hbar\frac{\partial \Psi}{\partial t} - q\frac{\partial f}{\partial t}\Psi = \left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}' + q\nabla f\right)^2 + q\varphi' - \mu_N\right]\Psi \tag{4.32}$$

or

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\left(\Psi e^{iqf/\hbar}\right) &= e^{iqf/\hbar}\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}' + q\nabla f\right)^2 + q\varphi'\right]\Psi \\ &= \left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}'\right)^2 + q\varphi'\right]\left(\Psi e^{iqf/\hbar}\right) \end{aligned} \tag{4.33}$$

where we have used that

$$e^{iqf/\hbar}\left(\frac{\hbar}{i}\nabla + q\nabla f\right)\Psi = \frac{\hbar}{i}\nabla\left(e^{iqf/\hbar}\Psi\right). \tag{4.34}$$

The Schrödinger equation in the transformed system is now on the same form as the original system:

$$i\hbar\frac{\partial \Psi'}{\partial t} = \left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}'\right)^2 + q\varphi'\right]\Psi' \tag{4.35}$$

with $\Psi' = e^{iqf/\hbar}\Psi$. Thus a Gauge transformation $\boldsymbol{A} \rightarrow \boldsymbol{A} + \nabla\chi$ imply a transformation $\phi \rightarrow \phi + q\chi/\hbar$ in the phase.

A Gauge invariant phase will be on the form

$$\phi_{\mathrm{GI}} = \phi - \frac{q}{\hbar}\int \boldsymbol{A}\cdot d\boldsymbol{r}, \tag{4.36}$$

as transformation $\phi \to \phi + q\chi/\hbar$ and $\boldsymbol{A} \to \boldsymbol{A} + \nabla\chi$ give

$$\begin{aligned}\phi_{\mathrm{GI}} &\to \phi + \frac{q}{\hbar}\chi - \frac{q}{\hbar}\int \left(\boldsymbol{A}+\nabla\chi\right)\cdot d\boldsymbol{r} = \phi - \frac{q}{\hbar}\int \boldsymbol{A}\cdot d\boldsymbol{r} + \frac{q}{\hbar}\chi - \frac{q}{\hbar}\chi \\ &= \phi - \frac{q}{\hbar}\int \boldsymbol{A}\cdot d\boldsymbol{r}.\end{aligned} \tag{4.37}$$

We have so far taken $\boldsymbol{A}$ to be zero so that $\phi_{\mathrm{GI}} = \phi$. However, now we must include the vector potential in the phase as the magnetic field is non-zero.

4.3.2 Bohr-Sommerfeld quantization condition

Using the same method as in equation (4.5), but with the Gauge invariant phase we get the quantization condition

$$\begin{aligned}2\pi n = \oint d\phi &= \int_L^R \pm\boldsymbol{k}^{\pm}\cdot d\boldsymbol{l} \pm \frac{e}{\hbar}\int_L^R \boldsymbol{A}\cdot d\boldsymbol{l} + \phi^R_{(eh)(he)} + \int_R^L \pm\boldsymbol{k}^{\mp}\cdot d\boldsymbol{l} \mp \frac{e}{\hbar}\int_R^L \boldsymbol{A}\cdot d\boldsymbol{l} + \phi^L_{(he)(eh)} \\ &= L(k_x^+ - k_x^-) + L\tan\theta(k_y^+ - k_y^-) - 2\eta \pm \left(\Delta\varphi + \frac{2e}{\hbar}\int_L^R \boldsymbol{A}\cdot d\boldsymbol{l}\right).\end{aligned} \tag{4.38}$$

Again, we let $k_x^+ \approx k_x^-$ and $k_y^+ \approx k_y^-$ and are left with

$$2\pi n = -2\eta \pm \left(\Delta\varphi - \gamma\right), \tag{4.39}$$

where we have defined

$$\gamma = -\frac{2e}{\hbar}\int_L^R \boldsymbol{A}\cdot d\boldsymbol{l}. \tag{4.40}$$

We have assumed the curvature of the electrons, due to the Lorentz force, to be much larger than the length of the junction, so that we can neglect the effect the field has on the trajectory of the electrons/holes.

4.3.3 ABS energy

The energy is then

$$E_{\mathbf{k}} = \Delta_0 \cos\eta = \Delta_0 \cos\left(\frac{\Delta\varphi}{2} - \frac{\gamma}{2}\right). \tag{4.41}$$

We see that when the the field is zero, i.e. $\gamma = 0$, we get the same expression as equation (4.7).

Chapter 5

Andreev Bound State Current in SNS-junction

We can use equation (2.32) and (2.37) to express the Josephson current in terms of the ABS energy and the phase difference.

$$\begin{aligned} I_x(\Delta\varphi) &= \sum_{k_y} \delta I(\boldsymbol{r},\boldsymbol{k}) \to \int dy \int \frac{dk_y}{2\pi} \delta I(\boldsymbol{r},\boldsymbol{k}), \\ I_y(\Delta\varphi) &= \sum_{k_x} \delta I(\boldsymbol{r},\boldsymbol{k}) \to \int dx \int \frac{dk_x}{2\pi} \delta I(\boldsymbol{r},\boldsymbol{k}), \end{aligned} \tag{5.1}$$

where we have defined

$$\delta I(\boldsymbol{r},\boldsymbol{k}) \equiv -\frac{2e}{\hbar} \tanh\left(\frac{E_{\mathbf{k}}}{2k_B T}\right) \frac{\partial E_{\mathbf{k}}}{\partial(\Delta\varphi)} \tag{5.2}$$

and I_y should be zero due to current conservation. The current density will be given as

$$\begin{aligned} j_x(x,y) &= \int \frac{dk_y}{2\pi} \delta I(\boldsymbol{r},\boldsymbol{k}) = \frac{k_F}{2\pi} \int d\theta_k \cos\theta_k \delta I(\boldsymbol{r},\boldsymbol{k}), \\ j_y(x,y) &= \int \frac{dk_x}{2\pi} \delta I(\boldsymbol{r},\boldsymbol{k}) = \frac{k_F}{2\pi} \int d\theta_k \sin\theta_k \delta I(\boldsymbol{r},\boldsymbol{k}). \end{aligned} \tag{5.3}$$

5.1 ABS current without barriers or applied field

In the case with no barriers or magnetic field we use equation (4.7) in equation (5.2) to obtain

$$\delta I = \frac{e\Delta_0}{\hbar}\sin\left(\frac{\Delta\varphi}{2}\right)\tanh\left(\frac{\Delta_0\cos(\Delta\varphi/2)}{2k_BT}\right), \tag{5.4}$$

which we notice is independent of position and θ_k. The total current is thus given as

$$I_x = k_F W\frac{e\Delta_0}{\pi\hbar}\sin\left(\frac{\Delta\varphi}{2}\right)\tanh\left(\frac{\Delta_0\cos(\Delta\varphi/2)}{2k_BT}\right). \tag{5.5}$$

5.2 ABS current with barriers

In the case with barriers we obtained the ABS energy in equation in (4.26), which give

$$\delta I = \frac{e\Delta_0}{2\hbar}\frac{\sin(\Delta\varphi)}{\sqrt{(\cos^2(\Delta\varphi/2)+\zeta)(\zeta+1)}}\tanh\left(\frac{\Delta_0}{2k_BT}\sqrt{\frac{\cos^2(\Delta\varphi/2)+\zeta}{\zeta+1}}\right). \tag{5.6}$$

Again the δI is independent of position and θ such that the total current is simply

$$I_x = k_F W\frac{e\Delta_0}{2\pi\hbar}\frac{\sin(\Delta\varphi)}{\sqrt{(\cos^2(\Delta\varphi/2)+\zeta)(\zeta+1)}}\tanh\left(\frac{\Delta_0}{2k_BT}\sqrt{\frac{\cos^2(\Delta\varphi/2)+\zeta}{\zeta+1}}\right). \tag{5.7}$$

5.3 ABS current with constant applied field

With no barriers, but magnetic field we find the current from the ABS energy in equation (4.41):

$$\delta I_k(\Delta\varphi) = \frac{e\Delta_0}{\hbar}\sin\left(\frac{\Delta\varphi}{2}-\frac{\gamma_k}{2}\right)\tanh\left(\frac{\Delta_0\cos\left(\frac{\Delta\varphi}{2}-\frac{\gamma_k}{2}\right)}{2k_BT}\right). \tag{5.8}$$

γ_k will be dependent on the magnetic field, and on the trajectory of the particle. We will consider the case where we have a constant field, B, in the normal region, and assume complete expulsion, due to the Meissner effect, in the superconducting region:

$$\boldsymbol{B} = B\left[\Theta(x+L/2)-\Theta(x-L/2)\right]\hat{z}. \tag{5.9}$$

We choose the gauge of the $\boldsymbol{A}$-field as

$$\boldsymbol{A} = -By\,[\Theta(x+L/2) - \Theta(x-L/2)]\,\hat{x}. \tag{5.10}$$

We can now use equation (4.40) to calculate γ by integrating along a path through the point (x_0, y_0) at an angle θ_k with the x-axis, as shown in figure !!! FIGREF !!!. This gives !!! See appendix maybe?!!!

$$\gamma_k = \frac{2L}{l_m^2}\left(y_0 - x_0 \tan\theta_k\right) \tag{5.11}$$

where $l_m = \sqrt{\hbar/eB}$ is the magnetic length. If we assume W to be much larger than L we can ignore the boundaries along the junction, and the current will be conserved such that there will be no x_0 dependence.

Chapter 6

Results

Chapter 7

Conclusion and outlook

Appendix A

Additional Information

A.1 Commutation relations

$$\left[c^{\dagger}_{\mathbf{k},\sigma}, c_{\mathbf{k}',\sigma'}\right]_{+} = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma,\sigma'} \tag{A.1a}$$

$$\left[c^{\dagger}_{\mathbf{k},\sigma}, c^{\dagger}_{\mathbf{k}',\sigma'}\right]_{+} = 0 \tag{A.1b}$$

$$\left[c_{\mathbf{k},\sigma}, c_{\mathbf{k}',\sigma'}\right]_{+} = 0 \tag{A.1c}$$

A.2 The pauli matrices

$$\sigma \tag{A.2}$$

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