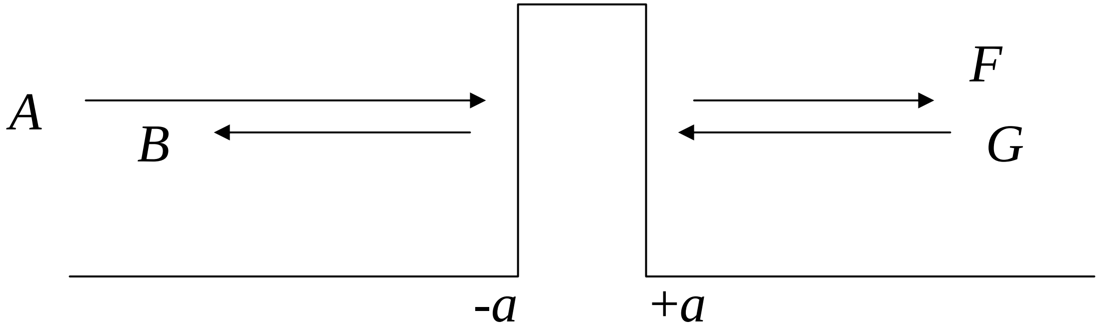
The story so far:

- Semiclassical transport pretty good at duplicating experimental results in regime of validity.
- Quantum coherence leads to interference effects that can dominate in samples ~ size of coherence length.
- Can think of conductance as transmission in this quantum limit.
- Specific example: when considering tunneling conduction through both single- and double-barrier structures.
- In that case, can use matrix to connect mode amplitudes on one side of sample to mode amplitudes on other side of sample.

Tunneling revisited: the scattering matrix

Slightly different way of formulating transmission problem.

Instead of **M** matrices relating amplitudes on different sides of sample, use **S** matrix - relates *incoming* amplitudes to *outgoing* ones.



$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{pmatrix} F \\ G \end{pmatrix} \longrightarrow \begin{pmatrix} B \\ F \end{pmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{pmatrix} A \\ G \end{pmatrix}$$

$$T_{LR}(E) = |S_{21}|^2 \qquad R_{LR}(E) = |S_{11}|^2$$

$$T_{RL}(E) = |S_{12}|^2 \qquad R_{RL}(E) = |S_{22}|^2$$

The scattering matrix

In asymmetric case, can have different velocities on L and R sides. Results become:

$$T_{LR}(E) = |S_{21}|^2 \frac{v_1}{v_2} \qquad\qquad T_{RL}(E) = |S_{12}|^2 \frac{v_2}{v_1}$$

Common way to deal with this: rescale each matrix element: $S_{nm} \to S_{nm}\sqrt{\dfrac{v_n}{v_m}}$

Result: **S** matrix is now *unitary*: $\mathbf{S}^{+}\mathbf{S} = \mathbf{S}\mathbf{S}^{+} = 1$

Conservation of probability is the result.

Landauer-Buttiker formalism

- A general approach to understanding conduction properties of small, (noninteracting) quantum coherent systems connected via contacts to *classical* (decohering) reservoirs that can serve as current sources/sinks or voltage (chemical potential) probes.
- Commonly used to analyze many molecular electronics experiments.
- Can be modified to include electron-electron interaction effects.
- Makes remarkable predictions confirmed by experiments, independent of the microscopic details of samples or measurements.

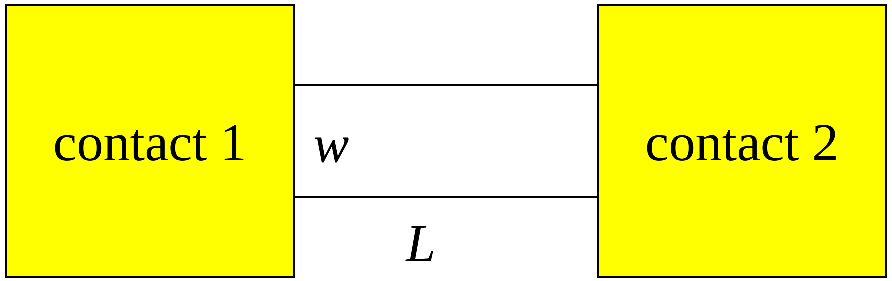
presentation after that of Datta and Ferry.

Ballistic conductors

Landauer (1959) started examining the question of a *ballistic* conductor.

Is the conductance of such an object actually *infinite*?

No - experimentally, limit of conductance remains finite even as w, $L \ll l$.



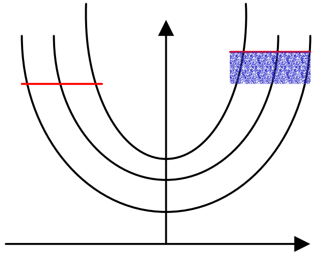
- 2-terminal conductance must be limited by *contact resistance*.

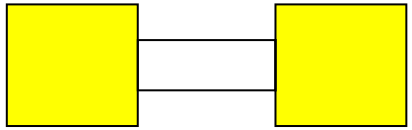
Calculating the current

Let's work assuming a narrow (1d) conductor, for ease.

Each transverse mode has some minimum energy:

$$\varepsilon_N = E_N(k=0)$$





Number of modes at energy E
$M(E)$ counts those with $\varepsilon_N < E$.

$$M(E) \equiv \sum_N \vartheta(E - \varepsilon_N)$$

Occupation probability of state in left reservoir = $f^+(E)$

Current carried left to right:

$$I^+ = \frac{e}{L}\sum_k v(E) f^+(E) M(E) \qquad \sum_k \to 2\times\frac{L}{2\pi}\int dk$$

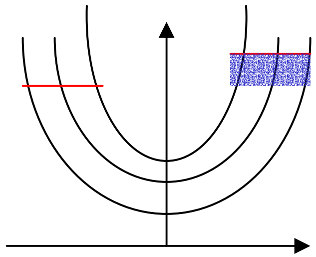
$$= \frac{e}{L}\sum_k \frac{1}{\hbar}\frac{\partial E}{\partial k} f^+(E)M(E) \qquad \to I^+ = \frac{2e}{h}\int_{-\infty}^{\infty} f^+(E)M(E)dE$$

Bottom line: current carried per mode per unit energy = $2e/h$

The 2-terminal Landauer formula

Net current results when chemical potential of L and R contacts are different! More occupied modes going L-R than R-L, for example.

$$I = \frac{2e}{h} M (\mu_1 - \mu_2) = \frac{2e^2}{h} M \underbrace{\frac{(\mu_1 - \mu_2)}{e}}_{V}$$



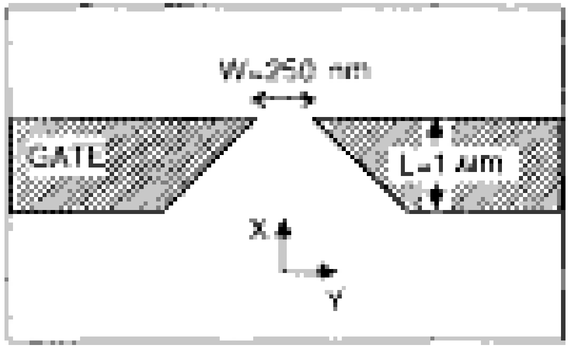
So, conductance of M channel ballistic conductor =

$$\boxed{G_c = \frac{2e^2}{h} M}$$

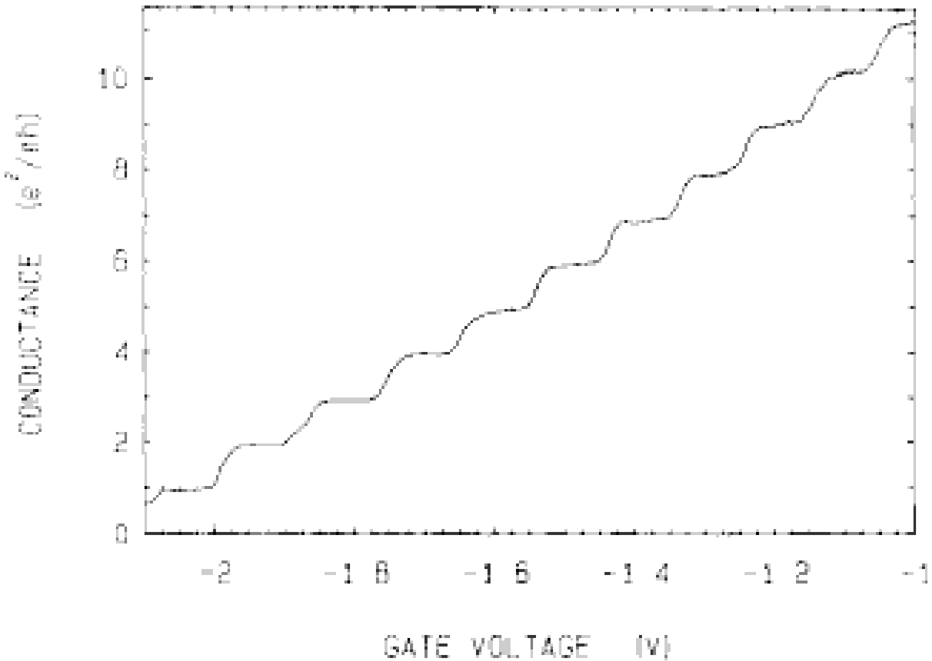
Since the conductor is *ballistic*, the resistance ($1/G$) must be a *contact resistance*, associated somehow with the interfaces between the contacts and the ballistic "waveguide".

Conductance quantization

The implication of the Landauer formula is, if we could deform a ballistic conductor to vary the number of modes with cutoff energies below E_F, we should see *steps* in the conductance.



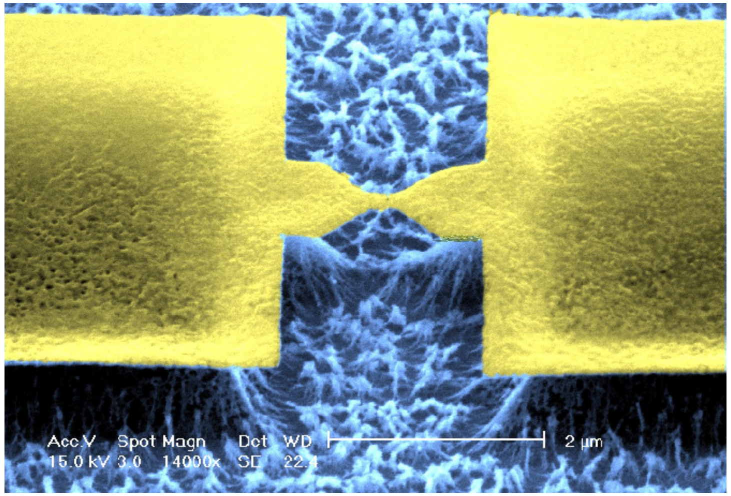
GaAs/AlGaAs 2deg
T = 0.6 K
n_{2d} = 3.6 x 10^{11} / cm^2



Van Wees *et al.*, PRL **60**, 848 (1988)

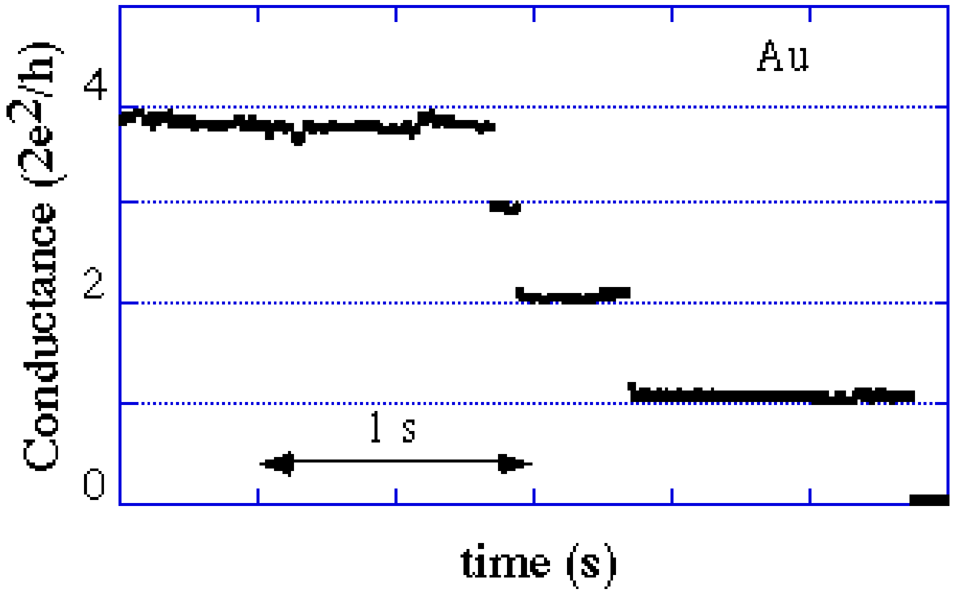
Conductance quantization

Can happen in the case of "real" metals, too:



From van Wees, Groningen

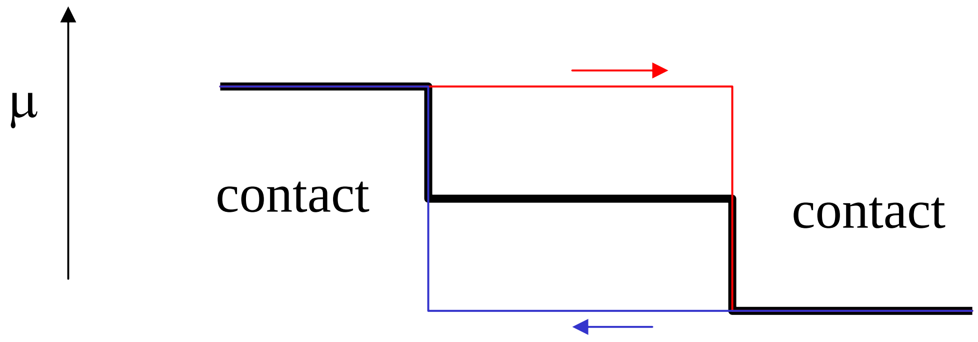
From Reed, Yale



Metal "break junction" – Au constriction narrows and breaks as wafer is bent.

- Result: conductance quantization as cross section approaches atomic scale ($d \rightarrow 1/k_F$).
- Quantum effects dominate even at room temperature!

Where is the voltage dropped?



- Right-moving carriers are at chemical potential of left side until they reach right contact.
- Left-moving carriers are at chemical potential of right side until they reach left contact.
- Bold curve represents average of chemical potential.
- Looks again like it’s a *contact* resistance.
- Actual *electrostatic* potential smeared out by screening….

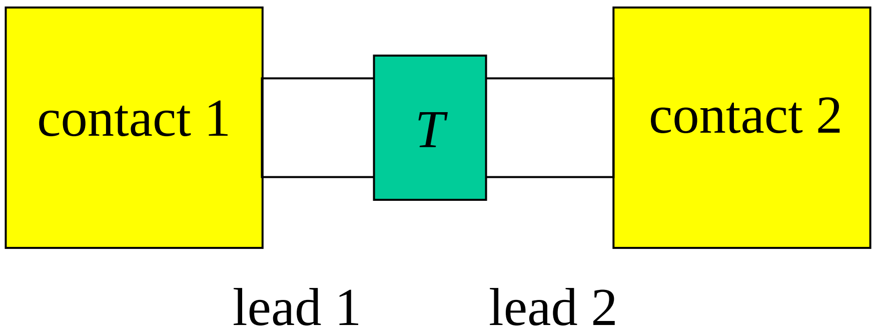
Reflectionless contacts

• Origin of contact resistance: contacts contain large number of modes, while only a few can get through the conductor.

• Assumes contacts are *reflectionless* - all the $+k$ carriers come from the left contact, and have chemical potential μ_1, regardless of what right contact is doing.

• Same for $-k$ carriers, only for right contact.

• Worth considering contacts, *leads*, and then the conductor, allowing reflections from conductor:



Nonballistic classical conductors: Ohm's Law?

If we join conductors together in series classically, we expect to find Ohm's law. Can we get this result?

Consider two conductors in series:

Direct transmission: T =(?) T_1T_2

This ignores multiple reflections. Carrier could bounce off 2nd conductor, then off 1st conductor, *then* be transmitted….

$$
\begin{aligned}
T_{12} &= T_1T_2 + T_1T_2R_1R_2 + T_1T_2R_1^2R_2^2 + \ldots \\
&= T_1T_2 \frac{1}{1-R_1R_2}
\end{aligned}
$$

Ohm's Law

Rewriting in terms of Ts, $$\frac{1-T_{12}}{T_{12}} = \frac{1-T_1}{T_1} + \frac{1-T_2}{T_2}$$

Implication for N identical scatterers:

$$T(N) = \frac{T}{N(1-T)+T}$$

If there are ζ scatterers per unit length, then

$$T(L) = \frac{L_0}{L+L_0} \qquad L_0 \equiv \frac{T}{\zeta(1-T)}$$ mean free path.

For large L, this gives Ohmic variation, as expected.

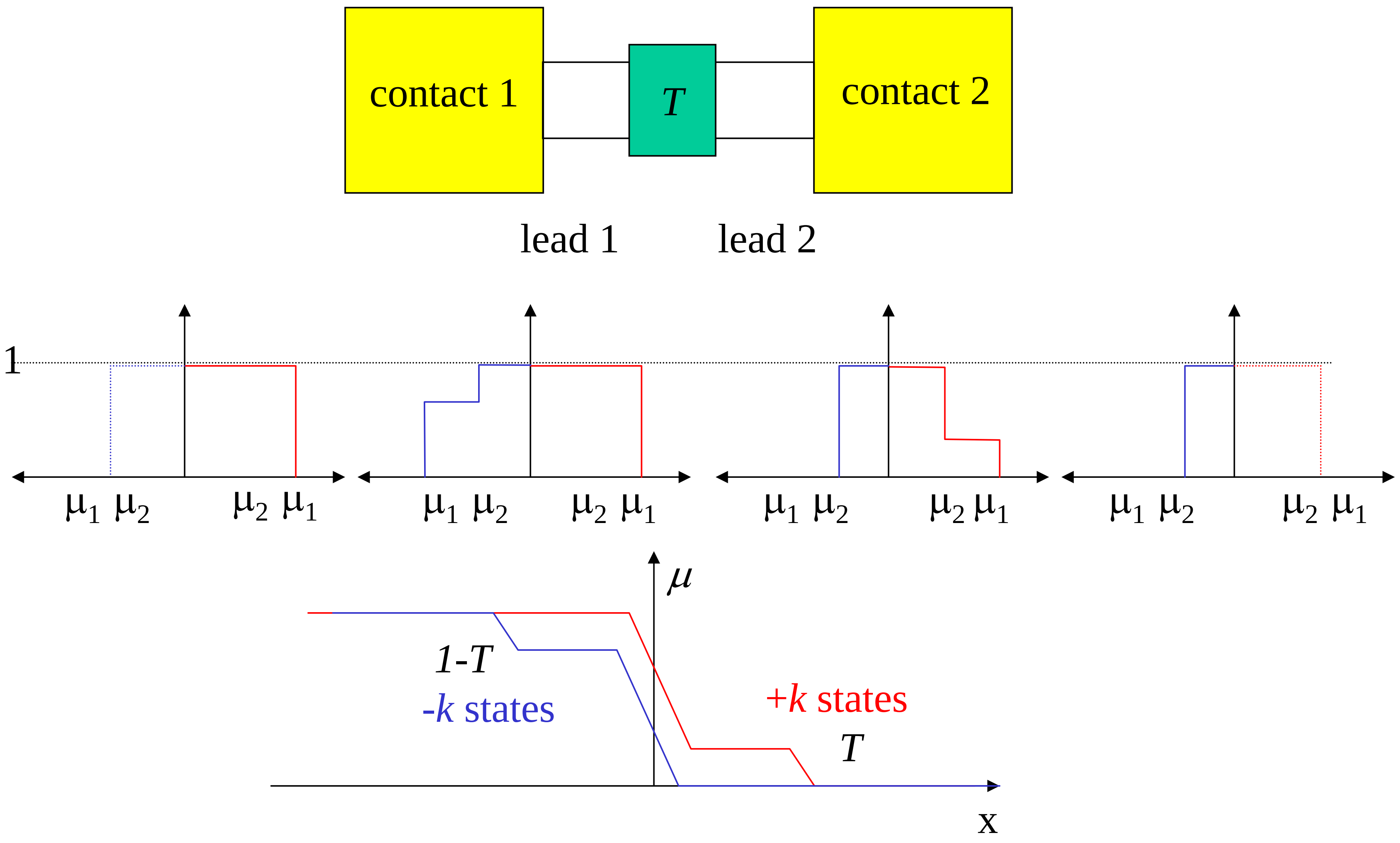
Since $(1\text{-}T)/T$ is additive, suggests that resistance per scatterer is proportional to that quantity for each scatterer.

Rewriting our Landauer formula,

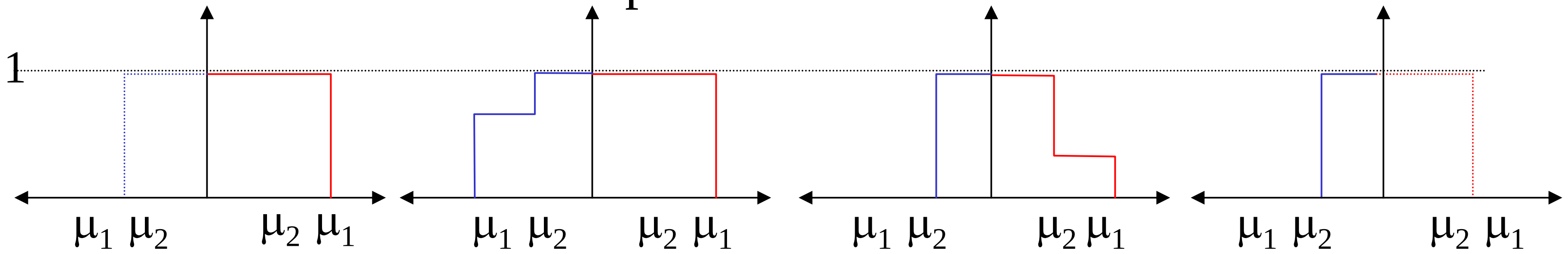
$$G^{-1} = \frac{h}{2e^2M}\frac{1}{T} = \frac{h}{2e^2M} + \frac{h}{2e^2M}\frac{1-T}{T}$$

contact piece scatterer piece

Energy distribution of electrons



Where is the heat dissipated?



We've assumed energy relaxation takes place without switching directions of carriers - not always true….

Nonthermal distributions thermalize on some energy relaxation length scale.

For case above, that happens in the *leads*.
For case of point contact, that happens in the *contacts*.

Main idea: get nonthermal electronic distributions; heat dissipation happens when electronic distributions can relax. That's why, *e.g.*, ballistically conducting nanotubes can handle high current densities!

The correct conductance formula….

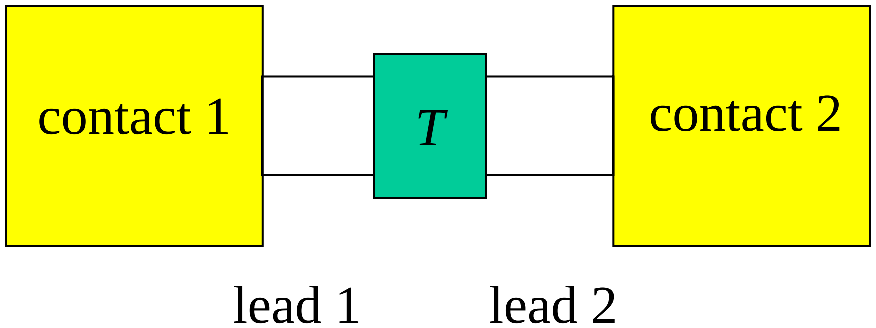
$$G^{-1} = \frac{h}{2e^2 M}\frac{1}{T} = \frac{h}{2e^2 M} + \frac{h}{2e^2 M}\frac{1-T}{T}$$

contact piece "device" piece

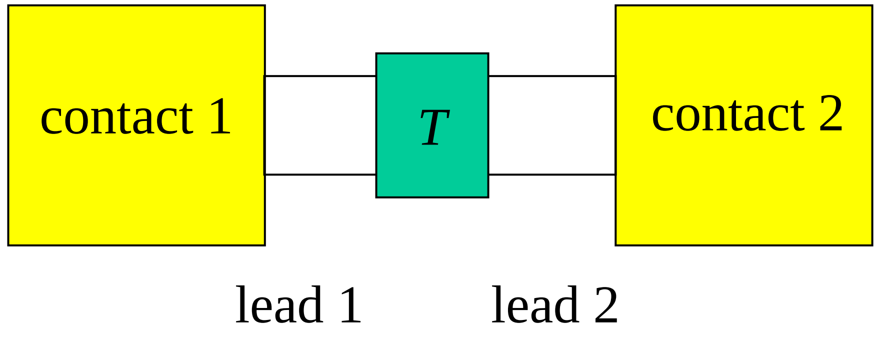
Separating resistances into contact and device pieces.

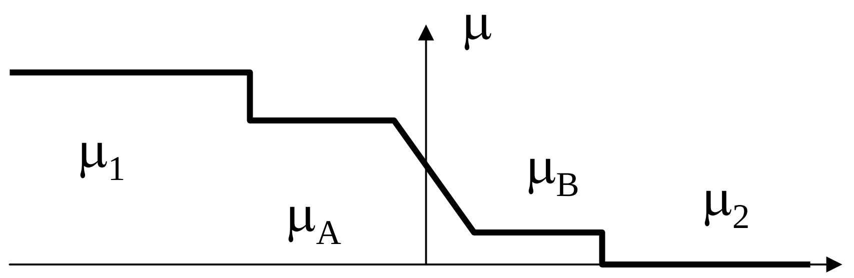
Note that if scattering is zero, $T = 1$ and the device contribution vanishes, as expected - a true ballistic conductor (just the conductor!) has infinite two-terminal conductance.

What's going on here?



The correct conductance formula





$$G_{device}^{-1} = \frac{h}{2e^2M}\frac{1-T}{T} = \frac{\mu_A - \mu_B}{I}$$

Chemical potential in *leads* differs from that in contacts due to *scattered* electrons….

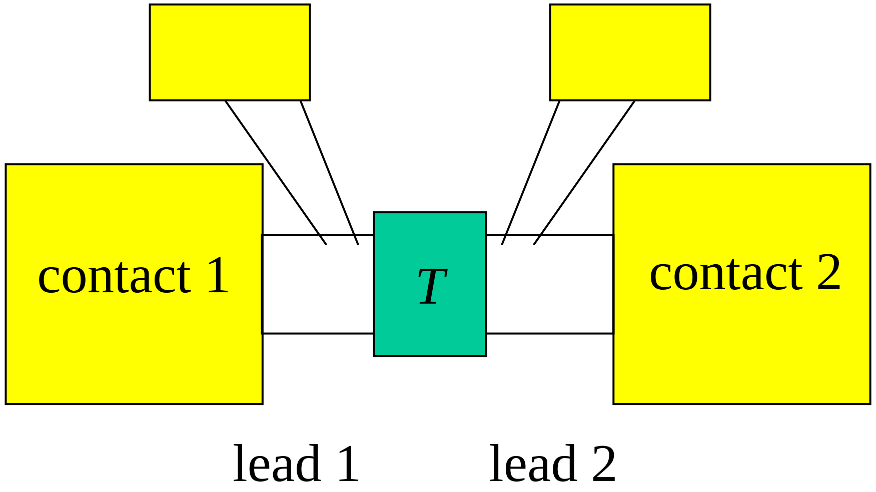
Two-terminal vs. four-terminal

If the resistance from the two-terminal Landauer formula is really a contact resistance, one might think of performing a 4-terminal measurement….

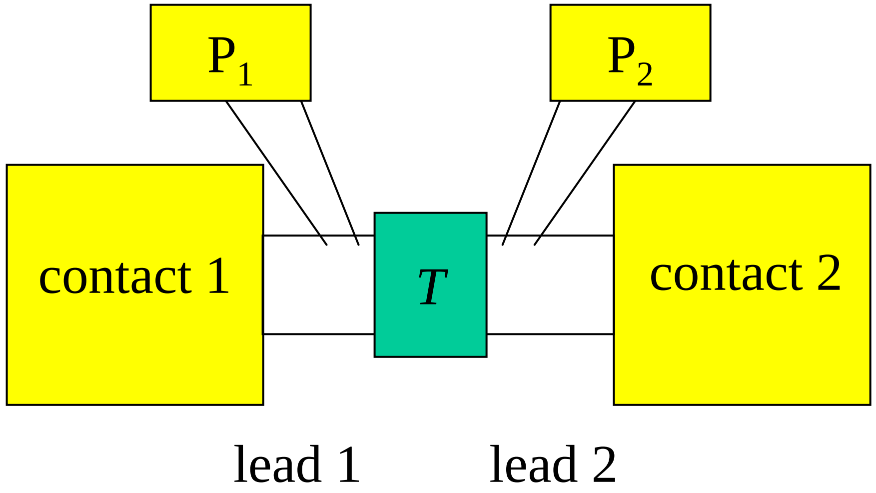
That is, can one actually sense the "device" resistance contribution, which should truly go to zero for a ballistic device?

Complications:

- Voltage probes can perturb currents in device
- Voltage probes may not couple to +*k* and -*k* modes equally.



Four-terminal probes

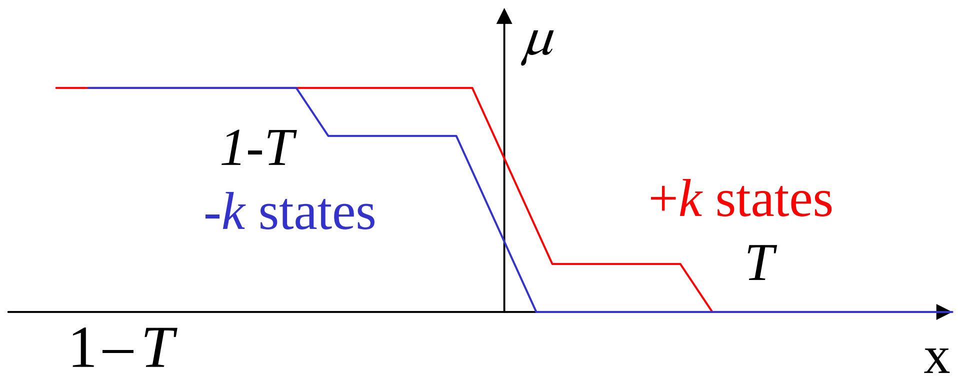


Voltage measured (ideal):

$$\mu_{P1} - \mu_{P2} = (1-T)(\mu_1 - \mu_2)$$

Current:

$$I = \frac{2e}{h} MT(\mu_1 - \mu_2)$$



4T resistance:

$$R_{4T} = \frac{(\mu_{P1} - \mu_{P2})/e}{I} = \frac{h}{2e^2 M} \frac{1-T}{T}$$

What if left probe only coupled to left-moving states, and vice-versa?

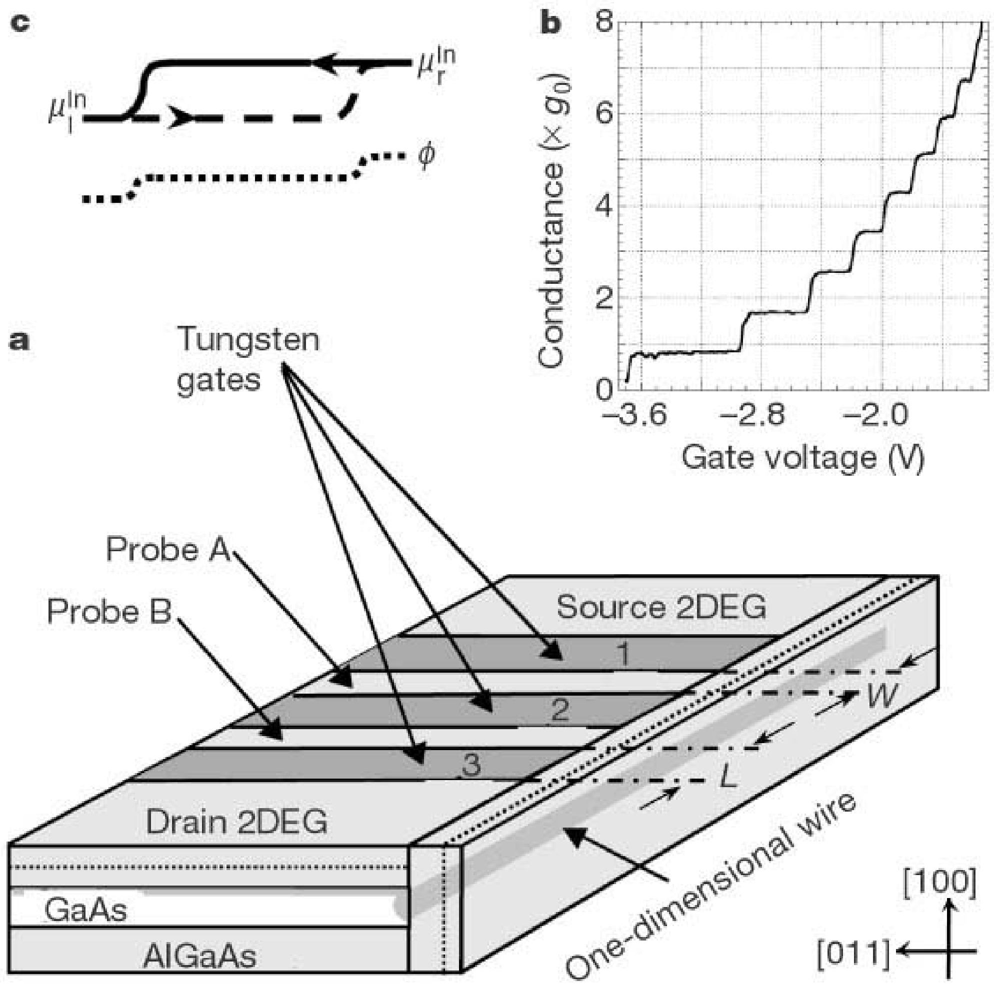
$$\mu_{P1} \approx (1-T)(\mu_1 - \mu_2),$$
$$\mu_{P2} \approx T(\mu_1 - \mu_2)$$

$$\longrightarrow \quad R_{4T} = \frac{h}{2e^2 M} \frac{1-2T}{T}$$

Could even be *negative*!

Two-terminal vs. four-terminal - experiment

de Picciotto *et al.*, *Nature* **411**, 51 (2001).



Cleaved-edge overgrowth technique to produce quantum wire with 2d contacts.

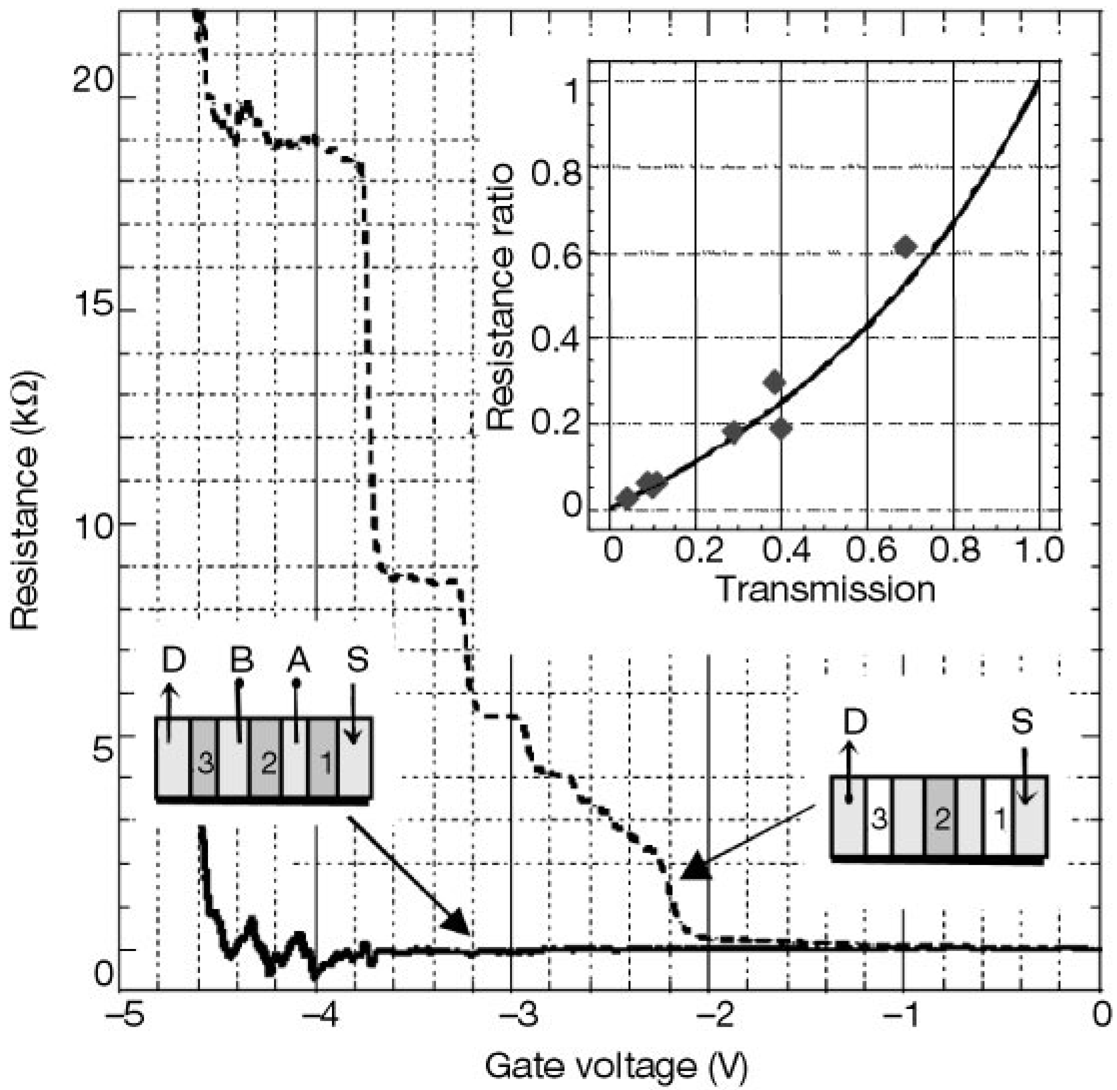
Gates used to create 1d wire regions at edge of wafer.

2-terminal conductance shows quantized conductance, but with $T \sim 0.8$.

Two-terminal vs. four-terminal - experiment

Can carefully tune invasiveness of voltage probes.

Comparison of 2-terminal vs. 4-terminal resistance shows that, for ideal voltage probe coupling, one really does measure a device resistance of zero in the ballistic case!



To summarize:

- Scattering matrix (relating ingoing and outgoing fluxes) is an alternative way of describing conduction of quantum scattering systems.
- Landauer formula: 2-terminal conductance of some quantum coherent system coupled to classical reservoirs is given by $(2e^2/h)\Sigma_M T$, where M is the number of channels and T is the transmission coefficient of each channel.
- Lack of inelastic scattering means nonthermal electronic distributions and subtleties in figuring out voltage drops.
- Finite 2T conductance of ballistic system is result of contact resistance.
- Proper 4T conductance of ballistic system really does approach infinity.

Next time:

- Accounting for Coulomb interactions
- Buttiker formula - multiterminal case
- Finite temperature and bias