Toy Example

Permutation Closed Testing with Sum-Based Statistics

Data

		G		
(1)	(2)	(3)	(4)	(5)
28.42	16.68	9.36	6.12	9.40
0.10	0.06	1.37	0.08	0.56
0.69	3.07	4.33	0.83	0.36
1.07	30.31	1.11	8.55	0.26
0.22	7.45	2.87	0.48	1.02
1.83	0.04	2.85	0.04	0.02
17.68	1.82	6.00	1.52	1.06
1.77	26.12	0.29	0.26	4.07
2.71	0.37	8.47	5.83	4.42
1.14	0.03	24.06	8.84	2.41

We test $S = \{5\}$ with level $\alpha = 0.2$

Analysis

d_S	D				R			
(5)	(4)	(3)	(2)	(1)				
0.00	0.00	0.00	0.00	0.00	0.00 (1)	0.00 (2)	0.00 (3)	0.00 (4)
-8.84	-6.03	-7.99	-16.62	-28.32	-6.03 (4)	-7.99 (3)	-16.62 (2)	-28.32 (1)
-9.04	-5.29	-5.02	-13.61	-27.72	-5.02 (3)	-5.29 (4)	-13.61 (2)	-27.72 (1)
-9.14	2.43	-8.25	13.63	-27.34	13.63 (2)	2.43 (4)	-8.25 (3)	-27.34 (1)
-8.38	-5.63	-6.49	-9.23	-28.19	-5.63 (4)	-6.49 (3)	-9.23 (2)	-28.19 (1)
-9.38	-6.08	-6.51	-16.64	-26.59	-6.08 (4)	-6.51 (3)	-16.64 (2)	-26.59 (1)
-8.34	-4.59	-3.36	-14.86	-10.74	-3.36 (3)	-4.59 (4)	-10.74 (1)	-14.86 (2)
-5.33	-5.85	-9.07	9.44	-26.65	9.44 (2)	-5.85 (4)	-9.07 (3)	-26.65 (1)
-4.98	-0.28	-0.89	-16.31	-25.71	-0.28 (4)	-0.89 (3)	-16.31 (2)	-25.71 (1)
-6.99	2.72	14.70	-16.65	-27.27	14.70 (3)	2.72 (4)	-16.65 (2)	-27.27 (1)

L_v and U_v are the 8-th ordered statistics of

$$d_{\tilde{V}} = d_S + \sum_{i=1}^{v} D_i \qquad\qquad u_v = d_S + \sum_{i=1}^{v} R_i$$

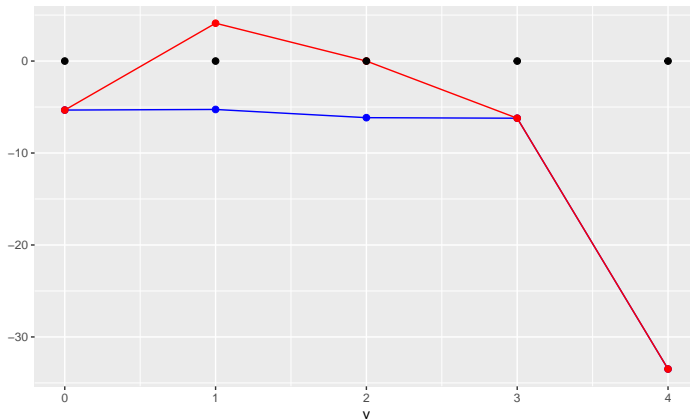| v	0	1	2	3	4
U_v	-5.33	4.11	0.00	-6.22	-33.49
L_v	-5.33	-5.26	-6.16	-6.22	-33.49
rej	T	?	?	T	T

Figure 1: Upper (red) and lower (blue) critical values and observed values (zero, black) by additional superset size v.

Branch and Bound – Lowest Statistic

The total space is partitioned according to the inclusion of 4.
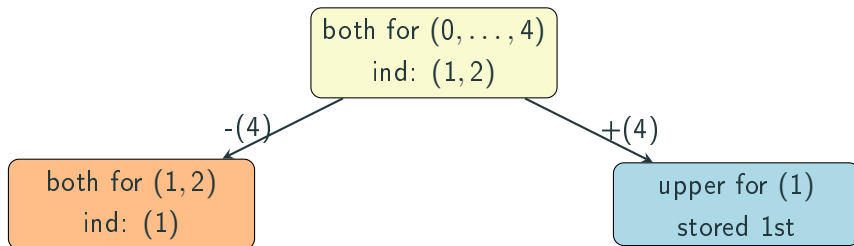
In both subspaces, U_v decreases.

- $\mathbb{S}_{-4}$: L_v may change, hence we examine both bounds
- $\mathbb{S}_{+4}$: L_v does not change, hence we examine U_v

For each node, we save:

- sizes v to be examined (when keeping an index, v decreases of 1 unit)
- $\mathbf{R}$ and the corresponding indices
- cumulative sums of $\mathbf{d}_S + \mathbf{d}_{\text{kept}}$ with $\mathbf{R}$ and $\mathbf{D}$

- We enumerate the two subspaces: $\mathbb{S}_{+4}$ is stored, and $\mathbb{S}_{-4}$ is examined (both bounds)
- we keep removing indices until we can close a node
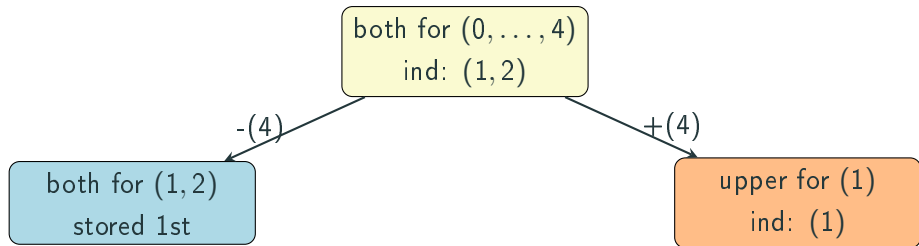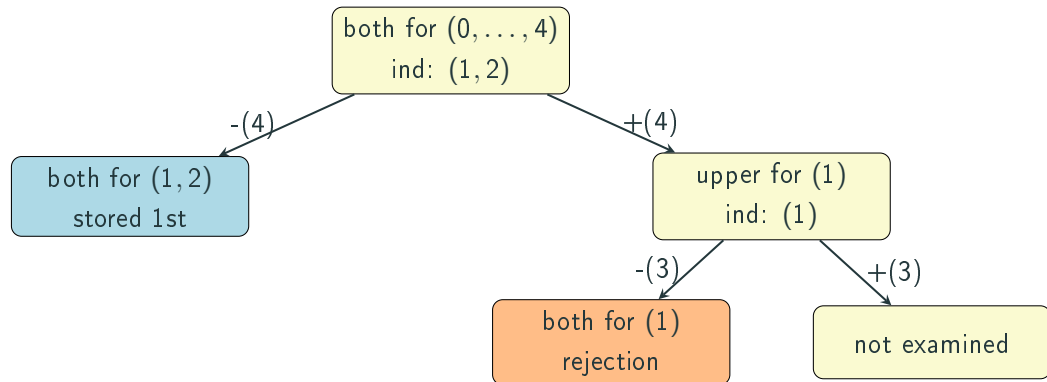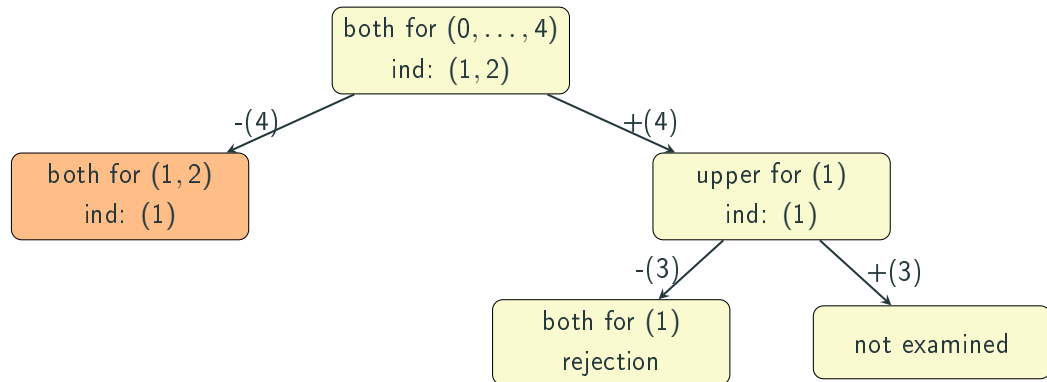- then we start again from the node that was stored last
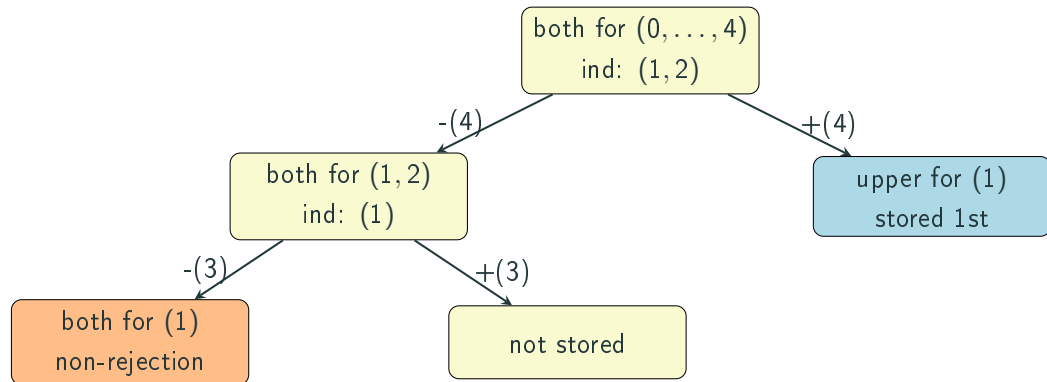
We start by examining U_v in $\mathbb{S}_{+4}$ (hence we cannot find any non-rejection).

both for $(0, \ldots, 4)$
ind: $(1, 2)$

$-(4)$

$+(4)$

both for $(1, 2)$
indecisive: (1)

upper for (1)
ind: (1)

$-(3)$

$+(3)$

$-(3)$

$+(3)$

both for (1)
non-rejection

not examined

both for (1)
rejection

not examined
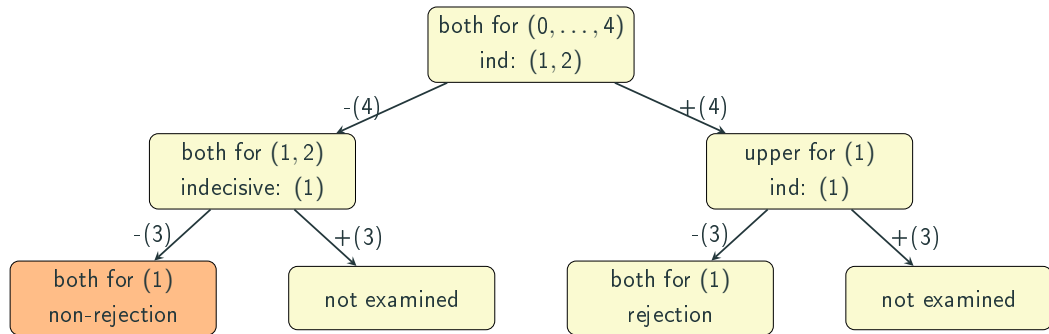
Branch and Bound - Highest Statistic

The total space may also be partitioned according to the inclusion of 1.

As in the previous case, U_v decreases in both subspaces.

- $\mathbb{S}_{-1}$: L_v does not change, hence we examine U_v
- $\mathbb{S}_{+1}$: L_v may change, hence we examine both bounds

In this case, it takes 3 steps in both cases.