

CTRP - A Toy Example

1 Data

Considering $f = 5$ predictors and $B = 10$ permutations, we simulate a $B \times f$ matrix of global test statistics:

G				
(1)	(2)	(3)	(4)	(5)
28.42	16.68	9.36	6.12	9.40
0.10	0.06	1.37	0.08	0.56
0.69	3.07	4.33	0.83	0.36
1.07	30.31	1.11	8.55	0.26
0.22	7.45	2.87	0.48	1.02
1.83	0.04	2.85	0.04	0.02
17.68	1.82	6.00	1.52	1.06
1.77	26.12	0.29	0.26	4.07
2.71	0.37	8.47	5.83	4.42
1.14	0.03	24.06	8.84	2.41

Assume that we want to test $S = \{5\}$ with significance level $\alpha = 0.20$.

2 Shortcut

v						
4						
3	F = {1, 2, 3, 4, 5}					
2	{1, 2, 5}	{1, 2, 3, 5}	{1, 2, 4, 5}	{1, 3, 4, 5}	{2, 3, 4, 5}	
1		{1, 3, 5}	{1, 4, 5}	{2, 3, 5}	{2, 4, 5}	{3, 4, 5}
0		{1, 5}	{2, 5}	{3, 5}	{4, 5}	
	S = {5}					

Table 1: Supersets of $S = \{5\}$, having sizes $|V| = 1 + v$ with $v = 0, \dots, 4$. The sets in bold are used to define the lower critical value L_v .

We define

- **D**, matrix of the centered test statistics in $F \setminus S$, where the indices appear in the order $(4, 3, 2, 1)$ (since $g_4 \leq g_3 \leq g_2 \leq g_1$);
- **R**, matrix obtained from **D** by sorting the elements within each row in decreasing order.

\mathbf{d}_S	\mathbf{D}				\mathbf{R}			
(5)	(4)	(3)	(2)	(1)				
0.00	0.00	0.00	0.00	0.00	0.00 (1)	0.00 (2)	0.00 (3)	0.00 (4)
-8.84	-6.03	-7.99	-16.62	-28.32	-6.03 (4)	-7.99 (3)	-16.62 (2)	-28.32 (1)
-9.04	-5.29	-5.02	-13.61	-27.72	-5.02 (3)	-5.29 (4)	-13.61 (2)	-27.72 (1)
-9.14	2.43	-8.25	13.63	-27.34	13.63 (2)	2.43 (4)	-8.25 (3)	-27.34 (1)
-8.38	-5.63	-6.49	-9.23	-28.19	-5.63 (4)	-6.49 (3)	-9.23 (2)	-28.19 (1)
-9.38	-6.08	-6.51	-16.64	-26.59	-6.08 (4)	-6.51 (3)	-16.64 (2)	-26.59 (1)
-8.34	-4.59	-3.36	-14.86	-10.74	-3.36 (3)	-4.59 (4)	-10.74 (1)	-14.86 (2)
-5.33	-5.85	-9.07	9.44	-26.65	9.44 (2)	-5.85 (4)	-9.07 (3)	-26.65 (1)
-4.98	-0.28	-0.89	-16.31	-25.71	-0.28 (4)	-0.89 (3)	-16.31 (2)	-25.71 (1)
-6.99	2.72	14.70	-16.65	-27.27	14.70 (3)	2.72 (4)	-16.65 (2)	-27.27 (1)

Let $k = \lceil (1-\alpha)B \rceil = 8$. The lower and upper critical values, L_v and U_v , are the k -th ordered statistics of

$$\mathbf{d}_{\tilde{V}} = \mathbf{d}_S + \sum_{i=1}^v \mathbf{D}_i \quad \mathbf{u}_v = \mathbf{d}_S + \sum_{i=1}^v \mathbf{R}_i.$$

The first column of \mathbf{R} having no positive elements has index $w = 3$. We start by computing both bounds for $v = 0, 1, 2$. No non-rejection is found, but the values $v = 1, 2$ lead to an indecisive outcome.

We proceed by examining $v > 2$, until we find either a rejection or a negative U_v . Since U_v becomes negative for $v = 3$, the supersets with $v = 3, 4$ are automatically rejected.

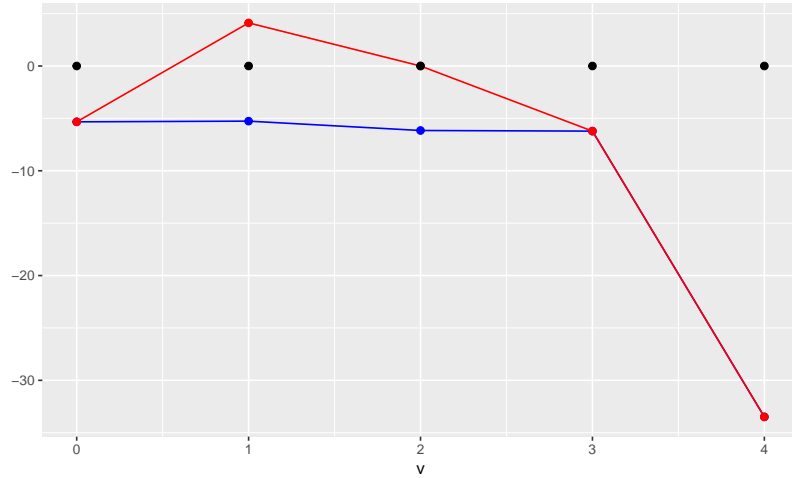


Figure 1: Upper (red) and lower (blue) critical values and observed values (zero, black) by additional superset size v . The bounds for $v = 4$ have not been computed in the analysis.

v	0	1	2	3	4
U_v	-5.33	4.11	0.00	-6.22	(-33.49)
L_v	-5.33	-5.26	-6.16	-6.22	(-33.49)
rej	T	?	?	T	T

3 Branch and Bound

3.1 Removal of the highest statistic.

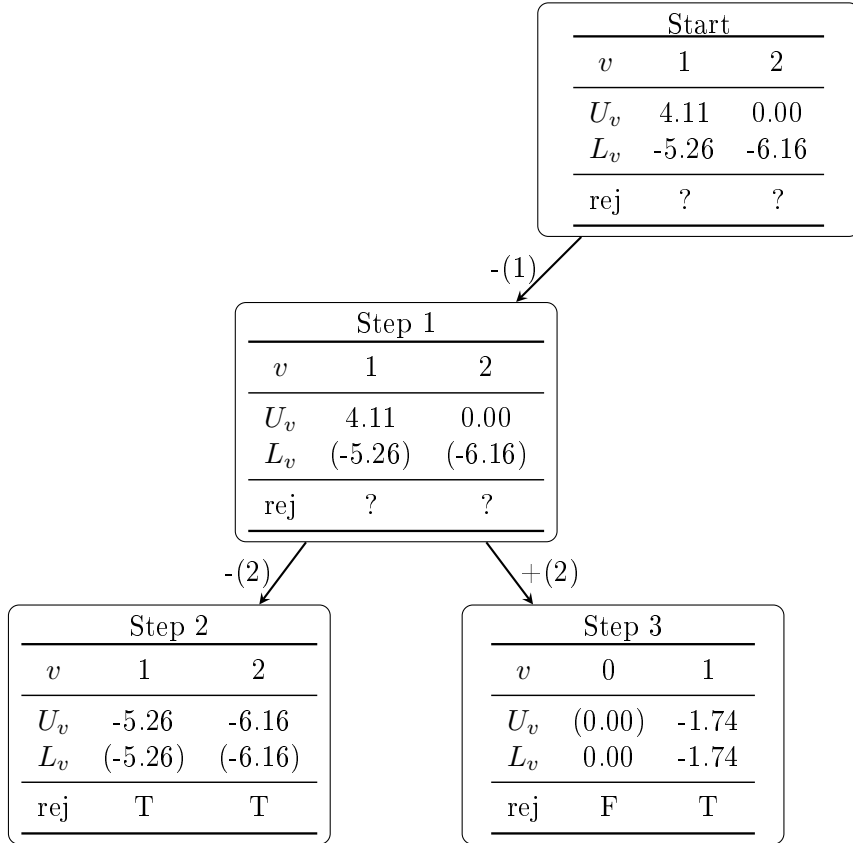
The last index in \mathbf{D} , $e = 1$, determines the branching rule. We explore first the subspace \mathbb{S}_{-1} , where we examine only U_v .

	\mathbb{S}_{-1}			\mathbb{S}_{+1}		
2	$\{2, 3, 5\}$	$\{2, 4, 5\}$	$\{3, 4, 5\}$	$\{1, 2, 5\}$	$\{1, 3, 5\}$	$\{1, 4, 5\}$
1	$\{2, 5\}$	$\{3, 5\}$	$\{4, 5\}$		$\{1, 5\}$	

Since the outcome is still indecisive for both sizes $v = 1, 2$, the subspace is partitioned again according to the inclusion of $e = 2$ (corresponding to the second highest statistic g_i).

	$\mathbb{S}_{-1,-2}$		$\mathbb{S}_{-1,+2}$	
2	$\{3, 4, 5\}$		$\{2, 3, 5\}$	$\{2, 4, 5\}$
1	$\{2, 5\}$	$\{3, 5\}$	$\{4, 5\}$	

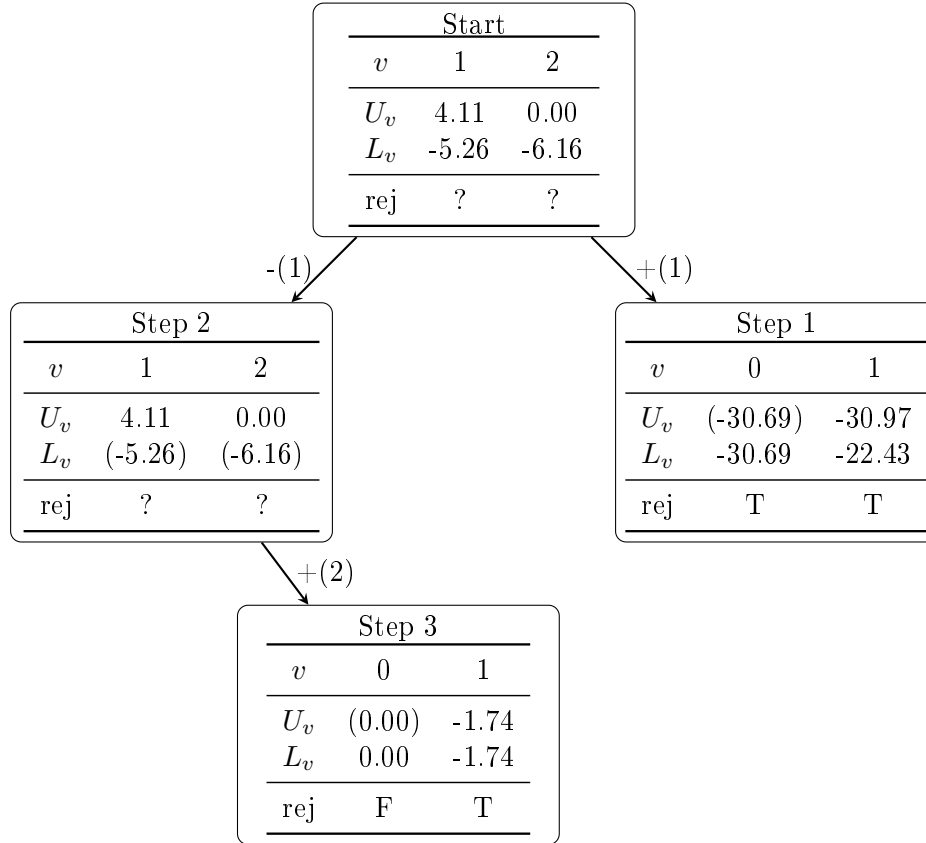
Since S is not rejected in $\mathbb{S}_{-1,+2}$, it is not rejected in the total space after 3 steps.



3.2 Keeping of the highest statistic.

The last index in \mathbf{D} , $e = 1$, determines the branching rule. We explore first the subspace \mathbb{S}_{+1} , where we examine both critical values.

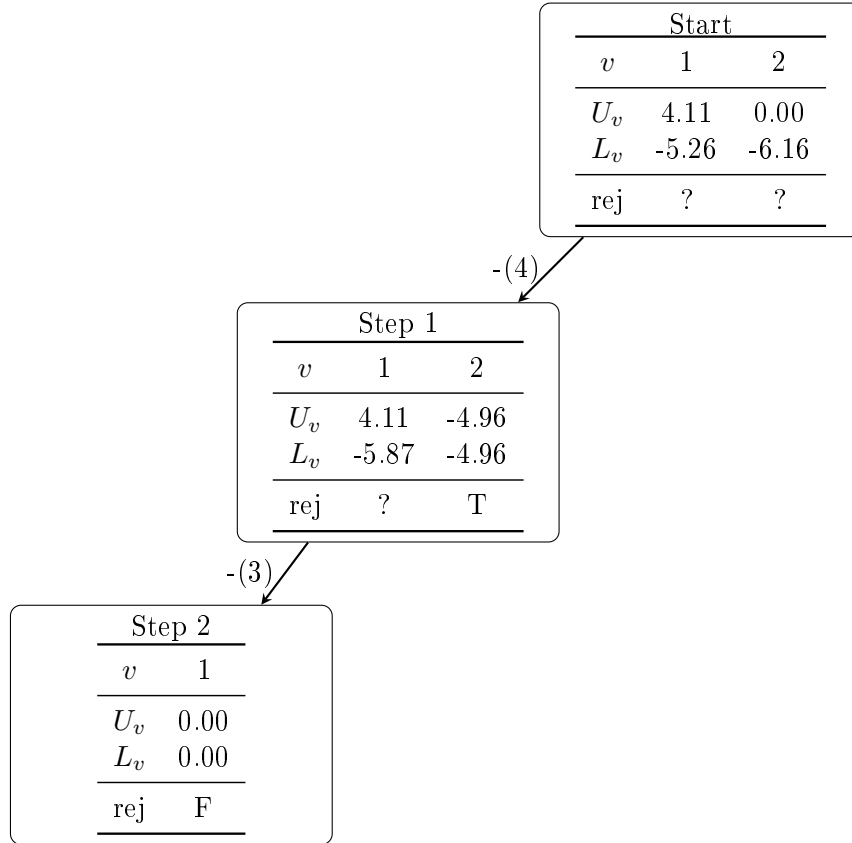
S is not rejected after 3 steps: it is rejected in \mathbb{S}_{+1} , but it is not rejected in $\mathbb{S}_{-1,+2}$.



3.3 Removal of the lowest statistic.

The first index in \mathbf{D} , $e = 4$, determines the branching rule. We explore first the subspace \mathbb{S}_{-4} , where we examine both critical values.

S is not rejected after 2 steps, where the subspaces \mathbb{S}_{-4} and $\mathbb{S}_{-4,-3}$ are explored.



3.4 Keeping of the lowest statistic.

The first index in \mathbf{D} , $e = 4$, determines the branching rule. We explore first the subspace \mathbb{S}_{+4} , where we examine only U_v .

After 2 steps, S is rejected in the subspace \mathbb{S}_{+4} . Subsequently, after 2 other steps, it is not rejected in \mathbb{S}_{-4} . In conclusion, it is not rejected after 4 steps.

