

# CTRP - A Toy Example

## 1 Data

Considering  $f = 5$  predictors and  $B = 10$  permutations, we simulate a  $B \times f$  matrix of global test statistics:

<b>G</b>				
(1)	(2)	(3)	(4)	(5)
28.42	9.36	9.40	16.68	6.12
0.10	1.37	0.56	0.06	0.08
0.69	4.33	0.36	3.07	0.83
1.07	1.11	0.26	30.31	8.55
0.22	2.87	1.02	7.45	0.48
1.83	2.85	0.02	0.04	0.04
17.68	6.00	1.06	1.82	1.52
1.77	0.29	4.07	26.12	0.26
2.71	8.47	4.42	0.37	5.83
1.14	24.06	2.41	0.03	8.84

Assume that we want to test  $S = \{3\}$  with significance level  $\alpha = 0.20$ .

## 2 Shortcut

The supersets of  $S$  have sizes  $1 + v$  with  $v = 0, \dots, 4$ :

v						
4						
3	<b>F = {1, 2, 3, 4, 5}</b>					
2	{1, 2, 3}	{1, 2, 3, 4}	{1, 2, 3, 5}	{1, 3, 4, 5}	{2, 3, 4, 5}	
1		{1, 3, 4}	{1, 3, 5}	{2, 3, 4}	{2, 3, 5}	{3, 4, 5}
0		{1, 3}	{2, 3}	{3, 4}	{3, 5}	
	<b>S = {3}</b>					

We define

- **D**, matrix of the centered test statistics in  $F \setminus S$ , where the indices appear in the order (5, 2, 4, 1) (since  $g_5 \leq g_2 \leq g_3 \leq g_4 \leq g_1$ );
- **R**, matrix obtained from **D** by sorting the elements within each row in decreasing order.

The lower and upper critical values,  $L_v$  and  $U_v$ , are the  $(1 - \alpha)$ -quantiles of

$$\mathbf{d}_{\tilde{V}} = \mathbf{d}_3 + \sum_{i=1}^v \mathbf{D}_i \qquad \mathbf{u}_v = \mathbf{d}_3 + \sum_{i=1}^v \mathbf{R}_i,$$

respectively.

$\mathbf{d}_S$ (3)	$\mathbf{D}$				$\mathbf{R}$			
(5)	(2)	(4)	(1)					
0.00	0.00	0.00	0.00	0.00	0.00 (1)	0.00 (4)	0.00 (2)	0.00 (5)
-8.84	-6.03	-7.99	-16.62	-28.32	-6.03 (5)	-7.99 (2)	-16.62 (4)	-28.32 (1)
-9.04	-5.29	-5.02	-13.61	-27.72	-5.02 (2)	-5.29 (5)	-13.61 (4)	-27.72 (1)
-9.14	2.43	-8.25	13.63	-27.34	13.63 (4)	2.43 (5)	-8.25 (2)	-27.34 (1)
-8.38	-5.63	-6.49	-9.23	-28.19	-5.63 (5)	-6.49 (2)	-9.23 (4)	-28.19 (1)
-9.38	-6.08	-6.51	-16.64	-26.59	-6.08 (5)	-6.51 (2)	-16.64 (4)	-26.59 (1)
-8.34	-4.59	-3.36	-14.86	-10.74	-3.36 (2)	-4.59 (5)	-10.74 (1)	-14.86 (4)
-5.33	-5.85	-9.07	9.44	-26.65	9.44 (4)	-5.85 (5)	-9.07 (2)	-26.65 (1)
-4.98	-0.28	-0.89	-16.31	-25.71	-0.28 (5)	-0.89 (2)	-16.31 (4)	-25.71 (1)
-6.99	2.72	14.70	-16.65	-27.27	14.70 (2)	2.72 (5)	-16.65 (4)	-27.27 (1)

Since the first column of  $\mathbf{R}$  having no positive elements has index  $w = 3$ , we start by computing  $L_v$  for  $v = 0, 1, 2$ .

No non-rejection is found, and thus we proceed by examining  $v > 2$ , until we find either a rejection or a negative  $U_v$ . Here  $U_v$  becomes negative for  $v = 3$ , hence the supersets with  $v = 3, 4$  are automatically rejected.

Finally, we determine the indecisive values by looking at  $U_v$ . Here they are  $v = 1, 2$ .

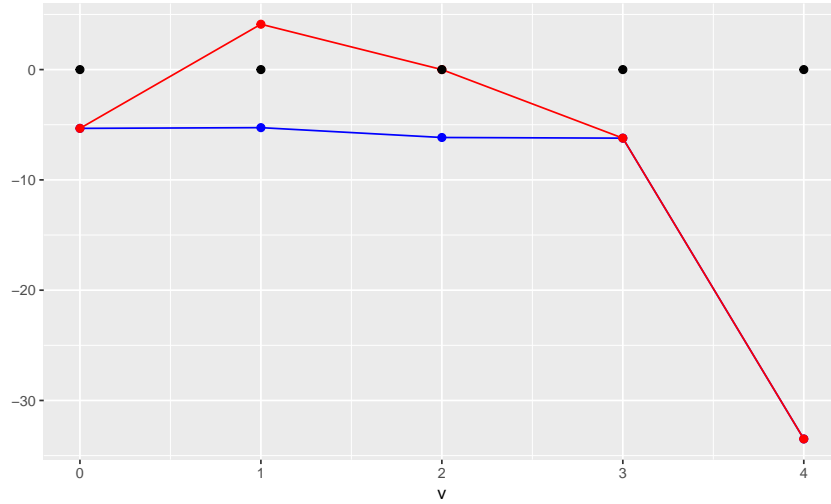


Figure 1: Upper (red) and lower (blue) critical values and observed values (zero, black) by additional superset size  $v$ . The bounds for  $v = 4$  have not been computed in the analysis.

$v$	0	1	2	3	4
$U_v$	-5.33	4.11	0.00	-6.22	(-33.49)
$L_v$	-5.33	-5.26	-6.16	-6.22	(-33.49)
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### 3 Branch and Bound

#### 3.1 Removal of the highest statistic.

The last index in  $\mathbf{D}$ ,  $e = 1$ , determines the branching rule. We explore first the subspace  $\mathbb{S}_{-1}$ , where the index is removed.

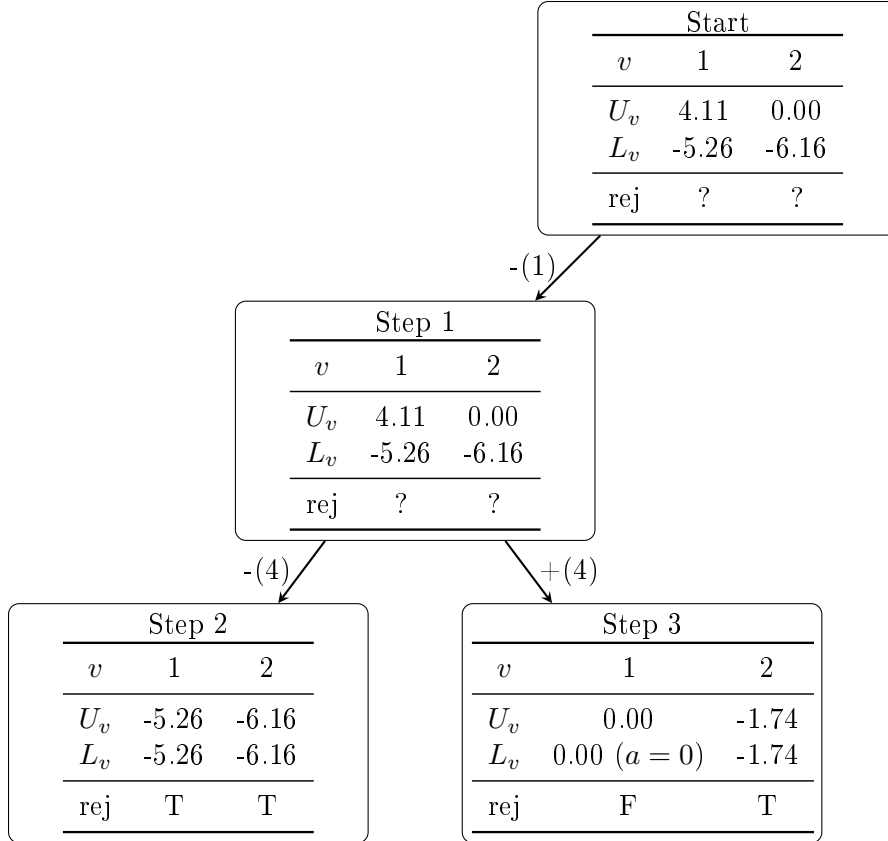
	$\mathbb{S}_{-1}$			$\mathbb{S}_{+1}$		
2	$\{2, 3, 4\}$	$\{2, 3, 5\}$	$\{3, 4, 5\}$	$\{1, 2, 3\}$	$\{1, 3, 4\}$	$\{1, 3, 5\}$
1	$\{2, 3\}$	$\{3, 4\}$	$\{3, 5\}$		$\{1, 3\}$	

Since the outcome is still indecisive for both sizes  $v = 1, 2$ , the subspace is partitioned again according to the inclusion of  $e = 4$  (corresponding to the second highest statistic  $g_i$ ).

	$\mathbb{S}_{-1,-4}$		$\mathbb{S}_{-1,+4}$	
2	$\{2, 3, 5\}$		$\{2, 3, 4\}$	$\{3, 4, 5\}$
1	$\{2, 3\}$	$\{3, 5\}$	$\{3, 4\}$	

$S$  is rejected in both subspaces. We proceed by exploring  $\mathbb{S}_{+1}$ , where  $S$  is not rejected. In particular, the superset  $V = \{3, 4\}$  is not rejected.

In conclusion,  $S$  is not rejected after 3 steps.

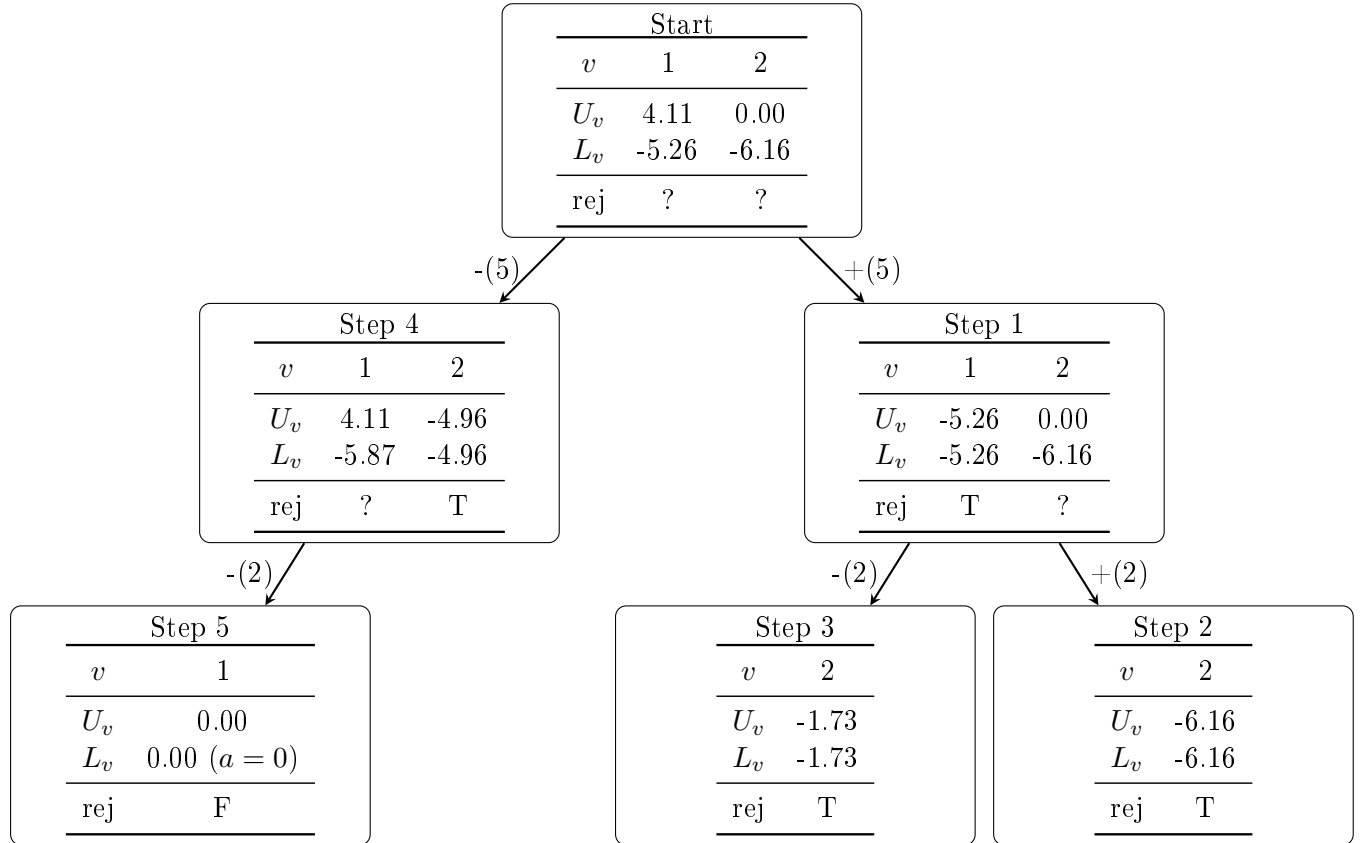


### 3.2 Keeping of the lowest statistic.

The first index in  $\mathbf{D}$ ,  $e = 5$ , determines the branching rule. We explore first the subspace  $\mathbb{S}_{+5}$ , where the index is kept.

After 3 steps,  $S$  is rejected in the subspace  $\mathbb{S}_{+5}$ . Subsequently, after 2 other steps, it is not rejected in  $\mathbb{S}_{-5}$ .

In conclusion,  $S$  is not rejected after 5 steps.



### 3.3 Removal of the lowest statistic.

The first index in  $\mathbf{D}$ ,  $e = 5$ , determines the branching rule. We explore first the subspace  $\mathbb{S}_{-5}$ , where the index is removed.

$S$  is not rejected after 2 steps, where the subspaces  $\mathbb{S}_{-5}$  and  $\mathbb{S}_{-5,-2}$  are explored.

