CTRP - A Toy Example

Data. Considering $f = 5$ predictors and $B = 10$ permutations, we simulate a $B \times f$ matrix of global test statistics:

G

(1)	(2)	(3)	(4)	(5)
28.42	9.36	9.40	16.68	6.12
0.10	1.37	0.56	0.06	0.08
0.69	4.33	0.36	3.07	0.83
1.07	1.11	0.26	30.31	8.55
0.22	2.87	1.02	7.45	0.48
1.83	2.85	0.02	0.04	0.04
17.68	6.00	1.06	1.82	1.52
1.77	0.29	4.07	26.12	0.26
2.71	8.47	4.42	0.37	5.83
1.14	24.06	2.41	0.03	8.84

Assume that we want to test $S = \{3\}$ with significance level $\alpha = 0.20$.

Shortcut. The supersets of S have sizes $1 + v$ with $v = 0, \dots, 4$:

v							
4				$\mathbf{F} = \boldsymbol{\{1,2,3,4,5\}}$			
3		$\{1,2,3,4\}$	$\{1,2,3,5\}$		$\{1,3,4,5\}$	$\boldsymbol{\{2,3,4,5\}}$	
2	$\{1,2,3\}$	$\{1,3,4\}$	$\{1,3,5\}$		$\{2,3,4\}$	$\boldsymbol{\{2,3,5\}}$	$\{3,4,5\}$
1		$\{1,3\}$	$\{2,3\}$		$\{3,4\}$	$\boldsymbol{\{3,5\}}$	
0				$\mathbf{S} = \boldsymbol{\{3\}}$			

We define

- $\mathbf{D}$, matrix of the centered test statistics in $F \setminus S$, where the indices appear in the order $(5, 2, 4, 1)$ (since $g_5 \leq g_2 \leq g_3 \leq g_4 \leq g_1$);
- $\mathbf{R}$, matrix obtained from $\mathbf{D}$ by sorting the elements within each row in decreasing order.

The lower and upper critical values, L_v and U_v, are the $(1 - \alpha)$-quantiles of

$$\mathbf{d}_{\tilde{V}} = \mathbf{d}_3 + \sum_{i=1}^{v} \mathbf{D}_i \qquad\qquad \mathbf{u}_v = \mathbf{d}_3 + \sum_{i=1}^{v} \mathbf{T}_i,$$

respectively.

$\mathbf{d}_S$	$\mathbf{D}$				$\mathbf{R}$			
(3)	(5)	(2)	(4)	(1)				
0.00	0.00	0.00	0.00	0.00	0.00 (1)	0.00 (4)	0.00 (2)	0.00 (5)
-8.84	-6.03	-7.99	-16.62	-28.32	-6.03 (5)	-7.99 (2)	-16.62 (4)	-28.32 (1)
-9.04	-5.29	-5.02	-13.61	-27.72	-5.02 (2)	-5.29 (5)	-13.61 (4)	-27.72 (1)
-9.14	2.43	-8.25	13.63	-27.34	13.63 (4)	2.43 (5)	-8.25 (2)	-27.34 (1)
-8.38	-5.63	-6.49	-9.23	-28.19	-5.63 (5)	-6.49 (2)	-9.23 (4)	-28.19 (1)
-9.38	-6.08	-6.51	-16.64	-26.59	-6.08 (5)	-6.51 (2)	-16.64 (4)	-26.59 (1)
-8.34	-4.59	-3.36	-14.86	-10.74	-3.36 (2)	-4.59 (5)	-10.74 (1)	-14.86 (4)
-5.33	-5.85	-9.07	9.44	-26.65	9.44 (4)	-5.85 (5)	-9.07 (2)	-26.65 (1)
-4.98	-0.28	-0.89	-16.31	-25.71	-0.28 (5)	-0.89 (2)	-16.31 (4)	-25.71 (1)
-6.99	2.72	14.70	-16.65	-27.27	14.70 (2)	2.72 (5)	-16.65 (4)	-27.27 (1)

Since the first column of $\mathbf{R}$ having no positive elements has index $w = 3$, we start by computing L_v for $v = 0, 1, 2$.

No non-rejection is found, and thus we proceed by examining $v > 2$, until we find either a rejection or a negative U_v. Here U_v becomes negative for $v = 3$, hence the supersets with $v = 3, 4$ are automatically rejected.

Finally, we determine the indecisive values by looking at U_v. Here they are $v = 1, 2$.

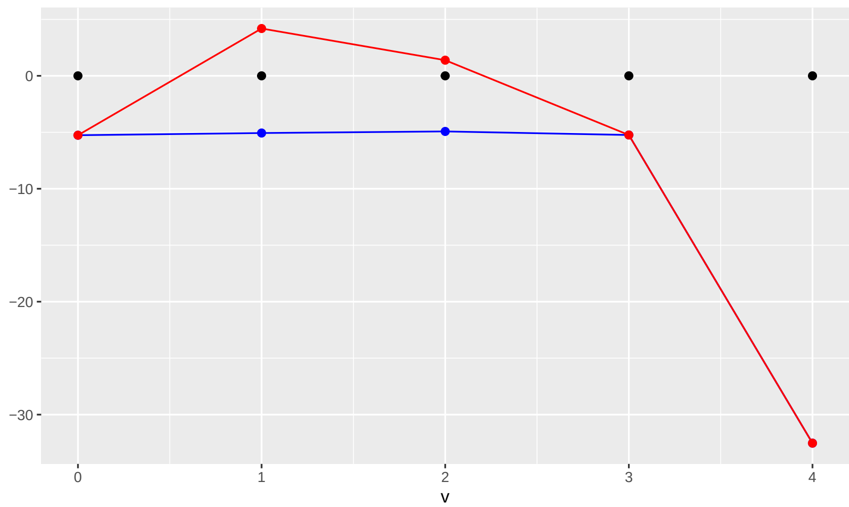

Figure 1: Upper (red) and lower (blue) critical values and observed values (zero, black) by additional superset size v. The bounds for $v = 4$ have not been computed in the analysis.

v	0	1	2	3	4
U_v	-5.26	4.19	1.38	-5.24	(-32.53)
L_v	-5.26	-5.06	-4.93	-5.24	(-32.53)
rejection	T	?	?	T	T

Branch and Bound when removing the highest statistic. The last index in $\mathbf{D}$, $i = 1$, determines the branching rule. We explore first the subspace where the index is removed.

	remove $S \subseteq V \subseteq F \setminus \{1\}$			keep $S \cup \{1\} \subseteq V \subseteq F$		
2	$\{2, 3, 4\}$	$\{2, 3, 5\}$	$\{3, 4, 5\}$	$\{1, 2, 3\}$	$\{1, 3, 4\}$	$\{1, 3, 5\}$
1	$\{2, 3\}$	$\{3, 4\}$	$\{3, 5\}$		$\{1, 3\}$	