CTRP - A Toy Example

1 Data

Considering $f = 5$ predictors and $B = 10$ permutations, we simulate a $B \times f$ matrix of global test statistics:

G

(1)	(2)	(3)	(4)	(5)
28.42	9.36	9.40	16.68	6.12
0.10	1.37	0.56	0.06	0.08
0.69	4.33	0.36	3.07	0.83
1.07	1.11	0.26	30.31	8.55
0.22	2.87	1.02	7.45	0.48
1.83	2.85	0.02	0.04	0.04
17.68	6.00	1.06	1.82	1.52
1.77	0.29	4.07	26.12	0.26
2.71	8.47	4.42	0.37	5.83
1.14	24.06	2.41	0.03	8.84

Assume that we want to test $S = \{3\}$ with significance level $\alpha = 0.20$.

2 Shortcut

v							
4				$\mathbf{F = \{1, 2, 3, 4, 5\}}$			
3		$\{1,2,3,4\}$	$\{1,2,3,5\}$		$\{1,3,4,5\}$	$\mathbf{\{2,3,4,5\}}$	
2	$\{1,2,3\}$	$\{1,3,4\}$	$\{1,3,5\}$		$\{2,3,4\}$	$\mathbf{\{2,3,5\}}$	$\{3,4,5\}$
1		$\{1,3\}$	$\{2,3\}$		$\{3,4\}$	$\mathbf{\{3,5\}}$	
0				$\mathbf{S = \{3\}}$			

Table 1: Supersets of $S = \{3\}$, having sizes $|V| = 1 + v$ with $v = 0, \ldots, 4$. The sets in bold are used to define the lower critical value L_v.

We define

- **D**, matrix of the centered test statistics in $F \setminus S$, where the indices appear in the order $(5, 2, 4, 1)$ (since $g_5 \leq g_2 \leq g_4 \leq g_1$);
- **R**, matrix obtained from **D** by sorting the elements within each row in decreasing order.

$\mathbf{d}_S$	$\mathbf{D}$				$\mathbf{R}$			
(3)	(5)	(2)	(4)	(1)				
0.00	0.00	0.00	0.00	0.00	0.00 (1)	0.00 (4)	0.00 (2)	0.00 (5)
-8.84	-6.03	-7.99	-16.62	-28.32	-6.03 (5)	-7.99 (2)	-16.62 (4)	-28.32 (1)
-9.04	-5.29	-5.02	-13.61	-27.72	-5.02 (2)	-5.29 (5)	-13.61 (4)	-27.72 (1)
-9.14	2.43	-8.25	13.63	-27.34	13.63 (4)	2.43 (5)	-8.25 (2)	-27.34 (1)
-8.38	-5.63	-6.49	-9.23	-28.19	-5.63 (5)	-6.49 (2)	-9.23 (4)	-28.19 (1)
-9.38	-6.08	-6.51	-16.64	-26.59	-6.08 (5)	-6.51 (2)	-16.64 (4)	-26.59 (1)
-8.34	-4.59	-3.36	-14.86	-10.74	-3.36 (2)	-4.59 (5)	-10.74 (1)	-14.86 (4)
-5.33	-5.85	-9.07	9.44	-26.65	9.44 (4)	-5.85 (5)	-9.07 (2)	-26.65 (1)
-4.98	-0.28	-0.89	-16.31	-25.71	-0.28 (5)	-0.89 (2)	-16.31 (4)	-25.71 (1)
-6.99	2.72	14.70	-16.65	-27.27	14.70 (2)	2.72 (5)	-16.65 (4)	-27.27 (1)

The lower and upper critical values, L_v and U_v, are the 8-th ordered statistics of

$$\mathbf{d}_{\tilde{V}} = \mathbf{d}_3 + \sum_{i=1}^{v} \mathbf{D}_i \qquad\qquad \mathbf{u}_v = \mathbf{d}_3 + \sum_{i=1}^{v} \mathbf{R}_i.$$

Since the first column of $\mathbf{R}$ having no positive elements has index $w = 3$, we start by computing L_v for $v = 0, 1, 2$.

No non-rejection is found, and thus we proceed by examining $v > 2$, until we find either a rejection or a negative U_v. Here U_v becomes negative for $v = 3$, hence the supersets with $v = 3, 4$ are automatically rejected.

Finally, we determine the indecisive values by looking at U_v. Here they are $v = 1, 2$.

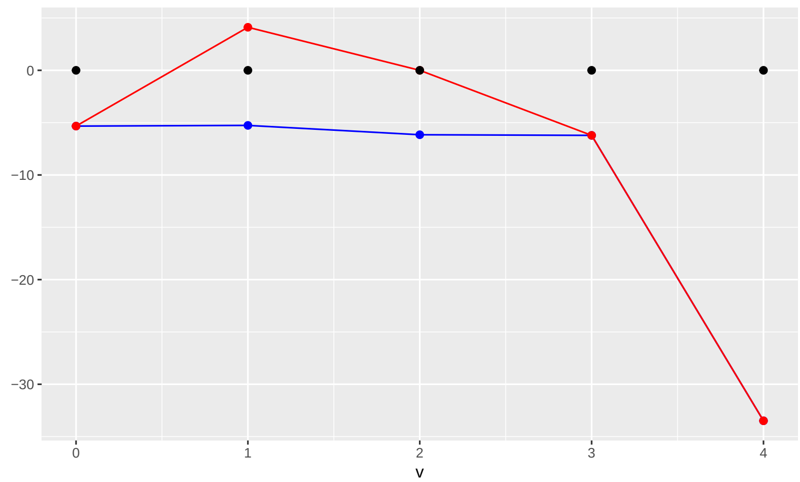


Figure 1: Upper (red) and lower (blue) critical values and observed values (zero, black) by additional superset size v. The bounds for $v = 4$ have not been computed in the analysis.

v	0	1	2	3	4
U_v	-5.33	4.11	0.00	-6.22	(-33.49)
L_v	-5.33	-5.26	-6.16	-6.22	(-33.49)
rej	T	?	?	T	T

3 Branch and Bound

3.1 Removal of the highest statistic.

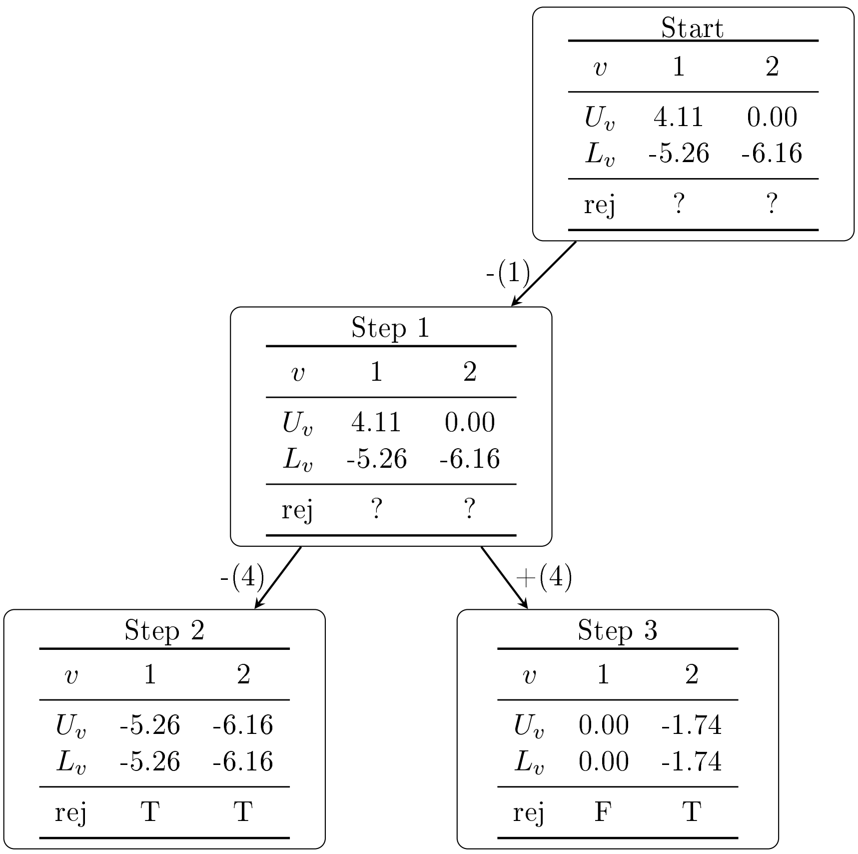
The last index in $\mathbf{D}$, $e = 1$, determines the branching rule. We explore first the subspace $\mathbb{S}_{-1}$, where the index is removed.

	$\mathbb{S}_{-1}$			$\mathbb{S}_{+1}$		
2	$\{2,3,4\}$	$\{2,3,5\}$	$\{3,4,5\}$	$\{1,2,3\}$	$\{1,3,4\}$	$\{1,3,5\}$
1	$\{2,3\}$	$\{3,4\}$	$\{3,5\}$		$\{1,3\}$	

Since the outcome is still indecisive for both sizes $v = 1, 2$, the subspace is partitioned again according to the inclusion of $e = 4$ (corresponding to the second highest statistic g_i).

	$\mathbb{S}_{-1,-4}$			$\mathbb{S}_{-1,+4}$		
2		$\{2,3,5\}$		$\{2,3,4\}$		$\{3,4,5\}$
1	$\{2,3\}$		$\{3,5\}$		$\{3,4\}$	

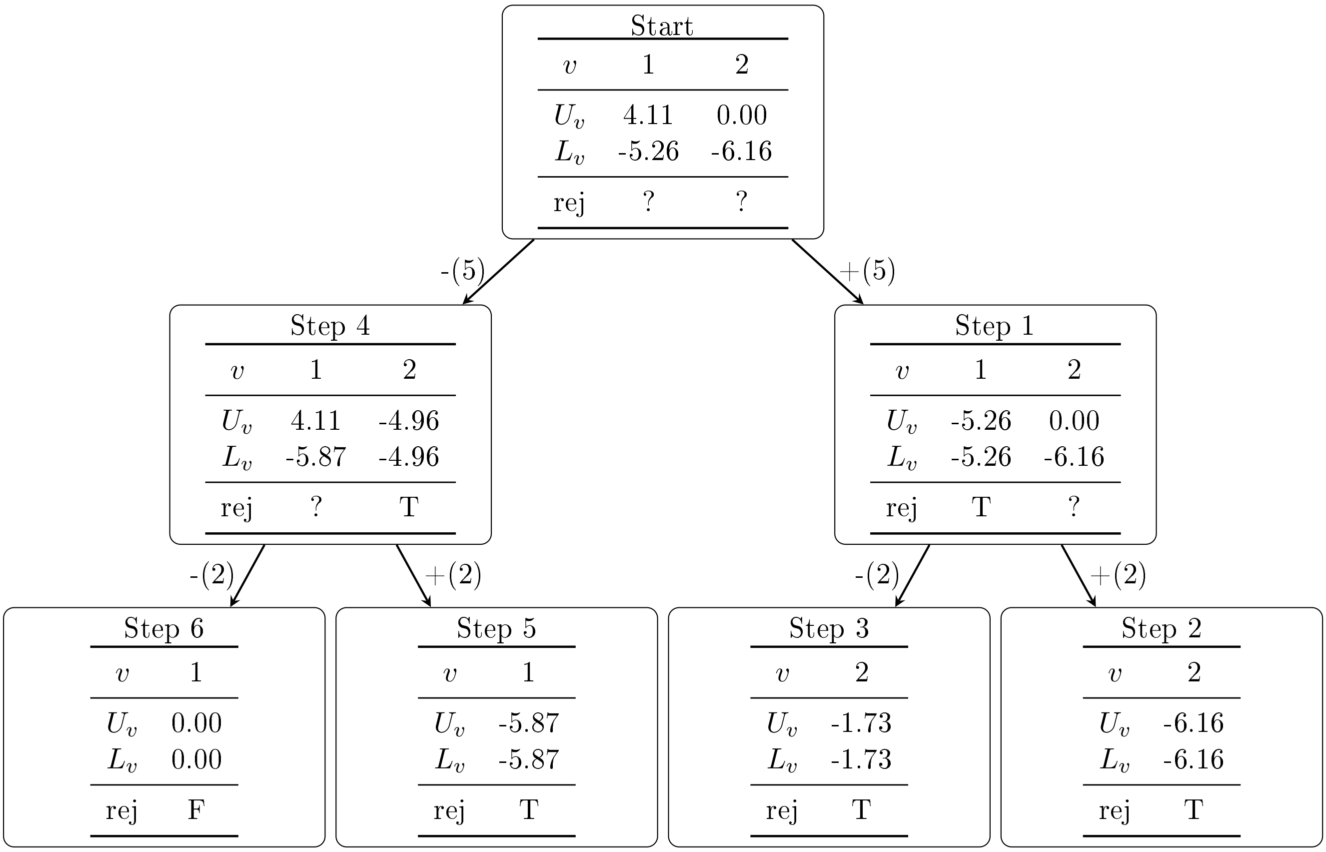
Since S is not rejected in $\mathbb{S}_{-1,+4}$, it is not rejected in the total spece after 3 steps.



3.2 Keeping of the lowest statistic.

The first index in $\mathbf{D}$, $e = 5$, determines the branching rule. We explore first the subspace $\mathbb{S}_{+5}$, where the index is kept.

After 3 steps, S is rejected in the subspace $\mathbb{S}_{+5}$. Subsequently, after 3 other steps, it is not rejected in $\mathbb{S}_{-5}$. In conclusion, it is not rejected after 6 steps.



3.3 Removal of the lowest statistic.

The first index in $\mathbf{D}$, $e = 5$, determines the branching rule. We explore first the subspace $\mathbb{S}_{-5}$, where the index is removed.

S is not rejected after 2 steps, where the subspaces $\mathbb{S}_{-5}$ and $\mathbb{S}_{-5,-2}$ are explored.

Start

v	1	2
U_v	4.11	0.00
L_v	-5.26	-6.16
rej	?	?

-(5)

Step 1

v	1	2
U_v	4.11	-4.96
L_v	-5.87	-4.96
rej	?	T

-(2)

Step 2

v	1
U_v	0.00
L_v	0.00
rej	F