Weekly HW #1

6) a. $E(N(t)) = \lambda t$ $\quad V(N(t)) = \lambda t$

$$E(N(t)) = \sum_{n=0}^{\infty} n \cdot P[N(t)=n]$$

$$\sum_{n=0}^{\infty} n \, \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

$$(\lambda t)\sum_{n=1}^{\infty} \frac{e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$

$$(\lambda t)\times 1 = \lambda t$$

$$\therefore E(N(t)) = \lambda t$$

Variance of poisson:

$$E(N^2(t)) = \sum_{n=0}^{\infty} n^2 \, P(N(t)=n)$$

$$\sum_{n=0}^{\infty} n^2 \, \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

$$(\lambda t)\sum_{n=1}^{\infty} n \, \frac{e^{\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$

$$\lambda t \sum_{n=1}^{\infty} (n-1+1) \, \frac{e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$

$$\lambda t\left(\sum_{n=1}^{\infty}(n-1)\frac{e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!} + \sum_{n=1}^{\infty} 1 \cdot \frac{e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}\right)$$

$$\lambda t\left(\lambda t \sum_{n=2}^{\infty} \frac{e^{-\lambda t}(\lambda t)^{n-2}}{(n-2)!} + \sum_{n=1}^{\infty} \frac{e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}\right)$$

$$(\lambda t)\left((\lambda t(1) + (1)\right)$$

$$= (\lambda t)^2 + \lambda t$$

$V(N(t)) = E(N^2(t)) - (E(N(t))^2)$

$(\lambda t)^2 + (\lambda t) - (\lambda t)^2$

$= \lambda t$

$\therefore V(N(t)) = \lambda t$

314074269

45131658

b. $P(N=18) = \dfrac{e^{-2}(\lambda)^n}{n!}$

$\dfrac{e^{-171}(171)^{18}}{18!}$

dpois$(171, 18) = 5.5019 \times 10^{-103}$ or

essentially $P = 0$ // ANS

c. $\lambda = n \cdot p$

$.95 \times 18 = 17.1$

$\sigma = \sqrt{\lambda} = \sqrt{17.1} = 4.3$ // ANS

d. ~~The method used in example 7.4 is likely more accurate but both contain 171 w/in the interval~~

The exact method gives a more precise answer so it is better for a single day. The central limit theorem applies to larger time periods ~~due~~ bc it gives a larger r value.

5. $\lambda = 171$

$P(x) = \lambda e^{-\lambda x} = 171 e^{-171x}$

$P(x \leq 153) = 171 \frac{e^{-171x}}{-171} \Big|_{x=0}^{153}$

$P(x \leq 153) = -1(e^{-171 \cdot 153} - e^{0}) = (1 - e^{-26163})$

a. $2050/12 = 170\ 5/6$

estimated 171 house fires / month

b. $\lambda = 171$

normal variation

$\frac{n}{\lambda} - 2\frac{\sqrt{n}}{\lambda} \leq X_1 + \ldots + X_n \leq n/\lambda + 2\sqrt{n}/\lambda$

$\cancel{P(\frac{e^{-\lambda}\lambda^x}{x!}}$, variation of ± 23

range = 148 to 195 for 1 month

$\sigma^2 \int_0^\infty (x - 1/\lambda)^2 \lambda e^{-\lambda x} dx$ (x12)

1776 → 2340 calls / year

$X \sim$ Poisson

c. The range for λ for 2050 calls (~~the~~ range of normal variation) is

139 to 190

$\frac{n}{171} - \frac{2\sqrt{n}}{171} \leq X_1 + \ldots X_n \leq \frac{n}{171} + \frac{2\sqrt{n}}{171}$

Poisson

$$P(\bar{X}=x) = \frac{e^{-\alpha t}(\alpha t)^x}{x!}$$

x	0	1	2	...
$P(x)$	$\frac{e^{-\alpha t}(\alpha t)^0}{0!}$	$\frac{e^{-\alpha t}(\alpha t)^1}{1!}$	–	–
$P(X \le x)$				

d. 29 years of data needed to obtain an estimate for λ w/ erro of $\pm$ 0.5 (chegg)

11.

AD type	detect	acquire	hit
low (guns)	.9	.8	.05
High (missiles)	.75	.95	.7

low, guns - 20/min - 1 min exposed

high, missiles - 3/min - 5 min exposed

high, missiles	low, guns
15 shots	20 shots
.498 = P(hit)	.036 = P(hit)
7.481/15 hits	.72/20 hits
safer	

a. the optimal flight path is flying high

b. 70% chance to destroy the target

P(target destroyed) = P(detect ∩ acquire ∩ hit ∩ destroy)

high = .498 · .7   P = 0.3486

low = .036 · .7   P = .0252

c. assume normal distribution
P(success) for one is .3486
P(success) for 3 is >1

∴ 3 bombers necessary for
mission success

d. sensitivity in R
The 0.7 value is relatively sensitive.
The number of bombers, however,
will need to be 3 or more
regardless of if the value p = 0.7
changes ~~der~~ due to the other
probability conditions of
detect, acquire, and hit.

e. P detect ↓ P ↓

Δ	p		P detect
.1	.6	>	.43225
.2	.5	>	.36575
.3	.4	>	.29925
.4	.3	>	.23275
.5	.2	>	.16625
.6	.1	>	.09975
.7	0	<	.03325

Unless the probabilities drop by 0.7,
the bomber will be more successful