2015 "Will Ask" Qual Questions

Radiative Processes

1. **Derive the total power and characteristic frequency of synchrotron radiation from a relativistic particle of mass m, charge e, and energy E moving in a magnetic field B. Use this to explain why synchrotron radiation is generally negligible for protons.**

   Synchrotron radiation is the energy radiated by a relativistic charged particle as it moves in a magnetic field.

   Let's consider the case of an electron moving in a uniform external magnetic field. The electron experiences a Lorentz force:

$$\mathbf{F} = e\,(\frac{\mathbf{v}}{c} \times \mathbf{B}) \tag{1}$$

   The equation of motion of the particle can thus be written

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(\gamma m \mathbf{v})}{dt} = \gamma m \frac{d\mathbf{v}}{dt} = e\,(\frac{\mathbf{v}}{c} \times \mathbf{B}) \tag{2}$$

   Notice that the electron's speed does not change, because the magnetic field can only act perpendicular to its motion. Thus, only the direction of the electron can change. From this, you can see that both the parallel and perpendicular components of the velocity (wrt the magnetic field) remain constant.

   The net result is that you have a constant magnetic field in which an electron is moving along a field line on a uniform helical path (or with zero velocity along a field line, a circular path) with a constant linear and angular speed. There is no force along a field line.

   The quantity $\frac{d\mathbf{v}}{dt}$ is acceleration. Solve for it:

$$\mathbf{a} = \frac{e(\mathbf{v} \times \mathbf{B})}{\gamma m c} \tag{3}$$

   And in the instantaneous inertial frame of the electron, the velocity is perpendicular to the magnetic field and we can regard the acceleration as the centripetal acceleration. $\frac{v^2}{r}$. Setting $v = r\omega$, we get a characteristic frequency at which the electron is spiralling, called the **gyrofrequency**:

$$\omega_g = \frac{eB}{\gamma m_e c} \tag{4}$$

This is, however, not the characteristic frequency that we observe. We must take into account a number of relativistic effects, including that power must be multiplied by γ^2, a combination of relativistic beaming and Doppler effect that turns a sinusoidal light curve into a series of sharp pulses. The resulting radiation spectrum peaks at $\gamma^3\omega_g$ which is three times the gyrofrequency.

The **relativistic Larmor formula** gives the power radiated from a single relativistic electron:

$$P = \frac{2}{3}\frac{\gamma^4 e^2 a^2}{c^3} \tag{5}$$

You can derive it via Lorentz transformations, taking the non-relativistic Larmor formula to hold true in the frame of the particle.

Using our expression for acceleration above, we get that the power radiated is

$$P = \frac{4}{3}\,\sigma_T\, c\,\gamma^2\,\beta^2\,u_B \tag{6}$$

The synchrotron radiation loss for a particle scales like its gamma. So the loss rate for an electron is over 10^{13} times the loss for a proton of the same energy. (I think that it's just because of the mass?)

2. **Explain the connection between detailed balance, the Einstein A and B relations, Kirchoffs law and the Milne relations, and give an example of their use to connect the bremsstrahlung emission spectrum and the free-free absorption coefficient.**

The Einstein relations link together atomic properties and thus macroscopic conditions (like thermal equilibrium) don't matter. Consider a two-level system, the top level characterized by L_2, g_2 and the bottom level characterized by L_1, g_1. The energy gap is $h\nu$. The different Einstein coefficients characterize the transition probability per unit time for three different mechanisms: A_{21} for spontaneous emission (no radiation field required), $B_{12}\bar{J}$ for absorption, and $B_{21}\bar{J}$ for stimulated emission.

$$\bar{J} \equiv \int_0^\infty J_\nu \phi(\nu) d\nu \tag{7}$$

Note that the B's are both proportional to the mean intensity of the radiation field.

The detailed balance equation says that the number of transitions into level 2 equals the number of transitions out of level 2. In other words, in thermal equilibrium, the number of transitions into an atomic state equals the number of transitions out of the state. It can be written as follows:

$$n_1 B_{12} \bar{J} = n_2 B_{21} \bar{J} + n_2 A_{21} \tag{8}$$

The Boltzmann Equation governing thermal equilibrium is

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{h\nu/k_B T} \tag{9}$$

Using this to solve for $\bar{J}$ we get

$$\bar{J} = \frac{A_{21}/B_{21}}{(g_1 B_{12}/g_2 B_{21}) e^{h\nu/kT} - 1}$$

This is equal to

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

This gives the Einstein relations:

$$\frac{A_{21}}{B_{21}} = \frac{2h\nu^3}{c^2} \tag{10}$$

$$g_1 B_{12} = g_2 B_{21} \tag{11}$$

Notice that the temperature dependence drops, so the Einstein relations hold even outside of thermal equilibrium.

Kirchoff's law for thermal equilibrium says that a good emitter is a good absorber, in thermal equilibrium. The moral of the story is that there is clearly a relationship between emission and absorption at a microscopic level.

$$j_\nu = \alpha_\nu B_\nu(T) \tag{12}$$

The **Milne Relation** governs the inverse process of photoionization, that is an ion recapturing an electron and emitting a photon. Let σ_{fb} be the cross-section for recapture. We assume complete thermal equilibrium and derive a result that will end up being independent of thermal equilibrium.

Thermal equilibrium dictates that the rate of radiative recombinations must equal the rate of photoionization.

Bremsstrahlung, or **free-free emission**, involves an electron whizzing by an ion. There is radiation emitted due to the acceleration of charge in the Coulomb field of another charge.

Remember, a good emitter is a good absorber. Using Kirchoff's Law,

$$j_\nu^{ff} = \alpha_\nu^{ff} B_\nu(T)$$

$$\frac{dW}{dtdVdv} = \epsilon_\nu^{ff} = 4\pi j_\nu^{ff} = 6.8 \times 10^{-38} T^{-1/2} Z^2 n_e n_i e^{h\nu/kT} \bar{g}^{ff}(\nu, T) \tag{13}$$

But you also have Kirchhoff's Law (thermal); both of these processes are happening because we're in thermal equilibrium. $S_\nu = B_\nu$, and $j_\nu = \alpha_\nu B_\nu$.

$$\alpha_\nu^{ff} = 3.7 \times 10^8 T^{-1/2} Z^2 n_e n_i \nu^{-3} (1 - e^{-h\nu/kT}) g^{ff} \tag{14}$$

In the Wien limit, you get $\alpha_\nu^{ff} \propto \nu^{-3} T_{-1/2}$. In the RJ limit, you get $\alpha_\nu^{ff} \propto \nu^{-2} T^{-3/2}$.

So, you preferantially absorb low-energy electrons.

3. Draw the energy levels of the hydrogen atom and identify which transitions are allowed. Which ones are in the visible part of the spectrum? Which level has no allowed decays, and what is its main decay mode?

Instrumentation

1. **Describe quantitatively the point spread function of a diffraction-limited optical telescope. Explain how diffraction spikes arise, and what determines their position and intensities. Under what circumstances will the PSF be broadened by atmospheric turbulence?**

Stars

1. **Explain what the Hayashi track is, and describe what types of objects live on it. Qualitatively explain how it arises and what assumptions are required for its derivation.**

2. **Estimate the temperature as a function of depth in the Suns convection zone. What is the temperature at the base of the convection zone?**