

Linear Systems Homework #2

1. Show that the rank of the following matrix is 2:

$$A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 0 & 1 & -1 \\ -1 & 1 & 0 & 1 \end{bmatrix}$$

2. Let $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times m}$. Show that the nonzero eigenvalues of AB are the same as those of BA .

3. Find the ranks and nullities and provide basis for the range and null spaces of the following matrices:

$$(a) \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 4 & 0 \\ 1 & 0 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & 4 & 1 \\ -1 & 1 & 0 & 0 \end{bmatrix} \quad (d) \begin{bmatrix} 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 11 & 0 & 0 & 0 \end{bmatrix}$$

5. Find the diagonal or Jordan canonical forms for the following matrices:

$$(a) \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \quad (b) \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix} \quad (d) \begin{bmatrix} 2 & 11 \\ 0 & 2 \end{bmatrix}$$

6. $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. (a) Find the induce matrix norm $\|M\|_{1/1}$ using the “column sum”. (b) Write a

Matlab code to search 100 points for calculating the $\|M\|_{1/1}$. (c) Find the induce matrix norm

$\|M\|_{\infty/\infty}$ using the “row sum”. (d) Write a Matlab code to search 100 points for calculating the

$\|M\|_{\infty/\infty}$. (Please attach your Matlab codes and results in your report.)

7. $\mathbf{v} = [v_1 \quad v_2 \quad \cdots \quad v_n]$. Please show that $\frac{1}{n} \|\mathbf{v}\|_1 \leq \|\mathbf{v}\|_2 \leq \|\mathbf{v}\|_1$

8. $G(s) = \frac{2s+3}{s^2+5s+6}$, please find the state equations of the system in (a) controllable canonical form, (b)

observable canonical form, (c) diagonal canonical form.