數學實作 Math Application Project 2
Fourier Series and Transforms

Rules:

- This project is to be completed in a group of $\leqslant 4$ students. One submission per group, uploaded onto e3. Please list all members' student ID.
- Submission includes: (i) A written report, (ii) Any associated coding or Excel files.
- Report must:
  - Be written in English
  - All texts and equations must be typed, all graphs computer-generated. Hand-writing or photographed papers are not accepted.
  - Report must use the same section titles, numbering format, font and font-size as this document. It must be easily readable.
  - Questions that must be answered have been highlighted in **blue** below.
- This project is due by 5pm, June 16 2023. Late submission = zero points awarded. Check whether your upload is successful. (Don't wait until the last minute, upload 1-2 days early).
- Points will be deducted if academic dishonesty is discovered, or if it is deemed you have addressed this assignment in a manner that conflicts with the spirit of learning.
- Ask for clarification if needed.

Project Goal:

To give you an intuitive feel for Fourier Series and Transforms through examples.

What Do You Need To Do:

Part 1: Matlab:

1. **You'll need to install Matlab for this exercise. NYCU IT has student license available.** https://jupiter.math.nycu.edu.tw/~smchang/matlab/nycu_matlab_2022.pdf
2. **You'll need to learn how to make basic scatter-plot or line-plot with Matlab. Some example:** https://www.mathworks.com/products/matlab/plot-gallery.html
3. **You'll also need to learn how to read in an *.mp3* file using Matlab (**https://www.mathworks.com/help/matlab/ref/audioread.html**) and to visualize the signal as a plot (**https://www.mathworks.com/help/matlab/import_export/read-and-get-information-about-audio-files.html**).**

Part 1: Synthetic Signals:

1. **Simple cosine signal:**
   a. Given a cosine signal of amplitude = 1 and frequency = 10Hz. Write a function $g(t)$ for this signal. Beware: $\cos(10t)$ is *not* 10Hz.
   b. Find the Fourier Transform of $g(t)$ by hand to obtain $\hat{g}(f)$, where f denotes frequency. Show step-by-step derivation.
   c. Plot $\hat{g}(f)$ as a function of f in Matlab. Does the plot correctly show a peak at 10Hz?
   d. From the attached example Matlab code, generate a discrete cosine signal of the same amplitude and frequency as (a). Take the DFT of the signal. Plot both the signal and its magnitude spectrum. Does it agree with your hand-derived solution?
2. **Phasing signal:**

a. Add a sine signal of identical amplitude and frequency to the signal in (1a). Show how the raw "signal" changed as a result.
b. Did the magnitude spectrum's peak change in frequency? Why?

Part 2: Audio Manipulation:

3. **Application: This is the fun part, illustrating the application of FFT in signal-processing. Particularly: How do you separate two superimposed "voices" based on their frequencies.**
    a. Two audio files from https://www.fisheries.noaa.gov/national/science-data/sounds-ocean have been included: (i) Dolphin, (ii) Whale + ocean noise.
    b. Write Matlab code that reads both audio and plot their signals. (See instruction in Part 1.3 above).
    c. Perform FFT on the signal. Plot the magnitude spectrum of each audio file. Describe any differences (Dolphin should have higher freq).
    d. To simulate an *.mp3* where the Dolphin and Whale were recorded simultaneously, add the their signals together. This is performed by element-to-element summation of the two signals' arrays. (Note: since the *.mp3* files have different lengths, you may need to repeat the shorter signal). Plot this new signal and its spectrum. Compare to (3b, 3c).
    e. Can you identify which region of the frequency-domain belong to the Dolphin, the Whale and the ocean background noise, respectively?
    f. Bonus: Try to separate the signal in (5d-e) into ***Dolphin vs. Whale vs. Background*** in the frequency domain, then use inverse-FFT to reconstruct the time-domain audio files. Save these files and play. Was the audio manipulation successful? (I.e. did you clearly isolate Dolphin, Whale and Background sounds using FFT?) Some reference knowledge: https://medium.com/swlh/noise-removal-for-a-better-fast-fourier-transformation-284918d4250f