Homework

1. For the following system:

$$\dot{x}_1 = -x_1 + x_2 + u$$
$$\dot{x}_2 = -2x_2 + 0.1u$$
$$\mathrm{y} = x_1 + 0.1x_2$$

   (a) Please find the controllability matrix and observability matrix of above system.
   (b) Please check if the system is controllable and/or observable?
   (c) Find the balance realization of the above system.
   (d) Find the controllability grammian and observability grammian of the system after the balance realization.
   (e) Do the order reduction of the system based on the balanced realization technique. Please show the state space model of the reduced-order system.
   (f) Show the bode plots of the original system and reduced-order system in the same plot.

(Hint: you may use Matlab functions to complete this homework. But, if you do so, please turn in your Matlab scripts along with the plots.)

2. For the following system, design a state feedback controls using estimated states

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = 2x_1 + 3x_2 + u$$
$$\mathrm{y} = x_1 + x_2$$

   (a) Find the state feedback gain and observer gain such that the overall system has the eigenvalues at (-10, -15, -20, -20).
   (b) Write down the state equations for the overall system.
   (c) Using Matlab to simulate and draw the step response of the open loop system and close loop system on the same plot.

3. Create a 3rd order system that is unstabilizable. And explain why it is unstabilizable.