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1.

$\dot{x}_1 = -x_1 + x_2 + u$

$\dot{x}_2 = -2x_2 + 0.1u$

$y = x_1 + 0.1x_2$

$$\Rightarrow \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}1\\ 0.1\end{bmatrix}}_{B}u$$

$$y = \underbrace{\begin{bmatrix}1 & 0.1\end{bmatrix}}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad D=0.$$

a)
b) controllibility matrix $= [B \;\; AB]$ ← 2^{nd} order sys.

$AB = \underset{2\times2}{\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}}\underset{2\times1}{\begin{bmatrix}1\\ 0.1\end{bmatrix}} = \underset{2\times1}{\begin{bmatrix}-0.9\\ -0.2\end{bmatrix}}$

$$= \begin{bmatrix}1 & -0.9\\ 0.1 & -0.2\end{bmatrix} ※$$

而 $\text{rank}\left(\begin{bmatrix}1 & -0.9\\ 0.1 & -0.2\end{bmatrix}\right) = 2$ ← 故 full rank $\Rightarrow$ rank $= n = 2$

$\Rightarrow$ controllable ※.

gauss elimation

$\begin{pmatrix}1 & -0.9\\ 0.1 & -0.2\end{pmatrix} \rightarrow \begin{pmatrix}1 & -0.9\\ 0 & 1\end{pmatrix}$ ← rank = 2.

pivot

observability matrix $\begin{bmatrix}C\\ CA\end{bmatrix} = \begin{bmatrix}1 & 0.1\\ -1 & 0.8\end{bmatrix}$ ※

$CA = \underset{1\times2}{\begin{bmatrix}1 & 0.1\end{bmatrix}}\underset{2\times2}{\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}}$

$= \begin{bmatrix}-1 & 0.8\end{bmatrix}$

$\text{rank}\left(\begin{bmatrix}1 & 0.1\\ -1 & 0.8\end{bmatrix}\right)$

$= \text{rank}\left(\begin{bmatrix}1 & 0.1\\ 0 & 1\end{bmatrix}\right)$

pivot

$= 2$ (full rank)

$\Rightarrow$ observable ※

c) 由 Matlab 做
d)
e)
f)

2.

$\dot{x}_1 = x_2$

$\dot{x}_2 = 2x_1 + 3x_2 + u$

$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = x_1 + x_2$

$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

用 observer estimated states to feedback

(a) 求 K (state feedback gain), L (observer feedback gain) $\Rightarrow$ overall sys. eigenvalues: -10, -15, -20, -20 s.t. $\hookrightarrow n=4$.

K: 影響2个 eigenval. L: 2个. → 共4个 eigenvals.

$$\begin{bmatrix} \dot{x}_{2\times1} \\ \dot{\hat{x}}_{2\times1} \end{bmatrix}_{4\times1} = \begin{bmatrix} A & -BK \\ LC & A-LC-BK \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} v$$

Separation Thm $\Rightarrow$ K, L 可分開 design 不互相affect.

$u = -K\hat{x} + v \xrightarrow{\text{放入}} \dot{x} = Ax + B(-K\hat{x} + v)$

$$\dot{\hat{x}} = A\hat{x} + B(\underbrace{-K\hat{x}+v}_{u}) + L(\overset{Cx}{\cancel{y}} - C\hat{x})$$

observer estimate; correction term.

$= (A-LC)\hat{x} + B(-K\hat{x}+v) + LCx$

$= (A-LC-BK)\hat{x} + Bv + LCx$

↓ similiar transformation

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v$$

$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$

eigenvalues for control (poles) $\Rightarrow \det(sI - (A-BK))$

$$\Rightarrow \det \begin{vmatrix} s & -1 \\ k_1-2 & s+k_2-3 \end{vmatrix} = 0$$

$(s)(s+k_2-3) + k_1 - 2 = 0$

$\Rightarrow s^2 + (k_2-3)s + (k_1-2) = 0$ ⇐下頁

$$(A-BK) = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} - \underset{2\times1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \underset{1\times2}{\begin{bmatrix} k_1 & k_2 \end{bmatrix}} = \begin{bmatrix} 0 & 1 \\ 2-k_1 & 3-k_2 \end{bmatrix}$$

similiar transformation　T: non-singular

$\bar{x} = Tx$, Let $\overset{\text{estimation error}}{e} = x - \hat{x}$ $\Rightarrow \dot{\bar{x}} = \overset{\text{new A}}{TAT^{-1}}\bar{x} + \overset{\text{new B}}{TB}v$

$y = Cx = \underset{\text{new C}}{CT^{-1}}\bar{x}$

↓ new A B C for 新的座標表示

$$\underset{\text{新}}{\begin{bmatrix} x \\ e \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \underset{\text{原本的}}{\begin{bmatrix} x \\ \hat{x} \end{bmatrix}}$$

續

$s^2+(k_2-3)s+(k_1-2)=0 \longleftrightarrow (s+10)(s+15)=0$

↖ given eigenvals at -10 & -15

要design $K=[k_1\ k_2]$ 使 poles在 desirable places.

$s^2+25s+150=0$

$k_2-3=25 \Rightarrow k_2=28$

$k_1-2=150 \Rightarrow k_1=152$

$\Rightarrow K=[152\ \ 28]$ #

$\det(sI-(A-LC))=0 \longleftrightarrow (s+20)^2=0$

$s^2+40s+400=0$

$A-LC=\begin{bmatrix}0&1\\2&3\end{bmatrix}-\underbrace{\begin{bmatrix}l_1\\l_2\end{bmatrix}}_{2\times1}\underbrace{[1\ \ 1]}_{1\times2}$

$=\begin{bmatrix}-l_1&1-l_1\\2-l_2&3-l_2\end{bmatrix}$

$\Rightarrow \begin{vmatrix}s+l_1 & l_1-1\\ l_2-2 & s+l_2-3\end{vmatrix}$

$=(s+l_1)(s+l_2-3)-(l_1-1)(l_2-2)$

$=s^2+(l_1+l_2-3)s+l_1(l_2-3)$

$\quad -l_1l_2+2l_1+l_2-2$

$\begin{cases}l_1+l_2-3=40 \\ -l_1+l_2-2=400\end{cases} \Rightarrow \begin{matrix}l_1=-179.5\\ l_2=222.5\end{matrix} \Rightarrow L=\begin{bmatrix}-179.5\\222.5\end{bmatrix}$ #

b) overall state Eqns 如同上頁所述

$$\begin{bmatrix}\dot{x}\\ \dot{\hat{x}}\end{bmatrix}=\begin{bmatrix}A & -BK\\ LC & A-BK-LC\end{bmatrix}\begin{bmatrix}x\\ \hat{x}\end{bmatrix}+\begin{bmatrix}B\\B\end{bmatrix}v,\quad y=[C\ \ 0]\begin{bmatrix}x\\ \hat{x}\end{bmatrix}$$

而 汙用 similiar transformation 用另一S表示法.

$$\begin{bmatrix}\dot{x}\\ \dot{e}\end{bmatrix}=\begin{bmatrix}A-BK & BK\\ 0 & A-LC\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix}+\begin{bmatrix}B\\0\end{bmatrix}v,\quad y=[C\ \ 0]\begin{bmatrix}x\\ e\end{bmatrix}$$

2. C) Matlab 如附件

3. 3^{rd} order sys. 要 Unstabilizable

⇒ unstabilizable system 即為有 unstable modes (states) 在 uncontrollable subspace

因此無法以 control feedback 使它們 stable 下來

⇒ unstabilizable

⇒ Kalman canonical form 先將 controllable states & uncontrollable states 分開，在 Uncontrollable part 設計一个 unstable state

(假設 sys. 在 continuous time ⇒ $\lambda_i > 0$)

(因現在以微分方程表示 sys.)

$$\begin{bmatrix} \dot{\bar{X}}_c \\ \dot{\bar{X}}_{uc} \end{bmatrix} = \begin{bmatrix} \bar{A}_c & \bar{A}_{12} \\ 0 & \bar{A}_{uc} \end{bmatrix} \begin{bmatrix} \bar{X}_c \\ \bar{X}_{uc} \end{bmatrix} + \begin{bmatrix} \bar{B}_c \\ 0 \end{bmatrix} u$$

為 0，因為 $\bar{X}_{uc}$ 不可受 input affect，而 $\bar{X}_c$ 有受 input affect

⇒ 故 $\bar{X}_{uc}$ 亦不可亦 $\bar{X}_c$ affect，否則会有間接的影響，使 $\bar{X}_{uc}$ 受 input u 的影響，不符合 uncontrollable 的定義

假設最簡單的情況

⇒ $[\bar{A}_c]_{2\times2}$ $[\bar{A}_{uc}]_{1\times1}$

3^{rd} order

$\bar{A}_c$ (eig<0 stable)

$$\begin{bmatrix} [\dot{\bar{X}}_c]_{2\times1} \\ [\dot{\bar{X}}_{uc}]_{1\times1} \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [\bar{X}_c]_{2\times1} \\ [\bar{X}_{uc}]_{1\times1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

↳ 1个 uncontrollable state ($\lambda = 1 > 0$)

Unstabilizable system

整个 sys. 的 $A = \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

上三角 matrix

→ eig(A) = −2, −3, 1

1>0 blow up.

有 unstable state 在 uncontrollable subspace，

故無法以 feedback 使改變其 pole 位置而穩定下來

(亦可由 Eqn 看出 ⇒ $\dot{\bar{X}}_{uc} = 1 \cdot \bar{X}_{uc}$

⇒ Sol. $\bar{X}_{uc} = e^{1t}\bar{X}_{uc}(0)$，隨 $t \to \infty$, $e^t \to \infty$ unstable blow up.