HW 1

① a)

(1) $\frac{2s+3}{s^2+3s+2} = \frac{\cancel{A}\,1}{s+2} + \frac{\cancel{B}\,1}{s+1}$ $\quad A(s+1)+B(s+2) = 2s+3$

$A = 1$

$B = 1$

$\mathcal{L}^{-1}\{ \frac{1}{s+2} + \frac{1}{s+1} \}$

$= e^{-2t} + e^{-t}$

$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

知 $\mathcal{L}\{e^{-at}\cos\omega t\}$

$= \frac{s}{(s+a)^2+\omega^2}$

(2) $\frac{s+2\zeta}{s^2+2\zeta s+1} = \frac{s}{s^2+2\zeta s+1} + \frac{2\zeta}{s^2+2\zeta s+1}$

$s^2+2\zeta\omega_n s+\omega_n^2$

↳ where

$\omega_n = 1$

ζ damping ratio

$\mathcal{L}^{-1} \Rightarrow = e^{-\zeta t}\cos(\omega_d t) + 2\zeta e^{-\zeta t}\sin(\omega_d t)$.

$\omega_d = \sqrt{1-\zeta^2}$

↳ $\omega_d = \underset{1}{\omega_n}\sqrt{1-\zeta^2}$

(3) $f(t) = (3e^{-t} + 4e^{-3t})u(t)$

$\mathcal{L}\{f(t)\} = \frac{3}{s+1} + \frac{4}{s+3}$.

(4)

$f(t) = 5e^{-3t}(u(t-2)) = 5e^{-3(t-2)} \cdot e^{-6}u(t-2)$

$\mathcal{L}\{f(t)\} = 5e^{-6}\,\mathcal{L}\{e^{-3(t-2)}u(t-2)\} = \frac{5e^{-6}\cdot e^{-2s}}{s+3}$

(知 $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$)

(5)

$f(t) = t(u(t)-u(t-2)) + 2(u(t-2)-u(t-10))$

$= tu(t) - (t-2)\cdot u(t-2) - 2u(t-10)$

$\mathcal{L}\{f(t)\} = \frac{1}{s^2} - \frac{e^{-2s}}{s} - \frac{2e^{-10s}}{s}$

知

$\mathcal{L}\{u(t-10)\}$

$= \mathcal{L}\{\underset{f(t-a)}{1}\cdot u(t-10)\}$

$= e^{-10s}\cdot\frac{1}{s}$

(6)

$$f(t) = (t-2)(u(t-2) - u(t-4)) + 2u(t-4)$$

$$= (t-2)u(t-2) - (t-4)u(t-4)$$

$$\mathcal{L}\{f(t)\} = \frac{e^{-2s}}{s^2} - \frac{e^{-4s}}{s^2}$$

2. a) $f(t) = 10e^{-3t}\cos 5t\, u(t)$

$$\mathcal{L}\{f(t)\} = 10 \cdot \frac{s+3}{(s+3)^2+25}$$

b) $f(t) = e^{-5t} + 2te^{-5t} + t^2e^{-5t}$

Laplace transform 為 linear 的 ⇒ 每項各別取 Laplace 再相加

$$\mathcal{L}\{f(t)\} = \frac{1}{s+5} + \frac{2}{(s+5)^2} + \frac{2}{(s+5)^3}$$

知 $\mathcal{L}\{e^{-at}\sin\omega t\}$, $t \ge 0$

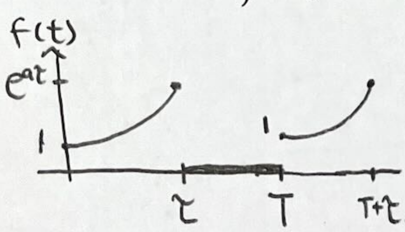
$$= \frac{\omega}{(s+a)^2+\omega^2}$$

$\mathcal{L}\{e^{-at}\cos\omega t\}$, $t \ge 0$

$$= \frac{s+a}{(s+a)^2+\omega^2}$$

$$\mathcal{L}\{t^2\} = \frac{2}{s^3}$$

c) 週期 fn., signal 每次皆 delay T time.



已知 $\mathcal{L}\{e^{at}(u(t) - u(t-\tau))\}$

$$= \frac{1}{s-a} - \frac{e^{-\tau s}}{s-a}$$

而為週期 fn ⇒ $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \cdot \frac{1-e^{-\tau s}}{s-a}$

$$= \frac{1-e^{-\tau s}}{(1-e^{-sT})(s-a)}$$

4. Show $\overset{N \text{ steps advance}}{Z\{f(k+N)\}} = Z^N F(z) - \sum_{j=1}^{N} z^j f(N-j)$

$f(k) = (f(0), f(1), \dots)$
sequence

$f(k+N) \triangleq (f(N), f(N+1), \dots)$.
N steps advance (left shift)

def: $\underline{F(z)} = Z\{f(k)\} \triangleq \sum_{k=0}^{\infty} f(k) z^{-k} = f(0) + f(1)z^{-1} + \underline{f(2)z^{-2} + f(3)z^{-3} + \dots}$

$$\Rightarrow \underbrace{Z\{f(k+1)\}}_{= \sum_{k=0}^{\infty} f(k+1) z^{-k}} = f(1) + f(2)z^{-1} + \underline{f(3)z^{-2} + f(4)z^{-3} + \dots}$$

$$= f(1) + z\left(\underline{f(2)z^{-2} + f(3)z^{-3} + f(4)z^{-4} + \dots}\right).$$

$$= f(1) + z\left(F(z) - f(0) - f(1)z^{-1}\right).$$

$$= \cancel{f(1)} + \underline{zF(z) - zf(0)} - \cancel{f(1)}$$

同理. $Z\{f(k+2)\} = \sum_{k=0}^{\infty} f(k+2) z^{-k} = f(2) + f(3)z^{-1} + f(4)z^{-2} + \dots$

$$= f(2) + z\left(\underline{f(3)z^{-2} + f(4)z^{-3} + \dots}\right)$$

$$= f(2) + z\left(zF(z) - zf(0) - f(1) - f(2)z^{-1}\right)$$

$$= z^2F(z) - z^2f(0) - zf(1)$$

因比 $Z\{f(k+N)\} = \sum_{k=0}^{\infty} f(k+N) z^{-k} = f(N) + f(N+1)z^{-1} + f(N+2)z^{-2} + \dots$

$$= f(N) + z\left(f(N+1)z^{-2} + f(N+2)z^{-3} + \dots\right).$$

$$= f(N) + z\left(z^{N-1}F(z) - z^{N-1}f(0) - z^{N-2}f(1) - \dots - f(N)z^{-1}\right)$$

$$= z^N F(z) - z^N f(0) - z^{N-1} f(1) - \dots - zf(N-1)$$

$$= z^N F(z) - \sum_{j=1}^{N} z^j f(N-j)$$

5.

$k = 0, 1, 2, \ldots$

$$Z\{e^{-akT}\}$$

$$= \sum_{k=0}^{\infty} e^{-akT} \cdot z^{-k} = \sum_{k=0}^{\infty} (e^{aT} \cdot z)^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{e^{aT} \cdot z}\right)^{k}$$

無窮等比級數

$r = 公比 = \frac{1}{e^{aT} \cdot z} < 1$

if $r < 1$ 則 $= \frac{1}{1 - \frac{1}{e^{aT} \cdot z}} = \frac{e^{aT} \cdot z}{e^{aT} \cdot z - 1}$

$$= \frac{z}{z - e^{-aT}}$$