Project 1 – Nuclear Gradient

Rules:

- This project is to be completed in a group of ⩽ 4 students. One submission per group, uploaded onto e3. Please list all members' student ID.
- Submission includes: (i) A written report, (ii) Any associated coding or Excel files.
- Report must:
    - Be written in English
    - All texts and equations must be typed, all graphs computer-generated. Hand-writing or photographed papers are not accepted.
    - Report must use the same section titles, numbering format, font and font-size as this document. It must be easily readable.
    - Questions that must be answered have been highlighted in **blue** below.
- This project is due by 5pm, May 10 2021. Late submission = zero points awarded. Check whether your upload is successful. (Don't wait until the last minute, upload 1-2 days early).
- Points will be deducted if academic dishonesty is discovered, or if it is deemed you have addressed this assignment in a manner that conflicts with the spirit of learning.
- Ask for clarification if needed.

Overview:

This project seeks to demonstrate the connections between scalar-field and vector-field, as well as how to apply them in engineering and scientific situations. This project may require coding or creative use of Excel and other graphing software. You're free to choose your own method as long as it is appropriate. (Hand-drawn graphs are not acceptable).

Section 1: A Conversation with ChatGPT:

1. **Make an account to use ChatGPT**.
2. Ask ChatGPT about the Manhattan Project in 1945. **Show screencap of your conversation.**
3. Discuss with ChatGPT: what are some concepts of engineering mathematics that were used during the Manhattan Project. In particular, discuss how linear algebra and vector calculus were used. **Show screencap of your conversation.**
4. Based on your conversation, **provide an example** of how specific concepts in linear algebra and vector calculus could be used to model physical phenomena in nuclear engineering. Your example should be extremely precise, and specifically states: _____ concept, with _____ equation, can be used to model ____ variable in _____ phenomenon, and [how]. (You may not use the same example in the next section).

Section 2: Modeling of Temperature Field:

Figure 1 shows a common impression of what an atomic explosion is expected to look like. But Figure 1 occurs long after detonation, after the hot gas and plume from the explosion has had the chance to travel upwards into the sky. Figure 2 shows how an explosion look like *immediately* after detonation. The surface of the explosion's "fireball" is nearly spherical, as expected. This is because the heat from the explosion travels outward in all directions spherically. The fireball has a very sharp and distinct surface: outside the fireball (black) the air is cold and not yet disturbed, while inside the fireball (white) are the high temperature gases from the explosion.

In this exercise, we'll look how to model temperature distribution $T(x, y, z)$ inside the fireball as a scalar-field, and how to derive equations describing the shape of the fireball surface using vector calculus and linear algebra.

Figure 1. A common impression of atomic explosion. Source. Atomicheritage.org.

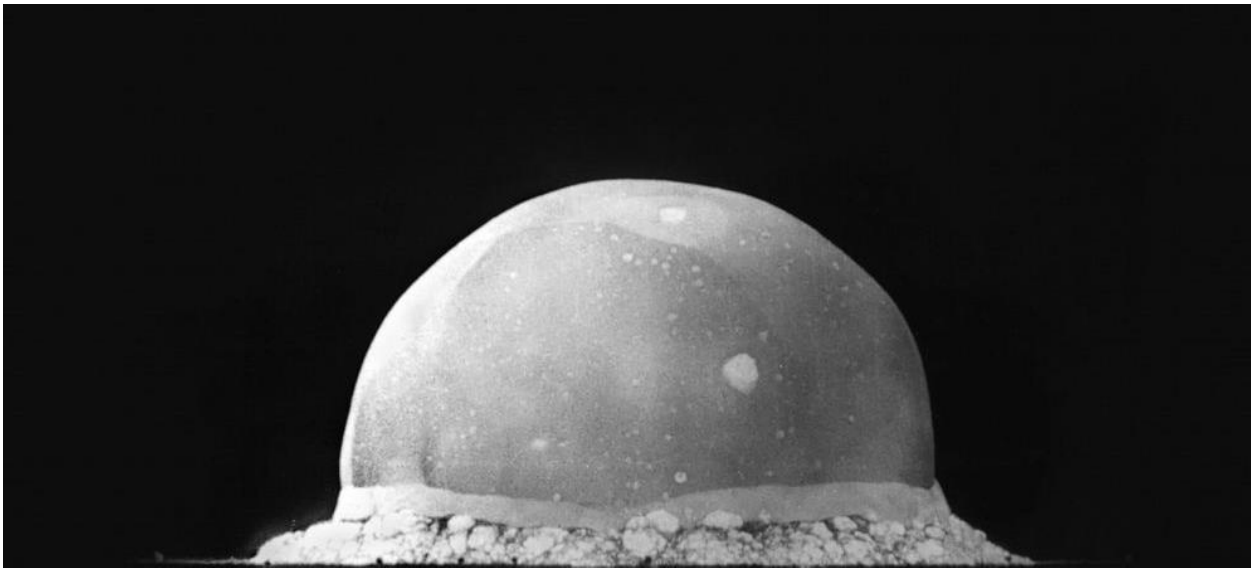

Figure 2. Image of an atomic explosion tested during the Manhattan Project in 1945. Source: U.S. Department of Energy.

1. **Defining your scalar field:**
    - Assume that the explosion's center is located at $\vec{r}_{center} = [0,0,0]$.
    - Let temperature at the center be: 1,000,000 *K*
    - Let temperature at a radius of 1000*m* in any direction from the center be: 10,000*K* (i.e. your field has spherical symmetry).
    - **Come up with an equation** that describes the 3D temperature-field (i.e. a scalar field $T(x, y, z)$), which satisfy the three constraints given above.
    - Hints: Be creative. You can use equations that are physically accurate as researched from papers, or equations that is only roughly correct. Your equation just needs to accept (x, y, z) as input and produce T as output, which satisfy the given constraints above, while not producing results that

are too ridiculous. E.g. you can consider using a 3D Gaussian/normal distribution as a starting point. (E.g. see Figure 3).

- **Describe the properties of your equation, why you choose it, and how you tune it to have the correct values.**

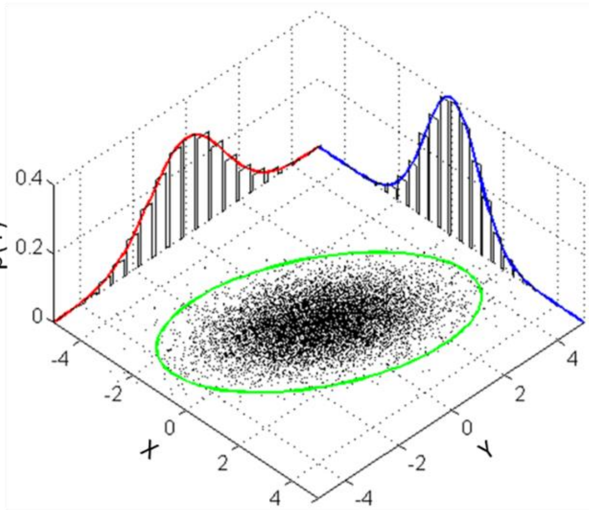

Figure 3. Example of a 2D multivariate Gaussian distribution. This function has a high value in the center, and the values decrease with distance from the center. It can conceivably be used to describe temperature distribution inside a fireball. (You'll need to use a 3D version of this). Source: Wikipedia.

2. **Finding the level surface:**
    - Using your equation, **find the radius R_{15000} where T = 15,000 K**
    - The equation $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ describes a sphere.
    - If your temperature-field equation is correct, the following equation should describe the *level surface* in your temperature-field that correspond to T = 15,000 K (see Figure 4):
    $$x^2 + y^2 + z^2 = R_{15000}^2$$
    - **Plot the level surface on a graph.**

3. **Surface normal via tangents of curves:**
    - **Write an equation** that describes a circle (i.e. a closed-path circular curve) which is tangent to the R_{15000} level surface along its entire length. The circle may have any orientation. (See Circle 1 in Figure 4).
    - **Write an equation** that describes a second circle, oriented in a different direction. (See Circle 2 in Figure 4).
    - Find the point $\vec{r}_{intersect}$ where Circle 1 and 2 intersect (choose any one of two intersections).
    - **Calculate the tangent vectors of Circle 1 and Circle 2 at $\vec{r}_{intersect}$.** Your calculation should be based on differentiating your circles' equations. Show steps in calculation.
    - **Calculate the unit normal vector of the R_{15000} level surface at $\vec{r}_{intersect}$** using the two tangent vectors. Show step-by-step derivation.
    - **Plot the following on a graph:** The level surface, Circles 1 and 2, the intersection point, the two tangent vectors, and the unit normal vector.

4. **Surface normal via gradient:**
    - There is an alternative way to finding surface normal. Gradients are pointed in the directional of level surfaces' normal vectors.
    - **Calculate the gradient** of your temperature-field equation.
    - **Find the value of the gradient field at $\vec{r}_{intersect}$.**
    - **Normalize to get unit normal vector.**

5. **Conclusion:**
    - **Plot** the normal vectors found using (i) tangents of circles and (ii) gradient on the same graph.
    - **Comment** on whether they are the same.

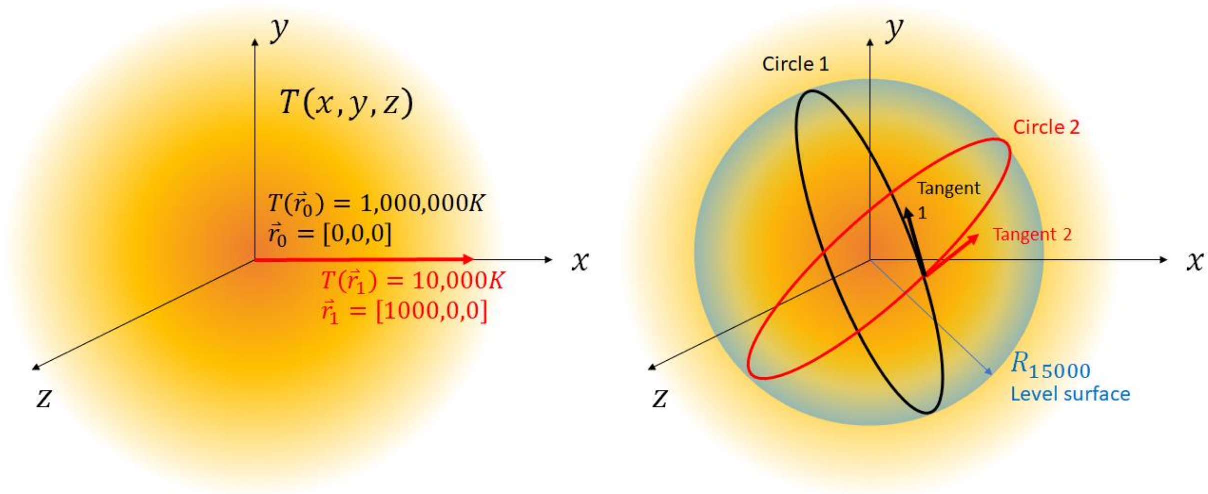

Figure 4. *Left:* Temperature-field using 3D Gaussian distribution. *Right*: The level-surface, circles on surface and tangents at intersection of the circles.