Project 2 – System of ODE's and Phase Plane

Logistics:

- This project is to be completed in a group of ⩽ 4 students. One submission per group, uploaded onto e3. Please list all members' student ID.
- Submission includes: A written report.
- Report must:
  - Be written in English
  - All texts and equations must be typed, all graphs computer-generated. Hand-writing or photographed papers are not accepted.
  - Report must use the same section titles, numbering format, font and font-size as this document. It must be easily readable.
  - Questions that must be answered have been highlighted in blue below.
- This project is due by 5pm, Jan 12 2021. Late submission = zero points awarded (Don't wait until the last minute, upload 1-2 days early).
- Points will be deducted if academic dishonesty is discovered, or if it is deemed you have addressed this assignment in a manner that conflicts with the spirit of learning.
- Ask for clarification if needed.

Section 1: Observing ODE in Real Life:

What you need to do:

1. Pick an engineering application or daily-life scenario where systems of ODE can be used to model some behavior. This application must have a system of three linear 1st-order, homogeneous ODE, with constant coefficients. The application must *not* be an exact repeat of examples in class.

2. In the report, complete the following:
   1. Describe your chosen scenario in words. Include a clearly-drawn schematic (示意圖) of your scenario.
   2. Describe the physics/principles at work in this scenario. Include the equations of any physics models (e.g. Newton's 2nd law or conservation of mass).
   3. Show step-by-step: how you derive a system of three linear 1st-order, constant-coefficient, homogeneous ODE to model your scenario.
   4. Note: your coefficients must have realistic numbers and units. Describe how you get these numbers and units.
   5. Write your system of ODE into vector/matrix form.

Section 2: Solving the ODE via Eigenvalue Technique:

1. Solve your system of ODE using the eigenvalue technique. In the report:
   i. Find the eigenvalues (λ_i) and eigenvectors ($\vec{k}^{(i)}$) of your system of ODE. Show step-by-step process.
   ii. Provide the general solution in the form:
   $$\vec{y} = C_1\vec{k}^{(1)}e^{\lambda_1 t} + C_2\vec{k}^{(2)}e^{\lambda_2 t} + C_3\vec{k}^{(3)}e^{\lambda_3 t}$$
   iii. Derive equations that show the relationship between your general solution's unknown constants (C_1, C_2, C_3) with respect to initial conditions: $\vec{y}(0)$.

2. Explore the particular solutions. In the report:
   i. What are some realistic initial conditions $\vec{y}(0)$ for your scenario? (Give realistic ranges of values for $y_1(0), y_2(0), y_3(0)$.
   ii. What is a realistic time-scale (Δt) for your system to exhibit interesting behaviors? (E.g. given some initial condition, how long before the system's "action" approach equilibrium or some steady oscillation?) Describe your logic.
   iii. In separate plots, show $y_1(t), y_2(t), y_3(t)$ from $t = 0$ to Δt, for 10 different initial conditions that cover the ranges described in (2.2.i). *Use the same colors for the same initial conditions between all three plots. Use a good-looking color scheme like transition from blue-to-orange or green-to-red instead of random colors. See this for example: https://www.mathworks.com/help/matlab/ref/colormap.html *.