Linear Systems Homework #3

1. Find the diagonal, Jordan canonical, or complex Joran forms $\left(\begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}\right)$ for the following matrices:

   (a) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 2 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 11 \\ 0 & 2 \end{bmatrix}$

2. For each of the following matrices, compute e^{At} and A^k using any method.

   (a) $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ (b) $A = \begin{bmatrix} 2 & 11 \\ 0 & 3 \end{bmatrix}$ (c) $A = \begin{bmatrix} -3 & 2 \\ -2 & -3 \end{bmatrix}$

   (d) $A = \begin{bmatrix} \sigma & \omega & 0 & 0 & 0 \\ -\omega & \sigma & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 \end{bmatrix}$ (e) $A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$

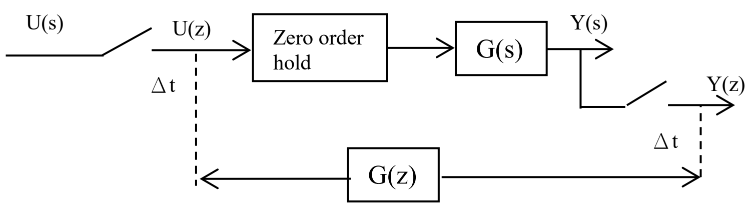
3. For the following system, (a) show the analytical solution of $\mathrm{y(t)}$; (b) use Matlab to plot the $\mathrm{y(t)}$ solution from (a); (c) use "ode45" in Matlab to simulate the response of the system with different initial conditions from (a) and then plot $\mathrm{y(t)}$ in the same plot with (b).

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot 1, \; y = \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

4. For the following system, (a) show the analytical solution of $\mathrm{y(k)}$; (b) use Matlab to plot the $\mathrm{y(k)}$ solution from (a); (c) write a "for loop" to do the iteration in Matlab to simulate the response of the system with different initial conditions from (a) and then plot $\mathrm{y(t)}$ in the same plot with (b).

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.24 & -1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot 1, \; y(k) = \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

5. Consider the following system, $\mathrm{G(s)} = \frac{125}{s^3+6s^2+30s+125}$,



(a) Assuming the sampling time is $\frac{2\pi}{50}$, please **hand-derive** the $\mathrm{G(z)}$ using the ZOH-table, Laplace transformation table, Z-transformation table.

(b) Using the Matlab functions, "c2d" and the method "zoh", to find each $\mathrm{G(z)}$ for the sampling time of $\frac{2\pi}{50}, \frac{2\pi}{100}, \frac{2\pi}{15}$, respectively. Furthermore, specify the poles and zeros of each $\mathrm{G(z)}$.

(c) Using the Matlab functions to draw the bode plot of $\mathrm{G(s)}$, and $\mathrm{G(z)}$ from (b) with different sampling time on the same plot.

(d) Assuming the sampling rate is $\frac{2\pi}{50}$, please **hand-derive** the $G_1(z)$ by doing the "Tustin" transformation method on $\mathrm{G(s)}$. Also, **hand-derive** the $G_2(z)$ by doing the discrete time approximation using $\dot{y}(t) = \frac{y(k+1)-y(k)}{\Delta t}$.

(e) Using the Matlab functions to draw the bode plot of $\mathrm{G(z)}, G_1(z), G_2(z)$ (all have the same sampling time of $\frac{2\pi}{50}$) on the same plot.

(f) Assuming $\mathrm{u(t)}$ is the following and zero initial conditions of the system, please using Matlab to simulate the following response and draw them on the same plot: $\mathrm{y(t)}$ of the continuous system; $\mathrm{y(k)}$ of the $\mathrm{G(z)}, G_1(z)$, and $G_2(z)$.