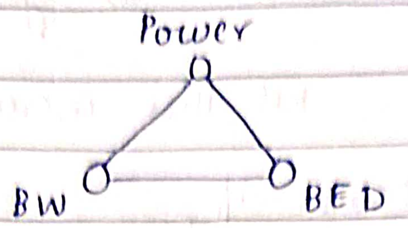
15/3/23.

# Line Coding.




For good communication, we need

Power ↓

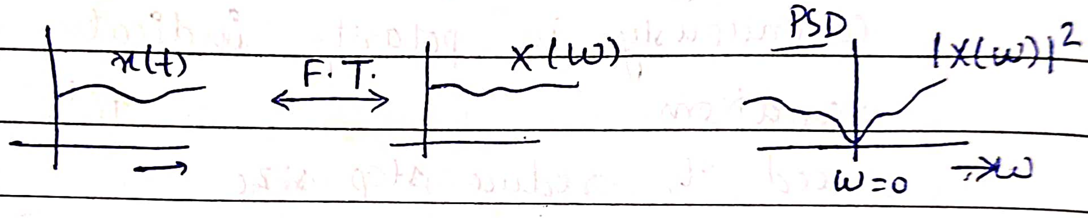
BW ↓

BED ↓

(1) <u>Transmission BW</u> - a line code should ~~refer~~ preferably be of lower BW / BW as minimum as possible.

(2) <u>Power Efficiency</u> - A line code for a given BW and error detection probability, should consume or required transmission power as small as possible

(3) <u>Favourable PSD (Power spectral Density)</u> - A line code demands 0 PSD at $\omega = 0$.




(4) <u>Error detection and Correction</u> - A line code should have provision of error detection & possibly correction.

↓

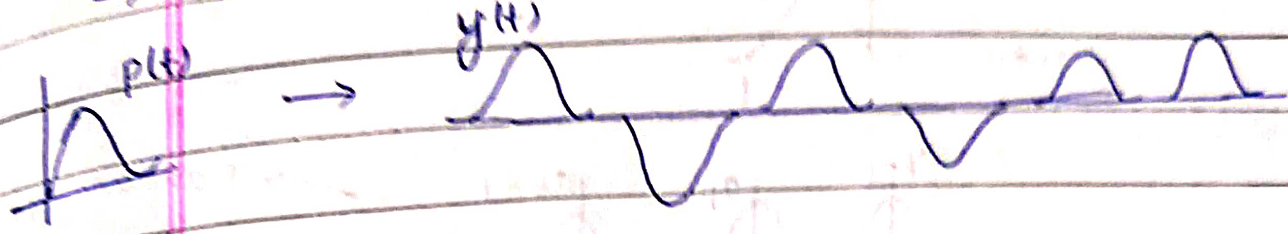
Bipolar ✓

ON-OFF polar - X

(5) Timing information.

(6) Transparency - which provides precise extraction irrespective of the bits.

→ PSD (Pulse spectral Density).




pulse transmitted.

1 0 1 0 1 1 . .

↓

$T_b$ -

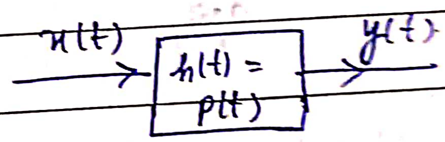
(Rate), $\boxed{R_b = 1/T_b}$

→ $\boxed{S_y(\omega) = |Y(\omega)|^2}$

→ If $a_k = 1, -1$ → Polar

$a_k = 0, 1, -1$ → Bipolar.

$a_k$ → random.




LTI system.

$y(t) = x(t) * h(t)$

$|Y(\omega)|^2 = |X(\omega)|^2 |H(\omega)|^2$.

$\cancel{S_y(\omega) = S_x(\omega) |P(\omega)|^2}$.

$S_y(\omega)$ = PSD of $y(t)$

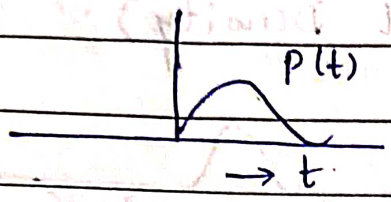
$S_x(\omega)$ = PSD of $x(t)$.

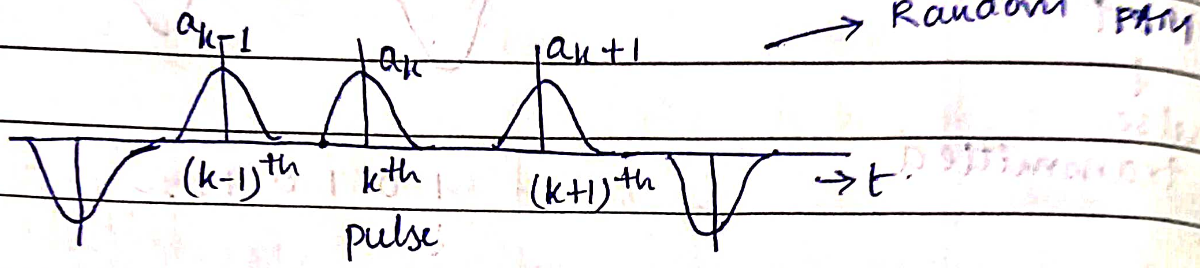
⇒ $\boxed{S_y(\omega) = S_x(\omega) \cdot P^2(\omega)}$.

20/3/23.

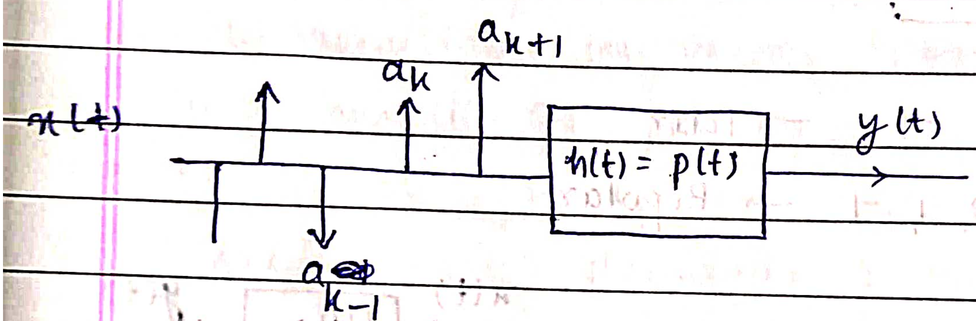
PSD of line coding

→ let us assume that there is binary data transmission at rate $R_b$ such that $R_b = 1/T_b$.




→ $a_k$'s are random and can take values, $-1, 0, +1$.




If the system is LTI,

$$y(t) = x(t) * h(t)$$

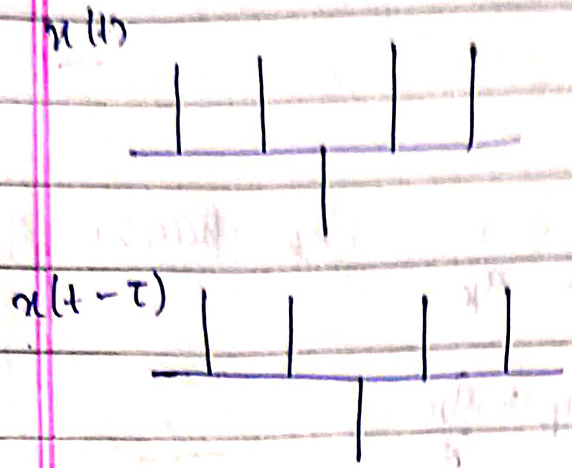
$$\Rightarrow Y(\omega) = X(\omega) \cdot H(\omega)$$

Now, squaring

$$\Rightarrow |Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$$

$$\Rightarrow \underbrace{S_y(\omega)}_{\text{PSD of } y(t)} = \underbrace{S_x(\omega)}_{\text{PSD of } x(t)} \cdot P^2(\omega).$$

# Autocorrelation




(Autocorrelation)

$$R_{xx}(\tau) = \int x(t)\cdot x(t-\tau)\,dt$$

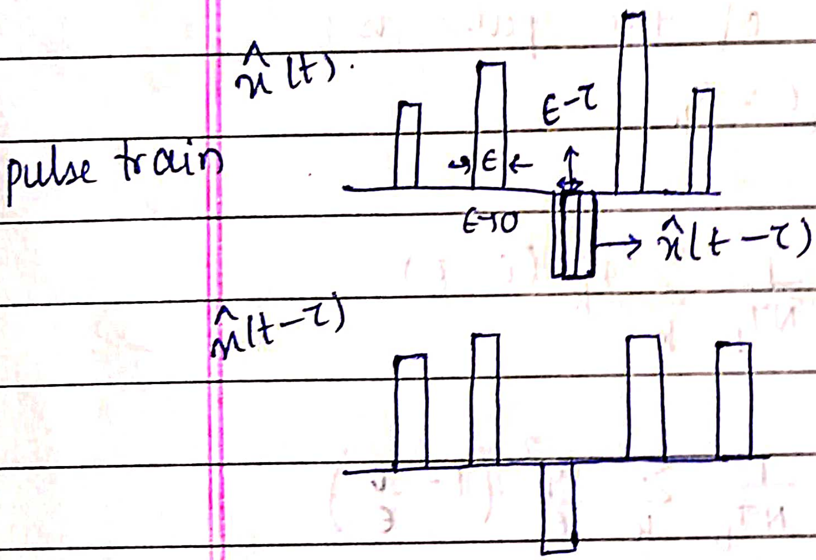
$= 0$

How to make $R_{xx}(\tau) \neq 0$

→ to get autocorrelation func

& then to calculate BW.

→ Impulse train is considered to be as a pulse train of width $\epsilon \to 0$.




$$R_x(\tau) = \lim_{\epsilon \to 0} R_{\hat{x}}(\tau)$$

$$R_{\hat{x}}(\tau) = \int \hat{x}(t)\,\hat{x}(t-\tau)\,dt$$

Case-(i) $\tau < \epsilon$.

→ Overlapping occurs.

Calculating over infinite Time interval, $(T \to \infty)$

$$R_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{x}(t)\cdot\hat{x}(t-\tau)\,dt$$

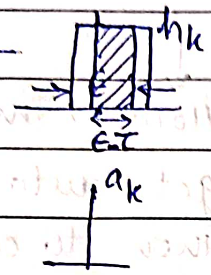
→ with $T \to \infty$, then $N \to \infty$ (infinite no. of pulses)

$$R_x^*(\tau) = \lim_{N\to\infty} \frac{1}{NT_b} \sum_k h_k \cdot h_k(\epsilon - \tau).$$

area $= h_k \cdot h_k(\epsilon-\tau)$

$T = NT_b$.

$h_k$ = height of the $k^{th}$ pulse.




$h_k\epsilon = a_k$.

$\Rightarrow h_k = \frac{a_k}{\epsilon}$.

→ impulse is looked as pulse of infinitesimals of small width, $\epsilon \to 0$

The strength of the pulse is,

$$h_k\epsilon = a_k.$$

$$\rightarrow R_x^*(\tau) = \lim_{N\to\infty} \frac{1}{NT_b} \sum_k \boxed{h_k^2}(\epsilon-\tau)$$

$$= \lim_{N\to\infty} \frac{1}{NT_b} \sum_k \frac{a_k^2}{\epsilon}\left(1 - \frac{\tau}{\epsilon}\right)$$

$\left(\because R_0 = \underset{N\to\infty}{Lt} \frac{1}{N} \sum_k a_k^2\right)$

$$= \frac{1}{\epsilon T_b} \cdot R_0 \cdot \left(1 - \frac{\tau}{\epsilon}\right).$$

$$\Rightarrow R_x^*(\tau) = \frac{1}{\epsilon T_b} \cdot R_0 \left(1 - \tau/\epsilon\right). = \boxed{\frac{R_0}{\epsilon T_b}\left(1 - \tau/\epsilon\right)}$$

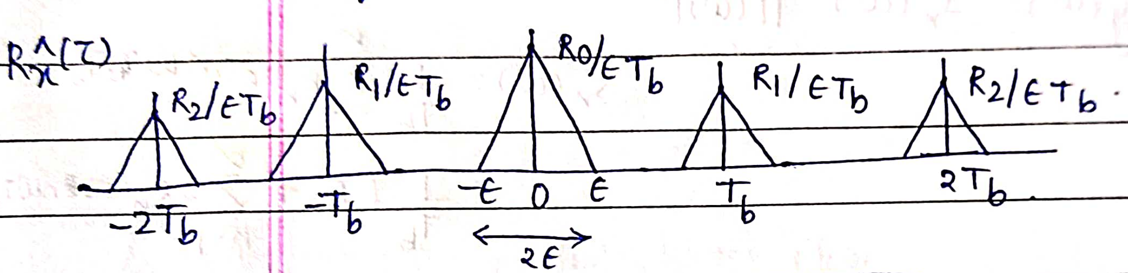
Case-(ii) :- $T > \epsilon$

The current pulse will overlap the next immediate pulse.

then, $R_1 = \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_k a_k \cdot a_{k+1}$

→ Autocorrelation is an even func. of $\tau$.

→ Convolution of two rect. func. gives a triangular func.




$R_2 = \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_k a_k a_{k+2}$

$$\boxed{R_x^{\wedge}(\tau) = \frac{R_n}{\epsilon T_b}\left(1 - \frac{|\tau|}{\epsilon}\right)}$$

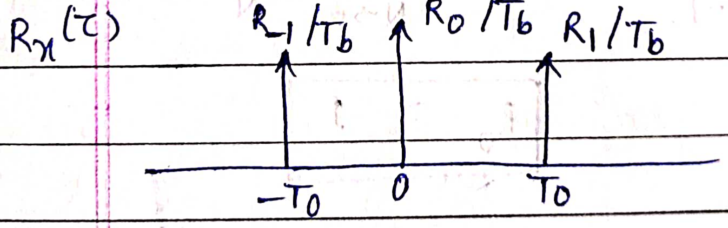
→ rectangular pulse.

$R_x(\tau) = ?$

→ Now, pulses can be thought of as an impulse train

→ Area of triangle $= \frac{1}{2} \times 2\epsilon \times \frac{R_0}{\epsilon T_b}$

$= R_0 / T_b$.




$$\boxed{R_x(\tau) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n \, \delta(t - nT_b)}$$

F.T of $R_x(\tau)$;

$\Rightarrow$ $$F.T[R_x(\tau)] = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} R_n \, e^{-jn\omega T_b}$$

It is an even func. so, $|R_n| = |R_{-n}|$

$\Rightarrow$ $$F.T[R_x(\tau)] = \frac{1}{T_b}\left[R_0 + \sum_{n=1}^{\infty} 2R_n \cos n\omega T_b\right]$$

Now, $S_y(\omega) = S_x(\omega)\ |P(\omega)|^2$.

$$S_x(\omega) = \frac{1}{T_b}\sum_{-\infty}^{\infty} R_n \, e^{jn\omega T_b}$$

$$= \frac{1}{T_b}\left(R_0 + \sum_{n=1}^{\infty} 2R_n \cos n\omega T_b\right)$$

$\Rightarrow$ Polar

1 +ve

0 -ve

$R_0, R_1, R_2, \ldots$

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k^2 . \qquad (a_k^2 = 1).$$

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\sum a_k \cdot a_{k+1}$$

$$= \lim_{N\to\infty} \frac{1}{N}(1+1+\cdots+1) \text{ N times}$$

$$= \lim_{N\to\infty} \frac{1}{N} \times N.$$

$$\boxed{R_0 = 1.}$$

$a_k \cdot a_{k+1}$ can be 1 or -1.

$1 \cdot 1$

$-1 \cdot 1$

$1 \cdot (-1)$

$(-1) \cdot (1)$

$$\Rightarrow R_1 = \lim_{N\to\infty} \frac{1}{N} \cdot \sum a_k \cdot a_{k+1}$$

$$= \lim_{N\to\infty} \frac{1}{N} \left[ \frac{N}{2}(1) + \frac{N}{2}(-1) \right]$$

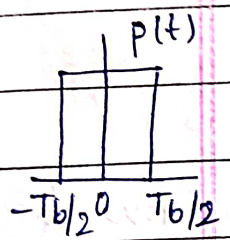
$$= 0.$$

so, $R_n = 0$ for $n \geq 1$

For polar, $S_y(\omega) = S_x(\omega) \cdot |P(\omega)|^2.$

$$= \frac{1}{T_b} |P(\omega)|^2 \quad (\because R_0 = 1).$$

$\rightarrow$

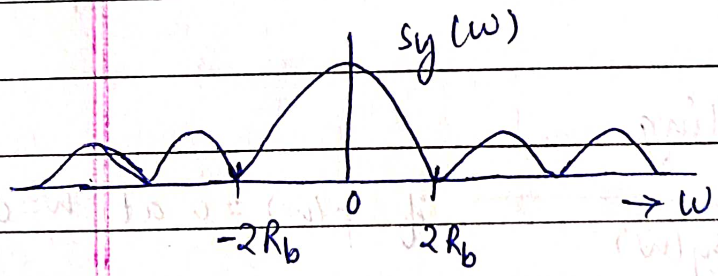



$$p(t) = \Pi . \left( t/T_b \right)$$

$$\text{sinc}\, \omega T_b = \frac{\sin \omega T_b}{\omega T_b}$$

$$\Rightarrow P(\omega) = \frac{T_b}{2} \cdot \text{sinc}\left( \frac{\omega T_b}{4} \right)$$

$$\Rightarrow |P(\omega)|^2 = \frac{T_b^2}{4} \text{sinc}^2\left( \frac{\omega T_b}{4} \right)$$




$$\frac{\omega T_b}{4} = \pi$$

$$\Rightarrow \omega = \frac{4\pi}{T_b} = 2\left(2\pi/T_b\right)$$

$$\Rightarrow \omega = 2\pi\left(\frac{2}{T_b}\right)$$

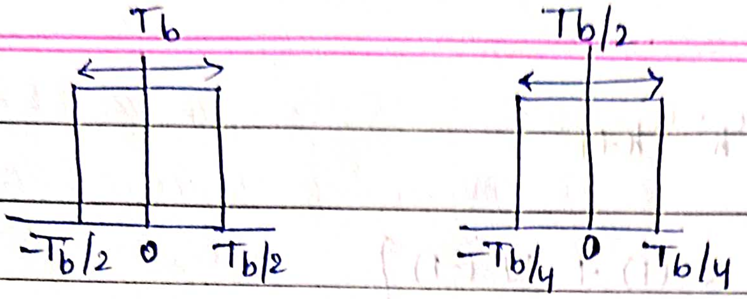
$$\Rightarrow \omega = 2\pi . (2R_b)$$

$\rightarrow$ Nyquist requirement, $BW = \frac{R_b}{2}$ (rate)

for $2R_b \rightarrow$ 4 times of nyquist req.

piglet. hu. ×

(mtlb? )
pig ka baccha?
cutee....




Disadv :-

→ polar signalling is not BW efficient.
It requires 4 times of nyquist BW req.

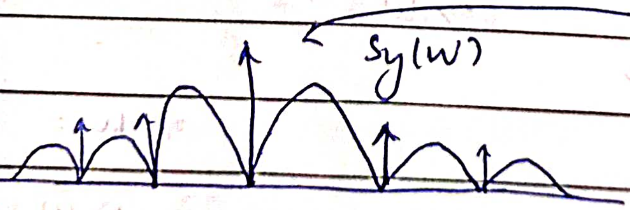
→ At $w=0$, PSD $\neq 0$. No favourable PSD.

→ It can't detect error. Error detection is not possible.

adv :-

→ It is power efficient. For same BW & Detection probability, polar signalling requires less power than bipolar / ON-OFF signalling.
ON-OFF req. twice the power as polar.

ON-OFF signalling.




if $p(w)=0$ at $w=0$

Func. is symm. abt X-axis.

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

$$= \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(0)\right] = \boxed{1/2}$$

dildo

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\sum_{k} a_k \cdot a_{k+1} = 1/4$$

$$R_2 = 1/4$$

$$R_n = 1/4, \quad n \geq 1.$$

Now, $S_x(\omega) = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} R_n \cdot e^{-jn\omega T_b}$

$\rightarrow \delta\left(\omega - \frac{n\omega}{T_b}\right).$

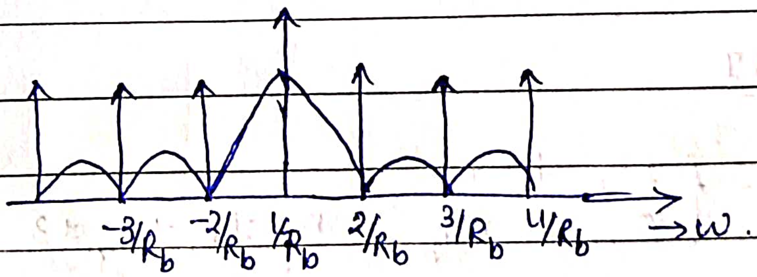
$$= \frac{1}{T_b}\left(R_0 + \boxed{\sum_{\substack{n=-\infty \\ n\neq 0}}^{n=\infty} R_n \cdot e^{-jn\omega T_b}}\right)$$

$$= \frac{1}{T_b}\left[\frac{1}{2} + \frac{1}{4}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-jn\omega T_b}\right]$$

$$= \frac{1}{T_b}\left[\frac{1}{4} + \frac{1}{4}\sum_{n=-\infty}^{\infty} e^{-jn\omega T_b}\right]$$

$\left(\frac{1}{2} = \frac{1}{4} + \frac{1}{4}\right)$ including $n=0$, $R_0 = 1/4$.

→




ON-OFF Contains sync. function, and is discrete.

It is polar + random.

and the discrete one is a periodic function

and anna is really pretty. uncomprehendable-ly

→ PSD comparison.

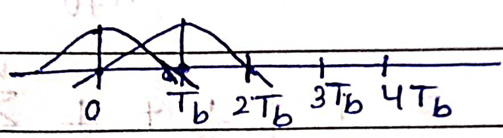
→ statement derivation.

24/03/23

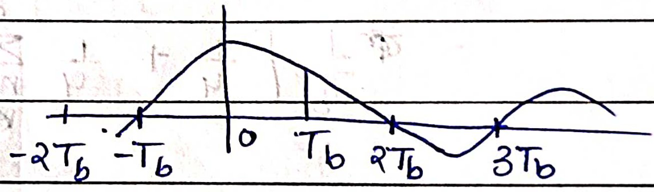
$$S_y(\omega) = S_x(\omega) \cdot |P(\omega)|^2$$

| | | |
|---|---|---|
| ↓ | ↓ | ↘ |
| P.S.D of Tx signal. | PSD of line code | pulse shape |

(1) Zero ISI.

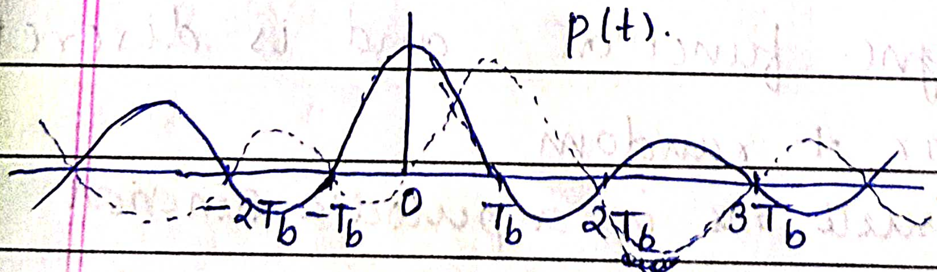



(2) Control ISI.




<u>Zero ISI.</u>

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = nT_b \end{cases} \quad n = \pm 1, \pm 2, \pm 3.$$

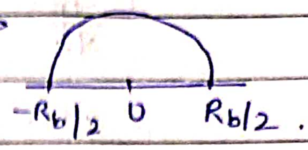



p(t).

$R_b \rightarrow R_b/2$.

kya interesting hai??

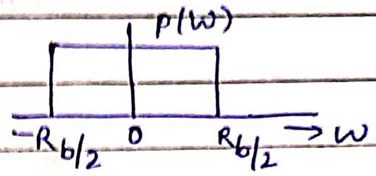
→ We need a pulse p(t), that will satisfy the time domain and freq. domain. p(w).




↓

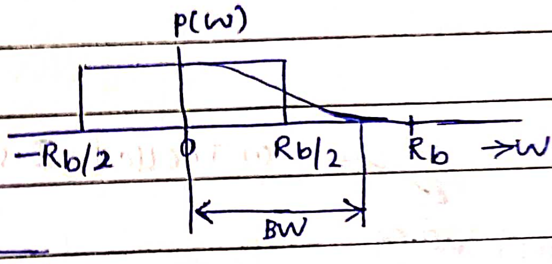
Only one pulse - sinc pulse

$p(t) = \text{sinc}(\pi R_b t)$




$P(w) = \frac{1}{R_b}\Pi\left(\frac{w}{2\pi R_b}\right)$

→ Σ over n is non-converging series. → Problem




→ Solution :- $B_T = W_b + W_x$

$r = \frac{\text{excess BW}}{\text{theoritical min. BW}}$

↓

roll-off factor

$r = \frac{W_x}{R_b/2}$

$\Rightarrow R_b \cdot r = 2W_x \Rightarrow W_x = \frac{r \cdot R_b}{2}$.

$B_T = \frac{R_b}{2} + \frac{rR_b}{2}$.

→ $P(w) = \frac{1}{R_b}\cos^2\left(\frac{w}{4R_b}\right) \cdot \Pi\left(\frac{w}{4\pi R_b}\right)$

→ when $W_x = R_b/2$ then $r = 100\%$

~~kaat~~ aur nhi bas 10:23 mine TnT

gandi kaua? crow.

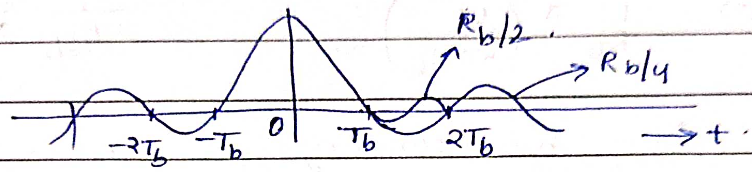
tum ho tum gaati ho na isliye tum kaua. ingale angle X yes.

niceee sorry Spelling toh aati nhi

$$p(\omega) = \frac{1}{R_b}\cos^2\left(\frac{\omega}{4R_b}\right)\cdot \pi\left(\frac{\omega}{4\pi R_b}\right)$$

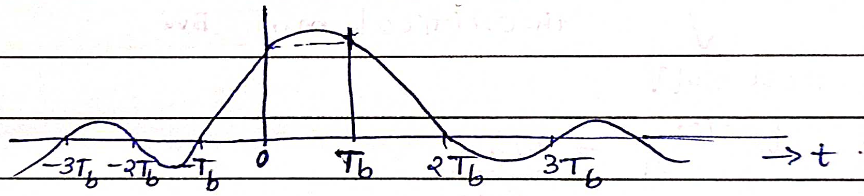
$\Downarrow$

$$p(t) = \frac{\sin(\pi R_b t)}{\pi R_b t\,(1 - 4\pi R_b^2 t^2)}$$




Duobinary pulse

Controlled ISI. (Two succesive pulses interfere each other)




$$P(nT_b) = \begin{cases} 1, & n = 0, 1 \\ 0, & \text{otherwise} \end{cases}$$

1 1 $\rightarrow$ +2

1 0 $\rightarrow$ 0 $\quad$ $x(t)$ $\quad$ 1 0 1 1 0 1 0 0 1.

0 1 $\rightarrow$ 0 $\quad$ sample $\quad$ 0 0 2 0 0 0 −2 0

0 0 $\rightarrow$ −2

$\rightarrow$ there should be odd no. of zeroes b/w two amplitudes of full values with opposite sign.

→ there should be even no. of zeroes betn 2 amplitudes of full values with same polarity.

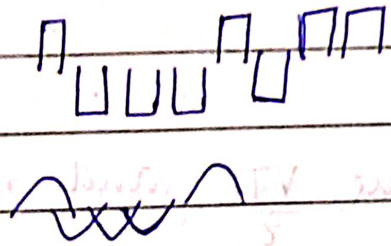
→ Error propagates in duobinary system. – Prblm.

<u>Differential Coding</u>

→ If bit at any time instant is '1', pulse to be the same as of previous one.

→ " " " " " " " '0', pulse to be opposite of previous one.

1 0 1 1 0 0 0 1



3|4|23.

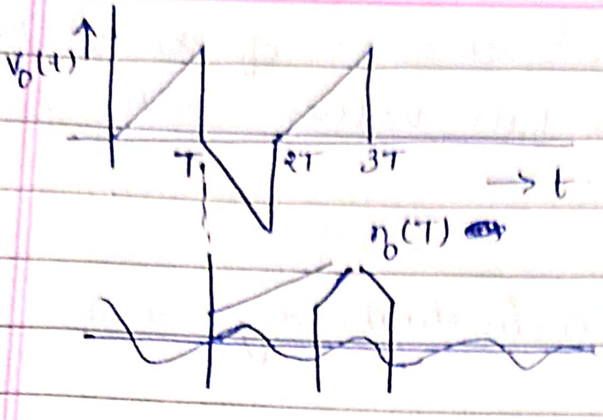
<u>Integrate & Dump.</u>

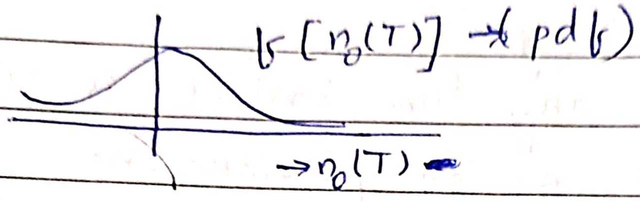
| | |
|---|---|
| 1 | $VT/\tau$ |
| 0 | $-VT/\tau$ |

$$v_0^2(T) = \frac{V^2T^2}{\tau^2}$$

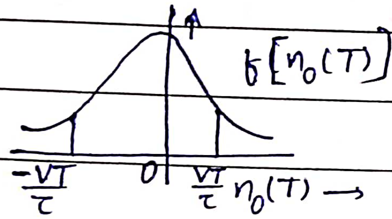
$$\boxed{n_0(T)} \qquad \sigma_n^2 = \frac{\eta T}{2\tau^2}$$

$$\left(\frac{S}{N}\right)_0 = \frac{V^2T^{\cancel{2}}}{\cancel{\tau^2}} \times \frac{2\cancel{\tau^2}}{\eta \cancel{T}} = \frac{2V^2T}{\eta} = \frac{2V^2T}{\eta}$$




$P_e$ - Probability of bit error.




→ if Gaussian, equal halves. ~~(intervals)~~

→ if the signal value, $\frac{VT}{\tau}$ and noise $n_0(T)$,

$n_0(T) > \frac{VT}{\tau}$

→ suppose, bit '0' was transmitted, so at the end of bit interval the signal value is $\frac{-VT}{\tau}$

Now, prob. of bit error while transmitting '0', $P(\epsilon|0)$ = area of the curve under the cylindrical region.

$$f[n_0(T)] = \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-\frac{n_0(T)}{2\sigma_n^2}}$$

$$\Rightarrow p(\epsilon/0) = \int_{VT/\tau}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{\frac{-n_0(T)}{2\sigma_n^2}}\, d[n_0(T)]$$

let $x = \dfrac{n_0(T)}{\sqrt{2}\,\sigma_n}$

$$\left(\sigma_n^2 = \frac{\eta T}{2\tau^2} \Rightarrow \sigma_n = \frac{1}{\tau}\sqrt{\frac{\eta T}{2}}\right)$$

| $x$ | $\infty$ | $\frac{VT}{\tau\sqrt{2}\,\sigma_n}$ |
|---|---|---|
| $n_0(T)$ | $\infty$ | $VT/\tau$ |

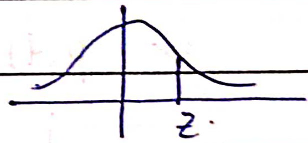
so, $x = \dfrac{VT}{\tau\sqrt{2}\,\sigma_n} = \dfrac{VT\,\cancel{\sqrt{2}}\,\cancel{\tau}}{\cancel{\tau}\,\cancel{\sqrt{2}} \times \sqrt{\eta T}} = \dfrac{V\sqrt{T}}{\sqrt{\eta}} = V\sqrt{T/\eta}$

Now, $$p(\epsilon/0) = \int_{V\sqrt{T/\eta}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-x^2}\, \sqrt{2}\,\sigma_n\, dx.$$

$$\left(\because\ dx = \frac{d(n_0 T)}{\sqrt{2}\,\sigma_n}.\right)$$

$$\Rightarrow \boxed{p(\epsilon/0) = \frac{1}{\sqrt{\pi}} \int_{V\sqrt{T/\eta}}^{\infty} e^{-x^2}\, dx}$$

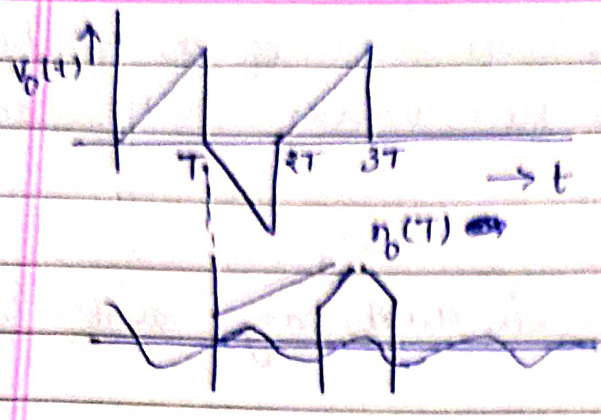
$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_0^y e^{-x^2/2}\, dx.$$

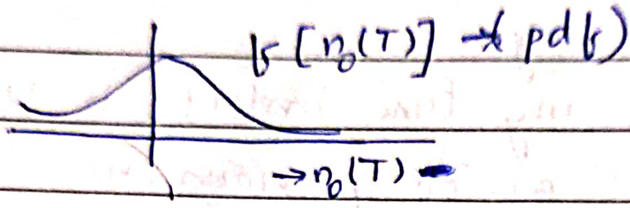


(error function) $$\boxed{erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2/2}\, dx}$$

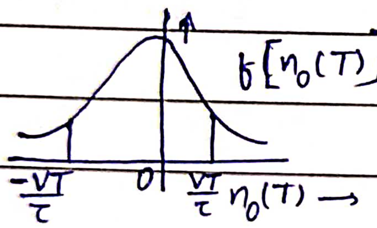
$$\boxed{erfc(z) = 1 - erf(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-x^2}\, dx}$$

(complementary error func. of z)




$P_e$ - Probability of bit error.




→ If Gaussian, equal halves. ~~(intervals)~~

→ if the signal value, $\frac{VT}{\tau}$ and noise $n_o(T)$,

$n_o(T) > \frac{VT}{\tau}$

→ suppose, bit '0' was transmitted, so at the end of bit interval the signal value is $\frac{-VT}{\tau}$.

Now, prob. of bit error while transmitting '0', $P(\epsilon|0)$ = area of the curve under the cylindrical region.

$$p[n_0(T)] = \frac{1}{\sqrt{4\pi\sigma_n^2}}\, e^{-\frac{n_0(T)}{2\sigma_n^2}}$$

$$\Rightarrow p(\epsilon/0) = \int_{VT/\tau}^{\infty} \frac{1}{\sqrt{4\pi\sigma_n^2}}\, e^{-\frac{n_0(T)}{2\sigma_n^2}}\, d[n_0(T)]$$

let $x = \dfrac{n_0(T)}{\sqrt{2}\,\sigma_n}$ $\left(\sigma_n^2 = \dfrac{\eta T}{2\tau^2}, \Rightarrow \sigma_n = \dfrac{1}{\tau}\sqrt{\dfrac{\eta T}{2}}\right)$

| | | |
|---|---|---|
| $x$ | $\infty$ | $\frac{VT}{\tau\sqrt{2}\sigma_n}$ |
| $n_0(T)$ | $\infty$ | $VT/\tau$ |

so, $x = \dfrac{VT}{\tau\sqrt{2}\sigma_n}$

$$= \frac{VT\,\cancel{\sqrt{2}}\,\cancel{\tau}}{\cancel{\tau}\,\cancel{\sqrt{2}} \times \sqrt{\eta T}} = \frac{V\sqrt{T}}{\sqrt{\eta}} = V\sqrt{T/\eta}$$

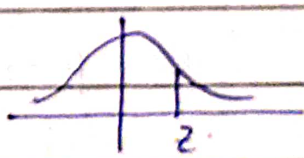
Now, $$p(\epsilon/0) = \int_{V\sqrt{T/\eta}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}}\, e^{-x^2}\, \sqrt{2}\,\sigma_n\, dx.$$

$$\Rightarrow \boxed{p(\epsilon/0) = \frac{1}{\sqrt{\pi}} \int_{V\sqrt{T/\eta}}^{\infty} e^{-x^2}\, dx}$$ ✓

$\left(\because dx = \dfrac{d(n_0 T)}{\sqrt{2}\,\sigma_n}\right)$

~~Imp~~

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_0^y e^{-x^2/2}\, dx.$$



(error function) $$\boxed{erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2/2}\, dx}$$

$$\boxed{erfc(z) = 1 - erf(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-x^2}\, dx}$$

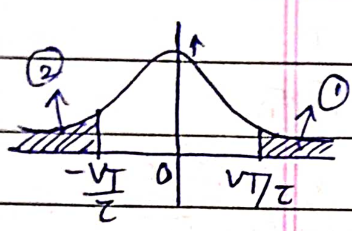
(complementary error func. of z)

$$p(\epsilon/0) = \frac{1}{2}\,\text{erfc}\left(V\sqrt{T/\eta}\right) = \frac{1}{2}\,\text{erfc}\left(\frac{V^2 T}{\eta}\right)^{1/2}$$

$$p(\epsilon/0) = \frac{1}{\sqrt{\pi}} \int_{V\sqrt{T/\eta}}^{\infty} e^{-x^2}\,dx$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{\pi}} \int_{V\sqrt{T/\eta}}^{\infty} e^{-x^2}\,dx$$

$$= \frac{1}{2}\,\text{erfc}\left(V\sqrt{T/\eta}\right)$$




$$= \frac{1}{2}\,\text{erfc}\left(\frac{V^2 T}{\eta}\right)^{1/2}$$

$$= \frac{1}{2}\,\text{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$

($E_s$ = Energy of signal)

$\Rightarrow$ $p(\epsilon/1)$ = area of the curve under shaded region 2

$$= \frac{1}{2}\,\text{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$

$\Rightarrow$ $p(t) = \sum_{i=0,1,\ldots} p(\epsilon/x_i) = p(0)\cdot p(\epsilon/0) + p(1)\cdot p(\epsilon/1)$

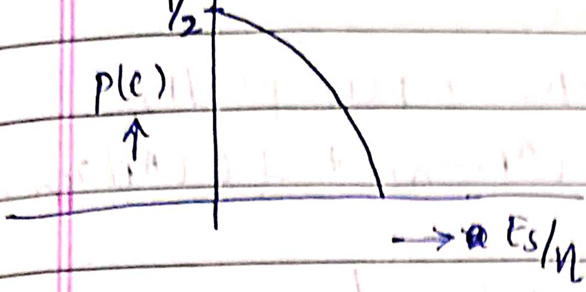
$\downarrow$

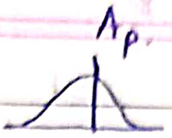
Total prob.

$p(0) = p(1) = 1/2$.

$$\Rightarrow p(t) = \frac{1}{2} \times \cancel{2}\,\frac{1}{\cancel{2}}\,\text{erfc}\left(E_s/\eta\right)^{1/2} = \frac{1}{2}\,\text{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$

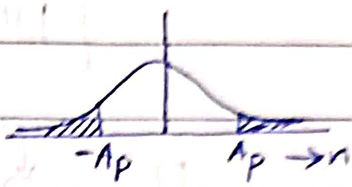
$\Rightarrow$




~~------~~ polar

$$p(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-n^2/2\sigma^2}$$

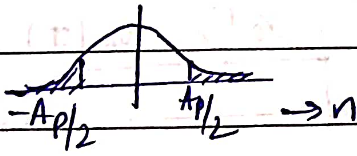


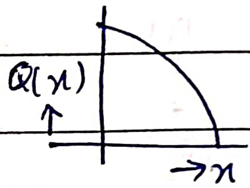

Polar.

$$p(E/0) = p(E/1) = Q(Ap/\sigma_n)$$

$$p(E) = Q(Ap/\sigma_n).$$

ON-OFF.




$$p(E/0) = Q(Ap/2\sigma_n)$$

$$p(E/1) = Q(Ap/2\sigma_n)$$

$$p(E) = \frac{1}{\cancel{2}} \times \cancel{2} Q(Ap/2\sigma_n)$$

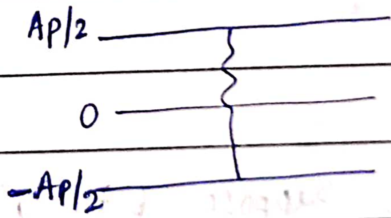
$$= Q(Ap/2\sigma_n)$$

Bipolar.

$$p(E/0) = p(|n| > Ap/2)$$

$$= p(n > Ap/2)$$

$$+ p(n < -Ap/2)$$




$$= Q(Ap/2\sigma_n) + Q(Ap/2\sigma_n)$$

$$p(E/1) = Q(Ap/2\sigma_n)$$

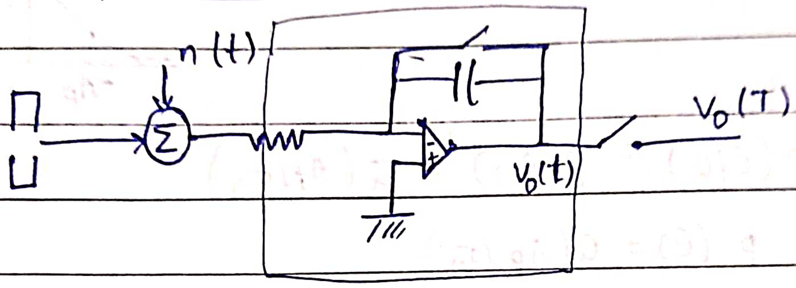
$$= 2Q(Ap/2\sigma_n)$$

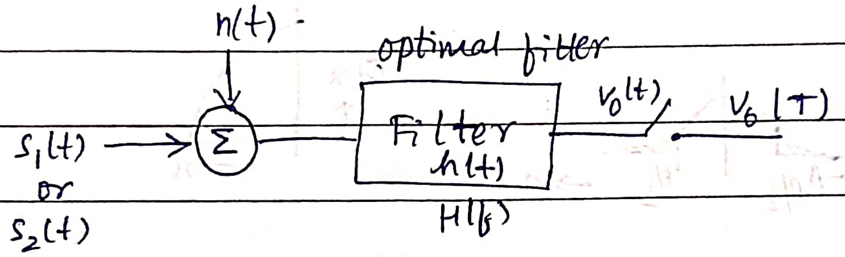
Total prob. of error,

$$p(t) = p(0)\cdot p(\epsilon/0) + p(1)\cdot p(\epsilon/1)$$

$$= \frac{1}{2}\cdot 2Q(A_p/2\sigma_n) + \frac{1}{2}Q(A_p/2\sigma_n)$$

$$\boxed{p(t) = 1.5\,Q(A_p/2\sigma_n)}$$

→ 

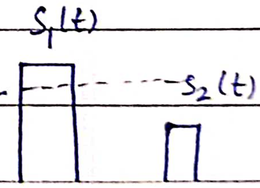




→ optimal filter implies $P_e$ to be minimum.

→ To characterize $h(t)$ or $H(f)$, so that $P_e$ to be minimum, $(P_e)_{min}$.

→ let '1' be represented by $s_1(t)$

'0' ⟶ $s_2(t)$



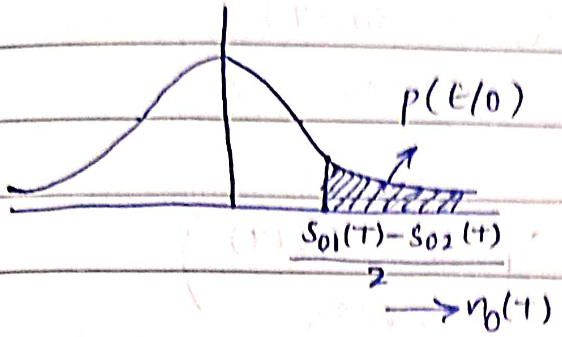

threshold at $\dfrac{S_{01}(T) + S_{02}(T)}{2}$

we assume, $s_1(t) > s_2(t)$

→ suppose bit '0' was transmitted, $S_{02}(T)$.

$$p(\epsilon/0) = \text{if } n_0(T) > \frac{S_{01}(T) + S_{02}(T)}{2} - S_{02}(T)$$

$$> \frac{S_{01}(T) - S_{02}(T)}{2}$$




$$p(\epsilon/0) = \int_{\frac{S_{01}(T)-S_{02}(T)}{2}}^{\infty} f[n_0(T)]\, d[n_0(T)]$$

$$= \frac{1}{\sqrt{2\pi\sigma_n^2}} \int_{\frac{S_{01}(T)-S_{02}(T)}{2}}^{\infty} e^{-\frac{n_0^2(T)}{2\sigma_n^2}}$$

$$x = \frac{n_0(T)}{\sqrt{2}\sigma_n}$$

| | | |
|---|---|---|
| $x$ | $\infty$ | $\frac{S_{01}(T)-S_{02}(T)}{2\sqrt{2}\sigma_n}$ |
| $n_0(T)$ | $\infty$ | $\frac{S_{01}(T)-S_{02}(T)}{2}$ |

$$\sqrt{2}\sigma_n\, dx = d[n_0(T)]$$

Now, $$p(\epsilon/0) = \int_{\frac{S_{01}(T)-S_{02}(T)}{2\sqrt{2}\sigma_n}}^{\infty} \frac{1}{\sqrt{\cancel{2}\pi\cancel{\sigma_n^2}}} e^{-x^2} \cdot \cancel{\sqrt{2}}\cancel{\sigma_n}\, dx.$$

✓ $$\Rightarrow p(\epsilon/0) = \frac{1}{\sqrt{\pi}} \int_{\frac{S_{01}(T)-S_{02}(T)}{2\sqrt{2}\sigma_n}}^{\infty} e^{-x^2}\, dx$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{S_{01}(T)-S_{02}(T)}{2\sqrt{2}\sigma_n}\right)$$

$$p(e/1) = \frac{1}{2}\,\text{erfc}\left(\frac{s_{01}(T) - s_{02}(T)}{2\sqrt{2}\,\sigma_n}\right).$$

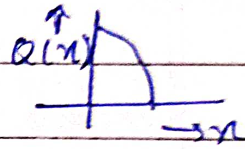
Total prob.

$$p(e) = \frac{1}{2}\,\text{erfc}\left(\frac{s_{01}(T) - s_{02}(T)}{2\sqrt{2}\,\sigma_n}\right).$$

$\left(\because\ p(e) = p(0)\cdot p(e/0) + p(1)\cdot p(e/1)\right)$

To make $p(e)$ as $[p(e)]_{min}$,

$$\gamma = \frac{s_{01}(T) - s_{02}(T)}{\sigma_n}$$

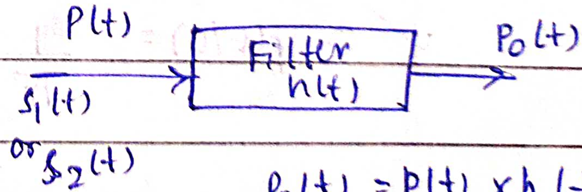



$(P_e)_{min} \rightarrow \gamma_{max}$ ..

$$P_e = \frac{1}{2}\cdot \text{erfc}\left(\frac{1}{2\sqrt{2}}\gamma\right) = \frac{1}{2}\,\text{erfc}\left(\frac{1}{8}\gamma^2\right)^{1/2}$$

$$(P_e)_{min} = \frac{1}{2}\,\text{erfc}\left(\frac{1}{8}\gamma^2_{max}\right)$$

$$\Rightarrow \gamma^2 = \frac{[s_{01}(T) - s_{02}(T)]^2}{\sigma_n^2}.$$




$p(t) = s_1(t) - s_2(t)$

$P_o(t) = p(t) \times h(t)$

$P_o(t) = s_{01}(t) - s_{02}(t).$

(F.T.) $P_o(f)$ in time domain, $P_o(t) = \int_{-\infty}^{\infty} P_o(f)\cdot e^{j2\pi f t}\,df.$

$P_0(T) = S_{01}(T) - S_{02}(T)$

$$\Rightarrow \quad P_0(T) = \int_{-\infty}^{\infty} H(f) \cdot P(f) \cdot e^{j2\pi f T} \, df$$

For noise, it will process the same way as signal.

$G_n(f)$ – PSD of I/P noise

$G_{no}(f)$ – PSD of O/P noise

$$\xrightarrow{G_n(f)} \boxed{H(f)} \xrightarrow{G_{no}(f)}$$

$$G_{no}(f) = G_n(f) \cdot |H(f)|^2$$

Output noise power, $\sigma_{no}^2 = \int_{-\infty}^{\infty} G_{no}(f) \, df$

$$\Rightarrow \quad \sigma_{no}^2 = \int_{-\infty}^{\infty} G_n(f) \, |H(f)| \, df$$

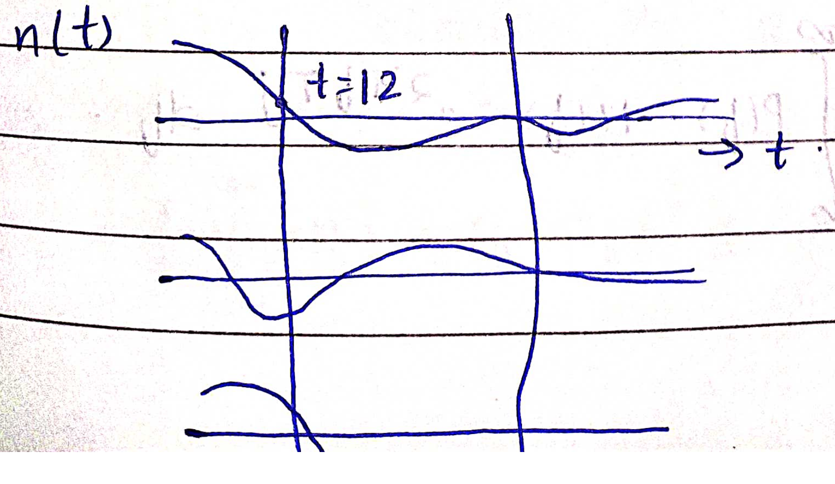
10/04/23.

Optimum filter $\rightarrow$ filter that minimizes $(P_e)$

$$p(t) \rightarrow \boxed{\begin{matrix} h(t) \\ H(f) \end{matrix}} \rightarrow P_n(t)$$

$(P_e)_{min}$

Random process is an extension of random variables (or) a connection of sampling functions.



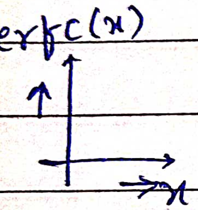
Random variable - func. of space, time.

→ The function values at any instant are independent of each other.

e.g:- temp at 12 p.m differ from that at 6 a.m.

→ Total sum of all the values gives an approximate /average result.

→ To find $(P_e)_{min}$ from $h(t)$ / $H(f)$




$$P_e = \frac{1}{2} \operatorname{erfc}\left[\frac{S_{01}(T) - S_{02}(T)}{2\sqrt{2}\,\sigma_0}\right]$$

$$P_e = \frac{1}{2} \operatorname{erfc} \frac{\gamma}{2\sqrt{2}} \qquad \because \gamma = \frac{S_{01}(T) - S_{02}(T)}{\sigma_0}$$

so, $(P_e)_{min}$, if $(\gamma)_{max}$.

$$p(t) = S_1(t) - S_2(t)$$

$$p_0(t) = S_{01}(t) - S_{02}(t)$$

$$P_0(t) = P(f) \cdot H(f)$$

$$p_0(T) = \int_{-\infty}^{\infty} P_0(t) \cdot e^{j2\pi fT}\, df$$

$$= \int_{-\infty}^{\infty} P(f) \cdot H(f)\, e^{2\pi fTj}\, df.$$

$G_{no}(f)$ = output noise PSD.
$G_n(f)$ = Input noise PSD

$$G_{no}(f) = G_n(f) \cdot |H(f)|^2.$$

→ To get $r_{max}$, we consider $r^2_{max}$.

$\sigma_0^2$ = output noise power.

$$= \int_{-\infty}^{\infty} G_{no}(f)\, df$$

$$= \int_{-\infty}^{\infty} G_n(f) \cdot |H(f)|^2\, df.$$

Now, $$r^2 = \frac{[s_{01}(T) - s_{02}(T)]^2}{\sigma_0^2} = \frac{[P_0(T)]^2}{\sigma_0^2}$$

$$= \frac{\left| \int_{-\infty}^{\infty} P(f)\, H(f)\, e^{j2\pi fT}\, df \right|^2}{\int_{-\infty}^{\infty} G_n(f) \cdot |H(f)|^2\, df}.$$

Schwatz Inequality.

→ The product integration square of two functions should be ≤ product of square of individual func.

$$\left| \int_{-\infty}^{\infty} X(f)\, Y(f)\, df \right|^2 \le \int_{-\infty}^{\infty} |X(f)|^2\, df \int_{-\infty}^{\infty} |Y(f)|^2\, df$$

$$\frac{\left| \int_{-\infty}^{\infty} X(f)\, H(f)\, df \right|^2}{\int_{-\infty}^{\infty} |X(f)|^2\, df} \leq \int_{-\infty}^{\infty} |Y(f)|^2\, df$$

→ if $Y(f) = K\, X^*(f)$, then they are equal.

$X^*(f)$ → complex conj. of $X(f)$

$$X(f) = \sqrt{G_n(f)} \cdot H(f)$$

$$Y(f) = \frac{P(f) \cdot \cancel{H(f)} \cdot e^{j2\pi f T}}{\sqrt{G_n(f)} \cdot \cancel{H(f)}} = \frac{P(f) \cdot e^{j2\pi f T}}{\sqrt{G_n(f)}}$$

→ if $X(f) = K\, Y^*(f)$, then also equal

$$X(f) = \sqrt{G_n(f)} \cdot H(f)$$

$$\boxed{H(f) = \frac{k \cdot P^*(f)\; e^{-2\pi j f T}}{G_n(f)}}$$ → Required transfer function.

$$Y(f) = \frac{P(t) \cdot e^{j2\pi f T}}{\sqrt{G_n(f)}}$$

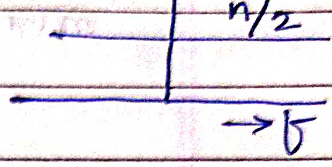
so, $$\gamma^2_{max} = k \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}\, df$$

$$|e^{j2\pi f T}|^2 = 1.$$

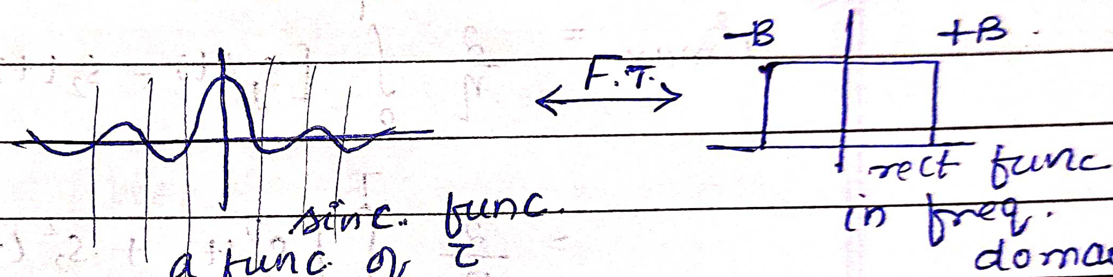
$$\boxed{\gamma^2_{max} = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}\, df}$$

→ Matched filter :- when the noise is white, the optimum filter is called matched filter.

$$H(f) = \frac{k \cdot P^*(f) \cdot e^{-2\pi j f T}}{\eta/2}$$




white - colour corresponding to all wavelength and frequencies.




$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{+2\pi f t j}\, df = \frac{2k}{\eta} \int_{-\infty}^{\infty} P^*(f)\, e^{-2\pi j f T} \cdot e^{+2\pi f t j}$$

$$= \frac{2k}{\eta} \int_{-\infty}^{\infty} p(t) \cdot e^{j 2\pi f (T-t)}\, df \quad (I.F.T.)$$

If filter is real

$P^*(f) = P(-f) = P$

$$= \frac{2k}{\eta} \cdot P(T-t)$$

$$= \frac{2k}{\eta} \left[ s_1(T-t) - s_2(T-t) \right].$$

$$h(t) = \frac{2k}{\eta}\left[s_1(T-t) - s_2(T-t)\right]$$

$$\sigma_0^2 = \int_{-\infty}^{\infty} G_{no}(f)\, df.$$

$$\gamma^2_{max} = \frac{2}{\eta}\int_{-\infty}^{\infty} |P(f)|^2\, df = \frac{2}{\eta}\int_{-\infty}^{\infty} p^2(t)\, dt.$$

$$= \frac{2}{\eta}\int_0^T p^2(t)\, dt.$$

$P_0(T)$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{S_{01}(T) - S_{02}(T)}{2\sqrt{2}\,\sigma_0}\right]$$

$$\gamma^2_{max} = \frac{2}{\eta}\int_0^T [s_1(t) - s_2(t)]^2\, dt.$$

$$= \frac{2}{\eta}\int_0^T \left[s_1^2(t) + s_2^2(t) - 2s_1(t)\cdot s_2(t)\right] dt$$

$$= \frac{2}{\eta}\left[\int_0^T s_1^2(t)\, dt + \int_0^T s_2^2(t)\, dt - 2\left(\int_0^T \sqrt{s_1(t)\cdot s_2(t)}\, dt\right)^2\right]$$

$$= \frac{2}{\eta}\left[E_{s1} + E_{s2} - 2E_{s12}\right]$$

$$E_{s12} = \int_0^T \left(\sqrt{s_1(t)\cdot s_2(t)}\right)^2 dt.$$

→ polar, $\boxed{s_1(t) = -s_2(t)}$

$$\boxed{E_{s1} = E_{s2} = -E_{s12} = E_s}$$

then, $\gamma^2_{max} = \frac{8E}{\eta}$.

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{s_{01}(T) - s_{02}(T)}{2\sqrt{2}\,\sigma_0}\right]$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2\sqrt{2}}\cdot\gamma\right]$$

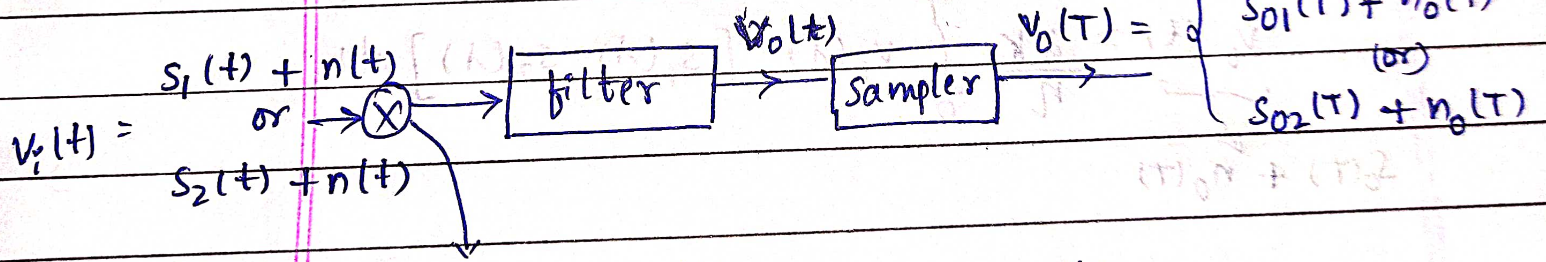
$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{8}\gamma^2\right]^{1/2}$$

$$(P_e)_{min} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{8}\gamma^2_{max}\right] = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{\cancel{8}} \times \cancel{8}\,\frac{E_s}{\eta}\right]^{1/2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_s}{\eta}}$$

1. Integrate and Dump
   $P_e$
2. Optimum Filter & find $H(f)$ / $h(t)$
3. Matched filter, $(P_e)_{min}$
4. Now, Correlation

Correlator.




$p(t) = s_1(t) - s_2(t)$

$$v_0(t) = \frac{1}{\tau}\int_{-\infty}^{\infty} v_i(t)\,\underbrace{[s_1(t) - s_2(t)]}_{p(t)}\,dt$$

$s_0(t) + n_0(t)$ , $\quad s_0(t) = \frac{1}{\tau}\int_{-\infty}^{\infty} s_i(t)\cdot p(t)\,dt$

$$n_0(t) = \frac{1}{\tau}\int_{-\infty}^{\infty} n(t)\cdot p(t)\,dt$$

$$s_0(T) = \frac{1}{\tau}\int_0^T s_i(t)\, p(t)\, dt \quad \& \quad n_0(T) = \frac{1}{\tau}\int_0^T n(t)\, p(t)\, dt$$

Matched filter,

$$h(t) = \frac{2k}{\eta}\left[s_1(T-t) - s_2(T-t)\right]$$

output of matched filter,

$$V_0(t) = \int_{-\infty}^{\infty} v_i(\lambda)\cdot h(t-\lambda)\, d\lambda$$

$$v_i(t) \longrightarrow \boxed{h(t)} \longrightarrow v_0(t)$$

$$V_0(t) = v_i(t) \cdot h(t)$$

$$v_0(t) = \int_{-\infty}^{\infty} v_i(\lambda)\cdot \frac{2k}{\eta}\left[s_1(T-t+\lambda) - s_2(T-t+\lambda)\right] d\lambda$$

Now, $h(t-\lambda) = \frac{2k}{\eta}\left[s_1(T-t+\lambda) - s_2(T-t+\lambda)\right]$

$$v_0(t) = \int_{-\infty}^{\infty} v_i(\lambda)\cdot h(t-\lambda)\, d\lambda \left(\frac{2k}{\eta}\right)$$

$$\underbrace{v_0(t)}_{s_0(T) + n_0(T)} = \frac{2k}{\eta}\int_0^T v_i(\lambda)\cdot\left[s_1(\lambda) - s_2(\lambda)\right] d\lambda$$

$$s_0(T) = \frac{2k}{\eta}\int_0^T s_i(\lambda)\cdot\left[s_1(\lambda) - s_2(\lambda)\right] d\lambda$$

$$n_0(T) = \frac{2k}{\eta}\int_0^T n(\lambda)\left[s_1(\lambda) - s_2(\lambda)\right] d\lambda$$