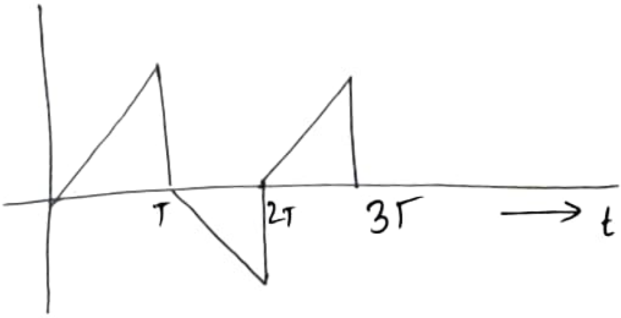
Integrate & Dump

3.04.202

$$V_0(T) = \frac{V_T}{\tau}$$

$$\sigma_n^2 = \frac{\eta T}{2\tau^2}$$

$$\left(\frac{S}{N}\right)_0 = \frac{V^2T^2}{\tau^2} \times \frac{2\tau^2}{\eta T} = \frac{2V^2T}{\eta}$$

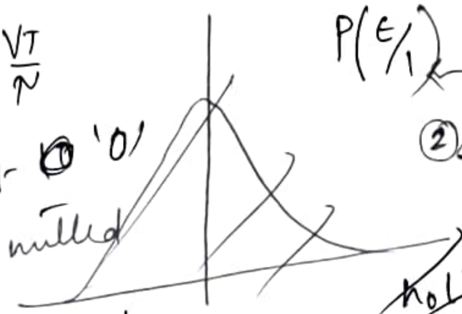


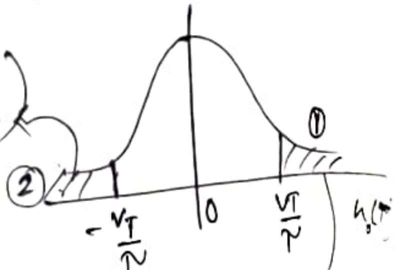
Calculate (P_e) probality of bit error.

[0 is the optimal threshold.]

if $n_0(T) > \frac{VT}{\tau}$

Suppose bit '0' was transmitted so at the end of the bit interval the signal value is $-\frac{VT}{\tau}$





$n_0(T)$

$\underbrace{P(E/0)}$

~~Probability~~ probability of error if 0 was transm...

① mathematically

If

$$f[n_0(T)] = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{n_0^2(T)}{2\sigma_n^2}}$$

[we consider here that noise is having 0 mean. ($\mu = 0$)]

$\therefore P(\varepsilon/0)$ = probability of bit error when 0 was transmitted.

$$= \int_{\frac{VT}{2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{n_0^2(T)}{2\sigma_n^2}} \, d[n_0(T)].$$

Let $x = \frac{n_0(T)}{\sqrt{2}\,\sigma_n}$.

$x = \cancel{\frac{VT}{\sqrt{2}\sqrt{T}}}$

x	∞	$V\sqrt{T/\eta}$
$n_0(T)$	∞	$\frac{VT}{2}$

$x = \cancel{V\sqrt{T}}$

$dx \quad \frac{dn_o(T)}{\sqrt{2}\,\sigma_n} \Rightarrow \sqrt{2}\,\sigma_n\, dx = dn_o$

$$= \int_{V\sqrt{\frac{T}{n}}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-x^2} \sqrt{2}\,\sigma_n\, dx.$$

$$= \frac{1}{\sqrt{\pi}} \int_{V\sqrt{\frac{T}{n}}}^{\infty} e^{-x^2}\, dx.$$

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_{0}^{y} e^{-x^2/2}\, dx.$$

error func(z) $\quad erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2/2}\, dx$

$$erfc(z) = 1 - erf(z)$$
$$= \frac{2}{\sqrt{\pi}} \int_z^{\infty} e^{-x^2/2}\, dx$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{\pi}} \int_{\sqrt{T}/\sqrt{\eta}}^{\infty} e^{-x^2}\, dx .$$

$$= \frac{1}{2} \operatorname{erfc}\left(v\sqrt{\frac{T}{\eta}}\right) = P(\epsilon/0)$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{v^2 T}{\eta}\right)^{1/2}$$

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$

$P(\epsilon/1)$ = Shaded region 2 (area under the curve)

$$= \frac{1}{2} \operatorname{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$ → due to symmetry of the Gaussian distribution.

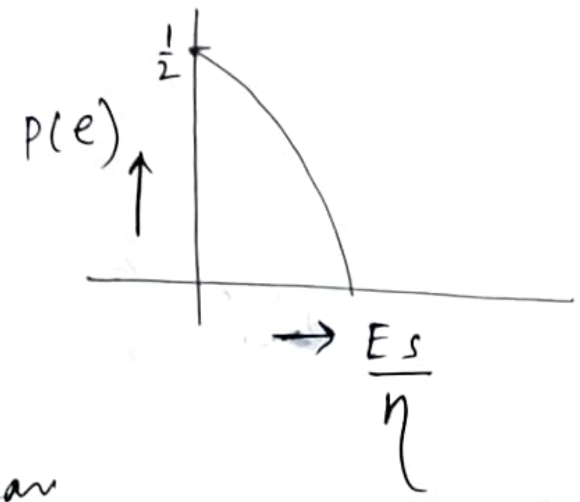
iii) $P(\epsilon)$

$$= \sum_{i=0,1} P(\epsilon/x_i) = P(0)\,P(\epsilon/0) + P(1)\,P(\epsilon/1)$$

If $P(0) = P(1) = 1/2$

$$\frac{1}{2} P(\epsilon/0) + \frac{1}{2} P(\epsilon/1)$$

$$= \frac{1}{2} \times \operatorname{erfc}\left(\frac{E_s}{\eta}\right)^{1/2}$$



Polar

$$P(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-x^2/2\sigma^2}$$

$$P(\epsilon/0) = P(\epsilon/1) = Q\left(\frac{A_p}{\sigma_n}\right)$$

$$P(\epsilon) = P(0)P(\epsilon/0) + P(1)P(\epsilon/1)$$

$$= \cancel{\tfrac{1}{2}} \times \cancel{2}\, Q\left(\frac{A_p}{\sigma_n}\right) = Q\left(\frac{A_p}{\sigma_n}\right)$$

On-off $\nearrow$ optimal.

[threshold $= \frac{1}{2}A_p$]

$$P(\epsilon/0) = Q\left(\frac{A_p}{2\sigma_n}\right)$$

[compare it with polar. This proves that polar is more immune]

$$P(\epsilon) = \frac{1}{2} \times 2Q\left(\frac{A_p}{2\sigma_n}\right)$$

$$= Q\left(\frac{A_p}{2\sigma_n}\right)$$

Bipolar

$$P(\epsilon/0) = P\left(|n| > \frac{A_p}{2}\right)$$

$$= P(n > A_p/2) + P\left(n < -\frac{A_p}{2}\right)$$

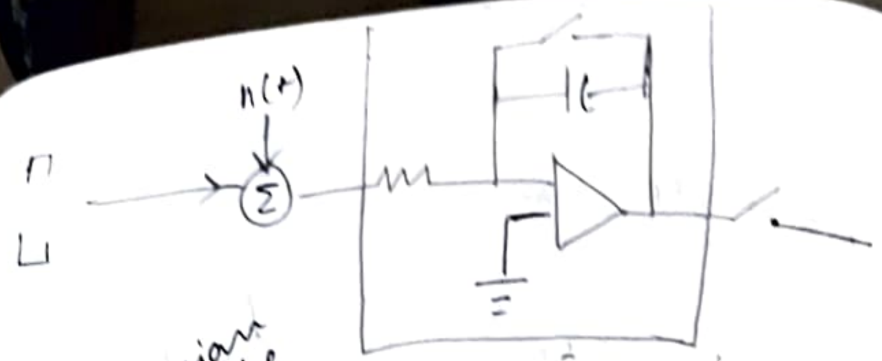
$$= Q\left(\frac{A_p}{2\sigma_n}\right) + Q\left(\frac{A_p}{2n}\right)$$

$$= 2Q\left(\frac{A_p}{2\sigma_n}\right).$$

$P(1) = P(0) = \frac{1}{2}$

$$P(\epsilon/1) \quad Q\left(\frac{A_p}{2\sigma_n}\right)$$

$$P(\epsilon) = \frac{1}{2}\left[2Q\left(\frac{A_p}{2\sigma_n}\right) + Q\left(\frac{A_p}{2\sigma_n}\right)\right] = \frac{3}{2}Q\left(\frac{A_p}{2\sigma_n}\right)$$



Gaussian noise

integrator is a low pass like filter.

Optimal filter (Probability of bit error is less/minimum)

$= \begin{cases} s_{01}(T) + n_0(T) \\ \text{or} \\ s_{02}(T) + n_0(T) \end{cases}$

To characterize filter $h(t)$ or $H(f)$ such that P_e is $(P_e)_{min}$.

$1 \longrightarrow s_1(t)$

$0 \longrightarrow s_2(t)$

Threshold

$= \frac{s_1(t) + s_2(t)}{2}$

Suppose '0' was transmitted $S_{o2}(t)$

$$P(\varepsilon/0) = \text{if } n_o(T) > \frac{S_{o1}(T)+S_{o2}(T)}{2} - S_{o1}(T)$$

$$> \frac{S_{o1}(T) - S_{o2}(T)}{2}.$$

$$P(\varepsilon/0) = \int f(n_o(T))\, dr\,(n_o(T)).$$

$$\frac{S_{o1}(T) - S_{o2}(T)}{2}$$

$$x = \frac{n_o(T)}{\sqrt{2}\,\sigma_n}.$$

$$\frac{1}{\sqrt{2\pi\sigma_n^2}} \int_{\frac{S_{o1}(T)-S_{o2}(T)}{2}}^{\infty} e^{-\frac{n_o^2(T)}{2\sigma_n^2}}$$

x	∞	$\frac{S_{o1}(T)-S_{o2}(T)}{2\sqrt{2}\,\sigma_n}$
$n_o(T)$	∞	$\frac{S_{o1}(T)-S_{o}(T)}{2}$

$$d(n_o(T)) = \sqrt{2}\,\sigma_n\, dx$$

$$\frac{1}{2}\,\frac{2}{\sqrt{\pi}} \int_{\frac{S_{o1}(T)-S_{o2}(T)}{2}}^{\infty} e^{-x^2}\, dx.$$

$$P(\epsilon/0) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{S_{01}(T) - S_{02}(T)}{2\sqrt{2}\,\sigma_n}\right)$$

$$= P(\epsilon/1)$$

$$P(e) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{S_{01}(T) - S_{02}(T)}{2\sqrt{2}\,\sigma_n}\right).$$

* $(P(e))_{min}$ Let us define a parameter

$$\frac{S_{01}(T) - S_{02}(T)}{\sigma_n} = \gamma$$

$$P(e) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\gamma}{2\sqrt{2}}\right).$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{8}\gamma^2\right)^{1/2}$$

For

$$\hookrightarrow [P(e)]_{min} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\gamma^2_{max}}{8}\right)$$

$$\gamma^2 = \frac{[S_{01}(T) - S_{02}(T)]^2}{\sigma_n^2}$$

$$p(t) = s_1(t) - s_2(t)$$

$$p_0(t) = s_{01}(t) - s_{02}(t)$$

$$p(t) \longrightarrow \boxed{\text{filter } h(t)} \longrightarrow p_0(t).$$

$$p_0(t) = p(t) * h(t)$$

$$P_0(f) = P(f)H(f)$$

$$p_0(t) = \int_{-\infty}^{\infty} P_0(f)\, e^{j2\pi f t}\, df.$$

$$p_0(T) = \int_{-\infty}^{\infty} H(f)\,P(f)\, e^{j2\pi f T}\, df$$

$$G_n(f) \longrightarrow \boxed{H(f)} \longrightarrow G_{no}(f)$$

PSD of white noise.

Output noise PSD $\hookrightarrow$ $G_{no}(f) = G_n(f)\,|H(f)|^2$

output noise power $\sigma_{n_0}^2 = \int_{-\infty}^{\infty} G_{no}(f)\, df$

$$= \int_{-\infty}^{\infty} G_n(f)\,|H(f)|^2\, df$$