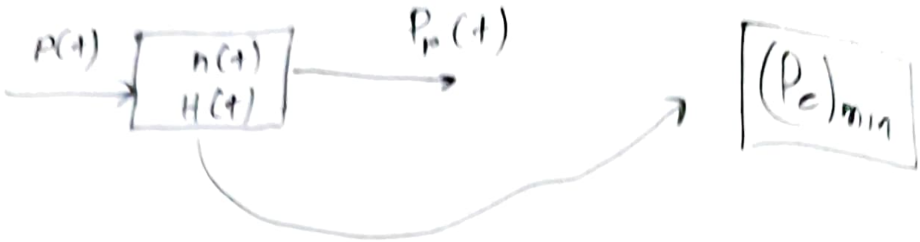
10/4/23



$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{s_{o1}(T) - s_{o2}(T)}{2\sqrt{2}\,\sigma_0}\right]$$

$(P_e)_{min}$ $\quad \gamma = \dfrac{s_{o1}(T) - s_{o2}(T)}{\sigma_0}$

if γ_{max}

$$p(t) = s_1(t) - s_2(t)$$

$$P_o(t) = s_{o1}(t) - s_{o2}(t)$$

$$P_o(f) = \int_{-\infty}^{\infty} P_o(t)\, e^{j2\pi ft}\, df$$

$$\boxed{P_o(f) = P(t)\, H(f)}$$

$$= \int_{-\infty}^{\infty} p(f)\, H(f)\, e^{j2\pi fT}\, df$$

$G_{no}(f)$: output noise PSD

$G_n(f)$: input noise PSD.

$$G_{no}(f) = G_n(f)\,|H(f)|^2$$

σ_0^2 = output noise power.

$$= \int_{-\infty}^{\infty} G_{no}(f)\,df$$

$$= \int_{-\infty}^{\infty} G_n(f)\,|H(f)|^2\,df$$

$$\gamma^2 = \frac{[S_{o1}(T) - S_{o2}(T)]^2}{\sigma_0^2}$$

$$= \frac{[P_o(T)]^2}{\sigma_0^2}$$

$$= \frac{\left|\int_{-\infty}^{\infty} P(f)\,H(f)\,e^{j2\pi f}\,df\right|^2}{\int_{-\infty}^{\infty} G_n(f)\,|H(f)|^2}$$

Schwartz Inequality :

$$\left| \int_{-\infty}^{\infty} X(f)\, Y(f)\, df \right|^2 \leq \int_{-\infty}^{\infty} |X(f)|^2\, df \int_{-\infty}^{\infty} |Y(f)|^2\, df$$

$$\Rightarrow \frac{\left| \int_{-\infty}^{\infty} X(f)\, Y(f)\, df \right|^2}{\int_{-\infty}^{\infty} |X(f)|^2\, df} \leq \int_{-\infty}^{\infty} |Y(f)|^2\, df$$

If $\boxed{Y(f) = k\, X^*(f)}$

$$\boxed{X(f) = \sqrt{G_n(f)}\; H(f)}$$

$$Y(f) = \frac{P(f)\, \cancel{H(f)}\, e^{j2\pi f T}}{\sqrt{G_n(f)}\; \cancel{H(f)}}$$

$$\therefore \boxed{Y(f) = \frac{P(f)\, e^{j2\pi f T}}{\sqrt{G_n(f)}}}$$

$$H(f) = \frac{k P^*(f)\, e^{-j2\pi f T}}{G_n(f)}$$

$$\gamma^2_{max} = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}\, df$$

Matched filter:

when the noise is white, the optimum filter is called matched filter.

$$H(f) = \frac{k\, P^*(f) e^{-2\pi j f T}}{\eta/2}$$

$$= \frac{2K}{\eta} P^*(f)\, e^{-j2\pi f T}$$

$$h(f) = \frac{2K}{\eta} \int_{-\infty}^{\infty} P^*(f)\, e^{-j2\pi f t}\, e^{j2\pi f t}\, df$$

If filter is real

$$P^*(f) = P(-f) = P(f)$$

$$n(t) = \int_{-\infty}^{\infty} H(f)\, e^{2\pi f t}\, df$$

$$= \frac{2k}{\eta} \int_{-\infty}^{\infty} P^*(f)\, e^{-2\pi f t}\, e^{j 2\pi f t}\, df$$

$$= \frac{2k}{\eta} \int_{-\infty}^{\infty} P(f)\, e^{j 2\pi f (t-\tau)}\, df$$

$$= \frac{2k}{\eta}\, P(T-t)$$

$$= \frac{2k}{\eta} \left[s_1(T-t) - s_2(T-t) \right]$$

$$n(t) = \frac{2k}{\eta} \left[s_1(T-t) - s_2(T-t) \right]$$

$$\sigma_0^2 = \int_{-\infty}^{\infty} G(f)\, |H(f)|^2\, df$$

$$\sigma_o^2 = \int_{-\infty}^{\infty} G_{n_o}(f)\, df$$

$$\gamma_{max}^2 = \frac{2}{\eta}\int_{-\infty}^{\infty} |P(f)|^2\, df$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{\overset{P_o(T)}{S_{o_1}(T) - S_{o_2}(T)}}{2\sqrt{2}\,\sigma_o}\right]$$

$$\gamma_{max}^2 = \frac{2}{\eta}\int_{-\infty}^{\infty} |P(f)|^2\, df$$

$$= \frac{2}{\eta}\int_{-\infty}^{\infty} P^2(f)\, df$$

$$= \frac{2}{\eta}\int_{0}^{T} P^2(t)\, dt$$

$$\gamma_{max}^2 = \frac{2}{\eta}\int_{0}^{T} P^2(t)\, dt = \frac{2}{\eta}\int_{0}^{T} [S_1(t) - S(t)]^2\, dt$$

$$= \frac{2}{\eta} \int_0^T s_1^2(t) + s_2^2(t) - 2s_1(t)s_2(t)\, dt$$

$$= \frac{2}{\eta} \left[\int_0^T s_1(t)^2 dt \right] + \int_0^T s_2^2(t)\, dt$$

$$- 2 \int_0^T \left(\sqrt{s_1(t)\, s_2(t)}\right)^2 dt$$

$$= \frac{2}{\eta} \left[E_{s_1} + E_{s_2} - 2E_{s_{12}} \right]$$

Polar case:

$$s_1(t) = -s_2(t)$$

$$E_{s_1} = E_{s_2} = -E_{s_{12}}$$

$$= E_s$$

$$\therefore \frac{2}{\eta} \left[E_s + E_s + 2E_s \right]$$

$$= \frac{8E_s}{\eta}$$

$$\gamma_{max}^2 = \frac{8E_s}{\eta}$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{s_{01}(T) - s_{02}(T)}{2\sqrt{2}\,\sigma_0}\right]$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2\sqrt{2}}\gamma\right]$$

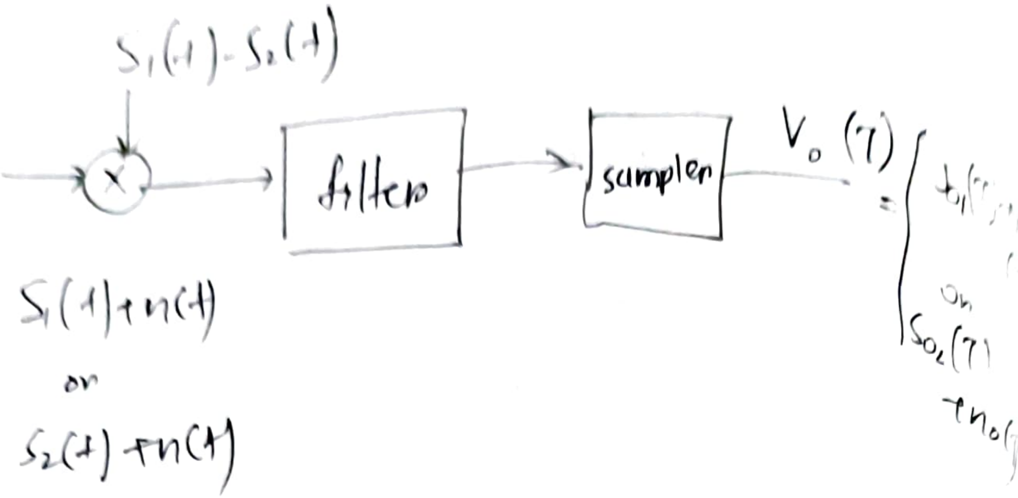
$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{8}\gamma^2\right]^{1/2}$$

$$(P_e)_{min} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{8}\gamma_{max}^2\right]$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{\cancel{8}} \times \cancel{8}\frac{E_s}{\eta}\right]$$

$$= \frac{1}{2}\,\mathrm{erfc}\left[\frac{E_s}{\eta}\right]^{1/2}$$

Correlator :



$$S_o(t) = \frac{1}{T} \int_{-\infty}^{\infty} S_i(t) \left[S_1(t) - S_2(t) \right] dt$$

&

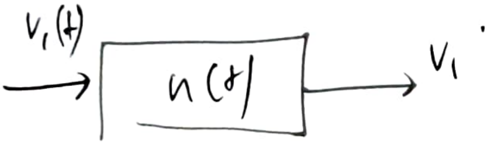
$$n_o(T) = \frac{1}{\tau} \int_{0}^{T} n(t) \left[S(t) - S(t) \right] dt$$

Matched filter :

$$h(t) = \frac{2k}{\eta} \left[S_1(T-t) - S_2(T-t) \right]$$

output of matched filter

$$V_o(t) = \int_{-\infty}^{\infty} v_1(\lambda)\, h(t-\lambda)\, d\lambda$$



$$\int_{-\infty}^{\infty} v_i(\lambda)\, h(t-\lambda)\, d\lambda$$

$$\Rightarrow v_o(t) = \int_{-\infty}^{\infty} v_i(\lambda) \frac{2k}{\eta} \left[s_1(T-t+\lambda) - s_2(T-t+\lambda) \right] d\lambda$$

$$h(t-\lambda) = \frac{2k}{\eta} \left[s_1(T-t+\lambda) - s_2(T-t+\lambda) \right]$$

$$v_o(T) = \frac{2k}{\eta} \int_{-\infty}^{\infty} v_i(\lambda) \left[s_1(\lambda) - s_2(\lambda) \right]$$

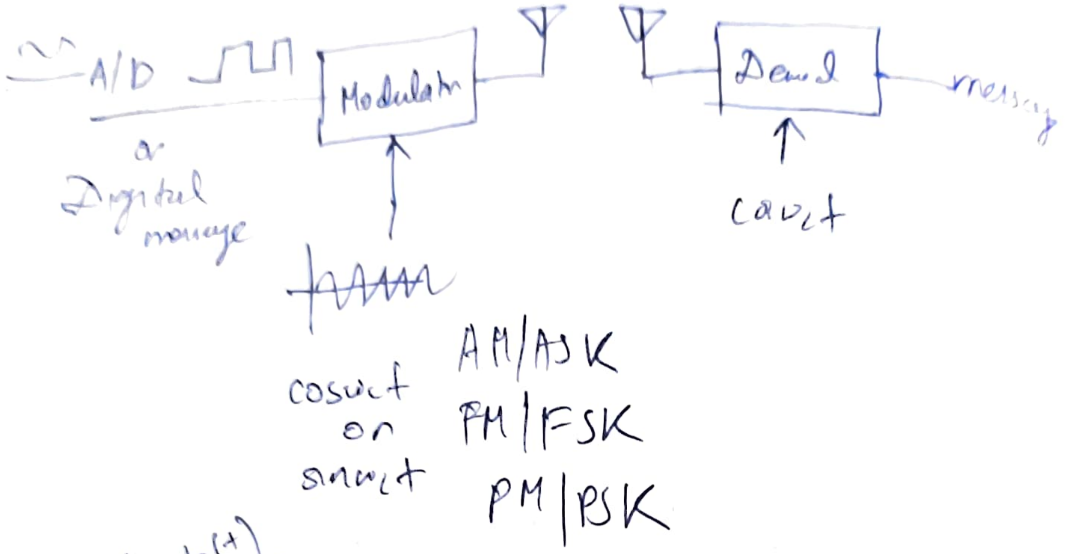
substitute

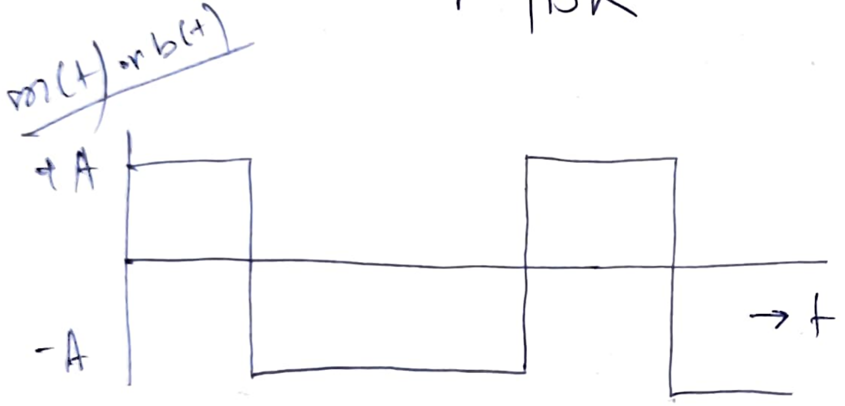
$t = T$

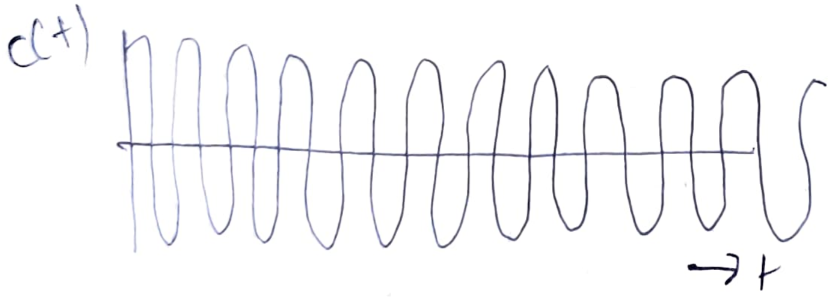
$$n_o(T) = \frac{2k}{\eta} \int_0^T n(\lambda) \left[s_1(\lambda) - s_2(\lambda) \right] d\lambda$$

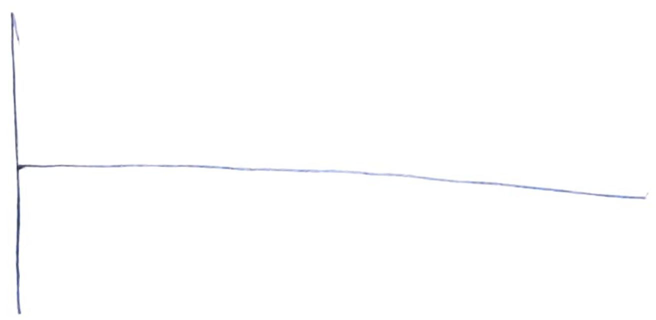
1) Digital Carrier Modulation : 11/04/22

2) Secured Comm



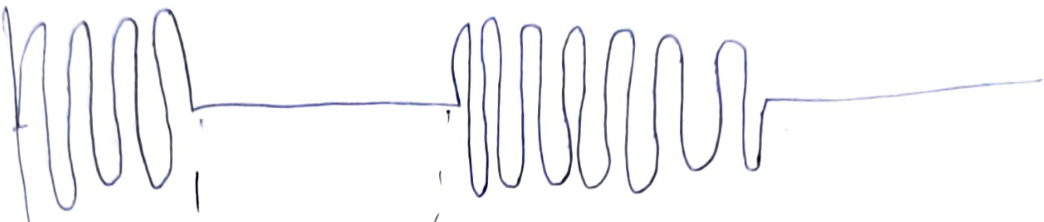




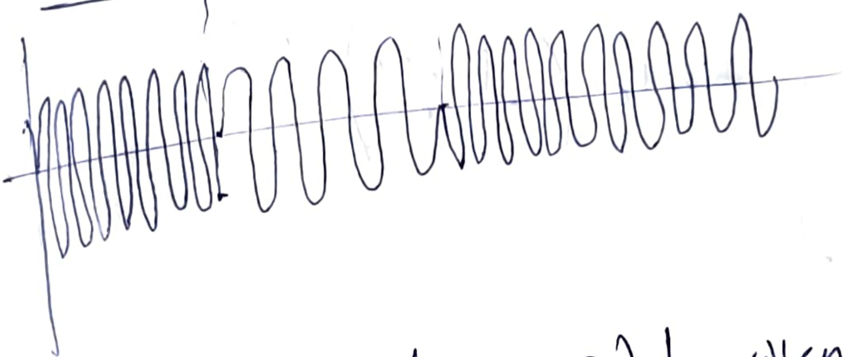


$V_{ASK}(t) = A\cos\omega_c t \cdot b(t)$

$b(t) = +V$
$\quad = 0, -V$

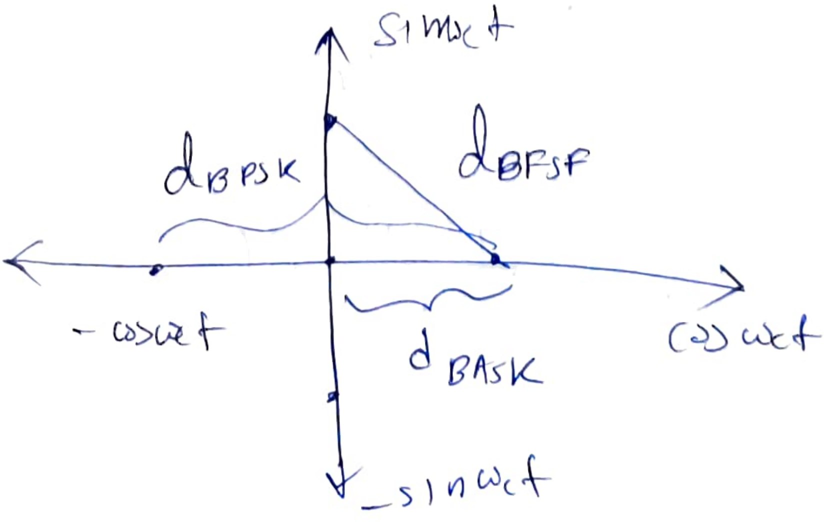


ASK :



$V_{FSK}(t) = A\cos(\omega_c + \Omega)t$ when $b(t)$ or $m(t) = +V$

$\quad\quad A\cos(\omega_c - \Omega)t$ $b(t)$ or $m(t) = -V$



BASK :

$s_1(t) = A\cos w_c t$

$$V_{BASK}(t) = A\cos w_0 t \quad b(t) = 1$$
$$= 0 \quad b(t) = 0$$

$s_2(t) = 0.$

$$p(t) = s_1(t) - s_2(t) = A\cos w_0 t$$

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{1}{8}\gamma^2\right)^{1/2}$$

$\Rightarrow \frac{1}{2}$

$P_{e\,min}$ when γ_{max}

$$\gamma_{max}^2 = \int_{-\infty}^{\infty} \frac{|P(f)|^2}{G_n(f)}$$

$$= \frac{2}{\eta}\int_{-\infty}^{\infty} \frac{|P(f)|^2}{1}\,df.$$

$$= \frac{2}{\eta}\int_0^T p^2(t)\,dt$$

$$= \frac{2}{\eta}\int_0^T A^2\cos^2\omega_0 t\,dt$$

$$= \frac{2}{\eta}\times A^2\left[\int_0^t dt + \int_0^t \cos 2\omega_0 t\,dt\right.$$

$$= \frac{A^2}{\eta}T$$

$$= \frac{2A^2T}{2\eta} = \frac{2E_s}{\eta}$$

$$(P_e)_{min} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{1}{8}\,\frac{2E_s}{\eta}\right)^{1/2}$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(0.25\,\frac{E_s}{\eta}\right)^{1/2}$$