15/3/23.

Line coding:

1) Transmission B.W: A line code should preferably be of lower B.W.
B.W should be as min. as possible.

2) Power efficiency: A line code, for a given B.W and error detection probability, should consume or require transmission power as small as possible.

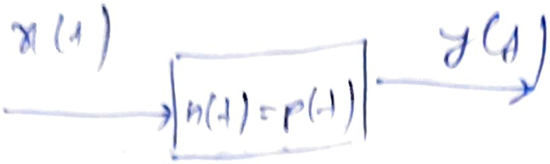
3) Favourable PSD: A line code demands zero PSD at $\omega = 0$

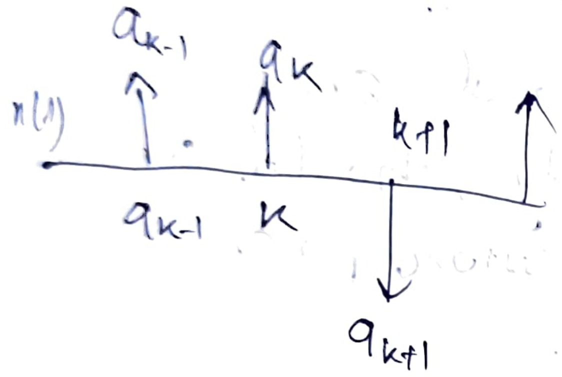
4) Error detection and correction:

5) Timing information

6) Transparency.

PSD:





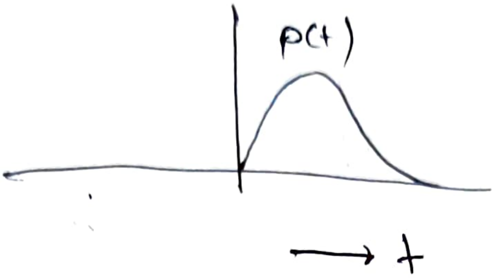
$$g(t) = x(t) * h(t)$$

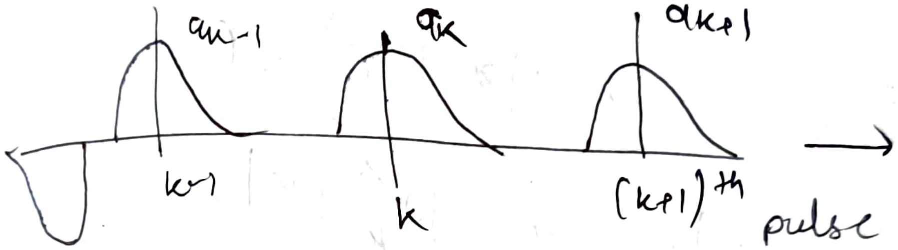
$$|Y(\omega)| = |X(\omega)||H(\omega)|^2$$

$$\cancel{S_y(\omega)} \quad S_\omega(Y) = S_x(\omega)|P(\omega)|^2$$

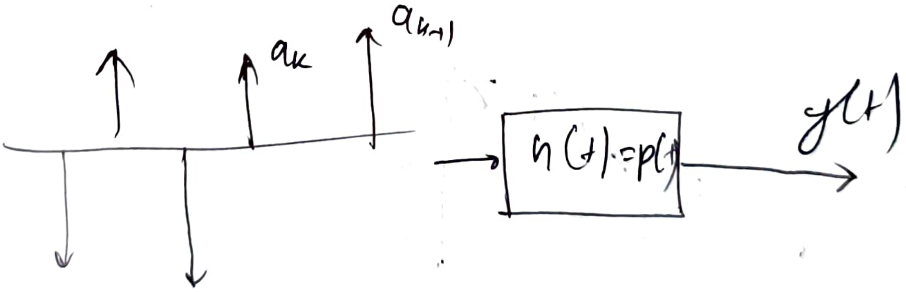
PSD of line codes :

Let us assume that there is a binary data transmission at rate R_b such that $R_b = \frac{1}{T_b}$





a_k's are random and take values $-1, 0, +1$

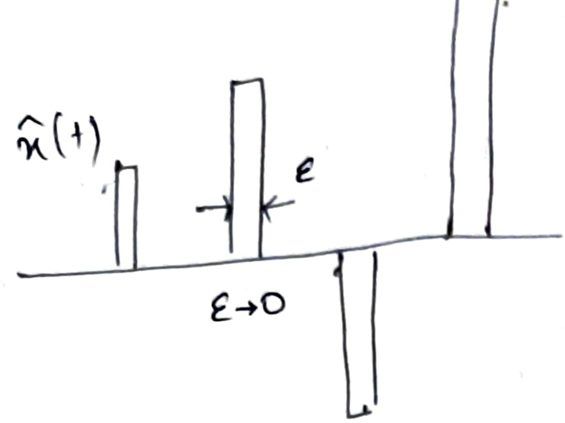


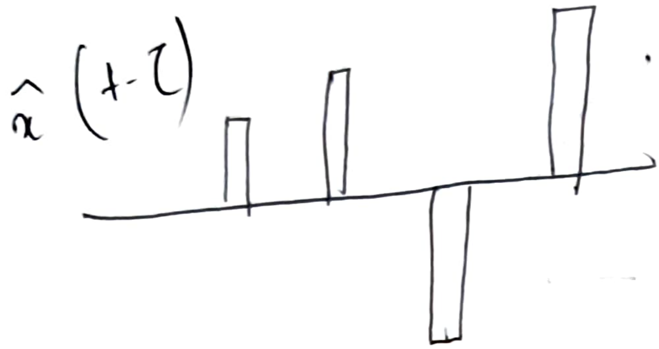
$y(t) = x(t) * h(t)$

$|Y(\omega)|^2 = |X(\omega)|^2 \, |H(\omega)|^2$

$$S_y(\omega) = \underline{S_n(\omega)}\, P^2(\omega)$$

↓ PSD   PSD of $x(t)$

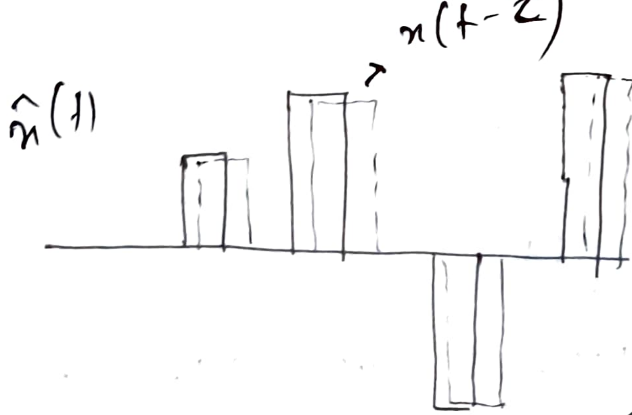




$$R_{xx}(\tau) = x(t)\,x(t-\tau)\,d\tau$$

$$R_x(\tau) = \lim_{\varepsilon \to 0} R_{\hat{x}}(\tau)$$

$$\boxed{R_{\hat{x}}(\tau) = \int \hat{x}(\tau)\,\hat{x}(t-\tau)\,dt}$$



$\tau < \varepsilon$

Case 1

$T = N(T_b)$

$$\hat{R_x}(\tau) = \lim_{T \to \infty} \int_{-T/2}^{T/2} \hat{x}(t)\,\hat{x}(t-\tau)\,dt$$

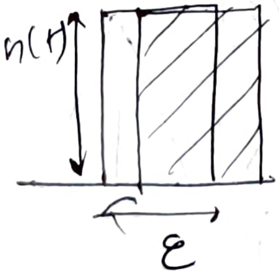
① $$\hat{R_x}(\tau) = \lim_{N \to \infty} \frac{1}{NT_0} \sum_k h_k^2 (\varepsilon - \tau)$$

i) impulse is looked as pulse of infinitesimal small width $\varepsilon \to 0$

$\Rightarrow$ strength of impulse

= area of pulse.

$a_k = h_k \varepsilon$



$\therefore$ $h_k \times \varepsilon = a_k$

$$h_k = \frac{a_k}{\varepsilon}$$

from ①:

$$\hat{R_x}(\tau) = \lim_{N \to \infty} \frac{1}{NT_b} \sum_k \frac{a_k^2}{\varepsilon}\left(1 - \frac{\tau}{\varepsilon}\right)$$

where

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

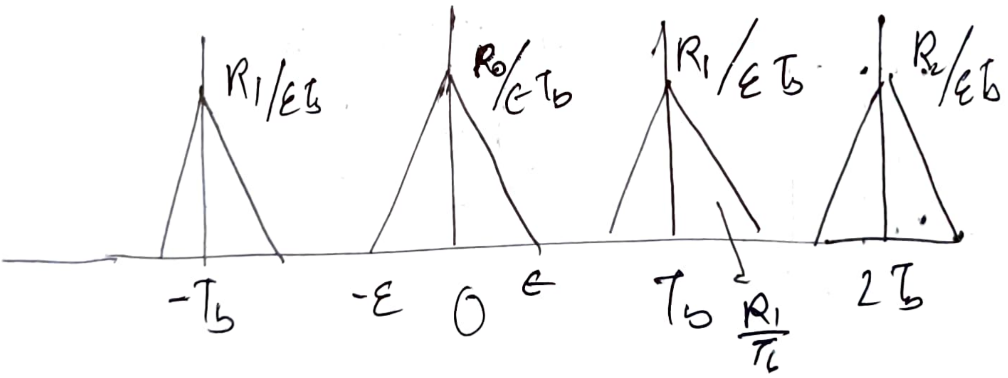
$$\hookrightarrow \quad R_n^{\wedge}(\tau) = \frac{R_0}{\epsilon T_b}\left(1 - \frac{\tau}{\epsilon}\right)$$

---

case (ii) $\tau > \epsilon$

↳ current pulse with overlap with next overlapped

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1}$$



$$\text{Area of pulse} = \frac{1}{\cancel{2}} \times \cancel{2\epsilon} \times \frac{R_0}{\cancel{\epsilon} T_b}$$

$$= \frac{R_0}{T_b}$$

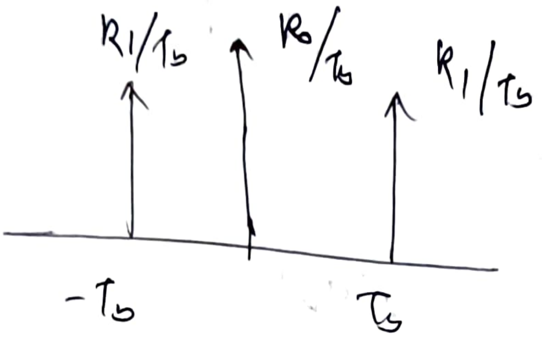
$$R_x^{\wedge}(\tau) = \frac{R_n}{\epsilon T_b} \sum a_k a_{k+n} \left(1 - \frac{|\tau|}{\epsilon}\right)$$

$$\Downarrow$$

$$R_x(\tau) = \frac{R_n}{\epsilon T_b}\left(1 - \frac{|\tau|}{\epsilon}\right)$$

?

$$\boxed{R_x(\tau) = \frac{1}{T_b} \sum_{-\infty}^{\infty} R_n \, \delta(t - nT_b)}$$



$$\boxed{S_y(\omega) = S_x(\omega) |P(\omega)|^2}$$

F.T. of $R_x(\tau)$

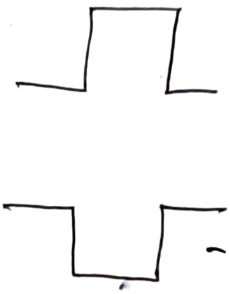
$$F_T[R_x(\tau)] = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-jn\omega T_b}$$

$$= \frac{1}{T_b}\left(R_0 + \sum_{n=1}^{\infty} R_n \cos 2n\omega T_b\right)$$

$$S_x(\omega) = \frac{1}{T_b} \sum R_n e^{-jn\omega T_b}$$

$$= \frac{1}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b\right]$$

<u>Polar Signal</u> : +1



0 -ve

R_0, R_1, R_2

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum a_k^2$$

$$= \lim_{N\to\infty} \frac{1}{N}[N]$$

$$= 1$$

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum a_k a_{k+1}$$

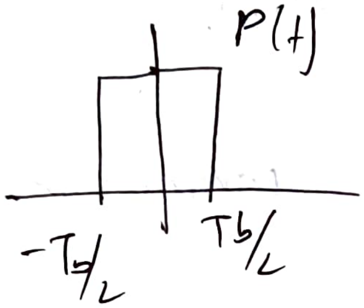
$$= \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right]$$

$$= 0 \qquad R_n = 0 \text{ for } n \geq 1$$

$$S_y(\omega) = S_x(\omega)\,|P(\omega)|^2$$

$$= \frac{R_0}{T_b}\,|P(\omega)|^2$$

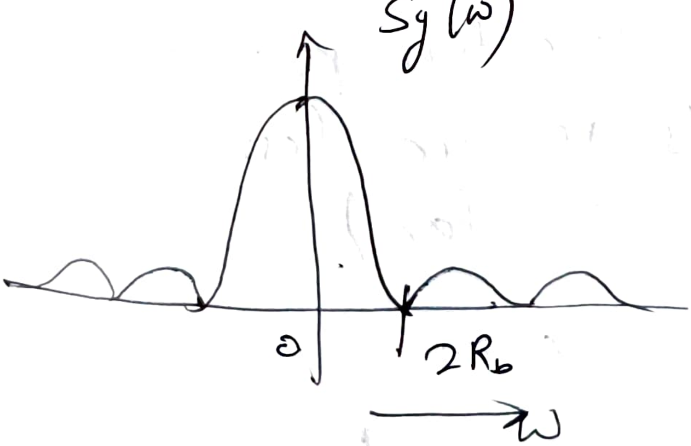
$$\boxed{S_y(\omega) = \frac{1}{T_b}\,|P(\omega)|^2}$$



$$p(t) = \Pi\left(\frac{t}{T_b}\right)$$

$$P(\omega) = T_b \sin\left(\frac{\omega T_b}{2}\right)$$

$$|P(\omega)|^2 = T_b^2\, \mathrm{sin}^2\mathrm{c}\left(\frac{\omega T_b}{2}\right)$$



Disadv: (of Polar)

1) Polar signal is not Bandwidth efficient.

4 times R_b

2) at $\omega = 0$ PSD $\neq 0$

No favourable PSD.

3) Error detection and correction is not possible

Adv:

1) Polar signal is power efficient.
(For same B.W. detection probability polar signals require less power than bipolar).

2) PSD can be made zero

Manchester coding

ON/OFF:

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k \varepsilon_k^2$$

$$= \frac{1}{N}\left[\frac{1}{2}(1) + \frac{1}{2}(0) \right]$$

$$= \frac{1}{2}$$

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1}$$

$$= \frac{1}{4}$$

$$R_n = \frac{1}{4} \quad n \geq 1$$

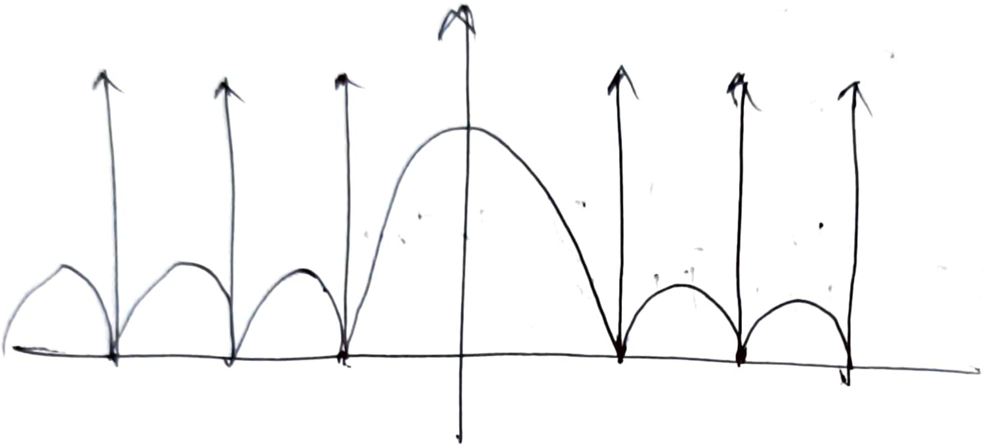
$$S_n(\omega) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-jn\omega T_b}$$

$$= \frac{1}{T_b}\left[R_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} R_n e^{-jnT_b} \right]$$

$$= \frac{1}{T_b}\left[\underbrace{\frac{1}{2}}_{\frac{1}{4}+\frac{1}{4}} + \frac{1}{4} \sum_{n=-\infty}^{\infty} e^{-jn\omega T_b} \right]$$

$$= \frac{1}{T_b}\left[1/4 + 1/4 \sum_{n=-\infty}^{n=\infty} e^{-jn\omega T_b} \right]$$

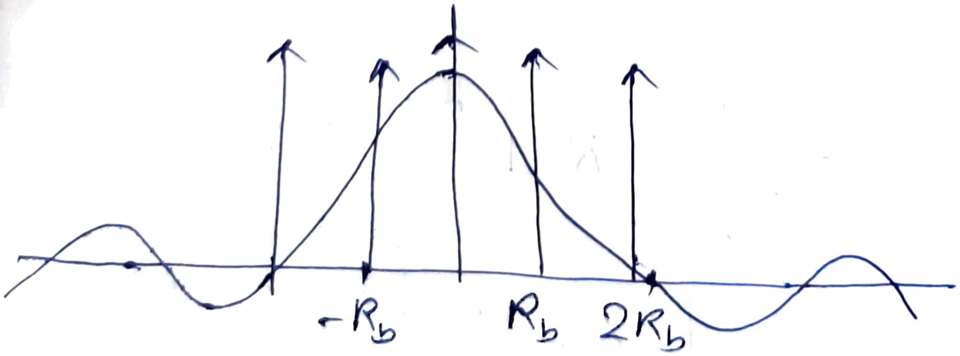
FT $\delta\left(\omega - \frac{n\omega}{T_b}\right)$



ON-OFF = Polar + Random

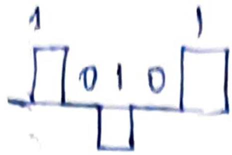
For on-off :

$$S_y(\omega) = \frac{1}{T_b} \operatorname{sinc}^2\left(\frac{\omega T_b}{4}\right)\left[\frac{1}{4T_b} + \frac{1}{4}\sum_{n=-\infty}^{\infty} \delta\left(n - \frac{n}{T_b}\right.\right.$$



ON-OFF = polar + periodic

Bipolar :

10101 

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

$$= \lim_{N\to\infty} \frac{1}{N} \left[N/2 (0) + N/2 (1) \right]$$

$$= 1/2$$

In Bipolar

$a_k \to 1$ or 0

$1 \to$ +ve or -ve

$0 \to$ non ~~polar~~ pulse

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k \cdot a_{k+1}$$

$$= \lim_{N\to\infty} \frac{1}{N} \left[\frac{3N}{4} (0) + N/4 (-1) \right]$$

$$\boxed{R_1 = -1/4}$$

a_k	a_{k+1}	
0	0	$\to 0$
0	1	$\to 0$
1	0	$\to 0$
1	1	$\to (-1)$

$$R_2 = \lim_{N\to\infty} \frac{1}{N}\sum_k a_k a_{k+2}$$

a_k	a_{k+1}	a_{k+2}	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	$\rightarrow$
1	1	0	
1	1	1	$\rightarrow +1$

$$\lim_{N\to\infty} \frac{1}{N}\left[\frac{6}{8}N(0) + \frac{1}{8}N(1) + \frac{1}{8}N(-1)\right]$$

$$= 0$$

for $n > 1$

$R_n = 0$

$R_0 = 1/2$

$R_1 = -1/4$

$R_n = 0 \quad n > 1$

$$S_y(\omega) = \frac{S_n(\omega)}{T_b}|P(\omega)|^2$$

$$= \frac{1}{T_b}\left[R_0 + 2\sum_{n=1}^{\infty} R_n \cos n\omega T_b\right]|P(\omega)|^2$$

$$= \frac{1}{T_b}\left[\frac{1}{2} - \frac{1}{2}\cos\omega T_b\right] P(\omega)^2$$

$$= \frac{1}{2T_b}\left[1 - \cos\omega T_b\right] |P(\omega)|^2$$

$$= \frac{1}{\cancel{2}T_b} \times \cancel{2}\sin^2\frac{\omega T_b}{2} |P(\omega)|^2$$

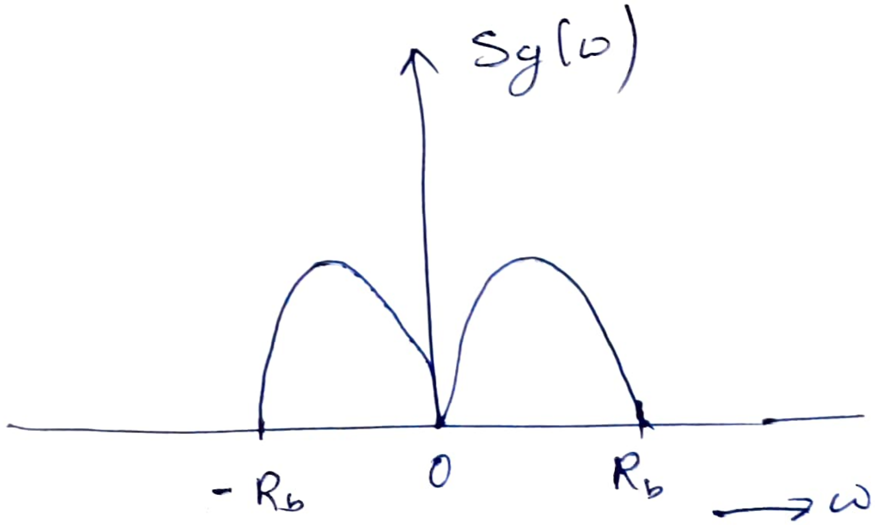
$$= \frac{1}{T_b}\sin^2\left(\frac{\omega T_b}{2}\right)|P(\omega)|^2$$

→ Bandwidth is reduced.

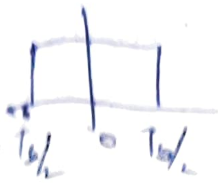
$\sin\left(\frac{\omega T_b}{2}\right) = 0$ and $\sin(\pi)$

$$\therefore \frac{\omega T_b}{2} = \pi$$

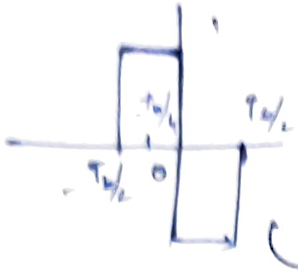
$$\Rightarrow \omega = \frac{2\pi}{T_b} = R_b$$



Manchester.



$$= \Pi\left(\frac{t-0}{T_b}\right)$$



$$p(t) = \Pi\left(\frac{t+T_b/4}{T_b/2}\right) - \Pi\left(\frac{t-T_b/4}{T_b/2}\right)$$

$$P(\omega) = \text{sinc}(\ )e^{-j\omega T_b/4} - \text{sinc}(\ )e^{j\omega T_b/4}$$

$$= \{\text{sinc}()\}\{e^{-j\omega T_b/4} - e^{j\omega T_b/4}\}$$

$$S_y(\omega) = \frac{1}{T_b} S_n(\omega)\,|P(\omega)|^2$$

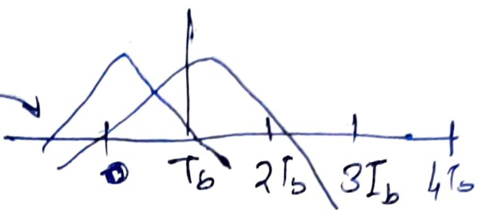
24/3/22

$$S_y(\omega) = S_n(\omega)\,|P(\omega)|^2 \rightarrow \text{pulse shape.}$$

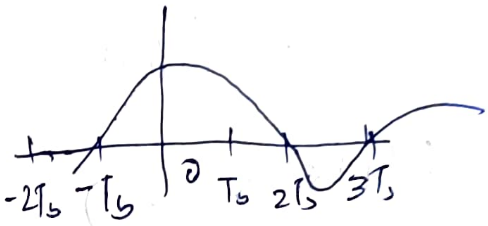
↳ PSD of line code.

↑ PSD of Tx signal

1) Zero ISI



2) Controlled ISI

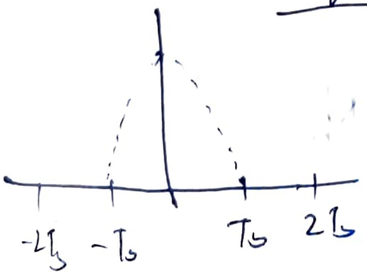


Zero ISI :-

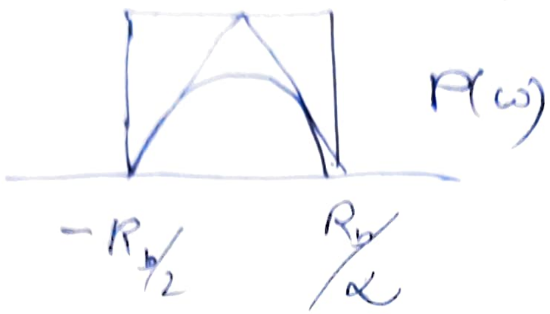
$$p(t) = \begin{cases} 1 & t = 0 \quad \cancel{nT_b \quad n = 0, \pm 1, \pm 2} \\ 0 & t = nT_b, \quad n = \pm 1, \pm 2 \ldots \end{cases}$$

↳ Requirement to solve ISI.

Theoritical req of $B.W = R_b/2$.



In Frequency Domain



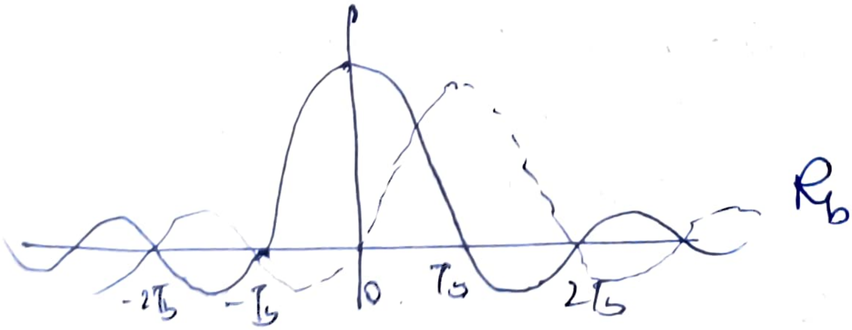
Duality:

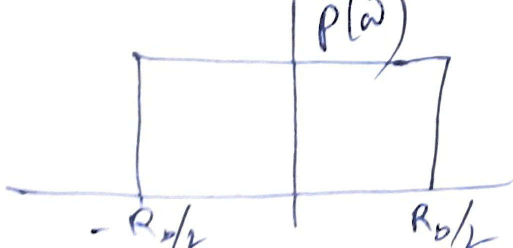
$$g(t) \Leftrightarrow G(\omega)$$

$$G(t) \Leftrightarrow g(-\omega)$$

$$p(t) = \text{sinc}(\pi R_b t)$$

$$P(\omega) = \frac{1}{R_b} \Pi\left(\frac{\omega}{2\pi R_b}\right)$$

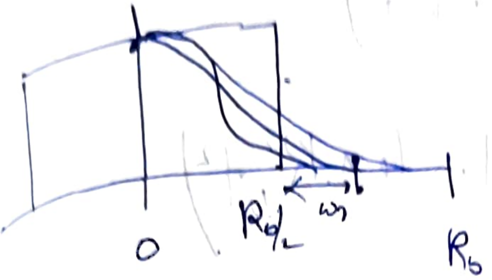


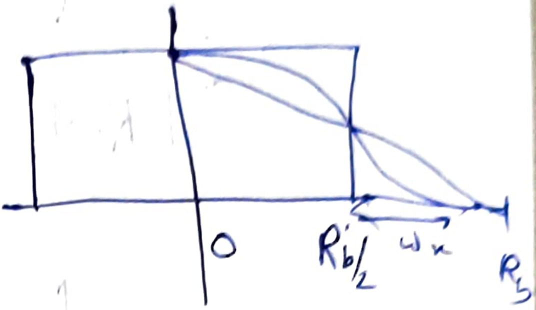


$B_T = W_b + W_x$

$r = \dfrac{\text{Excess B.W}}{\text{Theoritical min. BW}}$

↑ roll off factor





$$\text{Roll off factor } (r) = \frac{W_x}{R_b/2}$$

$$W_x = \frac{r \times R_b}{2}$$

$$B_T = W_b + W_x$$

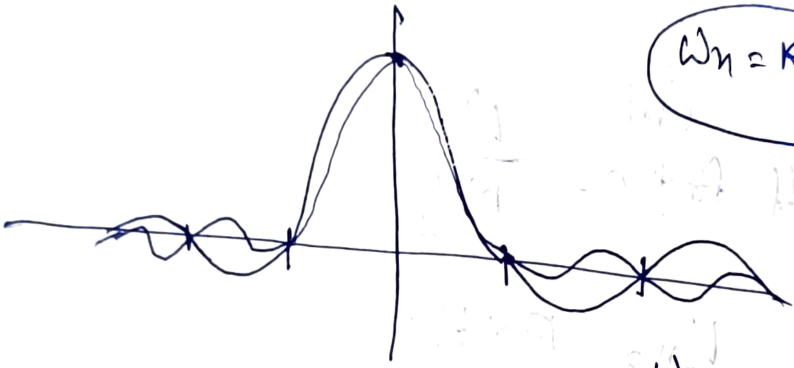
$$= \frac{R_b}{2} + \frac{r R_b}{2}$$

$$= \frac{R_b}{2}(1 + r)$$

$$\boxed{P(w) = \frac{1}{R_b} \cos^2\left(\frac{w}{4R_b}\right) \, \pi \left(\frac{w}{4\pi R_b}\right)}$$

$$P(\omega) = \frac{1}{R_b} \cos^2\left(\frac{\omega}{4R_b}\right) \Pi\left(\frac{\omega}{4\pi R_b}\right)$$

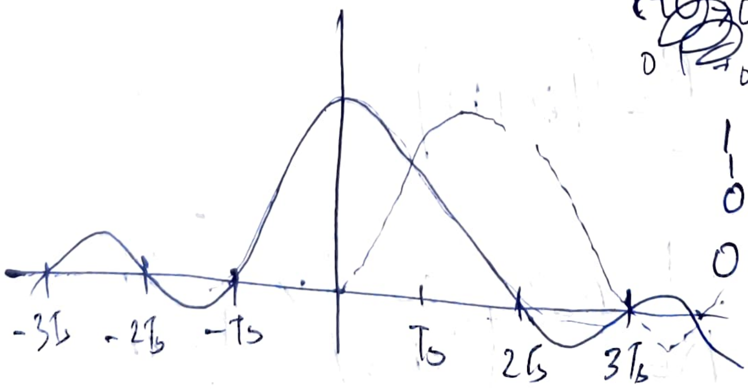
$$p(t) = \frac{\sin(\pi R_b t)}{\pi R_b t\,(1 - 4\pi R_b^2 t^2)}$$



$W_n = R_b/2$

$W_n = R_b/4$

$$P(nT_b) = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$



1 1 → +2
1 0 → 0
0 1 → 0
0 0 → −2

$x(t) =$ 1011010 01 ...

sample = 002 000 -2 0 ...

$\hookrightarrow$ 1101001

Error Det $n(t)$

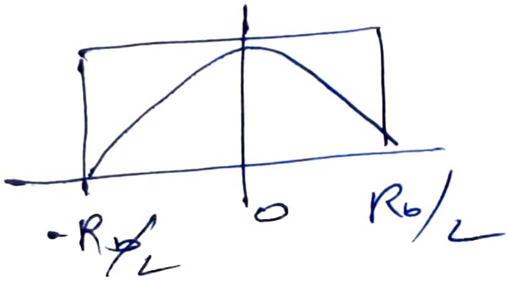
→ There should be odd no of zeroes within two samples of full value and opposite polarity.

→ an even no of zeroes " two full value sample of ~~even~~ same polarity.

Duobinary (pulse) signal gives error detection.

$$p(t) = \frac{\sin \pi R_b t}{1 - R_b^2 t^2}$$

$$p(\omega) = \frac{2}{R_b} \cos\left(\frac{\omega}{2R_b}\right) \operatorname{rect}\left(\frac{\omega}{2\pi R_b}\right)$$



Differential Coding :

10110001

→ If bit at any time instant is '1' pulse to be the same to that one of the present one

→ If bit at any time instant is 0, pulse to opposite ,,