3/1/23

What is meant by communication?

Communication means to convey message from one place to another which are some distance away.

Electrical Communication: When communication i.e. transform of message is accomplished by electrical means i.e through exchange of electrical signal.

Advantages of Electrical Communication:

i) Overcome the distance problem.

ii) Large volume of information can be exchanged.

iii) Electrical communications are cheap and reliable. (economic)

iv) Energy crisis problem can be avoided.

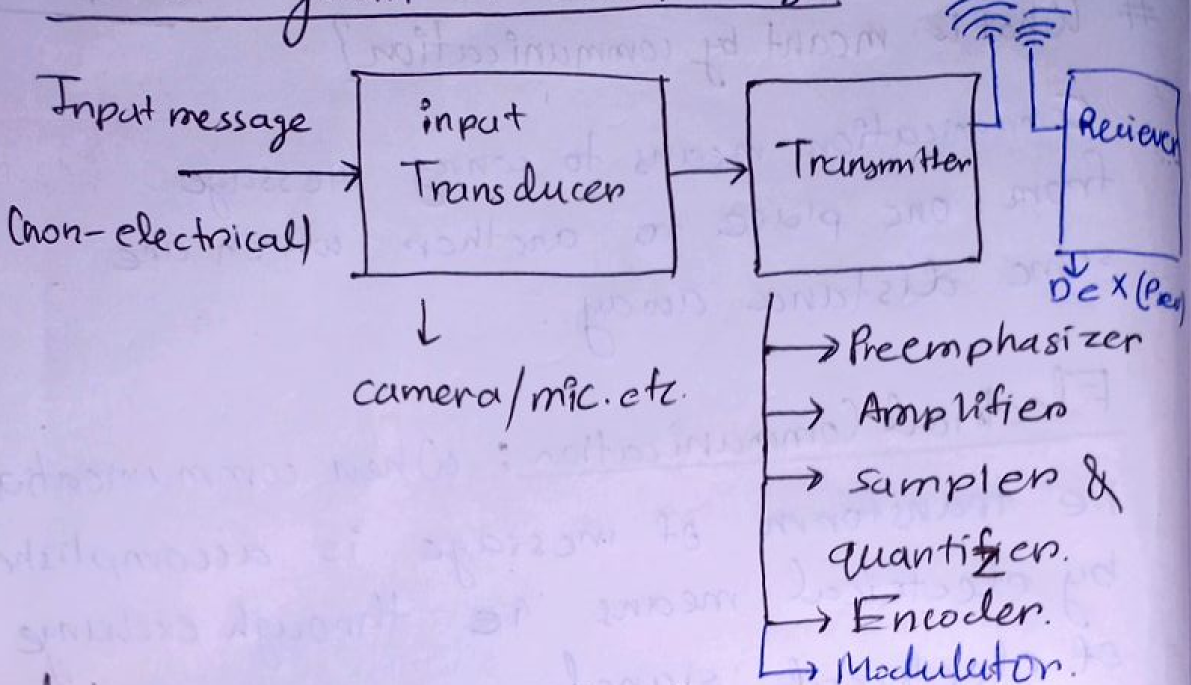
Componenents of Comm.ⁿ sys.:

(i) Transmitter

(ii) Receiver

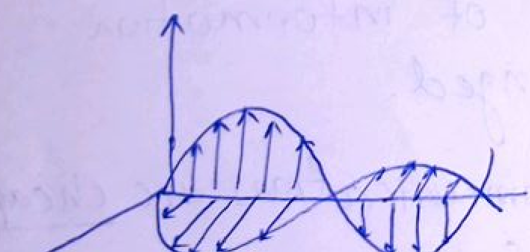
(iii) Channel

Block Diagram of Commⁿ Sys:



Antenna: Responsible for transmittion and reception of E.M. Waves.

Tx → Transmitting Antenna
Rx → Recieving Antenna



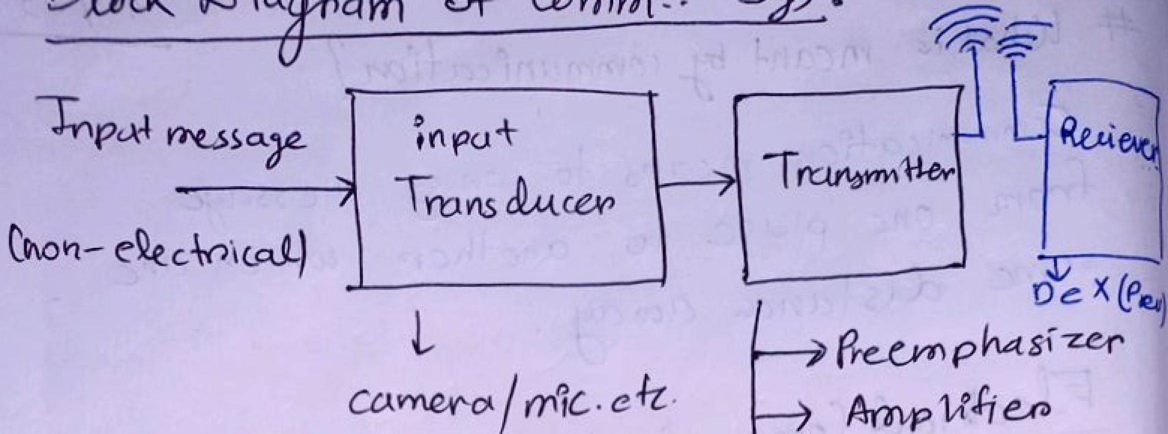
E.M. Wave

Size of Antenna depends on frequency of E.M. Wave.

Size of Antenna $= \dfrac{\lambda}{10}$ → 1/10th of the wavelength

low freqⁿ → high size of antenna
↳ Not feasible
↓
so higher

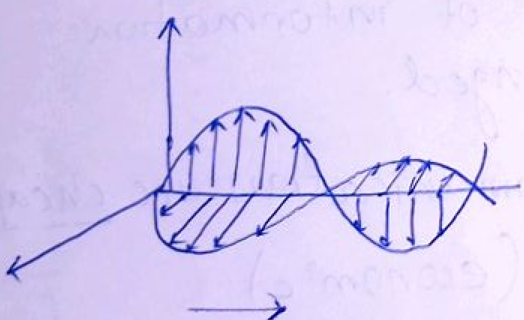
Block Diagram of Commⁿ Sys:



- → Preemphasizer
- → Amplifier
- → sampler & quantizer.
- → Encoder.
- → Modulator.

Antenna: Responsible for transmittion and reception of E.M. Waves.

Tx → Transmitting Antenna
Rx → Recieving Antenna



E.M. Wave

Size of Antenna depends on frequency of E.M. Wave.

Size of Antenna = $\frac{\lambda}{10}$ → 1/10th of the wavelength

low freqⁿ → high size of antenna
↳ Not feasible
↓
so higher frequency is used.

Hearable frequency range => 20 → 20kHz.

But, $f \cdot \lambda = c$

$3\times10^3 \times \lambda = 3\times10^8$

$\boxed{\lambda = 10^5}$

$\lambda/10 = 10^4 \rightarrow$ 10km antenna ✗

↓

not possible

So, <u>Modulation is reqd.</u>

(message signal)

$m(t)$ → ⊗ → $m(t)\left\{\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right\}$

modulated signal.

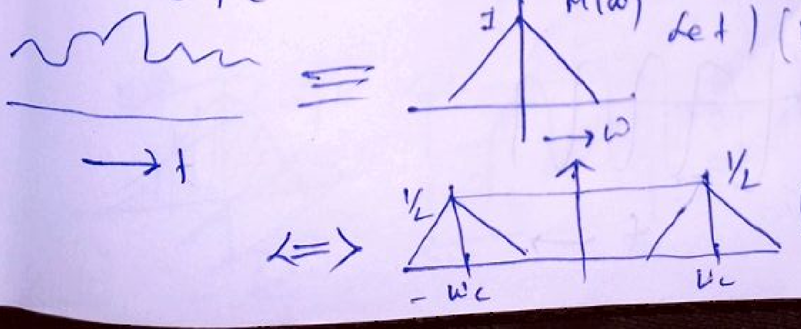
$\cos\omega_c t$

$$\therefore = \frac{1}{2}\left\{m(t)e^{j\omega_c t} + m(t)e^{-j\omega_c t}\right\}$$

<u>Taking F.T.</u>: (Frequency Domain Description)

signal is frequency shifted by ω_c

$m(t)\,e^{-j\omega_c t}$



→ Msg signal

Let) (Baseband signal)

(Bandpass signal)

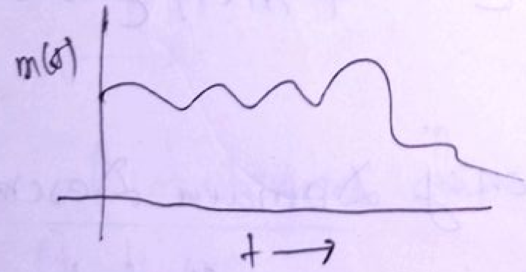
Q.

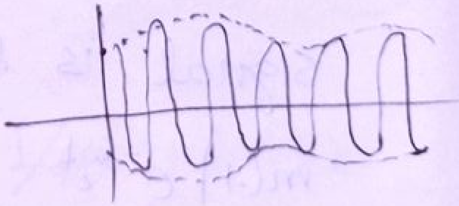
Modulation is a systematic alteration of some characteristic amplitude, phase or frequency of a high frequency signal called carrier signal according to the function of message signal.

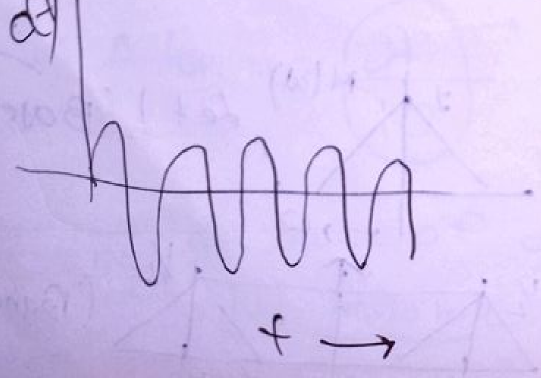
#

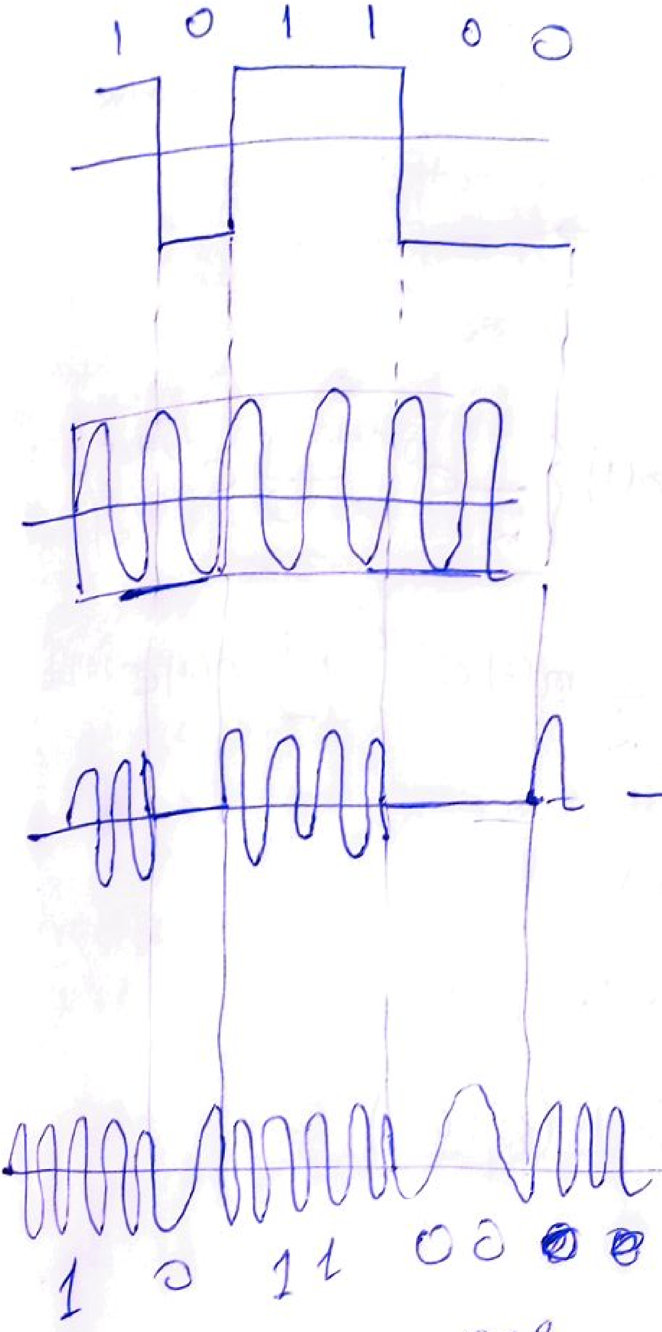
Carrier modulation is the process by which some characteristics (amp, phase, freq). of a high freq signal, carrier wave is varied in accordance to the baseband singal and modulating signal.

$$c(t) = A\cos(\omega_c k + \theta_c)$$



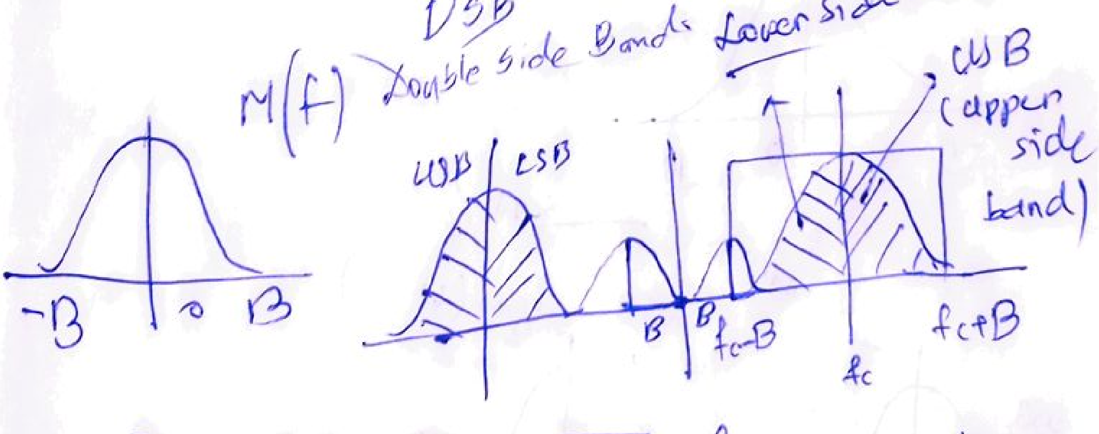






→ BASK (Binary Amplitude Shift Keying)

Frequency modulation



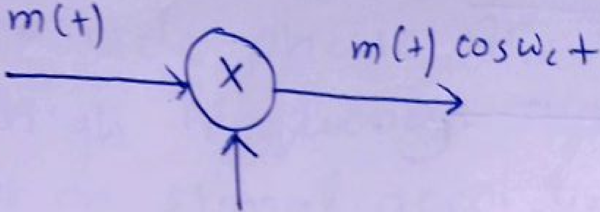
$\boxed{f_c \geq 2B}$ → for seperation of message.

DSB-SC :

$m(t)$: message signal

$\cos\omega_c t$: carrier wave.

$m(t)\cos\omega_c t$: modulated wave



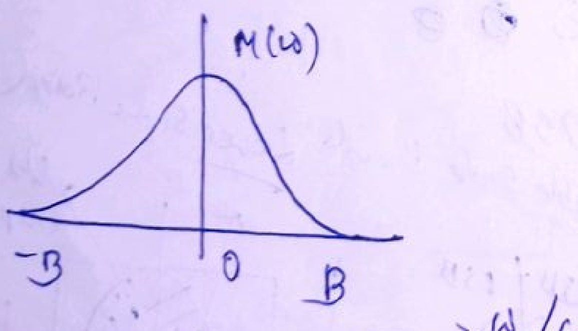
$$m(t)\cos\omega_c t = m(t)\left\{\frac{1}{2}e^{j\omega_c t} + \frac{1}{2}e^{-j\omega_c t}\right\}$$

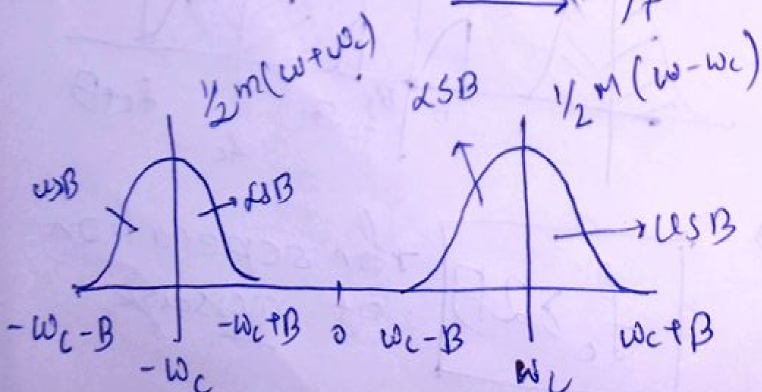
$$= \frac{1}{2}m(t)e^{j\omega_c t} + \frac{1}{2}m(t)e^{-j\omega_c t}$$

$$m(t) \Longleftrightarrow M(\omega)$$

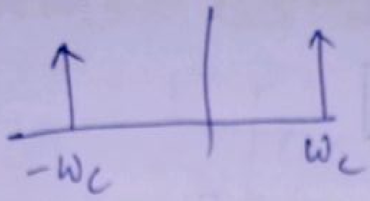
$$m(t)e^{j\omega_c t} \Longleftrightarrow M(\omega-\omega_c)$$

$$m(t)\,e^{-j\omega_c t} \Longleftrightarrow M(\omega+\omega_c)$$



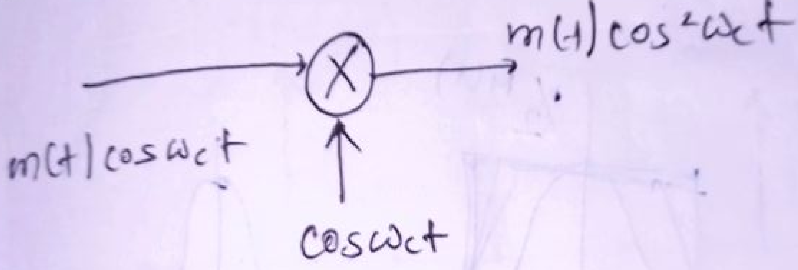


carrier signal in FDP :



No discrete carrier component in the modulated signal, ~~but~~ so it is called **Double-side Band Suppress Carrier.**

Demodulation Operation



$\boxed{m(t)\cos^2 \omega_c t}$ → can we get back m(t). if yes then how?

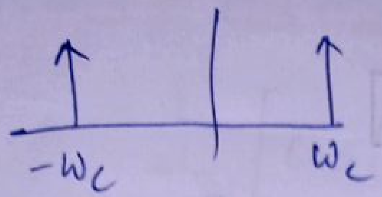
$$m(t)\cos^2 \omega_c t = \frac{1}{2} m(t)\, 2\cos^2 \omega_c t$$

$$= \frac{1}{2} m(t) \left\{ 1 + \cos 2\omega_c t \right\}$$

$$= \frac{1}{2} m(t) + \frac{1}{2} m(t) \cos 2\omega_c t$$

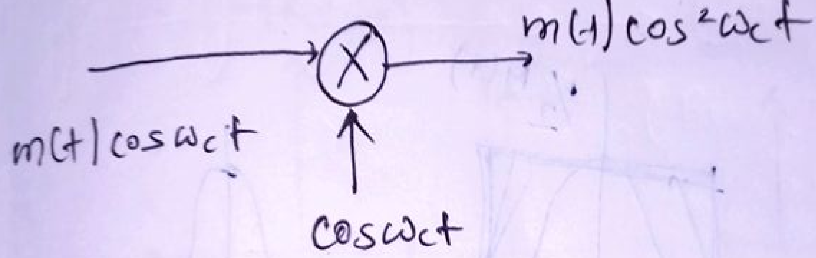
= Amplitude Scaled.

carrier signal in FDP :



discrete
No carrier component in the modulated signal, ~~but~~ so it is called Double-side Band Suppress Carrier.

Demodulation Operation



$\boxed{m(t)\cos^2\omega_c t}$ → can we get back $m(t)$. if yes then how?

$$m(t)\cos^2\omega_c t = \frac{1}{2}m(t)\,2\cos^2\omega_c t$$

$$= \frac{1}{2}m(t)\left\{1+\cos 2\omega_c t\right\}$$

$$= \frac{1}{2}m(t) + \frac{1}{2}m(t)\cos 2\omega_c t$$

= Amplitude Scaled.

$m(t)\cos^2\omega_c t = \frac{1}{2} m(t)\, 2\cos^2\omega_c t$

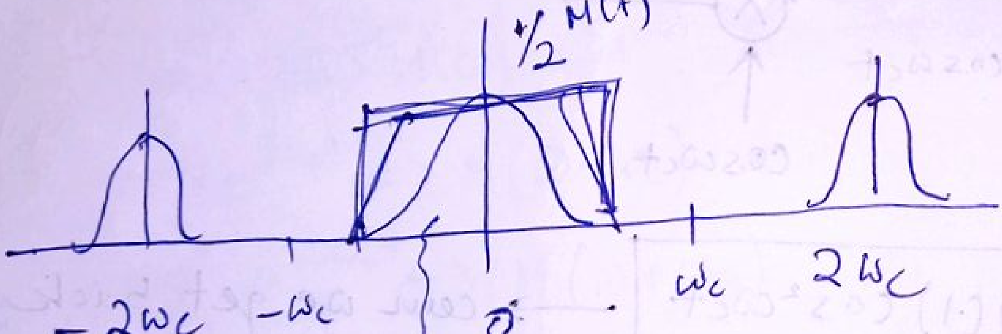
$\searrow$

$FT[m(t)\cos^2\omega_c t]$

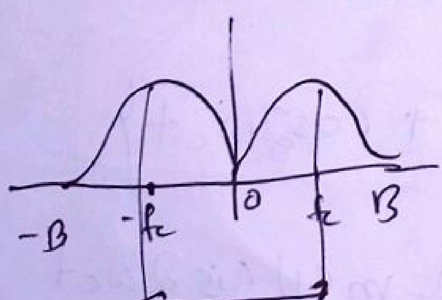
$= FT[\frac{1}{2} m(t)] + FT[\frac{1}{2} m(t)\cos^2\omega_c t]$

$= \frac{1}{2} M(\omega) + \frac{1}{2} M(\omega - 2c) + \frac{1}{2} M(\omega + 2\omega_c)$

FDD :



This can be separated by low pass filter.



$f_c \geq B$

$2f_c \geq 2B$ $\boxed{\omega_c \geq 2\pi B}$

let,

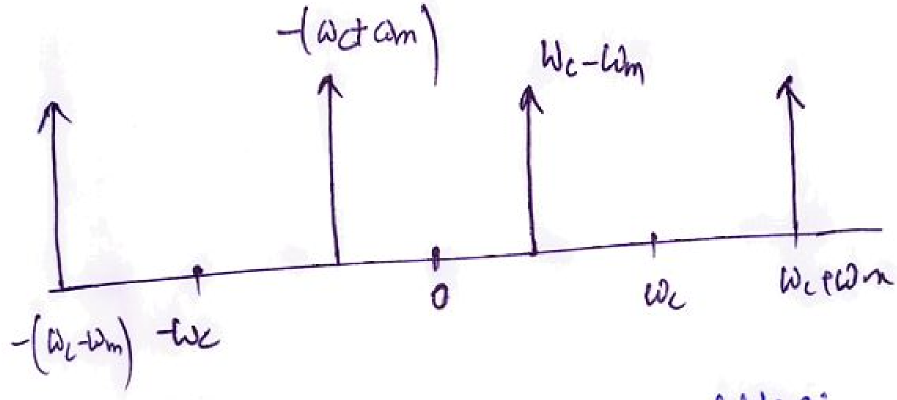
$$m(t) = \cos\omega_m t$$

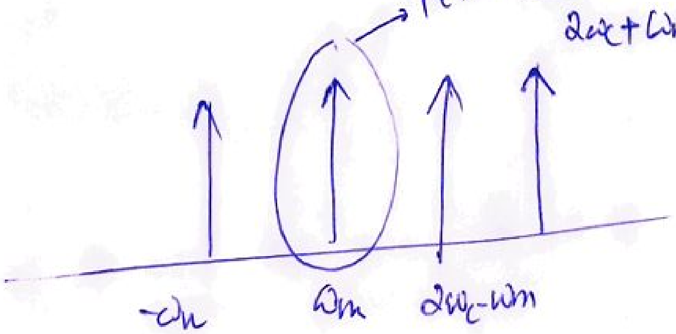
$$m(t)\cos\omega_c t = \cos\omega_m t \cdot \cos\omega_c t$$

$$= \frac{1}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$e^{j(\omega_c+\omega_m)t} + e^{-j(\omega_c+\omega_m)t}$$

FDD:





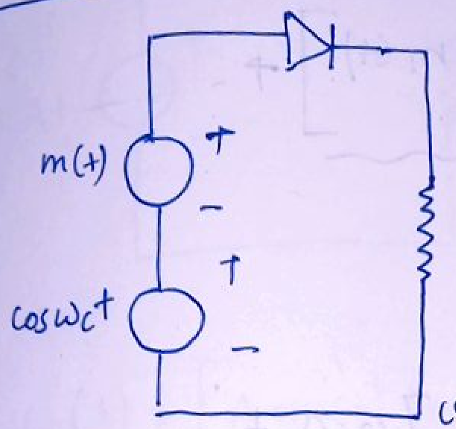
DSB-SC

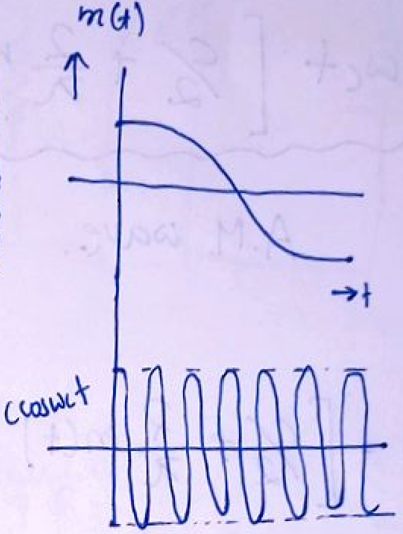
$m(t)\cos\omega_c t$

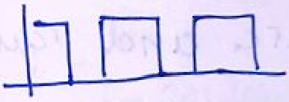
AM:

$[A+m(t)]\cos\omega_c t$

Generation of A.M. Wave:

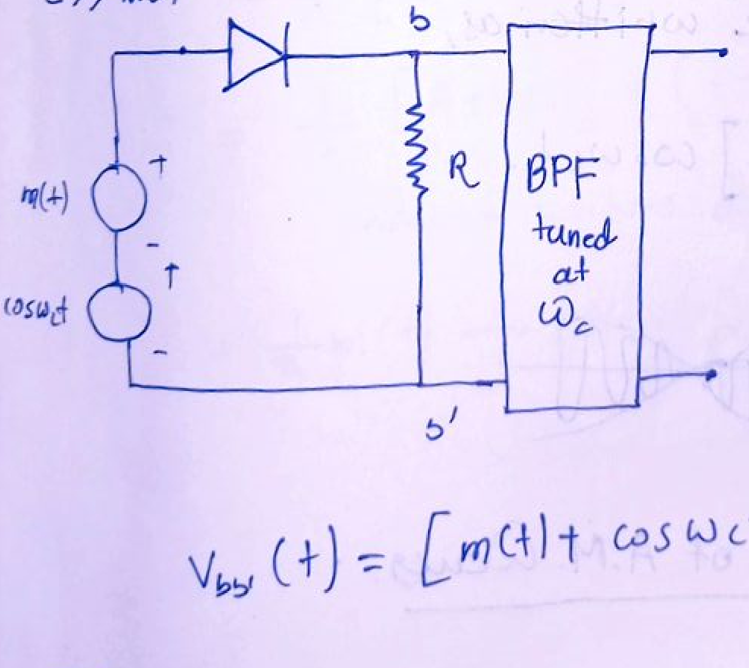






$m(t)+c\cos\omega_c t$

$c >> m(t)$



$$V_{bb'}(t) = [m(t)+\cos\omega_c t]\,.\,\omega(t)$$

switching fn.

F.T of $\omega(t)$:

$$\omega(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t \cdots\right]$$

$$V_{bb'}(t) = \left[m(t) + c\cos\omega_c t\right]\left[\frac{1}{2} + \frac{2}{\pi}\left\{\cos\omega_c t - \frac{1}{3}\cdots\right.\right.$$

$$= \frac{1}{2}m(t) + \boxed{\frac{2}{\pi}m(t)\cos\omega_c t + \frac{c}{2}\cos\omega_c t} + \frac{2c}{\pi}\cos^2\omega_c t \ldots$$

$$= \underbrace{\cos\omega_c t\left[\frac{c}{2} + \frac{2}{\pi}m(t)\right]}_{\text{A.M. wave.}} + \ldots$$

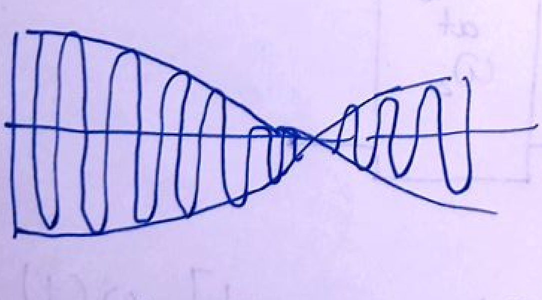
$$V_{cc'}(t) = \left[\frac{c}{2} + \frac{2}{\pi}m(t)\right]\cos\omega_c t$$

↳ BPF filters out all terms of higher harmonic and square term.

$V_{cc'}(t)$ can be written as,

$$\left[A + m(t)\right]\cos\omega_c t.$$

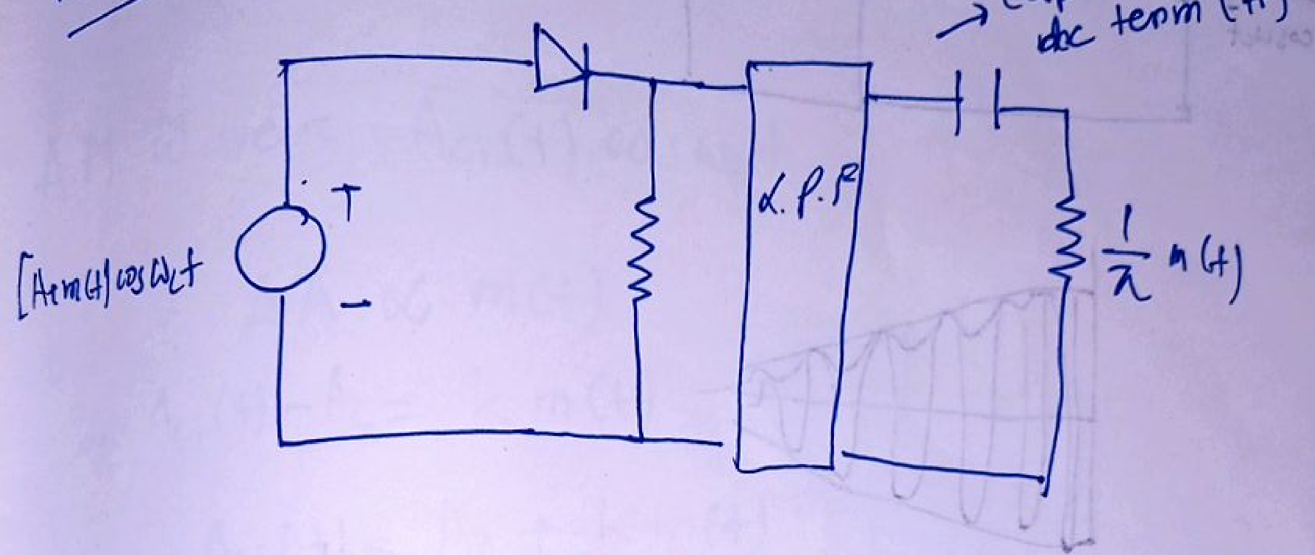
$\therefore$



Generation of A.M. Waves.

Demodulation of A.M. wave :

Rectifier detection :



$$V_R(t) = [A + m(t)]\, \omega(t)$$

$\omega(t)$: $\cos\omega_c t$

$$= [A + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}\left\{\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right\}\right]$$

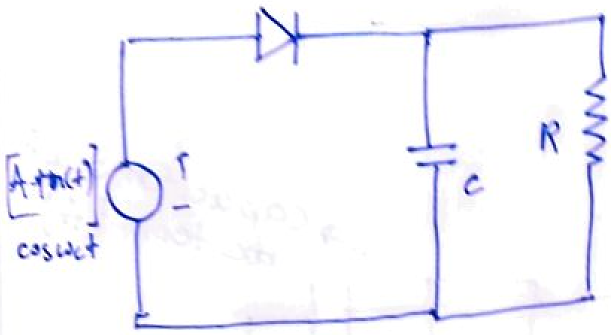
$$= \frac{2}{\pi}[A + m(t)][1 + \cos 2\omega_c t] + \cdots$$

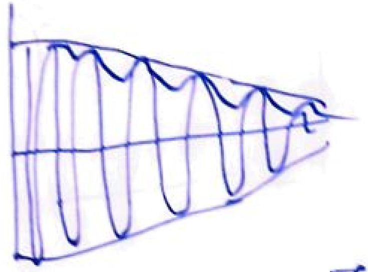
$$= \frac{1}{\pi}[A + m(t)] + (\cdots\cdots)$$

capacitor removes dc signal

$$= \frac{1}{\pi} m(t) \longrightarrow \text{After passing through L.P.F.}$$

Envelope Detection:





If $RC << \frac{1}{2\pi B}$ it will not properly follow envelope of message.

$RC \gg \frac{1}{\omega_c}$.

$< \frac{1}{2\pi B}$

↑ Bandwidth of message signal.

$$\boxed{\mu = \frac{A_m}{A_c}}$$

$A_m \rightarrow$ Amplitude of modulated.

$m(t) = \cos\omega_m(t)$

$F(t) = [1 + \mu\cos\omega_m(t)]\cos\omega_c t$

$m(t) = A_m \cos \omega_m (t)$

$c(t) = A_c \cos \omega_c (t)$

$E(t) \cos \omega_c t$

A.M. Wave $= A_{ci}(t) \cos \omega_c t$

$\Delta A \propto m(t)$

or $A_{ci}(t) - A_c = k\, m(t)$

or $A_{ci}(t) = A_c + k\, m(t)$

$\therefore$ A.M wave $= (A_c + \overset{1}{k}\, m(t)) \cos \omega_c t$

$= [A_c + m(t)] \cos \omega_c t$

$= [A_c + A_m \cos \omega_m (t)] \cos \omega_c t$

$= A_c \left[1 + \frac{A_m}{A_c} \cos \omega_m (t)\right] \cos \omega_c t$

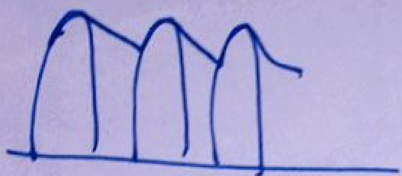
(making)

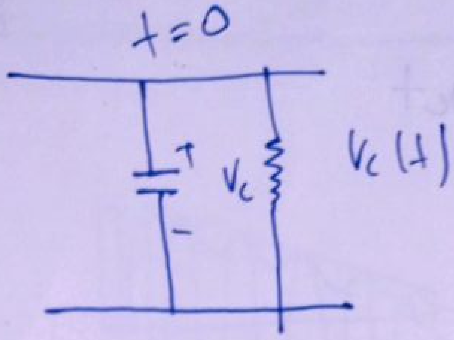
$= \overset{1}{A_c} (1 + \mu \cos \omega_m (t)) \cos \omega_c t$

$= [1 + \mu \cos \omega_m t] \cos \omega_c t$

$v_{AM}(t) = E(t) \cos \omega_c t$

$E(t) = 1 + \mu \cos \omega_m (t)$





$$\boxed{V_c(t) = V_c e^{-t/RC}}$$

$$V_c(t) = V_c\left[1 - \frac{t}{RC} + \frac{t^2}{(RC)^2} + \dots\right]$$

$$\approx V_c\left[1 - t/RC\right]$$

$$\frac{dV_c}{dt} \leq \frac{dE}{dt}$$

$$\left|\frac{dv_c}{dt}\right| = \left|-\frac{V_c}{RC}\right|$$

$$= \left|\frac{1}{RC}\right| \quad V_c = 1$$

$$\frac{dE(t)}{dt} = -\mu\omega_m \sin\omega_m(t)$$

$$\left|\frac{1}{RC}\right| \geq \mu \omega_m \sin \omega_m (t)$$

$$\Rightarrow \quad RC \leq \frac{1 + \mu \cos \omega_m (t)}{\mu \omega_m \sin \omega_m (t)}$$

(Normalising).

$\sin \omega (t)$

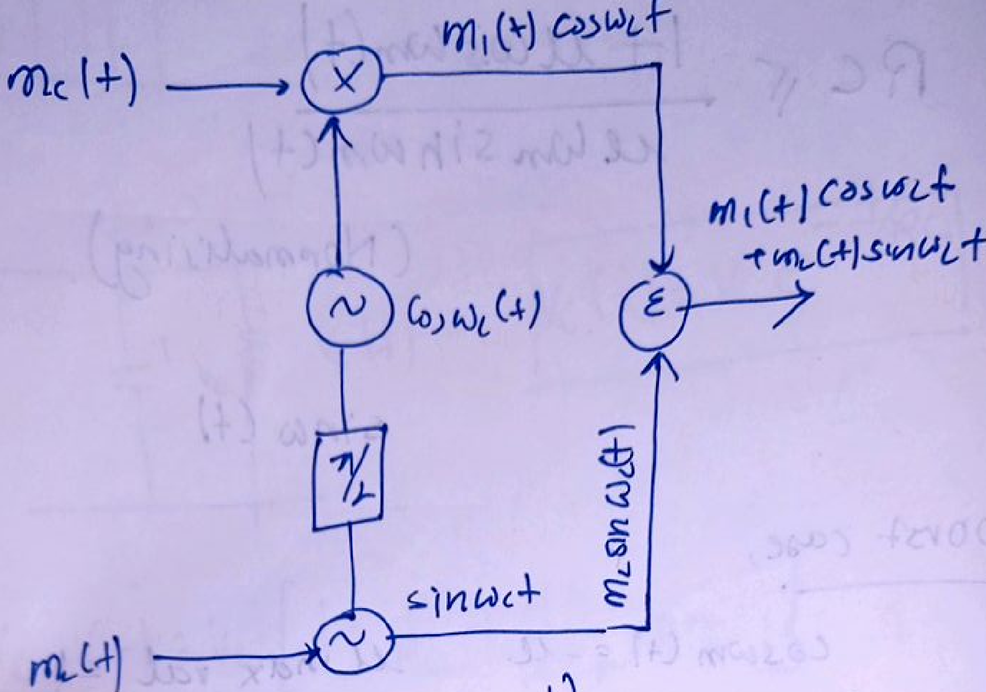
worst case,

$\cos \omega_m (t) = -\mu.$ $\qquad \mu$ max val $= 1$

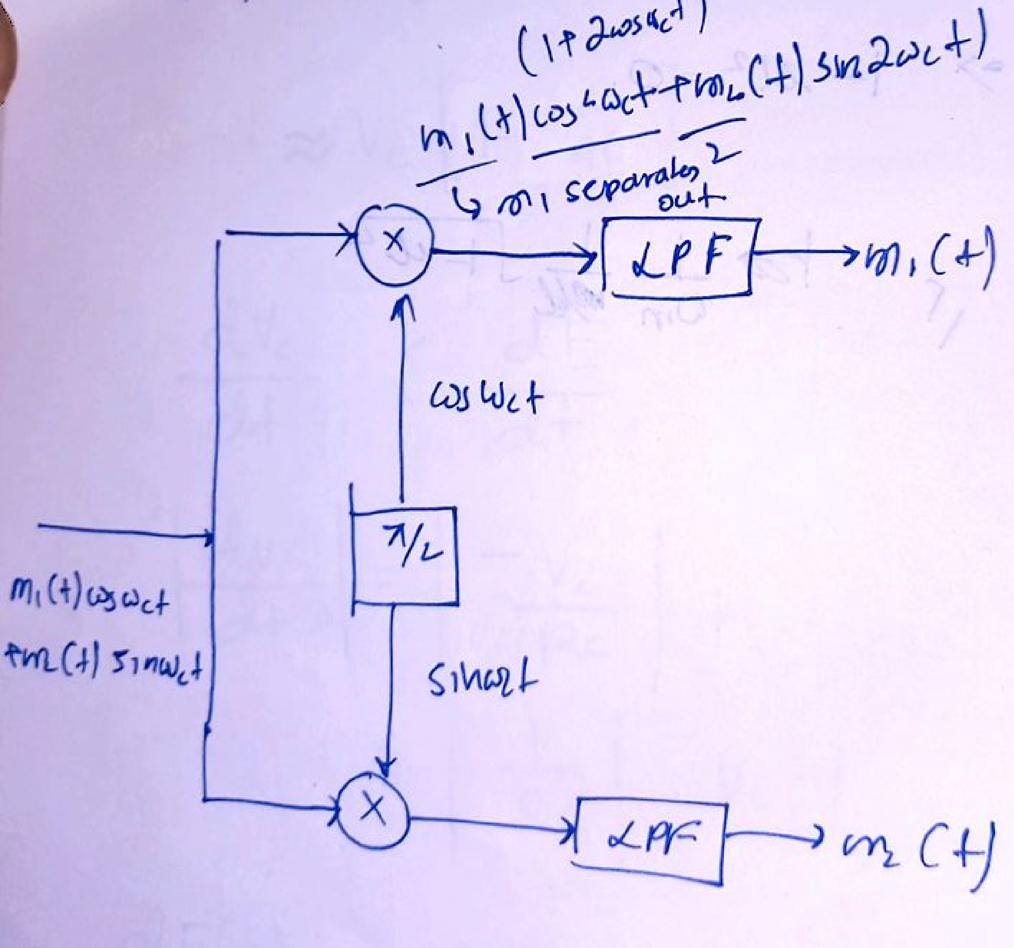
$\Rightarrow 1 - \mu^2 = 0.$

$$RC \leq \frac{1}{\omega_m} \frac{1}{\mu} \sqrt{1-\mu^2}$$

QAM:



m₁(t) cosωct
m₁(t) cosωct
+m₂(t) sinωct
cosωc(t)
π/2
sinωct
m₂ sin ωct



(1+2cosωct)
m₁(t) cos²ωct + m₂(t) sin2ωct/2
↳ m₁ separates out
LPF
m₁(t)
cos ωct
π/2
m₁(t) cos ωct
+m₂(t) sinωct
sinωct
LPF
m₂(t)

$V_c = Ee^{-t/RC}$

$= E(1 - t/RC)$ (After approximation)

$$\boxed{\frac{dv_c}{dt} = -\frac{E}{RC}}$$

$E(t) = A(1 + \mu \cos\omega_m t)$

$$\frac{dE}{dt} = -A\omega_m \mu \sin\omega_m t$$

$$\frac{dv_c}{dt} = -\frac{E}{RC} \geq \frac{dE}{dt}$$

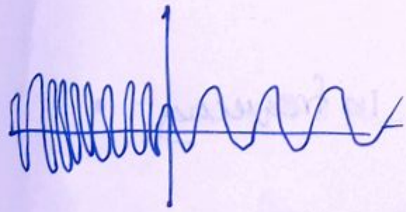
$$\boxed{\frac{A(1+\mu\cos\omega_m t)}{RC} \geq -A\omega_m \mu \sin\omega_m t}$$

Normalisation :

$\omega_i(t) = \omega_c t + k m(t)$

$\omega_c + K m_p$

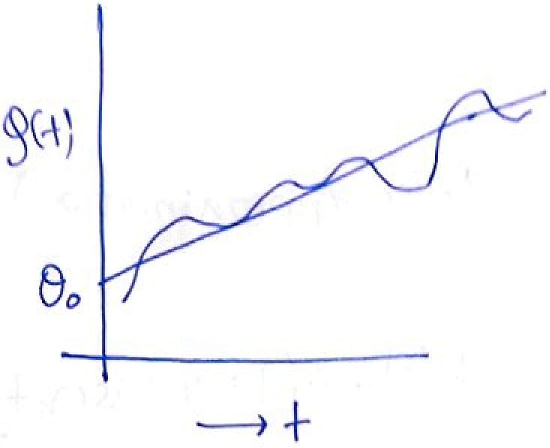
$\omega_c - K m_p$



Frequency Modulation comes under Angle Modln.

$\theta(t) = \omega_c t + \theta_0 \Rightarrow$ in $A\cos\theta t$

$\omega = \frac{d\theta}{dt} \rightarrow$ slope



Angle Modln

Phase Modln: Freq Modln

$$\psi_{PM}(t) = A\cos\theta(t)$$
$$= A\cos\left[\omega_c t + k_p m(t)\right]$$

Phase Mod^n : $\theta(t) = \omega_c t + k_p m(t)$

$$\omega_i = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

$$\Delta\omega = \underbrace{\omega_i(t)}_{\text{Instantaneous freq.}} - \omega_c = \text{change in frequency.}$$

$$\Delta\omega \propto m(t)$$

In F.M. derivation of carrier frequency is proportional to the message signal m(t)

$\Delta\omega \propto m(t)$

or $\Delta\omega = K_f\, m(t)$

$$\theta(t) = \int \omega_i(t)\,dt$$

$$= \int [\omega_c + K_f\, m(t)]\,dt$$

$$= \omega_c t + K_f \int m(t)\,dt$$

$$\therefore A\cos\theta(t) = A\cos\left[\omega_c t + K_f \int m(t)\,dt\right]$$

F.M. Signal

Block Diagram:

$$m(t) \longrightarrow \boxed{\int dt} \xrightarrow{\int m(\alpha)\,d\alpha} \boxed{PM} \longrightarrow FM$$

$$m(t) \longrightarrow \boxed{d/dt} \longrightarrow \boxed{F.M.} \longrightarrow PM.$$

If $m(t) = \cos\omega_m t$

$c(t) = A_c \cos\omega_c t$

Find $\varphi_{FM}(t)$

Angle Modulation or Exponential modulation

$$A \cos \theta(t)$$

$$\swarrow \quad \searrow$$

F.M.   P.M.

$$\varphi_{PM}(t) = A\cos\left[\omega_c t + R_p m(t)\right]$$

ω_c = carrier freq.

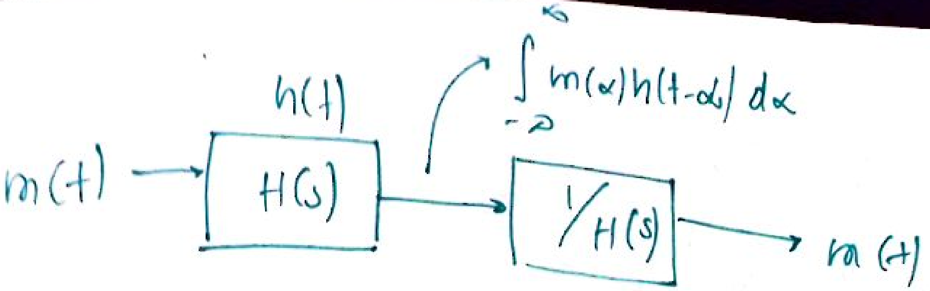
$m(t)$ = message signal

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

$$\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

$$m(t) \longrightarrow \boxed{\int dt} \xrightarrow{\int m(\alpha)\, d\alpha} \boxed{\text{P.M.}} \longrightarrow \text{F.M.}$$

$$m(t) \longrightarrow \boxed{\frac{d}{dt}} \xrightarrow{\frac{d\, m(t)}{dt}} \boxed{\text{F.M.}} \longrightarrow \text{P.M.}$$



$$h_{\infty}(t) = K_p \delta(t)$$

$$\int_{-\infty} m(\alpha)\, K_p\, \delta(t-\alpha)\, d\alpha$$

$$= m(\alpha)$$

Again, : $h(t) = K_f u(t)$

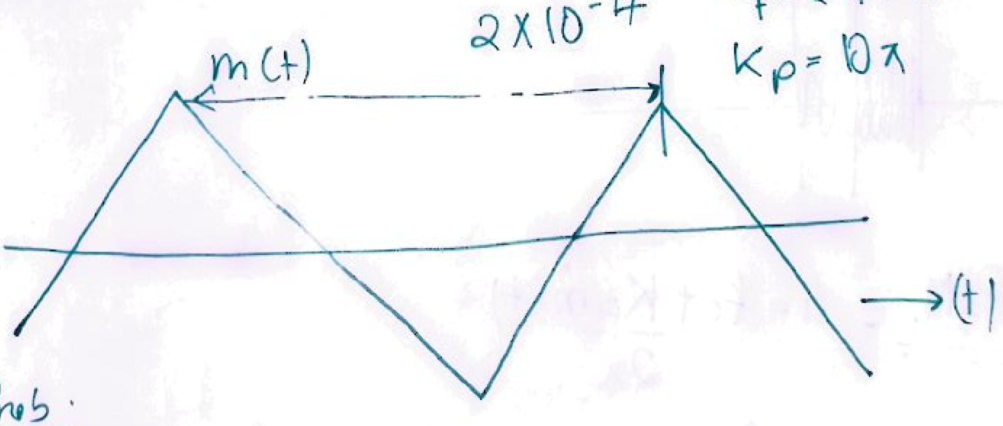
$$\int_{\alpha}^{\infty} m(\alpha)\, K_p\, (u(t-\alpha)$$

$$= \int_{\alpha}^{\infty} m(\alpha)\, K_p$$

Q)

$K_f = 2\pi \times 10^4$

$K_p = 10\pi$



Prob:

Carrier $f_c = 100$ MHz $= 10^8$ Hz

$$\omega_i = \omega_c + K_f m(t)$$

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

$$= 10^8 + 10^5 m(t)$$

$$= 10^8 + 10^5$$

$$= (100 + 0.1)\,10^6$$

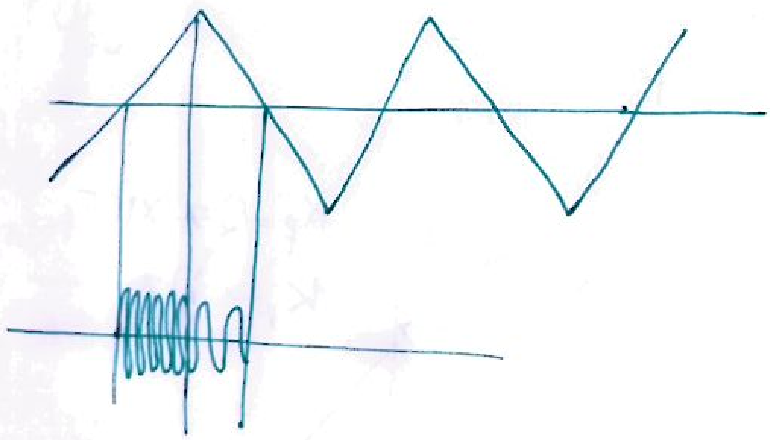
$$= 100.1 \text{ MHz}$$

$$(f_i)_{mean} = 10^8 + 10^5 m(t)\Big|_{min}$$

$$= 10^8 + 10^5(-1)$$

$$= 10^8 - 10^5$$

$$= 99.9 \text{ MHz}$$

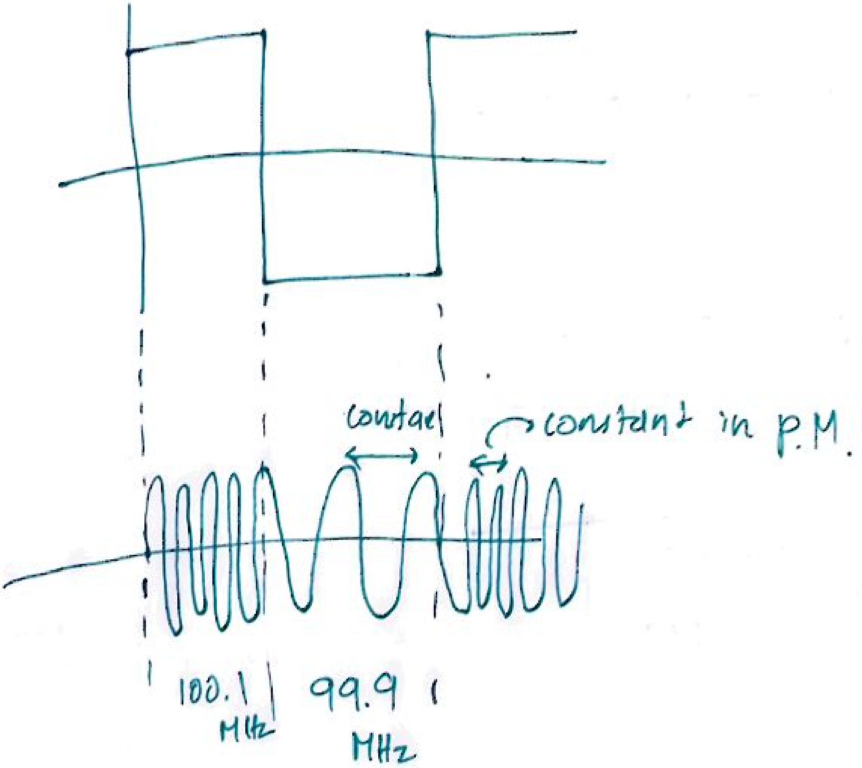


P.M. $= f_i = f_c + \dfrac{K_p}{2\pi}\dot{m}(t)$

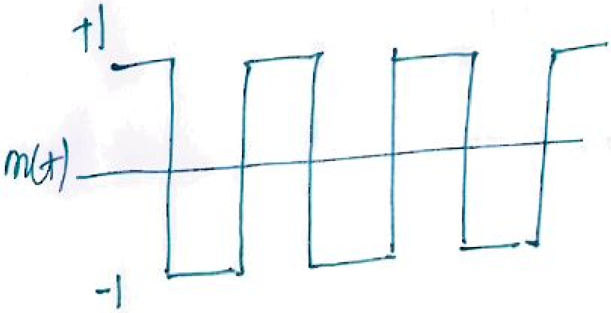
$$= 10^8 + \frac{5 \times 2}{10^{-4}}$$

$$= 10^8 + 10^5$$

$$\Rightarrow 10^6 \times 0.001$$



Problem 2:



$K_f = 2\pi \times 10^5$

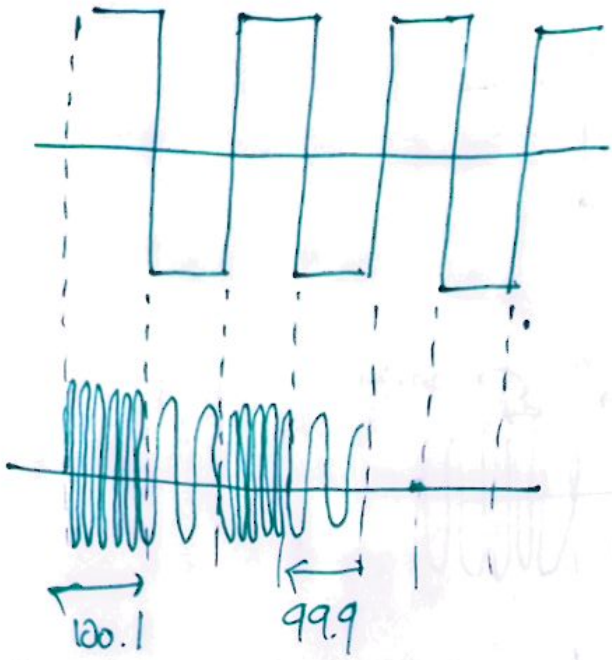
$K_p = \pi/2$

$$f_i = f_c + \frac{K_f}{2\pi} m(t)$$

$(f_i)_{max} = 100.1 \text{ MHz}$

$(f_i)_{mean} = 99.9 \text{ MHz}$

F.M.



FSK: Frequency Shift Keying

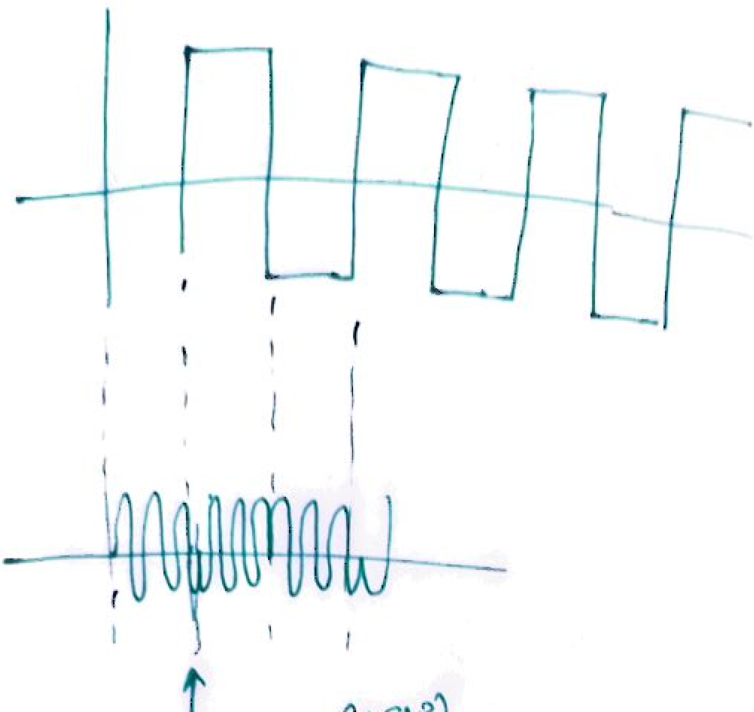
or, BSK: Binary " "

$$\psi_{PM}(t) = A \cdot \cos\left[\omega_c t + K_p m(t)\right]$$

$$= A \cdot \cos\left(\omega_c t + \frac{\pi}{2} m(t)\right)$$

$$= \begin{cases} A \sin \omega_c t, & m(t) = 1 \\ A \sin \omega_c t, & m(t) = -1 \end{cases}$$

P.M :



Phase change (180°) during transition.

Frequency remains unchanged

BPSK → Binary Phase Shift Keying

Bandwidth :

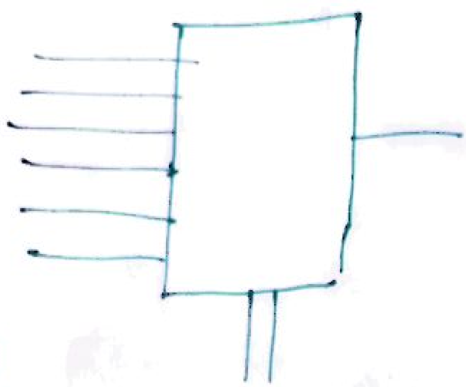
$$\omega_c + K_f\, m(t)$$

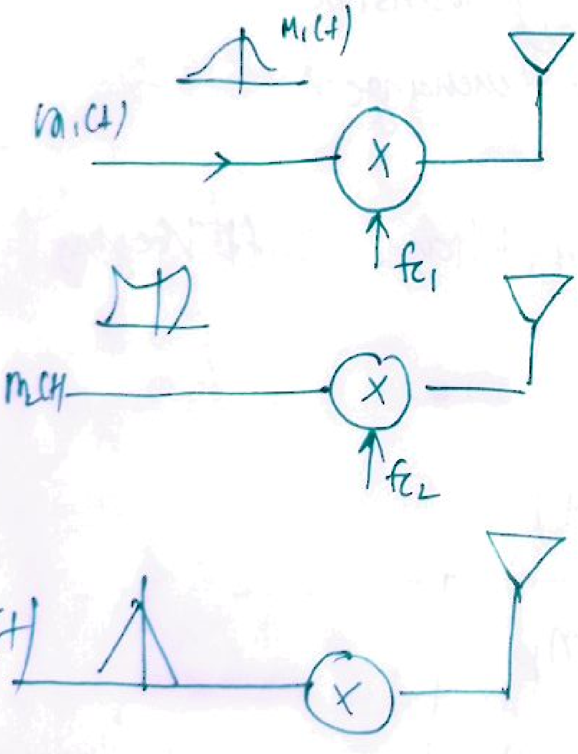
$$\boxed{\begin{aligned} f_i &= \omega_c + K_f\, m_p \\ f_i &= \omega_c - K_f\, m_p \end{aligned}}$$

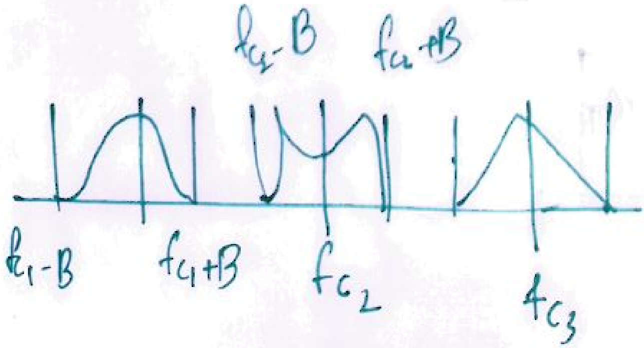
$$\boxed{B.W. = 2 K_p\, m_p}$$

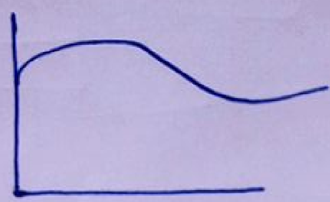
Multiplexing:

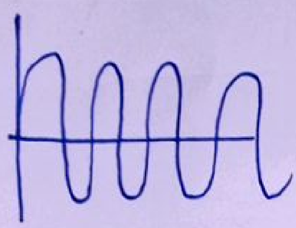
Multiplexing is the simultaneous translation of several message signals through a common channel











AM/

DSB - SC

DSW - C / AM

SSC

FM / PM

Pulse modulation

① PAM

② PTM

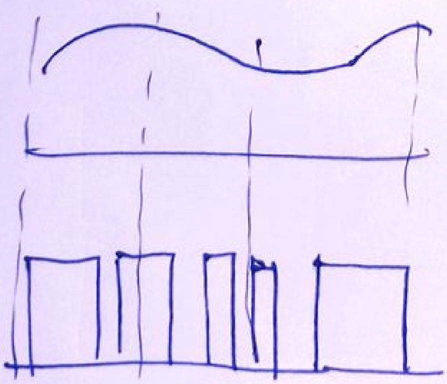
In PAM, the amplitudy of the pulse train (carrier signal) is in accordance with amplitude of the modulated signal

PTM

PWM

PLM

PDM

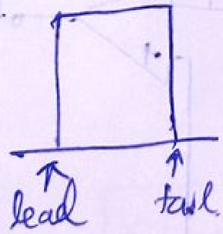


fixed amplitude and fixed modulation for all the pulses.

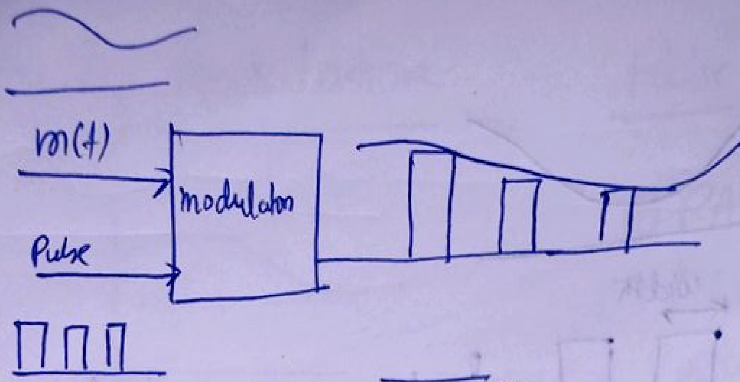
During of pulses same.

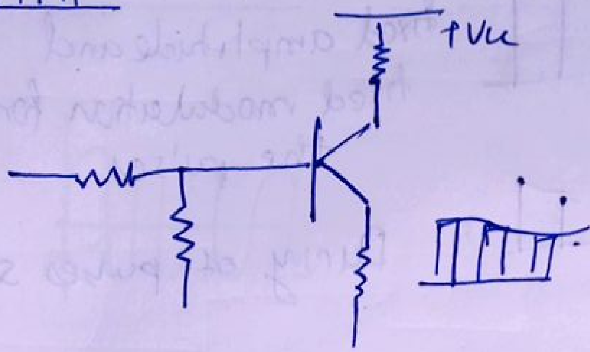
In PWM, the amplitude of pulses are same, but duration or length of pulses change.

<u>PPM</u>: Pulse Position Modulation

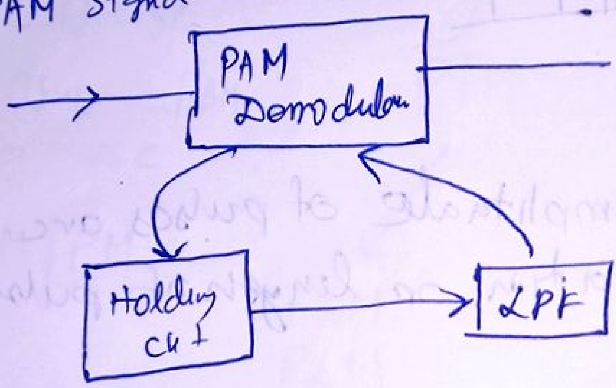


In PPM, both the amplitude and duration of pulses remains same (w.r.t) pulses of carrier pulse train. but the position (lead or tail) of the pulses gets changed.

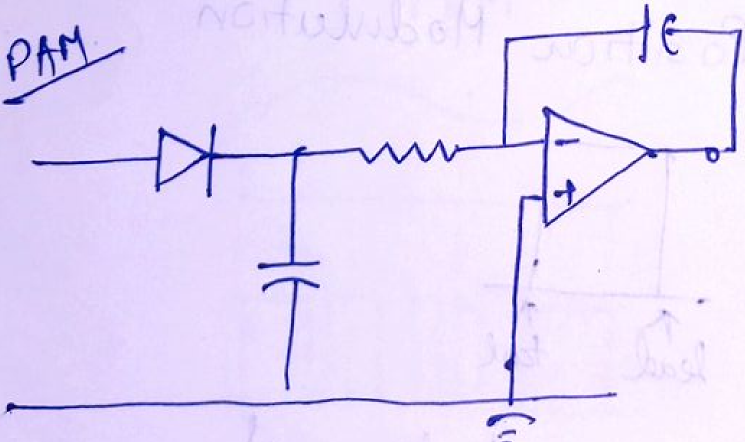


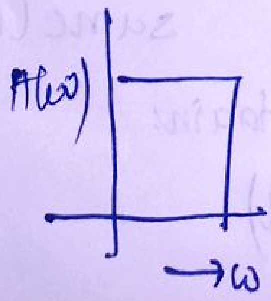


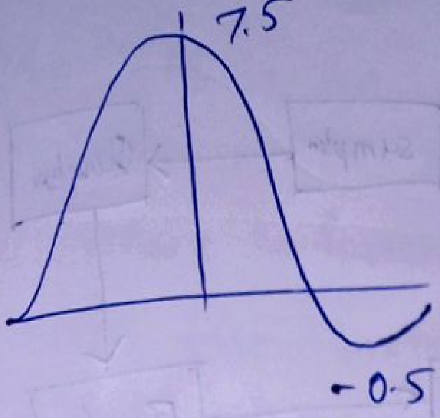
PAM signal

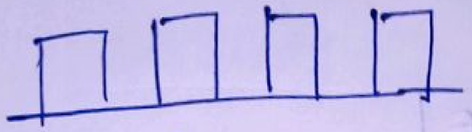


PAM

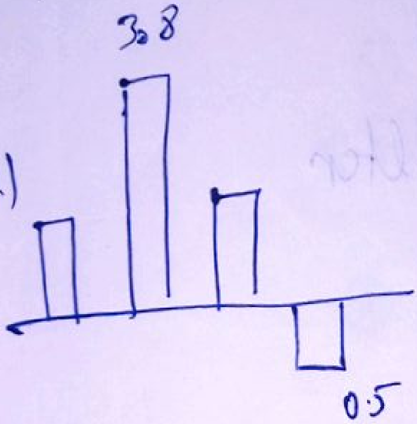








2.)



level

$-0.5 + 0.5 \rightarrow 0$

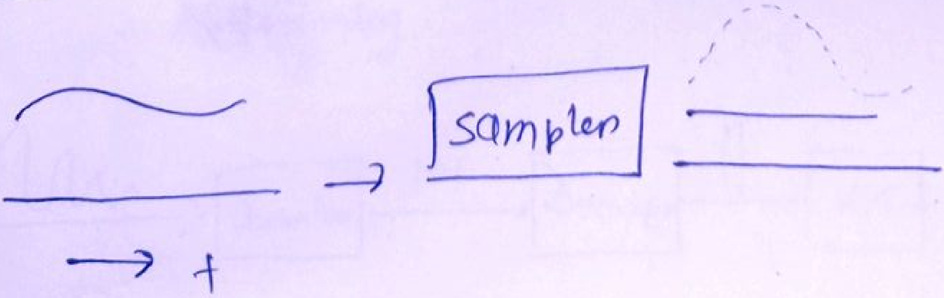
0.5 to $1.5 \rightarrow 1$

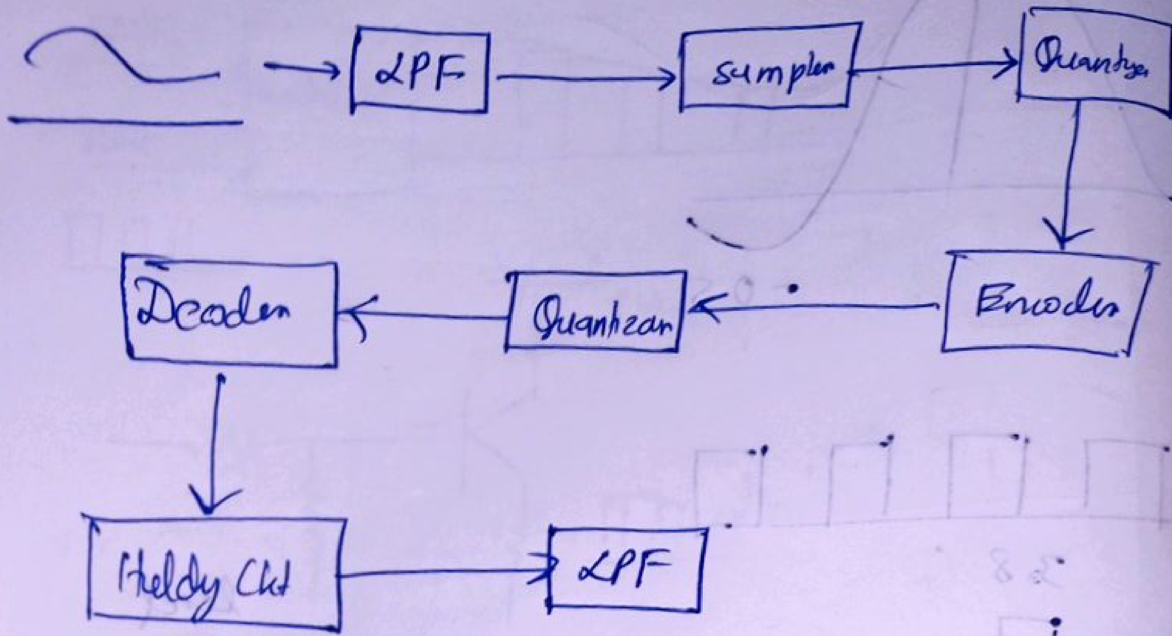
1.5 to $2.5 \rightarrow 2$

2.5 to $3.5 \rightarrow 3$

After quantisation

PDM:





LPF → Anti-aliasing filter

Reconstruction filter

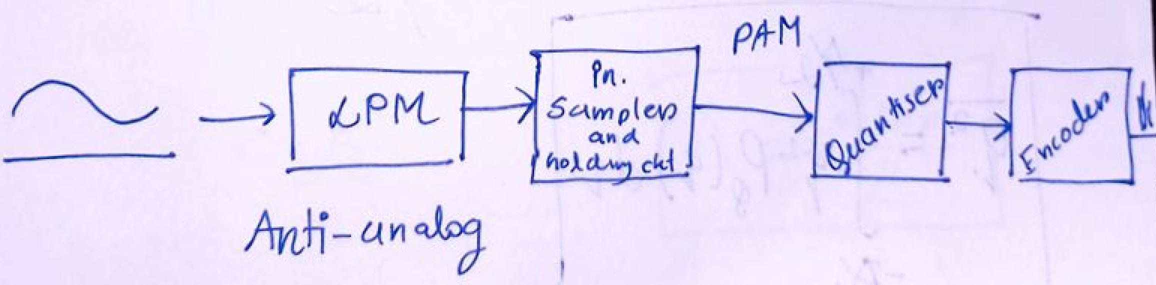
7/2/23

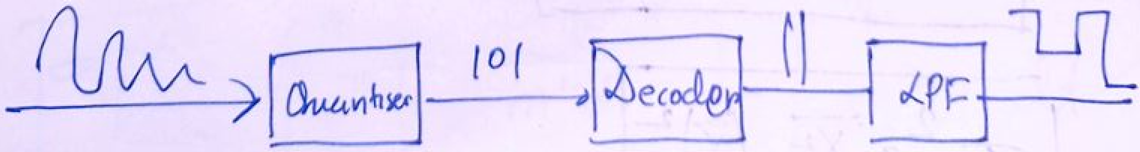
PCM: PCM is an analog-to-digital conversion of a special type where information contained in the instantaneous samples of an analog signal is converted to digital word, as a sequence of lists.

① Sampling => converts continous time to discrete time

② quantization => discretization on amplitude

③ Encoding → Binary
→ M-ary.

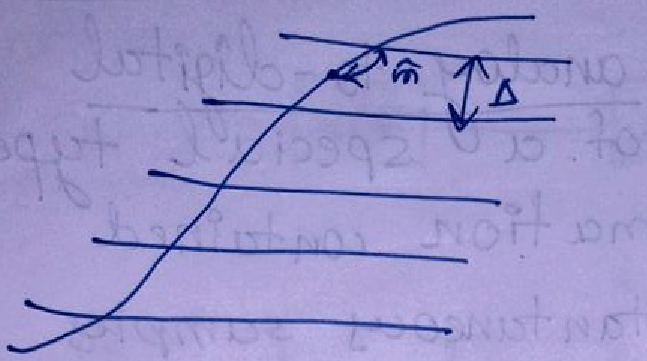


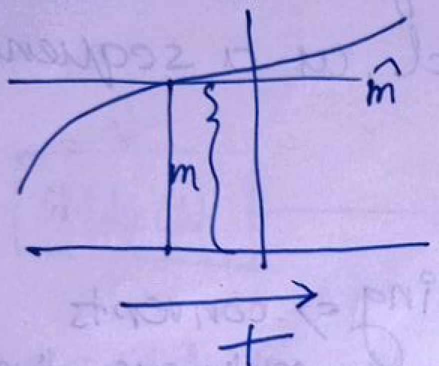


m = actual sample

$\hat{m}$ = quantized sample value

$e(1) = m \sim \hat{m}$





$q = m - \hat{m}$

$\downarrow$

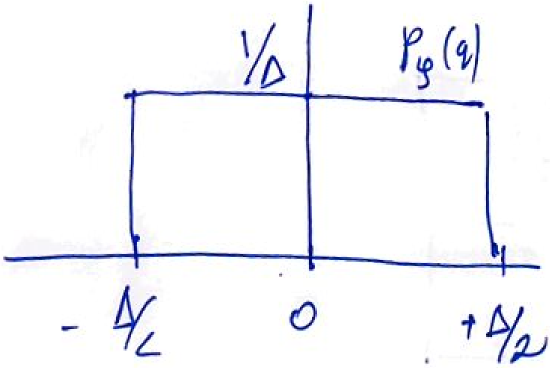
quantisation error.

error value may be +ve/-ve

$q \rightarrow$ r.v. can take any value from 0 to $\Delta/2$

$$\boxed{\overline{q^2} = \int_{-\Delta/2}^{+\Delta/2} q^2 P_q(q)\, dq}$$

$$\boxed{\sigma_x^2 = \overline{X^2} - (\overline{X})^2}$$



Probability of acquiring a value b/w $-\Delta/2$ to $\Delta/2$, having equal probability

$$\overline{q^2} = \int_{-\Delta/2}^{\Delta/2} q^2 \, \frac{1}{\Delta} \, dq$$

$$= \frac{1}{\Delta} \frac{q^3}{3}$$

$$\bar{q} = 0$$

$$= \frac{1}{\Delta} \left(\frac{\cancel{2}\Delta^3}{3 \times \cancel{8}_4} \right)$$

$$= \frac{\Delta^2}{12}$$

$$\boxed{\Delta = \frac{2m_p}{L}}$$

$$\boxed{\overline{q^2} = \frac{\Delta^2}{12}}$$

$$\widetilde{m^2(t)} = \frac{1}{T} \int_{-T/2}^{T/2} m^2(t) \, dt$$

$$\boxed{SNR = \frac{\widetilde{m^2(t)}}{\sigma_q^{\,2}}}$$

Signal to quantization Ratio

$$= \frac{\widetilde{m^2(t)}}{\Delta^2/12}$$

$$= \frac{\widetilde{m^2(t)}}{\frac{\cancel{4}\, m_p^2}{L^2 \times \cancel{12}\,3}}$$

$$= \frac{3L^2\, \widetilde{m^2(t)}}{m_p^2}$$

13/2/23

$$\sigma_q^2 = \overline{q^2} - (\bar{q})^2$$

$$= \frac{\Delta^2}{12}$$

$$= \frac{4m_p^2}{12L^2} \qquad \Delta = \frac{2m_p}{L}$$

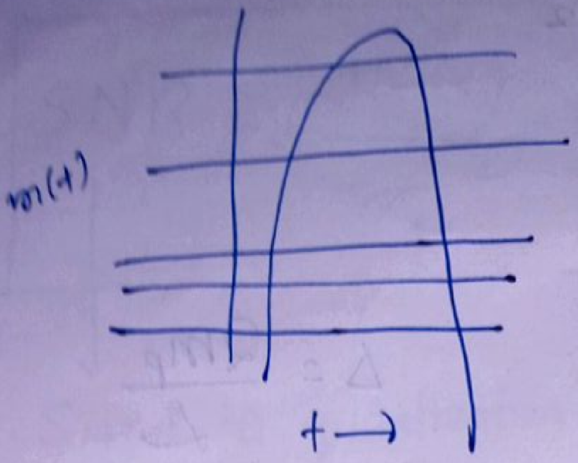
$$= \frac{1}{3}\frac{m_p^2}{L^2}$$

$$\frac{S}{N} = \frac{\overline{m^2(t)}}{\frac{1}{3}\frac{m_p^2}{L^2}} = \frac{3L^2\,\overline{m^2(t)}}{m_p^2}$$

Constant value of $\frac{S}{N}$ is desirable. However, the ~~sh~~ expression shows, S/N depends on signal amplitude. so, to have constant S/N we need variable stepsize 'Δ'. Large stepsize for large amplitude of signals, & small step-size for small ampl. of sngls.

$\frac{S}{N}$ varies up to $10^4 \approx 40\,dB$

DB = $10 \log_{10} x$



μ → law
A → law

$$y = \frac{1}{\ln(1+\mu)} \ln\left(1+\mu \frac{m}{m_p}\right)$$

$$0 \leq \frac{m}{m_p} \leq 1$$

USA / JAPan

A-law:

$$y = \begin{cases} \frac{A}{1+\ln A}\left(\frac{m}{m_p}\right) & 0 \leq \frac{m}{m_p} \leq 1 \\ \frac{1}{1+\ln A}\left(1+\ln \frac{Am}{m_p}\right) & \frac{1}{A} \leq m/m_p \leq 1 \end{cases}$$

Bandwidth of PCM :

no.f samples

① Sampling => $2B/s$ min

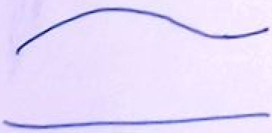
② Quantization => $L = 2^n$

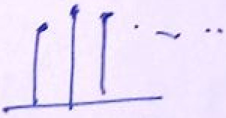
n no of bits.

=> No. of bits reqd to represent $2B$ samples are $\underline{2nB}$

=> No. of bits reqd to ~~rep~~ transmit in 1s second is $2nB$.

=> rate is $2nB$ / sec





$\rightarrow l$ 10

		ε_i
$m - \tilde{m}$ 1010 =>	0010	8
	1110	4
	1000	2

$\varepsilon_i = \tilde{m} - \hat{m}$

↑

to indicate bit position

$\varepsilon_i = (2^{-i}) F$

$$\boxed{\varepsilon_i = (2^{-i})(2m_p)}$$

This error occurs due to noise in the channel, so it is channel noise.

$$\overline{\varepsilon_i^2} = \sum_{i=1}^{n} \varepsilon_i^2 p(\varepsilon_i)$$

$$= \sum_{i=1}^{n} \varepsilon_i^2 p(\varepsilon)$$

$$= p(\varepsilon) \sum_{i=1}^{n} \varepsilon_i^2$$

$$\varepsilon_i^2 = p(\varepsilon) \sum_{i=1}^{n} \varepsilon_i^2$$

$$= p(\varepsilon) \sum_{i=1}^{n} 2^{-2i} (2m_p)^2$$

$$\boxed{\varepsilon_i^2 = 4m_p^2 p(\varepsilon) \sum_{i=1}^{n} 2^{-2i}}$$

$$= 4 m p^2 \, p(\varepsilon) \, 2^{-2} \{ 1 + 2^{-2} + 2^{-4} + \cdots \}$$

$$= 4 m p^2 \, p(\varepsilon) \, 2^{-2} \left\{ \frac{1(1-2^{-2m})}{1-2^{-2}} \right\}$$

$$= \frac{4 m p^2}{3} p(\varepsilon) \{$$

$(2^{-1} + 2^{-2}$

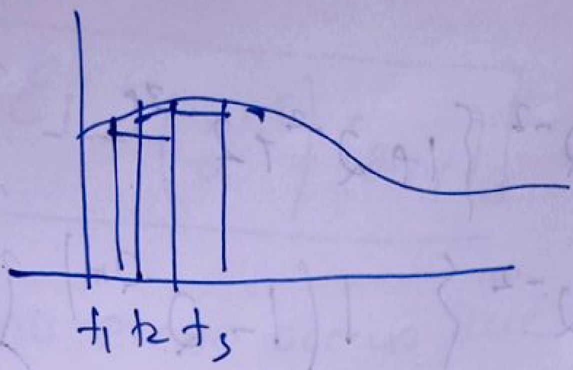
Mean :

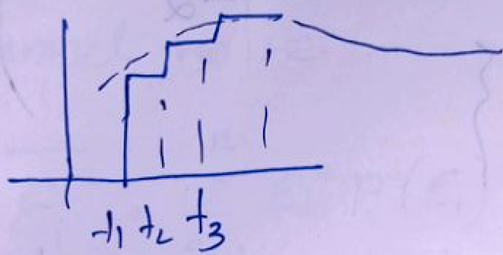
$$\overline{\varepsilon_i} = \sum_{i=1}^{n} \varepsilon_i \, p(\varepsilon_i) = p(\varepsilon) \sum_{i=1}^{n} \varepsilon_i$$

$$= p(\varepsilon) \{ 8 - 8 + 4 - 4 + 2 - 2 + \cdots \}$$

$$= 0$$

for same B.W. quantisation noise power of PCM > qn noise power of Diff





t	PCM	DPCM
t_1	x_1	x_1
t_2	x_2	$\Delta x = x_2 - x_1$

$\Delta x << x_2$

fix 'n' value, $L = 2^n$
↓
fixed

stepsize Δ = in PCM

$= \frac{\text{large}}{L}$

stepsize Δ_2 in DPM

$= \frac{\text{small amp}}{L}$

① For same BW., quantization noise power is more in PCM than DPCM

$\Rightarrow$ SNR is large in DPCM than PCM.

② for same SNR, ie, quantisation noise Power, B.W. in DPCM is lower than PCM.

$m(t) \rightarrow$ Taylor series expression.

$$m(t+T_s) = m(t) + \frac{T_s}{1!}\dot{m}(t) + \frac{T_s^2}{2}\ddot{m}(t) + \cdots$$

$$\simeq m(t) + \frac{T_s}{1}\dot{m}(t)$$

$m(kT_s + T_s)$

$m[(k+1)T_s]$

$$\dot{m}(t) = \frac{m[k] - m[k-1]}{T_s}$$

$t = k^{th}$ sample

$t = kT_s$

$$m[k+1] = m[k] + \cancel{T_s}\, \frac{m[k] - m[k-1]}{\cancel{T_s}}$$

$$m[k+1] = 2m[k] - m[k-1]$$

Ans: $\boxed{m[k+1] = 2m[k] - m[k-1]}$

$$m[k+1] = a_0 m[k] + a_1 m[k-1] + a_2 m[k-2] + a_3 m[k-3] + \cdots + a_N m[k-N]$$

If $m(t)$ has its all order derivatives to exist from Taylor's series, we write.

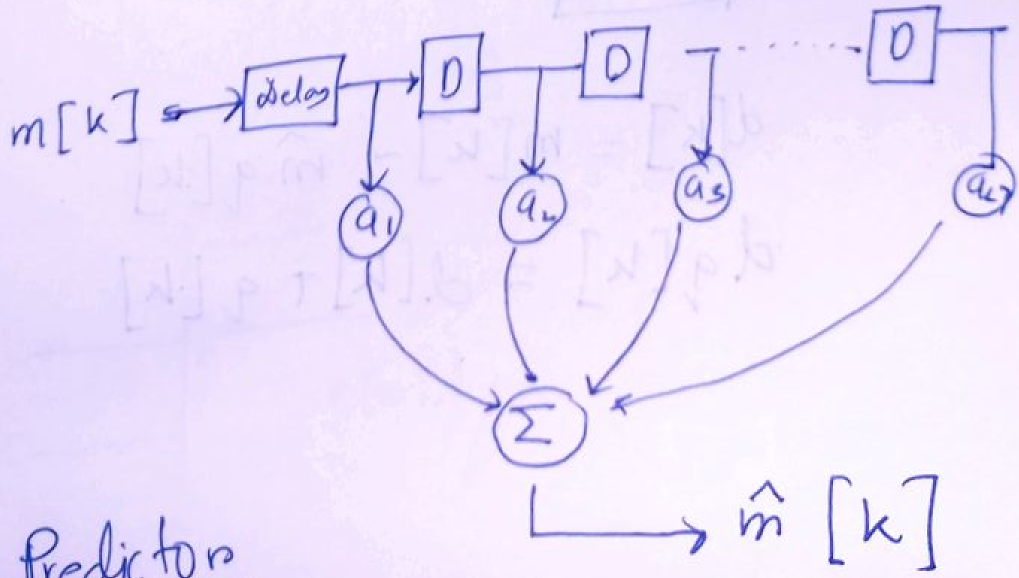
$$m(t+T_s) = m(t) + \frac{T_s}{1!}\dot{m}(t) + \frac{T_s^2}{2!}\ddot{m}(t) + \dots$$

$$\approx m(t) + m(t) - m(t-1)$$

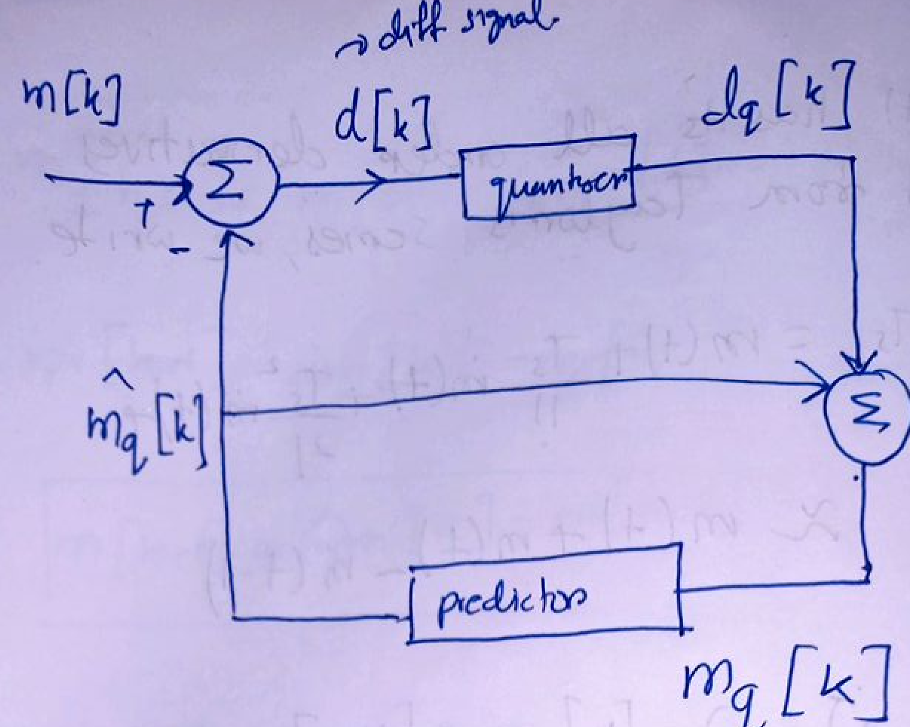
$$m(k+1) = 2m[k] - m[k-1]$$

$$m[k] \approx a_1 m[k-1] + a_2 m[k-2] + a_3 m[k-3] + \dots a_n m[k-N]$$

$$= \hat{m}[k]$$

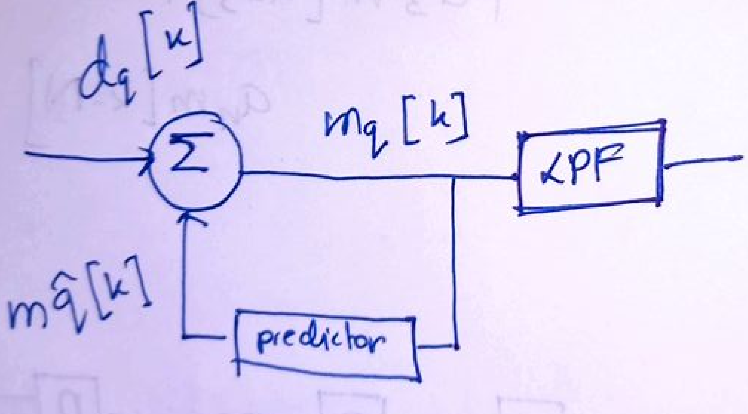


Predictor



$m_q[k]$: quantised version of k^{th} sample

$\hat{m}_q[k]$: estimated quantised value.



$$d[k] = m[k] - \hat{m}_q[k]$$

$$d_q[k] = d[k] + q[k]$$

$$m_q[k] = d[k] + \hat{m}_q[k]$$

$$= d[k] + q[k] + \hat{m}\, q[k]$$

$$= m[k] - \cancel{\hat{m}_q[k]} + q[k] + \cancel{\hat{m}_q[k]}$$

$$= m[k] + q[k]$$

$m(nT_s)$: n-th sample value.

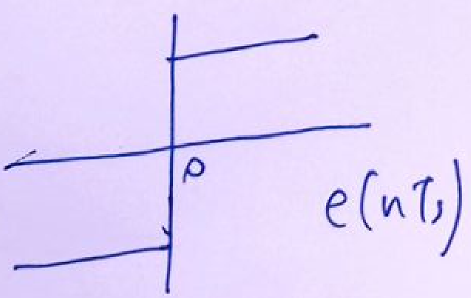
$m[(n-1)T_s] =$ $(n-1)$th quantised sample value.

$$e(nT_s) = m(nT_s) - m_q[(n-1)T_s]$$

↑ larger ↓ smaller

$$e_q(nT_s) = \Delta . \mathrm{Sgn}[e(nT_s)]$$

$+\Delta$



$$m_q(nT_s) = m_q[(n-1)T_s] + e_q(nT_s)$$