Linear Dynamical System (LDS)

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Perceptual decisions are made by comparing incoming sensory evidence to a criterion or category boundary. In a task with two response options (right vs. left), an observer will respond right when the evidence exceeds the criterion, and left otherwise. The criterion with which the sensory evidence is compared can change over time. There are two sources that can induce a shift in criterion:

- Systematic updating (choice history bias)
- Non-systematic updating (slow drifts)

The aim of the model is to disentangle these two sources causing the shifts in criterion. The model assumes that the choice on trial t follows a Bernoulli process

$$y_t \sim Bern(\theta_t) \tag{1}$$

with mean

$$E[y_t|s_t, c_t] = \theta_t \tag{2}$$

where θ_t represents the chance of a right response, and can be obtained by comparing the stimulus evidence s_t to the decision criterion c_t. The more s_t exceeds c_t, the higher the probability of a right response, and vice versa.

$$E[y_t|s_t, c_t] = \frac{1}{1 + e^{-(\beta_0 + \beta_s s_t + c_t)}} \tag{3}$$

A logistic link function ensures the scaling between 0 and 1. The parameter β_0 captures an overall response tendency (e.g. higher proportion of right responses over the course of the experiment) and the perceptual sensitivity of an observer is represented by β_s. The latter will dictate the steepness of the psychometric function. The decision criterion c_t can be rewritten as:

$$c_t = \beta_u u_t + \beta_d d_t \tag{4}$$

where u_t represents any systematic updating variable (e.g. the correctness of the previous response y_{t-1}) and d_t captures the slow drift. These latent drifts are assumed to follow a random walk process.

$$d_t = d_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_d) \tag{5}$$

Therefore, c_t can be rewritten as

$$c_t = \beta_u u_t + \beta_d (d_{t-1} + \epsilon_t) \tag{6}$$

Lastly, given all previous steps, we can redefine formula (3) as:

$$E[y_t | s_t, c_t] = \frac{1}{1 + e^{-(\beta_0 + \beta_s s_t + \beta_u u_t + \beta_d (d_{t-1} + \epsilon_t))}} \tag{7}$$