Technische Universität München

Department of Mathematics

Master’s Thesis

Fast Solvers for Batched Constrained Optimization Problems

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I hereby declare that this thesis is my own work and that no other sources have been used except those clearly indicated and referenced.

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Abstract

An under-explored problem in constrained convex optimization is efficiently solving many convex programs that share the same objective and constraint structure but differ in coefficients in each instance. Such problems appear, for example, in computational game theory and market design. Standard commercial solvers such as Gurobi and CPLEX for convex optimization excel at efficiently computing solutions for a single instance but are not able to leverage synergies to efficiently solve many problems of the same kind. The goal of this thesis is exploring possible solutions to the problem of batched constrained optimization, where individual program instances are usually fairly small but applying standard solvers is prohibitive because of lack of parallelism and unnecessary duplicated instantiation overhead. A major part of this thesis focuses on interior point methods for solving constrained optimization problems. The Primal-Dual Interior-Point Method is exploited in building fast converging algorithms with additional speed-up obtained by using computational power of a GPU.

Zusammenfassung

Bei einer in englischer Sprache verfassten Arbeit muss eine Zusammenfassung in deutscher Sprache vorangestellt werden. Dafür ist hier Platz.

Contents

Chapter 1

Introduction

1.1 Background: Quadratic Programming and Duality Theory

1.2 Background: LU, LDL^T Factorization

1.3 Background: Block Elimination using Schur Complement

Chapter 2

Primal-Dual Interior Point Methods

The primal-dual interior point methods are proven to be a class of algorithms that solves a wide range of optimization problems including linear-programming-problems, convex-quadratic-programming-problems, semi-definite-programming etc.[1]. They fall under the category of interior point methods and employ the Newton's method to solve optimization problems in an efficient manner. The research works that gave rise to the field of *Interior-Point Methods* and successively the *Primal-Dual Methods* started with the publication of Karmarkar's paper [2] which marked an important point in the history of optimization algorithms. This chapter walks through the basic building blocks of the primal-dual interior point methods, its evolution as a path-following algorithm, the Mehrotra's predictor-corrector approach and finally looks at the complexity analysis of these algorithms and compare it with other common optimization algorithms like the simplex method.

2.1 Newton's Method

The Newton's method finds the roots of a function by moving along search directions generated by the the liner approximation of the same function starting from a point in the functions domain. From the Taylor's theorem [3] the linear approximation of a function f which is differentiable at a point x_0 is given by:

$$\bar{f}(x) = f(x_0) + (x - x_0)f'(x_0)$$

The Newton Step Δx_n can be obtained by finding the root of this linear approximation of f as $f'(x_n)\Delta x_n = -f(x_n)$.In the matrix system the equivalent representation to obtain the newton step would be $J(x_n)\Delta x_n = -F(x_n)$ where $J(x_n)$ is the Jacobian of $F(x)$ at x_n. These newton steps are the search directions along which the the linear approximation of the function will be iteratively evaluated. The Newton Iterates are hoped to converge to the roots of the function $F(x)$[3].

2.2 Interior Point Methods and PDIPM's

2.3 Path-Following Algorithms

2.4 Mehrotra's Predictor Corrector Algorithm

2.5 Complexity Analysis of PDIPM's

Chapter 3

Results

Chapter 4

Conclusion

References

[1] Stephen J Wright. *Primal-dual interior-point methods*, volume 54. Siam, 1997.

[2] Narendra Karmarkar. A new polynomial-time algorithm for linear programming. In *Proceedings of the sixteenth annual ACM symposium on Theory of computing*, pages 302–311, 1984.

[3] Tom M Apostol. Mathematical analysis. 1964.