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Prediction of Financial Crises: Are critical slowing down indicators efficient ?

Sujet 6: Prévisions de crises financières

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Contents

Abstract

In this essay, we try to see whether financial crises can be predicted by critical slowing down indicators used as early warning signals. To do so, we consider four crises : Black Monday of 1987, the Asian Crisis, The Dot-Com Crash and the 2008 Financial Crisis. We study the fluctuations of time series on the AR(1) coefficient, the Mutual Information indicator, Standard Deviation and Skewness prior to the market collapses. We found through our study an evidence for critical slowing down before Black Monday 1987 but the analysis of other crises shows no significant statistical evidence for critical slowing down on the indicators.

1 Introduction

The contemporary era in which we live has been marked by a number of disastrous financial crises. For this reason, a lot of work has been done to learn more about the subject. In particular, it has led important scientists to undertake work on crisis prediction tools. These particular indicators are called Early Warning Signals (EWS). We could define them as “a group of statistical time-series signals which could be used to anticipate a critical transition before it is reached. EWSs are model-independent methods that have grown in popularity to support evidence of disease emergence and disease elimination” (Southall, Brett, Tildesley, & Dyson, 2021). They are in fact powerful and very interesting tools since they can be applied to a large number of fields such as ecology or biology for example.

Moreover, these indicators refer directly to the Critical Slowing Down theory, according to which when approaches a tipping point or a bifurcation, in particular due to an accumulation of shocks, it slowly returns to a state of equilibrium, and has more and more difficulties in absorbing shocks. Thus, it is before the arrival of the transition that everything is played out since some indicators such as autocorrelation, variance and others will have abnormal behaviors because they will be marked by a variation in their value.

The goal of this paper is to study the role of early warning signals in the prediction of financial crises. In order to answer this question, we decided to conduct our study by focusing on four major financial crises that have marked history: Black Monday of 1987, the Asian Crisis, the Dot-Com Crash and the 2008 Financial Crisis. We used different stock market index to conduct our study, each specifying a particular crisis. We then studied the fluctuations of time series on the AR(1) coefficient, the Mutual Information Indicator, Standard Deviation and Skewness prior before to the market collapses. In a last step, we tested the robustness of the parameters to see the strength of our analysis. We have obtained very mixed results regarding early warning signals. Only AR(1) and MI(1) seem to express a downward trend when approaching a critical transition. Therefore, there is little evidence to suggest that financial crises are preceded by a critical slowing down. Plus, of the four crises, only Black Monday crisis shows a critical slowing down before the crash.

Our paper is organized as follows; Section 2 deals with the theoretical part, i.e. it introduces the elements necessary for the construction and understanding of our study. Section 3 focuses the data used in our study. The fourth section is devoted to our methodology, in particular the econometric techniques used. Then, Section 5 is dedicated to the presentation of our results. Finally, the last section allows us to discuss the different results obtained.

2 Theory

We will now focus on the survey part, which is a very important component of our study since it helped building and guiding our work. Numerous works have already studied these phenomena, particularly with regard to financial crises. It is a subject at the heart of the current events which interests a large number of political decision makers since the optics of predicting financial crises would make it possible to avoid many catastrophes.

It is true that financial markets are exposed to changes throughout the years and actually go through critical transitions when these markets go from a normal state to a crisis state and vice versa (Gatfaoui, Nagot, & De Peretti, 2017). It happens when a set of indicators reaches a tipping point. First of all, in order to prevent risks leading to financial crises, it is essential to take action in advance (Coudert & Idier, 2018). The study of time series data including crisis episodes as well as a set of economic variables such as interest rates, or more generally real estate market indicators, or financial indicators will allow the prediction of potential crises (Tan & Cheong, 2014). Indeed, these different variables will release possible signals which, depending on their behavior, will determine whether or not the system is approaching a critical transition (Diks, Hommes, & Wang, 2019). When these indicators reach a certain threshold, it means that the phenomenon is likely to occur and therefore that the system is heading towards a tipping point (Nazarimehr, Jafari, Perc, & Sprott, 2020).

However, it is important to note that there is a trade-off when determining this particular threshold. There is indeed a trade-off with respect to the level of risk aversion. An optimal threshold must be chosen that balances sensitivity and specificity, i.e., a trade-off between missed crisis calls and false alarms (Aldasoro, Borio, & Drehmann, 2018). This is a relatively complicated task, but it can be accompanied by the AUROC criterion to determine the relevance of the indicator in question (Coudert & Idier, 2016).

Thus, as previously mentioned, the system goes through critical transitions over time when a set of indicators reaches a threshold limit. As a result, the system loses its effectiveness. This is when even small disturbances can cause the system to move towards a critical phase (Gatfaoui et al., 2017). When the system is heading towards a tipping point, there are several indicators that allow us to predict this phenomenon (Diks et al., 2019). First of all, there is the autocorrelation of the data, in particular the lag-1 autocorrelation, as this concerns the short term. Indeed, as the market is heading towards a tipping point, it becomes increasingly difficult for it to recover from disturbances and thus remains close to its previous state. The second indicator is the variance. The accumulation of shocks leads to a loss of resilience in the system and an increase in variance before the system changes state (Gatfaoui et al., 2017). Finally, the last phenomenon that we can note concerns the asymmetry of fluctuations that can occur when the system approaches the tipping point. These three indicators are then called Early Warning Signals (EWS) and allow us to anticipate the beginning of a major crisis.

More precisely, it is possible to describe the three states that a financial crisis goes through (Diks et al., 2019). During the first phase, the system is far from the bifurcation: a small variance and rapid fluctuations are observable. Then in the second phase, when the system is close to the bifurcation, the fluctuations are slower and larger, and the variance increases. In the third and last phase, the system is in an irreversible transition towards a new local minimum.

Moreover, the self-organizing behavior of a society can also be a precursor of a crisis (Moon & Lu, 2015). Indeed, the formation and collapse of a speculative bubble are the consequences of a herd behavior that appears during a systemic change. The latter the breaks the balance between the different entities and leads to an excessive influence of peers. This kind of behavior is quite visible on financial markets or during stock market crashes which are locally reinforced by this kind of phenomenon (Gresnigt, Kole, & Franses, 2015).

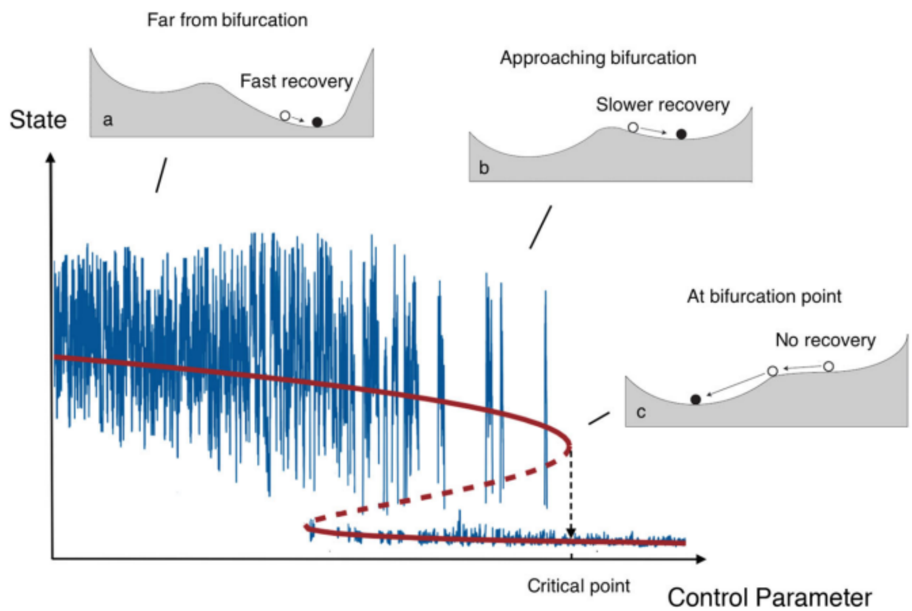


Figure 1: Main steps of a critical transition induced by a bifurcation (Diks et al., 2019)

3 Data

As previously mentioned, we used four major crises as described in Table 1. For each of them, we used one or two stock market index as summarized below. The following subsections focus on a brief description of data for each crisis.

Crisis	Critical Point	Time series
Black Monday	13 October 1987	S&P 500 index
Asian Crisis	01 October 1997	Hang Seng index
Dot-Com crash	24 March 2000	NASDAQ composite
2008 Financial crisis	12 September 2008	S&P 500 index
2008 Financial crisis	12 September 2008	VIX

Table 1: Summary of events and time series used in the analysis

3.1 Black Monday

For Black Monday, we used the S&P 500 index which is a stock market index based on 500 companies listed on the US stock exchanges.

Variable	N	Mean	Standard Deviation	Minimum	Maximum
Close	600	256.2305	35.2743004	187.52	336.77
High	600	257.8367333	35.5987258	187.76	337.89
Low	600	254.30	35.0312302	187.01	334.46

Table 2: Descriptive statistics for S&P 500 Index for Black Monday

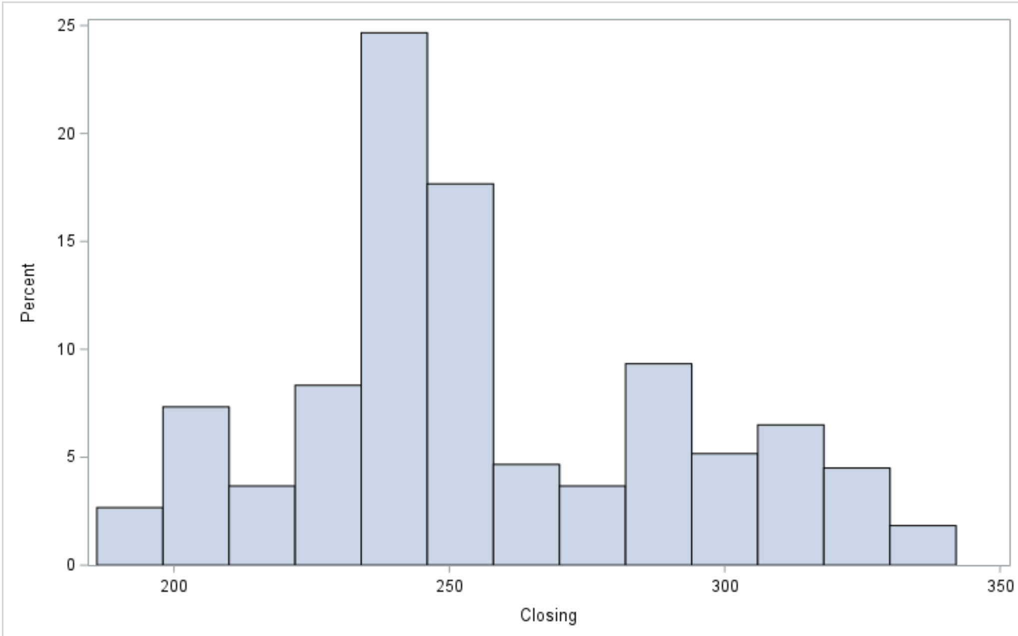


Figure 2: Frequency of Closing of S&P 500 Index (US$) for Black Monday

3.2 Asian Crisis

For the Asian crisis, we used the Hang Seng Index, which is the stock market index of the Hong Kong stock exchnage based on 45 companies that alone represent 65% of the market capitalization.

Variable	N	Mean	Standard Deviation	Minimum	Maximum
Close	600	11965.73	1817.16	8121.06	16673.27
High	600	11965.73	1817.16	8121.06	16673.27
Low	600	11965.73	1817.16	8121.06	16673.27

Table 3: Descriptive statistics for Hang Seng Index for Asian Crisis

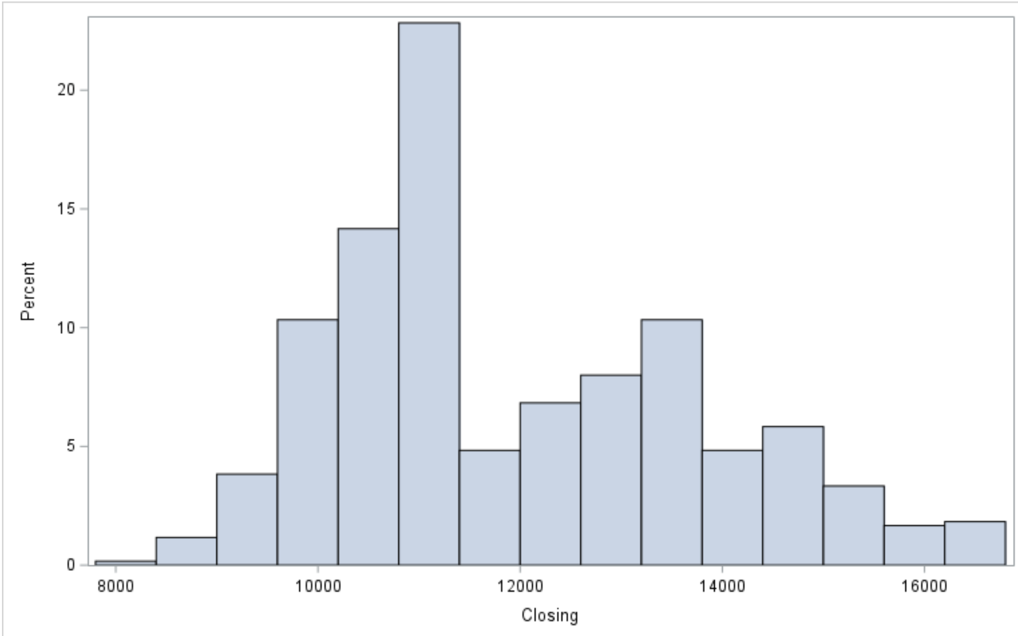


Figure 3: Frequency of Closing of Hang Seng index (HK$) for Asian Crisis

3.3 Dot-Com Crash

For the Dot-Com Crash, we used the NASDAQ Composite which is a stock market index based on all the companies in the world listed on the NASDAQ. It allows us to measure the performance of these companies.

Variable	N	Mean	Standard Deviation	Minimum	Maximum
Open	600	2797.54	921.9668966	1420.94	5060.34
High	600	2825.86	936.1652532	1420.94	5132.52
Low	600	2755.82	898.1278991	1357.09	5039.35
Close	600	2793.22	917.6142696	1419.12	5048.62

Table 4: Descriptive statistics for NASDAQ Composite for Dot-Com Crash

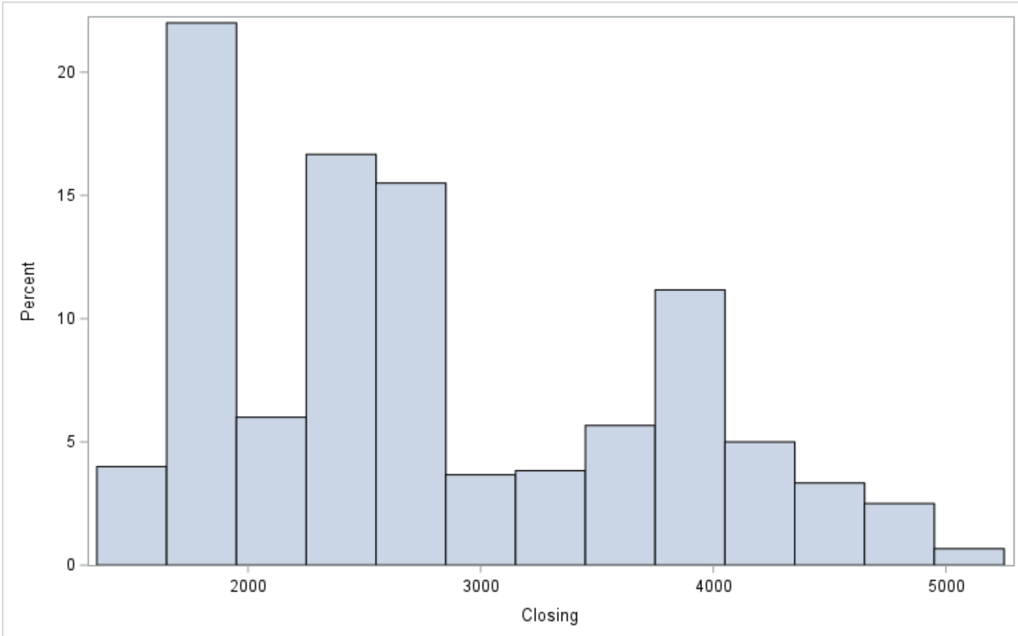


Figure 4: Frequency of Closing of NASDAQ Composite (US$) for Dot-Com Crash

3.4 2008 Financial Crisis

Finally, to study the 2008 Financial Crisis, we used the VIX index, which is a volatility index based in part on the S&P 500 index presented earlier. The latter is a measure of market risk.

Variable	N	Mean	Standard Deviation	Minimum	Maximum
Open	600	1332.99	199.0715765	763.45	1565.15
High	600	1343.49	194.9208313	801.20	1576.09
Low	600	1321.10	203.3450208	741.02	1555.46
Close	600	1332.55	199.3566178	752.44	1565.15

Table 5: Descriptive statistics for S&P 500 index for 2008 Financial Crisis

Variable	N	Mean	Standard Deviation	Minimum	Maximum
Open	600	24.3218667	14.6335258	9.89	80.74
High	600	25.5561667	15.6845376	10.06	89.53
Low	600	23.0870000	13.4785495	9.39	72.76
Close	600	24.2390667	14.5491261	9.89	80.86

Table 6: Descriptive statistics for VIX for 2008 Financial Crisis

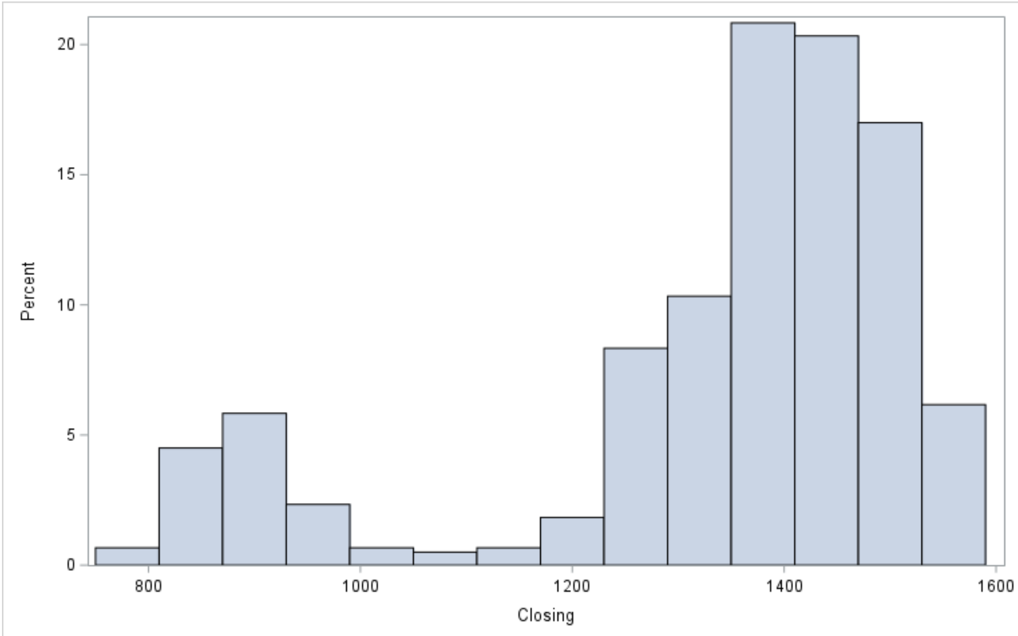


Figure 5: Frequency of Closing of S&P 500 Index (US$) for 2008 Financial Crisis

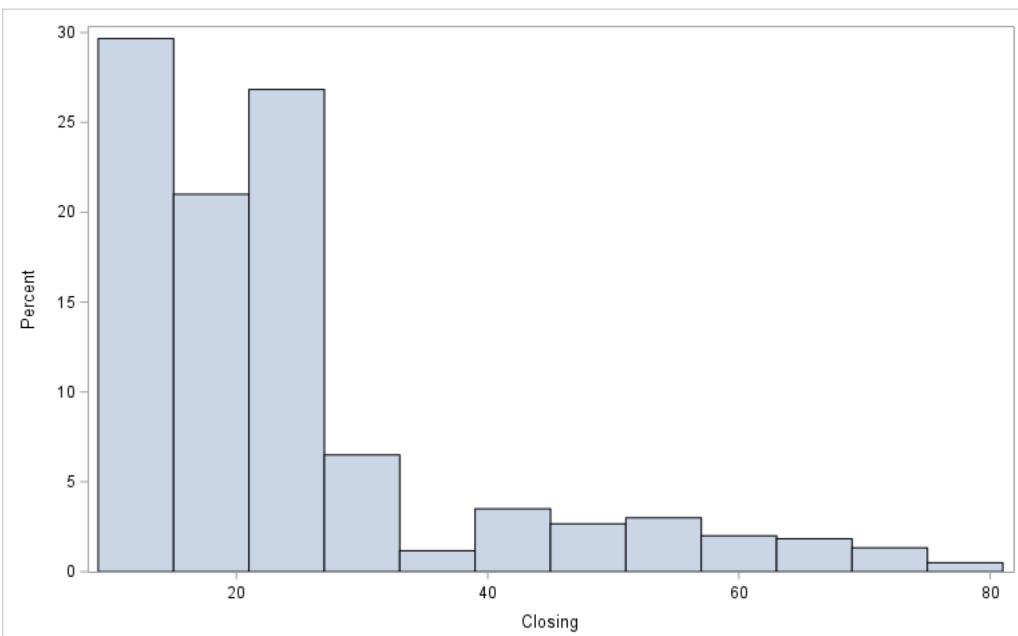


Figure 6: Frequency of Closing of VIX Index for 2008 Financial Crisis

4 Methodology

In this section we describe our methodology to obtain the results shown in the next section. We show how we remove the trend from a time series, then we detail the different formulas used in the calculation of the indicators and the Kendall's Tau.

4.1 Detrending

First, we have to remove the trend from the time series. To do so, we use a Gaussian filter. This method allows us to reduce the noise from the time series. Considering 500 days before the crisis, we apply the gaussian kernel filter presented in the following formula:

$$G(s) = \frac{1}{\sqrt{2\pi\sigma}} \times e^{\frac{-s^2}{2\sigma^2}},\ \sigma = 1 \tag{1}$$

The smoothing bandwidth is an important parameter, in this study we choose 10 trading days, in such a way that keeps the details of the fluctuations. Then the moving average is given by:

$$MA_t = \frac{\sum_{r=1}^{T} G(r-t) z_r}{\sum_{i=1}^{T} G(r-t)},\ t = 1, ..., T \tag{2}$$

where z_r is the logarithm of the index of the original series and T=500 days before the critical event. Thus we obtain the residuals by removing the moving average:

$$y_t = z_t - MA_t,\ t = 1, ..., T \tag{3}$$

We then obtain a stationary time series written $\{y_t\}_{t=1}^{T}$ that fluctuates by definition around 0. This series is used to compute several early warning indicators, namely the AR(1) coefficient, the lag-1 Mutual Information, the Standard Deviation and the Skewness. These indicators are determined using a moving estimation window across time allowing to inspect trends.

4.2 Indicators

4.2.1 AR(1) coefficient

As we approach a critical transition, AR(1) is expected to increase as the parameter approaches a critical value. This indicator is very useful to monitor critical slowing down. It can be estimated using a first-order autoregresssive model:

$$y_t = e^{-k\Delta t} y_{t-1} + \epsilon_t,\ \Delta t = 1 \tag{4}$$

where ϵ is a zero mean innovation, k indicates the magnitude of the recovery rate and $\lambda = e^{-k\Delta t}$ is the AR(1) coefficient. This coefficient is estimated by ordinary least square of the model :

$$y_t = \lambda y_t - 1 + \epsilon_t,\ t = j-n+1, ..., j,\ j = n, .., T \tag{5}$$

where n is the sliding window size. The estimation window size n is a very important parameter. Following different studies, we use $n = T/2 = 250$ and we demonstrate why it is the best size according to the smoothing bandwidth in Section 5.5. Indeed, taking a smaller window size with few observations make the estimation of the autocorrelation less precise.

4.2.2 Mutual Information (MI) indicator

This indicator is an extension of the AR(1) indicators that captures non-linear correlations. The MI measures the dependence between two random variables and quantifies the amount of information shared between them. The time delayed mutual information between a variable measured at time t and $t - \eta$ is given by:

$$MI(X_t, X_{t-\eta}) = \int p(x_t, x_{t-\eta})(\eta)log\frac{p(x_t, x_{t-\eta})}{p(x_t)p(x_t - \eta})\mathrm{d}x_t\mathrm{d}x_{t-\eta} \tag{6}$$

where the time η is the lag, $p(x_t)$ and $p(x_{t-\eta})$ are the marginal probability density functions and $p(x_t, x_{t-\eta})$ is the joint probability density function of the variable measured at time t and the same variable measured at time $t - \eta$. In this study this indicator estimation is based on the observations in the moving window.

4.2.3 Standard Deviation

When approaching a critical transition, the Standard Deviation is expected to increase. It is estimated within the window with the following formula:

$$\widehat{STD}_{.j} = \sqrt{\frac{1}{n-1}\sum_{t=j-n+1}^{j}(y_t - \hat{\mu}_j)^2},\ j = n, .., T \tag{7}$$

This indicator is more robust than AR(1) as an early warning signal.

4.2.4 Skewness

The skewness measures the asymmetry of the distribution of a random variable and is obtained by the following formula:

$$Skewness = \frac{\sum_{t=1}^{T}(y_t - \bar{y})^3}{(T-1)\sigma^3} \tag{8}$$

with σ the Standard Deviation and $\bar{y}$ the mean. A negative skew indicates that the tail is on the left side of the distribution and a positive skew on the right side. The skewnesss is expected to arise (by decreasing or increasing) before the occurence of a catastrophic bifurcation. Indeed, close to a tipping point, probability distributions may become highly asymmetric and lead to a non-null skewness.

4.3 Kendall's Tau

For each indicator we calculate the Kendall's Tau. It measures the correlation between the time-varying indicator and the time variable:

$$\hat{\tau} = \frac{C - D}{N},\ \hat{\tau} \in [-1, 1] \tag{9}$$

where C is the number of concordant pairs, D the nummber of discordant pair, and $N = n(n-1)/2$ is the total number of different pair combinations. If $|\tau|$ is close to 1, it suggests a strong trend.

In the next section we use this methodology on several crisis and analyse the results in order to know if the indicators are relevant to the approach of a critical transition. In order to facilitate explanation, we align the x-axis of the critical transition with 0 to clearly distinguish the days before and after it. As the random growth of the stock market indexes is in relative and not absolute terms, we take the logarithm of the data, thus allowing the linearization of the exponential growths present in the original time series, as well as the stabilization of the residuals to be analyzed.

5 Results

In this section, we are analyzing the results that we found to evaluate evidence for critical slowing down. To do so, we study the early warning signals before real critical transitions of 4 financial major crises : Black Monday 1987, the Asian Crisis, The Dot-com bubble and the 2008 Financial Crisis. We will then look at the robustness check to see the strength of our analysis.

5.1 Black Monday

Black Monday 1987 was on October 19 of 1987 and on this day, stock markets around the world crashed, starting in Hong Kong and spreading around Asia and in the US. This day was the biggest one-day percentage drop in the U.S stock market history. We chose the S&P 500 index to study this crisis, which suffered a decline of 20,4%.

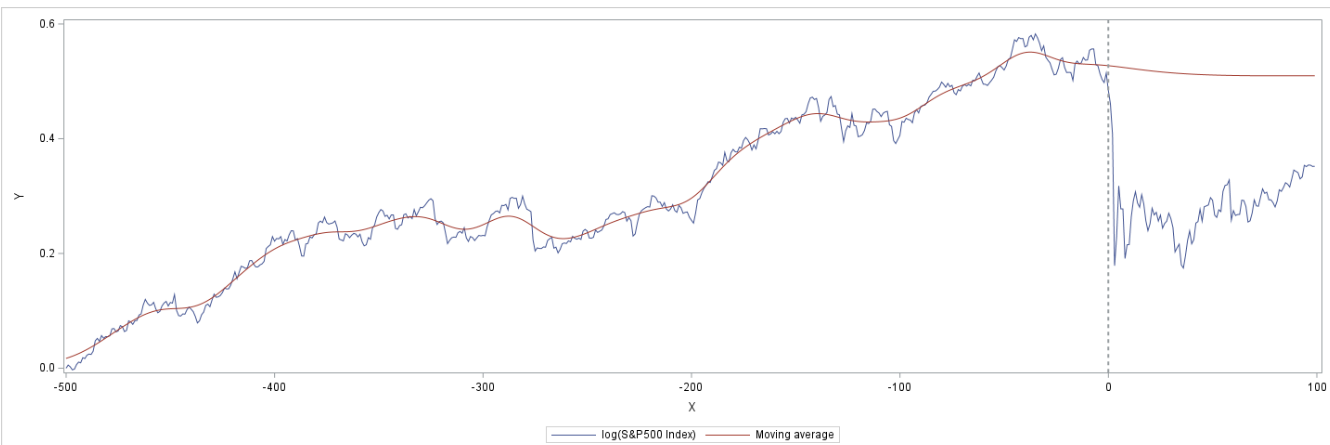


Figure 7: log(S&P 500 Index) and Smoothed time series

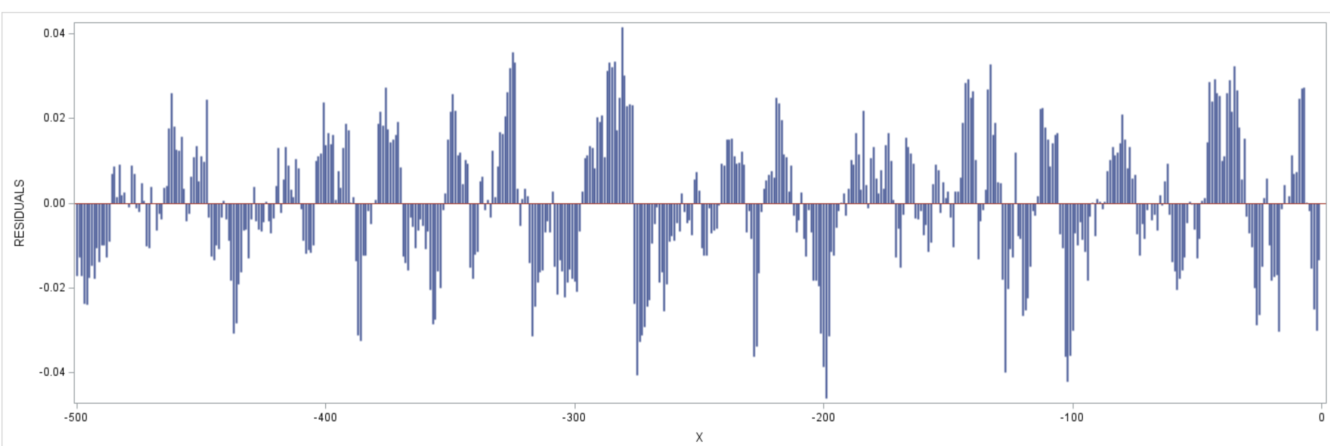
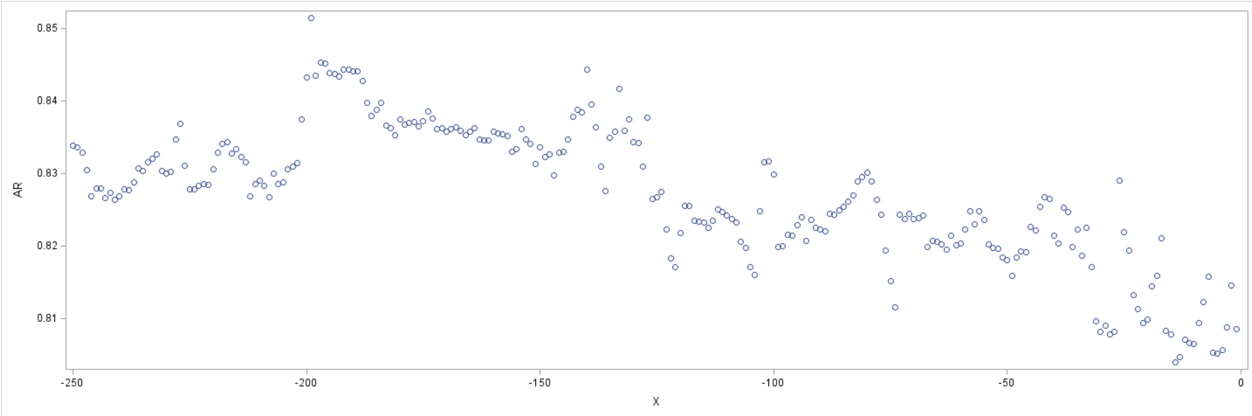
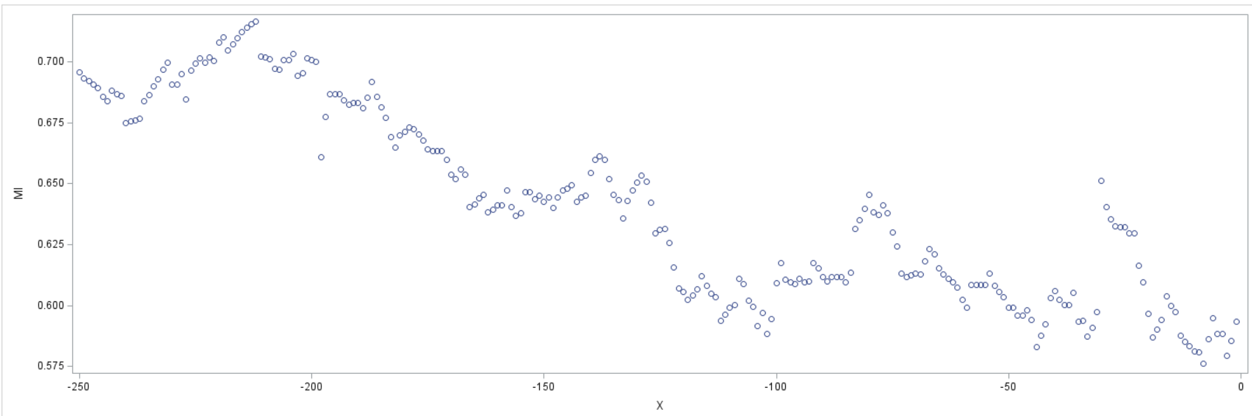


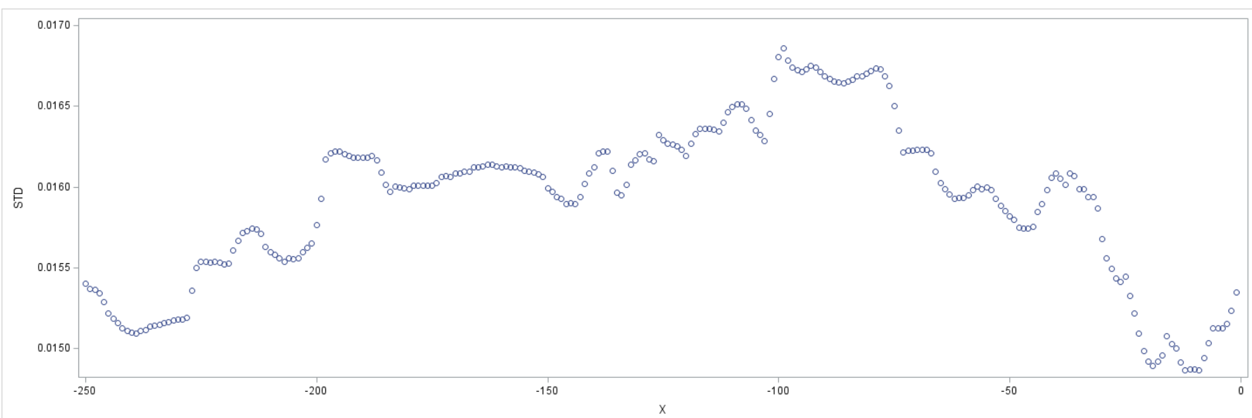
Figure 8: Residuals used to estimate the early warning signals



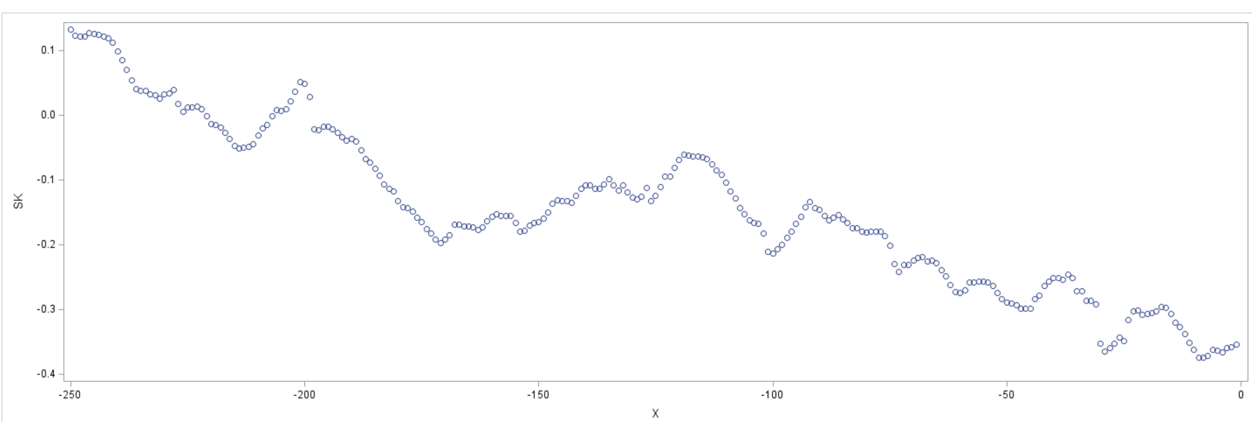
(a) AR(1)



(b) Mutual Information



(c) Standard Deviation



(d) Skewness

Figure 9: Indicators for Black Monday

The 6 figures above show early warning indicators for Black Monday 1987 crisis using the S&P 500 index, using 500 days before the crisis. The vertical dashed line in Figure 7 indicates the critical transition, which means the 13 of October 1987. We see in this graph the huge decline of the log(S&P500 index), as mentionned earlier. In fact, it declined of 20.4% in one day. The red line shows the smoothed time series.

Figure 9a, 9b, 9c represent the 3 early warning indicators used in our analysis : the auto-correlation(1), Mutual information(1) and Standard Deviation. They show a significant downward trend close to the bifurcation of the Black Monday crisis.

Figure 9d represents the skewness, which is another indicator that describes the instability properties in catastrophic bifurcations. The figure shows a decreasing skewness as we get closer to the tipping point, arriving almost at -0.4. This is an evidence for asymmetry of the probability distribution, that is here that more values are concentrated on the left side of the distribution graph.

	AR(1)	Mutual Information	Standard Deviation	Skewness
Time	-0.53349 (< .0001)	-0.70312 (< .0001)	0.11512 (0.0067)	-0.76006 (< .0001)

Table 7: Kendall Tau b Correlation Coefficients for Black Monday, N=250

The Kendall's rank correlation τ measures the degree of concordance between two pairs of ordinal variables. In the presence of critical slowing down, a significant upward trend is associated with a value of Kendall's tau close to zero, at least significantly positive. In the case of Black Monday here, we have significant negative values for AR(1), MI(1) and skewness. What is interesting to look at is the p-value associated with the Kendall's tau that is corrected for serial dependence. The trends of the three indicators are significant at the conventional 5% significance level.

5.2 Asian Crisis

The Asian Crisis occured in 1997 and got caused by the collapse of the currency exchange rate and hot money bubble in parts of Asia. To study the prediction of this crisis, we used the Hang Seng index.

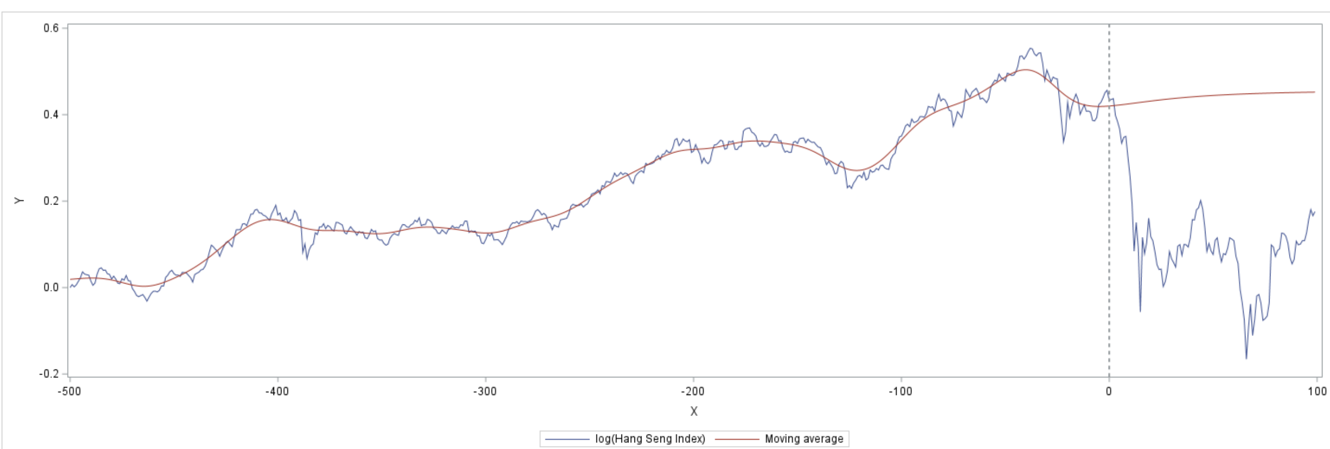


Figure 10: log(Hang Seng Index) and Smoothed time series

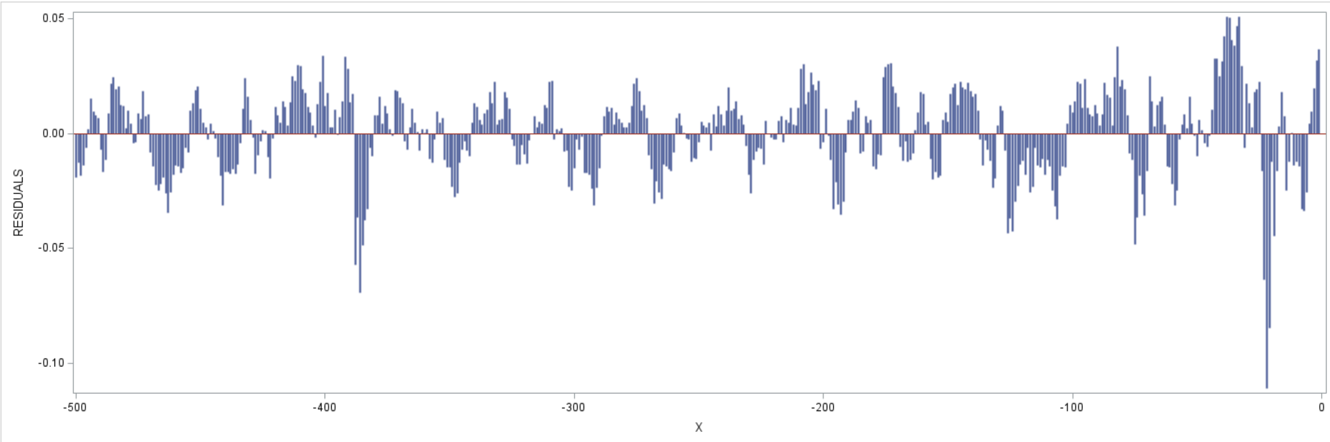


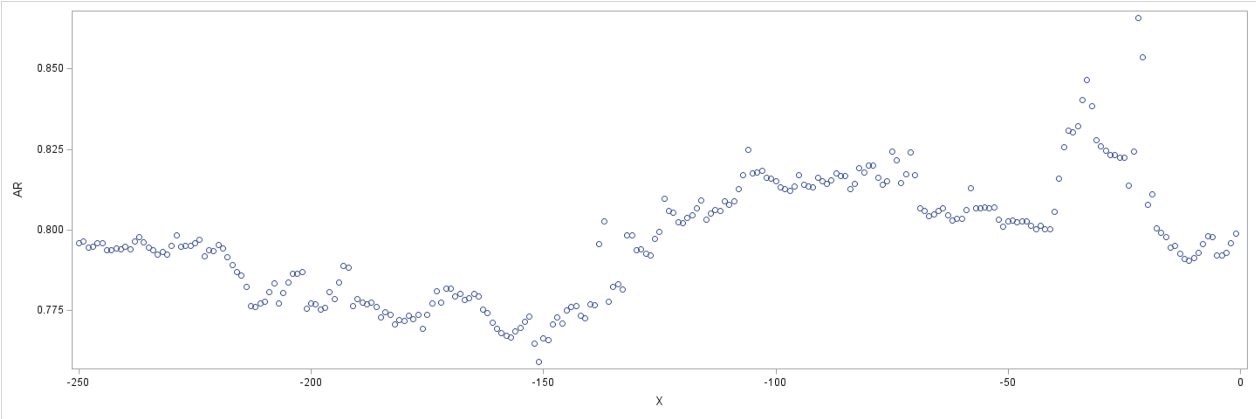
Figure 11: Residuals used to estimate the early warning signals

Figure 10 shows a huge decline in the log(Hang Seng) index after the critical transition, represented by the vertical dashed line. The results for the Asian Crisis are mixed. Indeed, there is an obvious positive trend in the Standard deviation indicator in Figure 12c. However, the AR(1) and the MI(1) indicators in Figure 12b and 12a show a downward trend before the critical transition of the crisis. Figure 12d shows a negative skewness close to the tipping point of the Asian crisis, while it was close to zero before the tipping point, which makes the difference interesting.

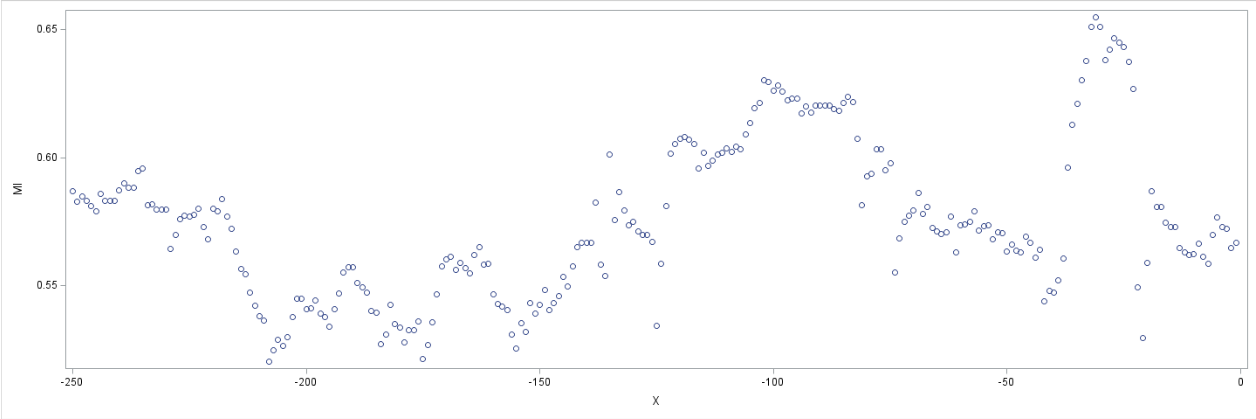
	AR(1)	Mutual Information	Standard Deviation	Skewness
Time	0.33802 ($<$ .0001)	0.19798 ($<$ .0001)	0.38994 ($<$ .0001)	0.22943 ($<$ .0001)

Table 8: Kendall Tau b Correlation Coefficients for Asian Crisis, N=250

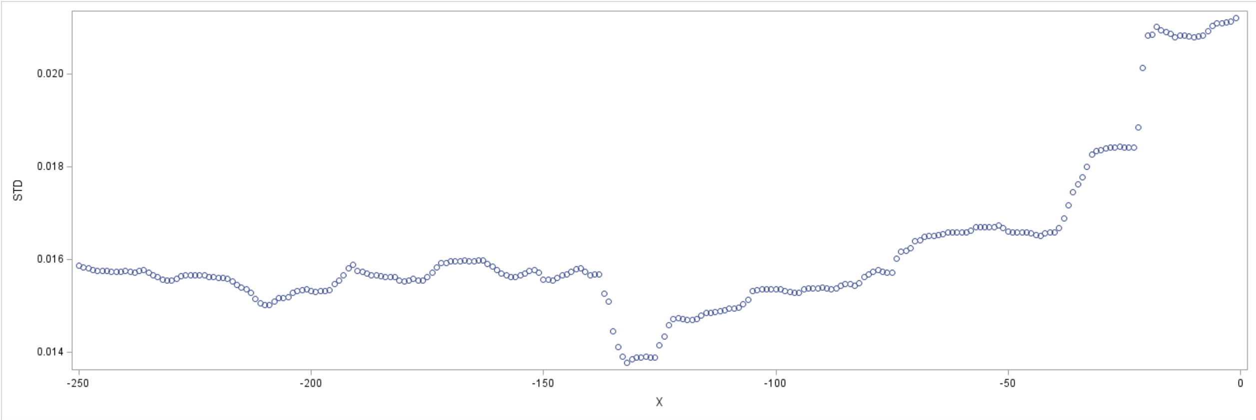
Concerning the Kendall's τ correlation for the Asian Crisis, they are all positive for the indicators in Table 8. Their p-values are < 0.0001, which means that the trends of the indicators for the Hang Seng Index are significant at a 5% significance level.



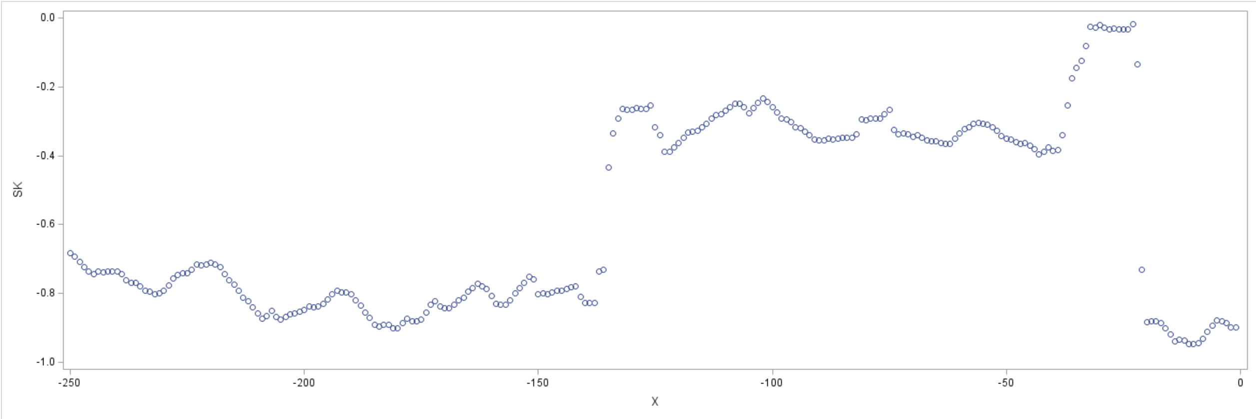
(a) AR(1)



(b) Mutual Information



(c) Standard Deviation



(d) Skewness

Figure 12: Indicators for Asian Crisis

5.3 Dot-com Crash

The Dot-Com Crash of 2000-2002 is a consequence of the Dot-Com bubble due to the commercialization of the Internet in 1995. We identify the beginning of the industry's rapid decline after the tech-heavy NASDAQ composite peaked on March 10th, 2000. That is why we chose the NASDAQ composite index to study the prediction of this crisis.

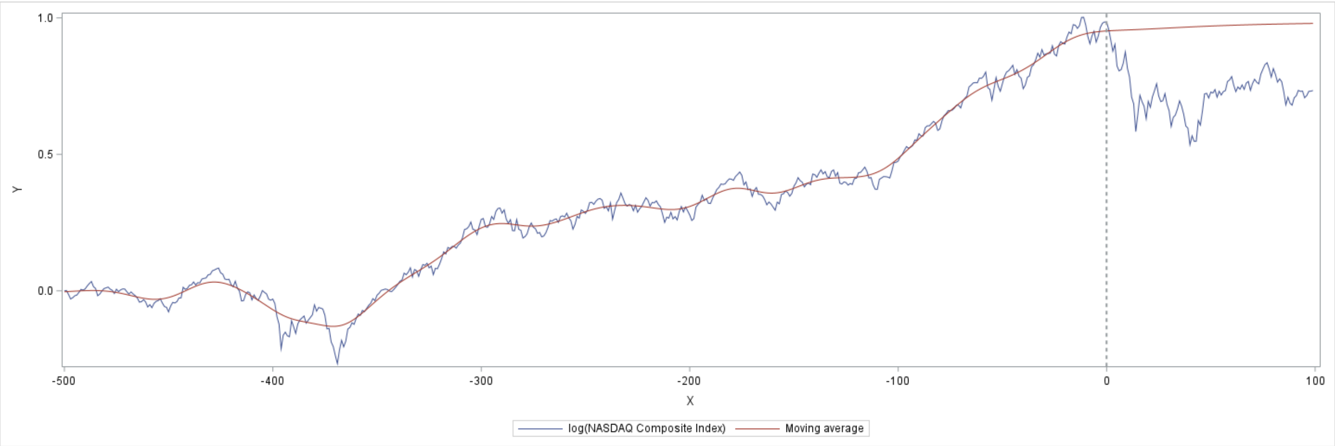


Figure 13: log(NASDAQ Composite) and Smoothed time series

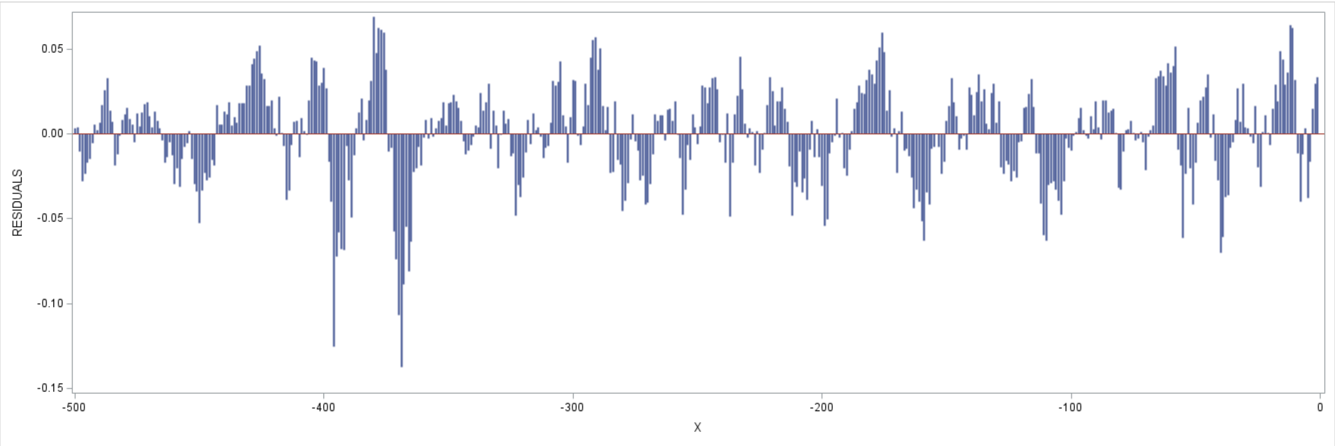
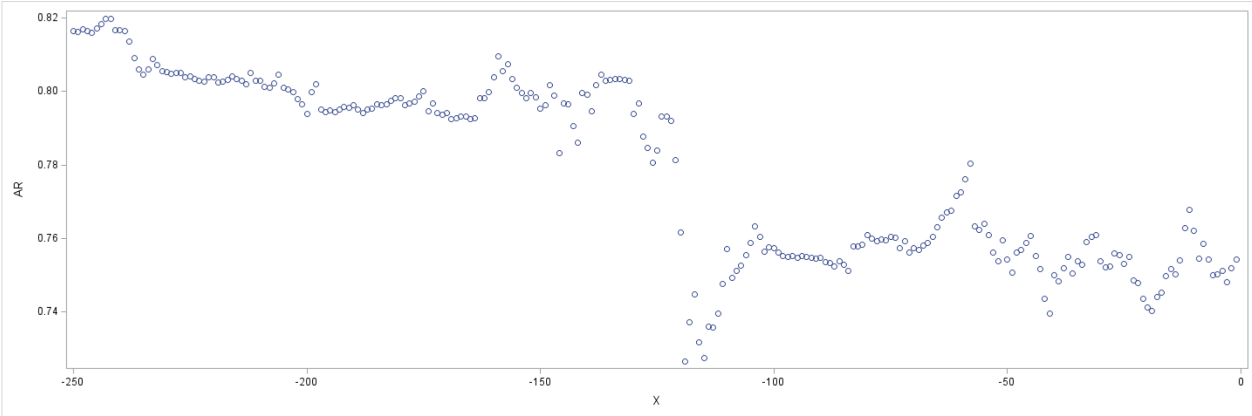


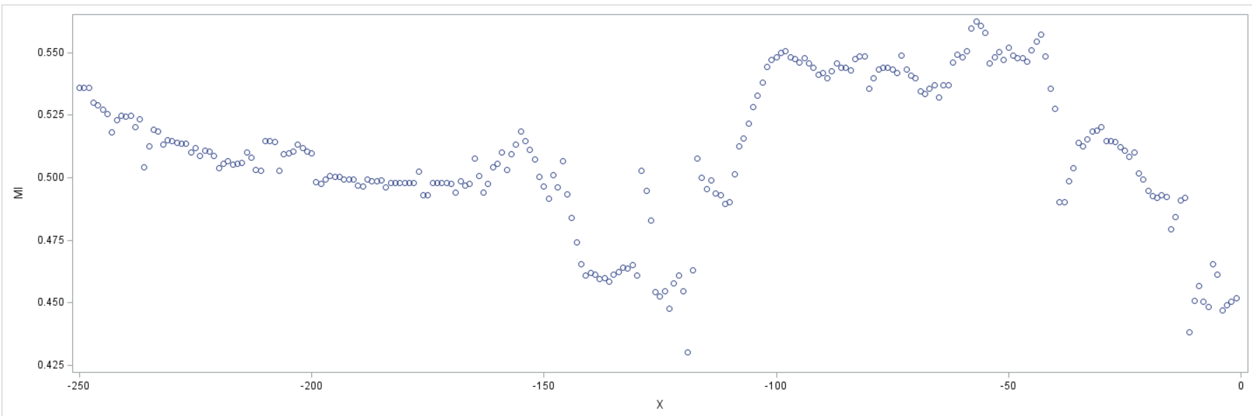
Figure 14: Residuals used to estimate the early warning signals

Figure 13 shows the same result as the previous crises, which means a decline in the index used to study the prediction of the crisis, right after the critical point of the bifurcation.

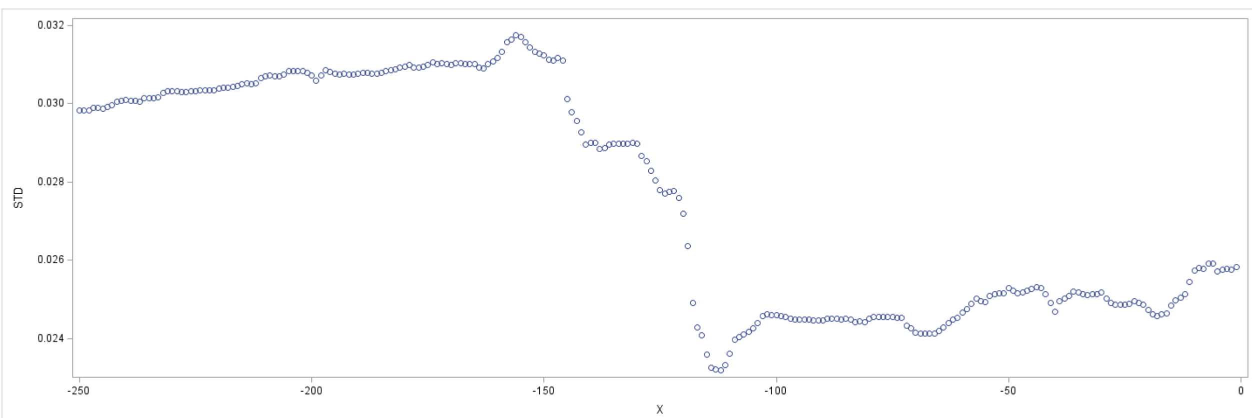
The result of the figures representing the early warning indicators (15a, 15b, 15c) are mixed because we observe an upward trend for the Standard Deviation and a downward trend for the Mutual Information and the Auto-correlation(1). We can notice a huge decline in the Standard Deviation around 150 days before the critical transition, as for the 2 others early warning signals. At this time, the skewness grew (around 125 days before the Dot-Com crash).



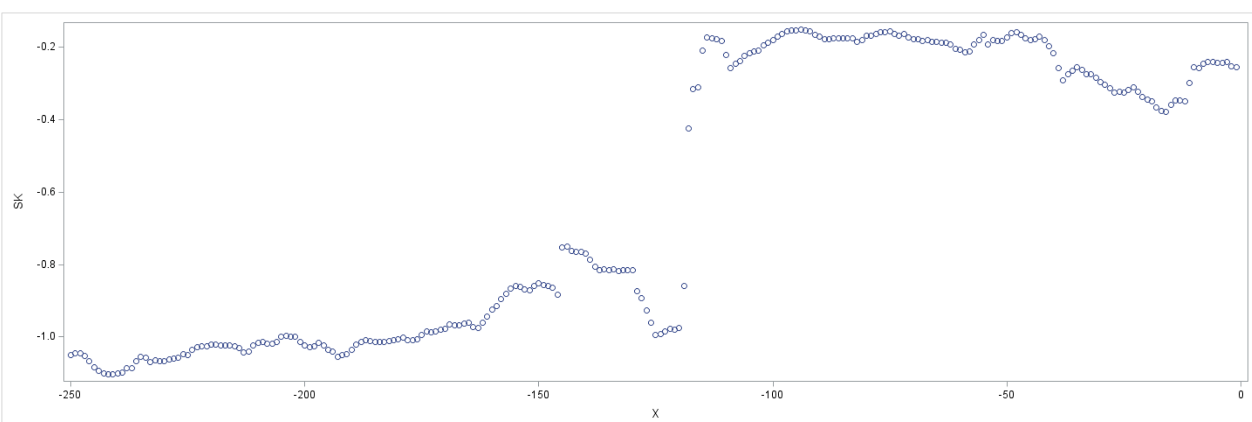
(a) AR(1)



(b) Mutual Information



(c) Standard Deviation



(d) Skewness

Figure 15: Indicators for the Dot-Com Crash

The skeweness associated to the Dot-Com crash in Figure 15d is increasing, meaning getting closer to zero as we approach the tipping point of the crisis. The probability distribution in this case is symmetric close to the crisis.

	AR(1)	Mutual Information	Standard Deviation	Skewness
Time	-0.64369 (< .0001)	-0.01724 (0.6850)	-0.31978 (< .0001)	0.62114 (< .0001)

Table 9: Kendall Tau b Correlation Coefficients for Dot-Com Crash, N=250

Table 9 shows the values and p-values associated with the Kendall's τ correlation coeffficient for the Dot-Com crash. The bootstrap p-values after taking account temporal dependence are less than 0.0001 for the AR(1), Standard deviation indicators and for the skewness. Therefore, the trends of these indicators are statistically significant at a 5% significance level. However, the p-value associated with the Kendall's τ for the Mutual Information indicator is 0.6850, so its trend is insignificant at any conventional level of significance.

5.4 2008 Financial Crisis

The 2008 Financial Crisis is known as the most severe financial crisis since the Great Depression in the 1930s. It was partly due to the busting of the US housing bubble in 2005-2006. We used two time series to study the prediction of this crisis : the S&P500 index and the VIX index, that both were impacted by the crisis.

5.4.1 S&P500 index

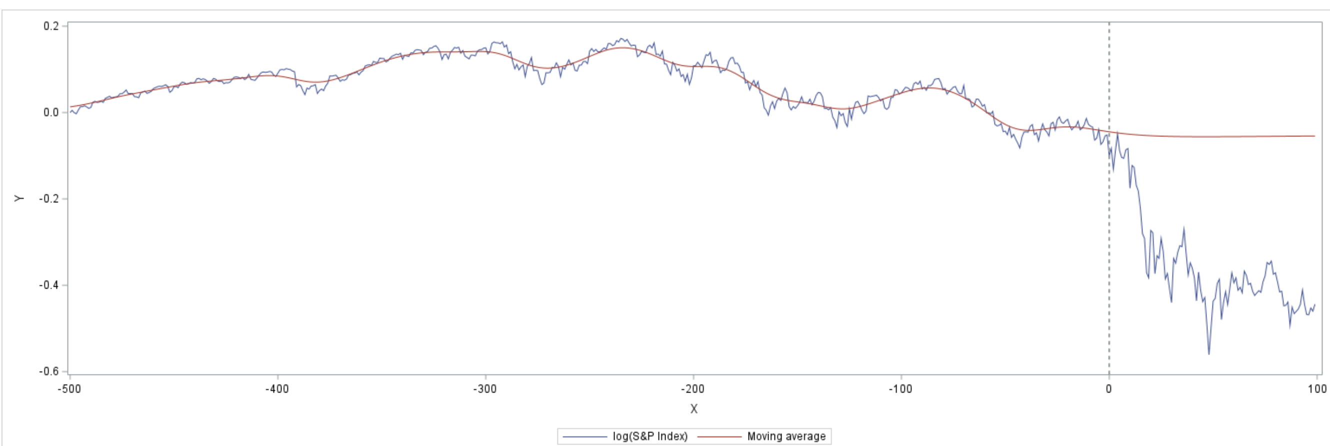


Figure 16: log(S&P 500 Index) and Smoothed time series

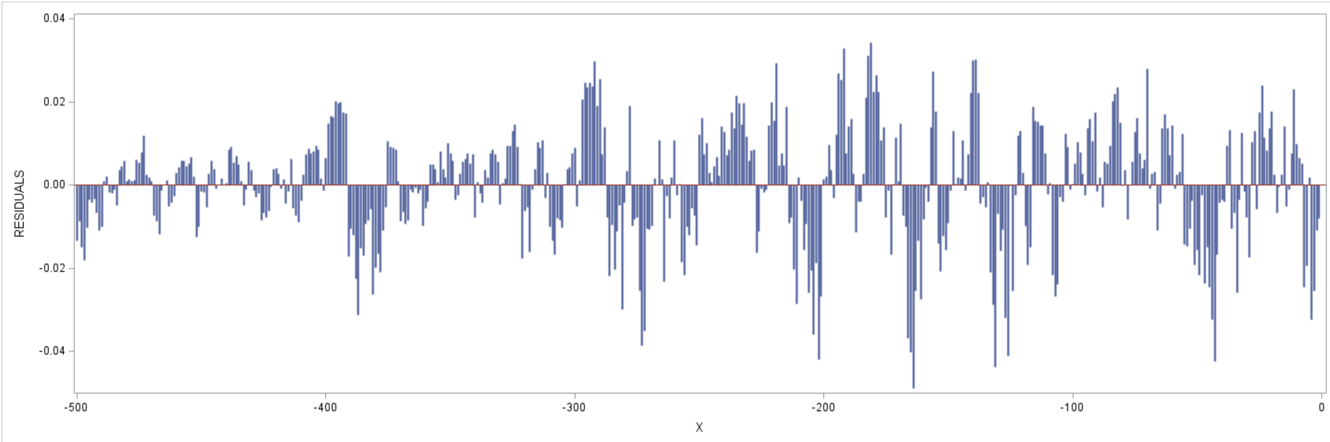


Figure 17: Residuals used to estimate the early warning signals

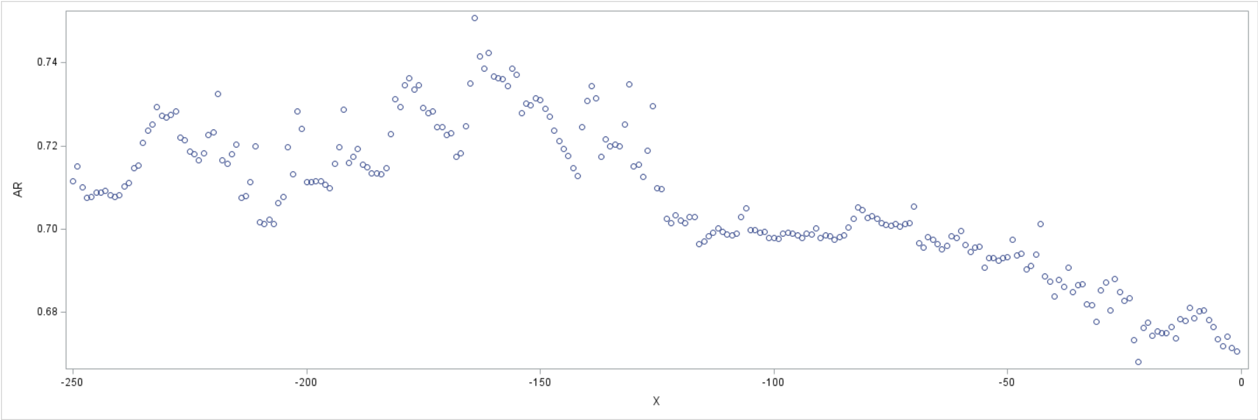
Figure 16 shows the fluctuations of the S&P500 index before and after the critical transition and shows the decline of the index after the vertical dashed line representing the critical point of the crisis.
The results of the figures representing the early warning indicators are mixed. Indeed, there is a significant strong upward trend for the standard deviation on Figure 18c and downward trends for the AR(1) and MI indicators on Figure 18a and 18b.

The skewness in Figure 18d using the S&P500 index decreases as we approach the tipping point of the 2008 Financial crisis, meaning that the probability distribution becomes highly negatively assymetric.

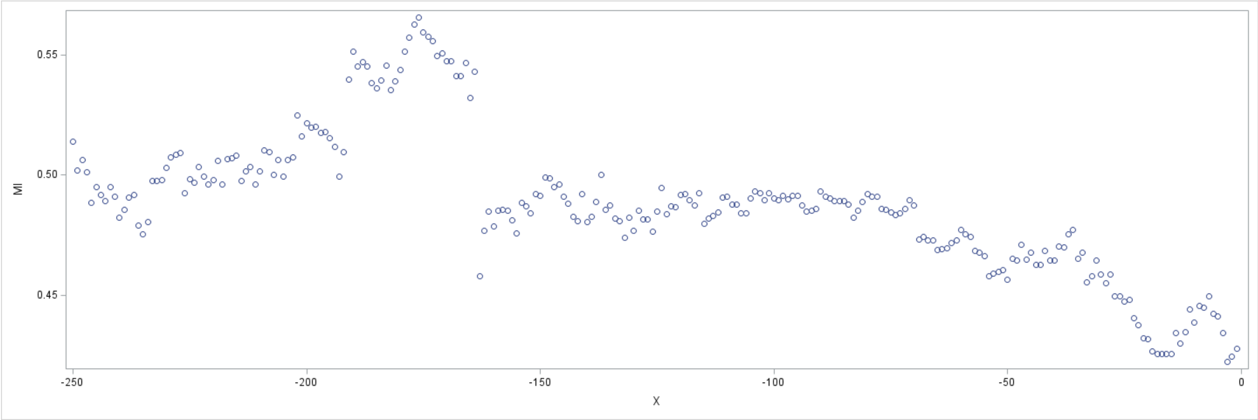
	AR(1)	Mutual Information	Standard Deviation	Skewness
Time	-0.59152 ($<$ .0001)	-0.57385 ($<$ .0001)	0.86043 ($<$ .0001)	-0.53921 ($<$ .0001)

Table 10: Kendall Tau b Correlation Coefficients for 2008 Financial Crisis (S&P 500), N=250

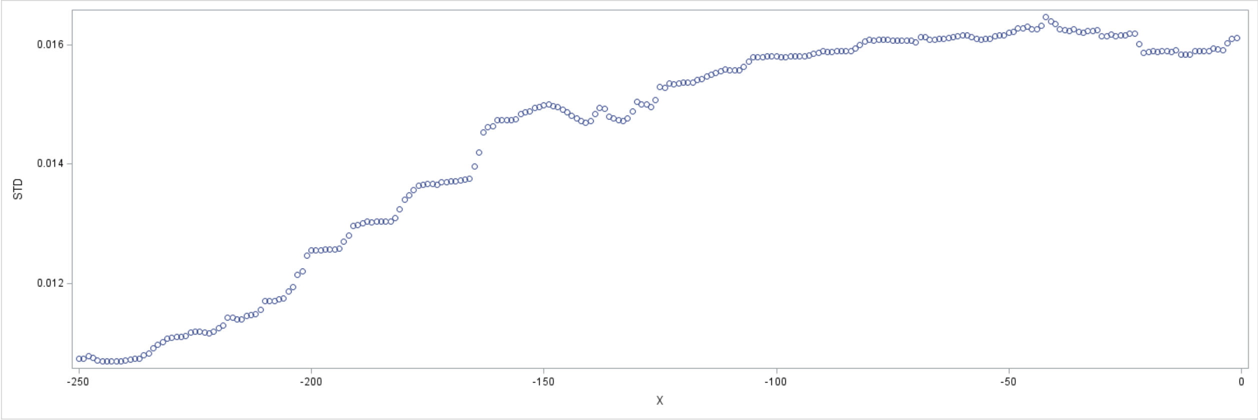
Table 10 relates the values and p-values of the Kendall's τ correlation coefficients for the Financial Crisis of 2008 with the S&P500 index. We can see in this table positive values for the Kendall's τ and the bootstrap p values taking into account the temporal dependence in the indicators are less than 0.0001. This means that the trends of the indicators of our analysis are all satistically significant at a 5% significance level.



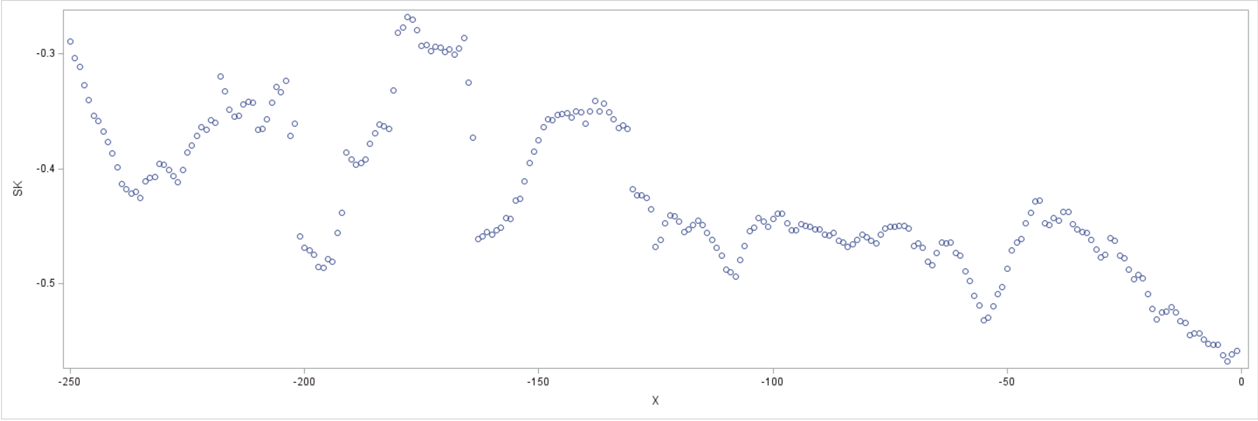
(a) AR(1)



(b) Mutual Information



(c) Standard Deviation



(d) Skewness

Figure 18: Indicators for 2008 Financial Crisis using S&P500 Index

5.4.2 VIX index

The results of the analysis of the 2008 Financial Crisis with the VIX index are quite different. Indeed, Figure 19 shows an increase in its values after the critical transition of the crisis (in the figure on the right of the vertical dashed line). The VIX is an estimated time series index of the volatility of the S&P 500 index over the next 30 days, which can explain this result of its log values and Moving Average.

The analysis of the VIX index and the early warning indicators chosen show a significant downward trend for the Standard Deviation on Figure 21c and a downward trend for the Mutual Information and the AR(1) on Figure 21b and 21a. As for most crises, the skewness of the VIX index in Figure 21d decreases, which means that the probability distribution is negatively skewed close to the tipping point.

	AR	MI	STD	SK
Time	-0.51544 (< .0001)	-0.15658 (0.0002)	-0.52540 (< .0001)	-0.84469 (< .0001)

Table 11: Kendall Tau b Correlation Coefficients for 2008 Financial Crisis (VIX), N=250

The values of Table 11 related to the Kendall's τ correlation coefficients for the Financial Crisis of 2008 with the VIX index are all negative. Moreover, the p-values associated are significant at a 5% significance for the 4 indicators, meaning that their trends are statistically significant.

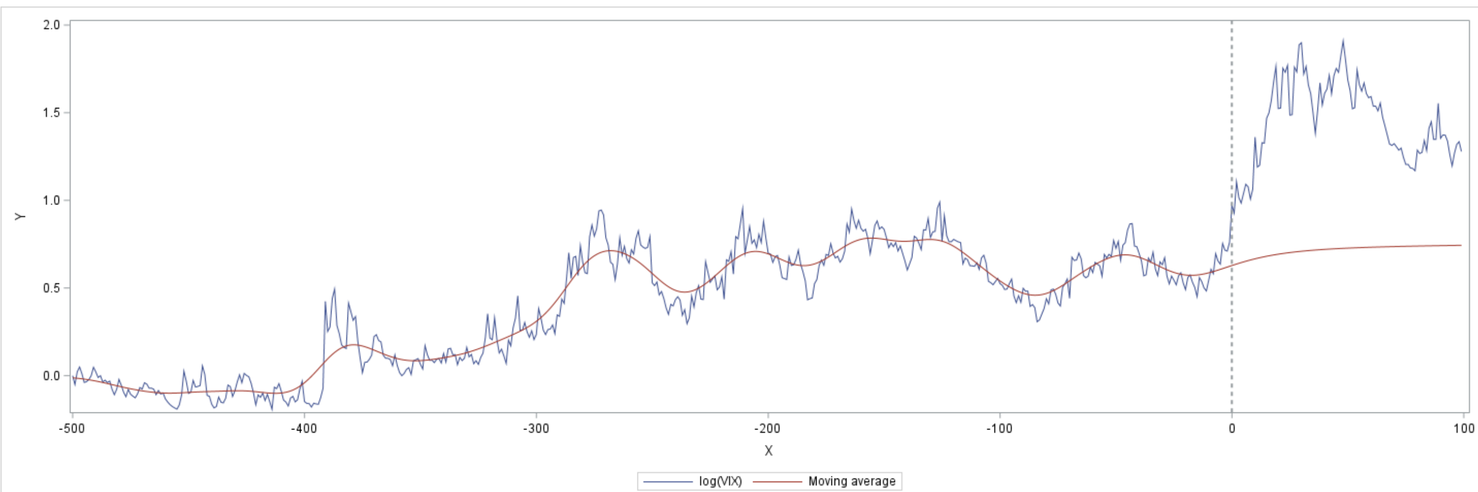


Figure 19: log(VIX) and Smoothed time series

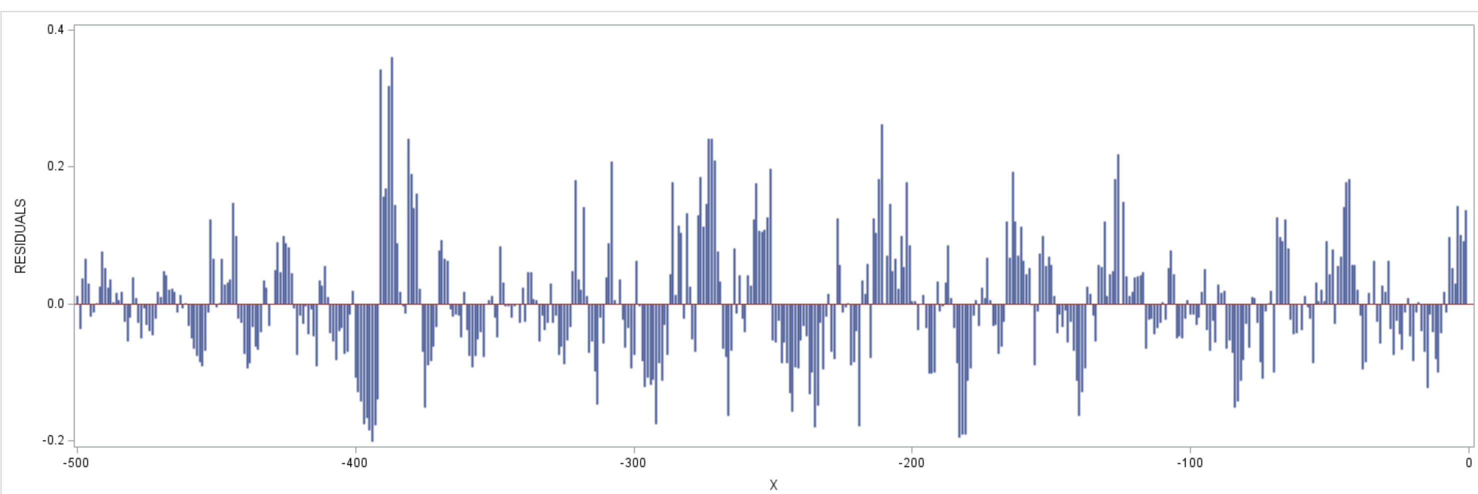
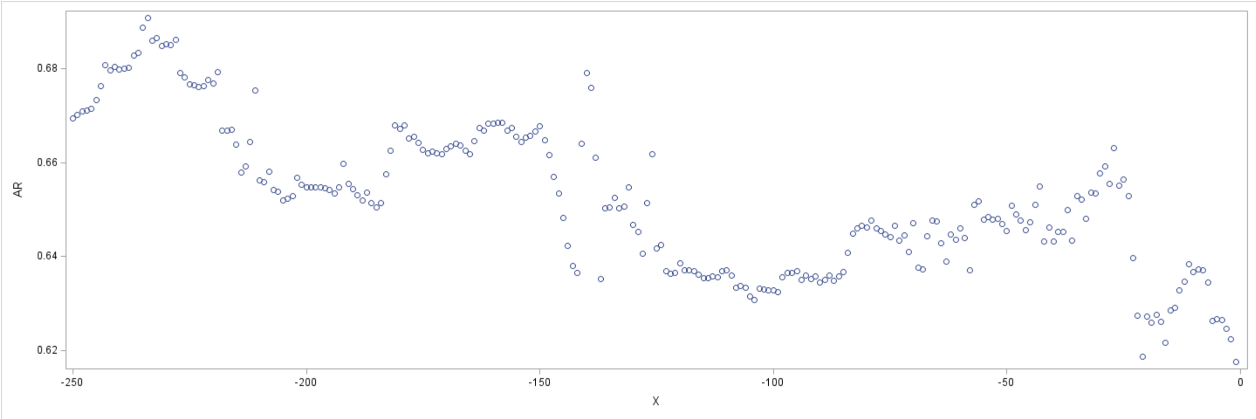
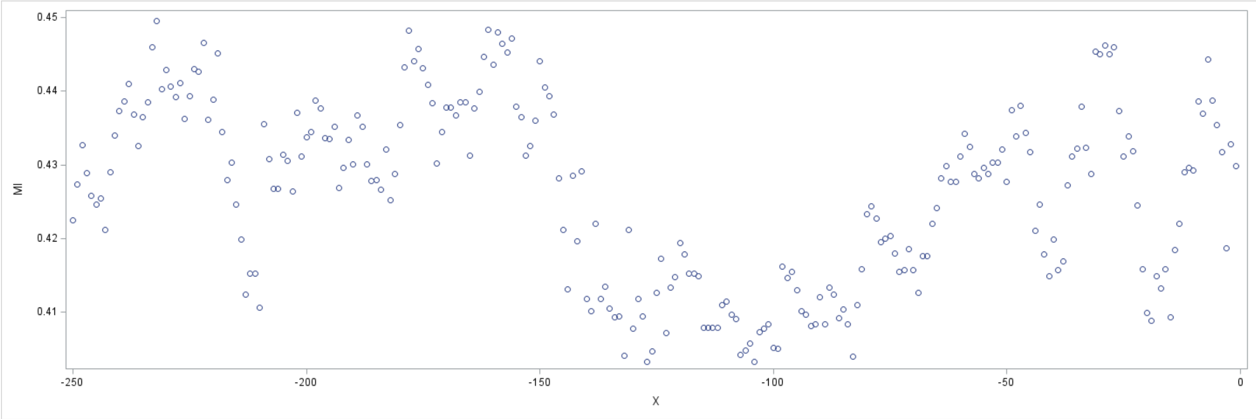


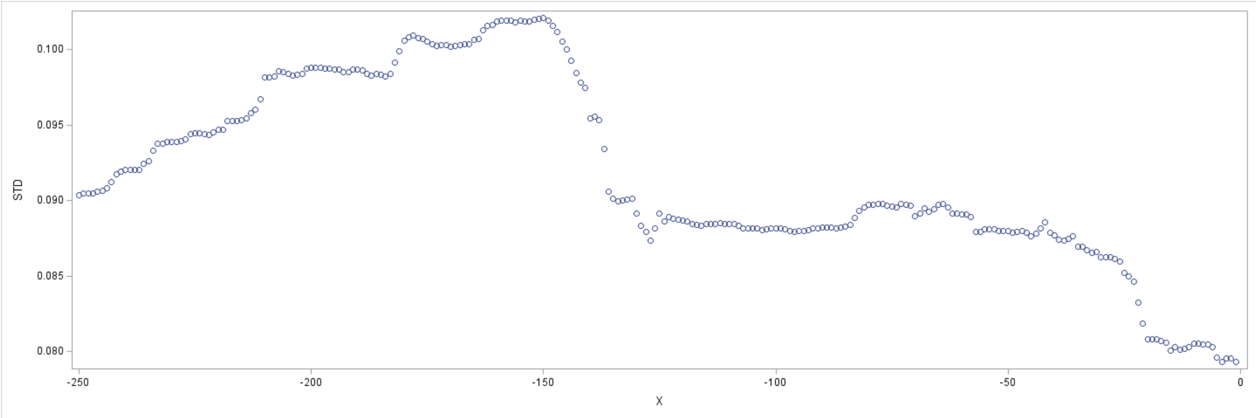
Figure 20: Residuals used to estimate the early warning signals



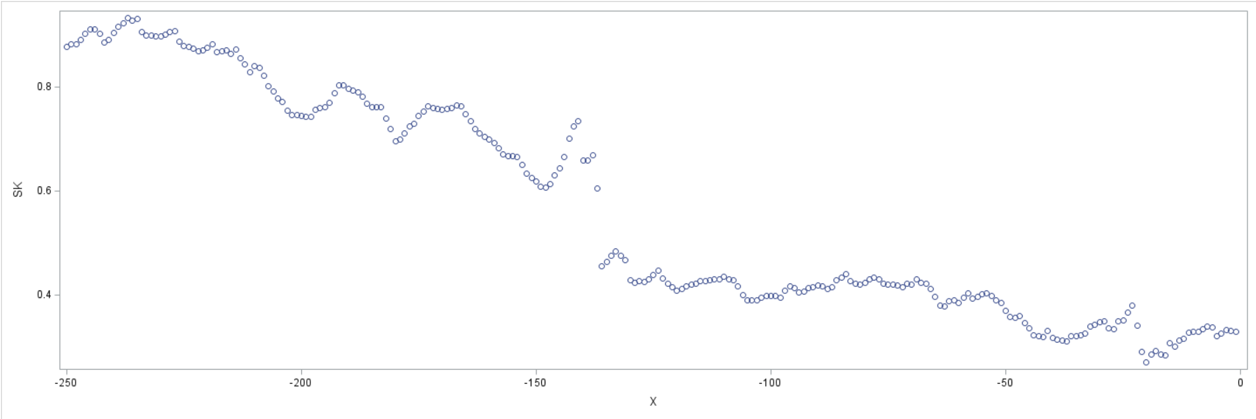
(a) AR(1)



(b) Mutual Information



(c) Standard Deviation

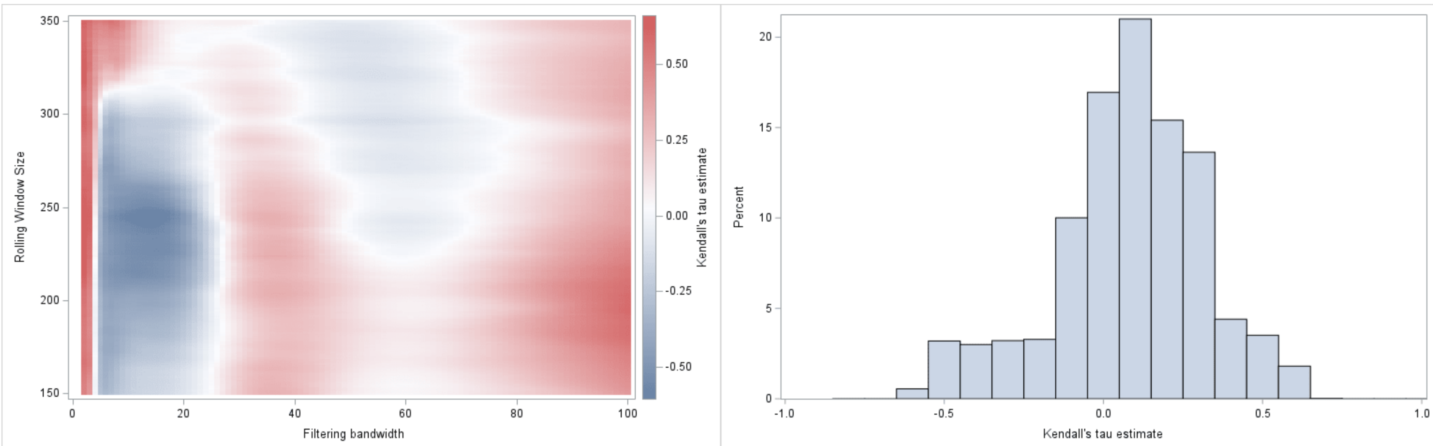


(d) Skewness

Figure 21: Indicators for 2008 Financial Crisis using VIX

5.5 Robustness of parameters

After running our analysis, it is very to run a test of robustness of the parameters. To do so, we used the observed trends of AR(1) indicators of the Black Monday crisis of 1987. The robustness is useful to study the trade-off of the size bandwith and of the estimation window size, that are crucial for the estimations and the detection of trends.



(a) Heatmap (b) Frequency

Figure 22: Test of robustness of parameters

Figure 22b is an histogram of Kendalll's tau values represented in Figure 22a. Both figures show the influence of the rolling window size and the filtering bandwith on the observed trends of AR(1) in the Black Monday crisis. The histogram shows evidence that the most positive values of Kendall's tau are on the contour plot. This test of robustness shows that the parameters we chose for our analysis really depends on the results we have found above. Moreover, we can see on the heatmap that the Kendall's Tau calculated with the values that we used for the sliding window size and the filtering bandwidth is located in the darkest areas, which means that these are good parameters that leads to strong negative trend.

6 Summary and Discussion

In this essay, we studied the role of early warning signals in the prediction of financial crises.

To conclude with the results we have found, we can notice that most of the crises have a negatively-skewed probability distribution close to the critical transition of the crisis, except for the Dot-Com crash that presents a skewness close to zero.

Most results from the early warning indicators chosen are mixed, but considering AR(1) and MI(1), they both show a downward trend close to the critical transition of crises. Concerning the standard deviation, there is little evidence that it is a good early warning indicator because some crisis show a significant upward trend, and others show a downward trend. So there is little evidence that financial crises are precedeed by critical slowing down.

On the four crises studied and the five time series examined, the only crisis that has evidence for critical slowing down is Black Monday of 1987 using as financial time serie the S&P500. In fact, it is the only crisis where the indicators shows a significant downward trend.

Then, it is difficult to conclude that the Critical Slowing down theory works for predicting crises using financial time series.

There are several reasons that can explain the fact that our analysis did not find evidence for critical slowing down for financial time series data.

The choice of smoothing bandwith, that is 10 trading days is an important parameter that plays on the results we have found.

Also, some studies used different variables for macroprudential policy (Coudert & Idier, 2016). They chose univariate indicators by adopting a signalling approach, and stood the idea that financial markets, credit and asset prices tends to move together cyclically. So they used variables such as credit, interest rates and analysid whether an indicator was relevant with the AUROC criterion.

We also decided in our essay to use a Gaussian filter to remove the trend from the time series, while we could have used another filter for the time series, such as the Kalman filter.

Finally, we did not take into account the herd behavior, that emerges due to the broken balance between autonomous behavior and peer influence (Moon & Lu, 2015). This herd behavior leads to the formation and collapse of speculative market bubble and it is an example of complex networks showing self-organization. Indeed, our analysis is based on one-dimensional systems while fiancial systems are way more complex. This does not enable us to really see the dynamics of financial systems and crises that result from it.

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