KLR modules in type $A_1^{(1)}$

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1 A couple of computations

Consider the unipotent cell of $\mathbb{C}[N]$ associated with the reduced word $(1,0,1,0)$. Then the $\bar{D}$ of the simple modules of the initial seed (i.e. the standard seed corresponding to the same reduced word) are given by

$$\frac{1}{\alpha_1} \quad \frac{1}{\alpha_1^2(\delta+\alpha_1)} \quad \frac{1}{\alpha_1^3(\delta+\alpha_1)^2(2\delta+\alpha_1)} \quad \frac{1}{\alpha_1^4(\delta+\alpha_1)^3(2\delta+\alpha_1)^2(3\delta+\alpha_1)}.$$

Conjecturally, when considering the reduced word $(1,0,1,0,\ldots,1,0)$ with n occurrences of 0 and 1 we will get fractions of the form

$$\frac{1}{\alpha_1^n(\delta+\alpha_1)^{n-1}\cdots((n-1)\delta+\alpha_1)}.$$