lyndon words

```
irreducible characters:
IC[1] = (q+q^-1)[1|2|2|3]+[1|2|3|2]+[2|1|2|3],
IC[2] = [2|1|3|2]+[2|3|1|2],
IC[3] = [2|1|2|3]+(q+q^-1)[2|2|1|3]+(q+q^-1)[2|2|3|1]+[2|3|2|1],
IC[4] = [1|2|3|2]+(q+q^-1)[1|3|2|2]+(q+q^-1)[3|1|2|2]+[3|2|1|2],
IC[5] = [2|3|2|1]+[3|2|1|2]+(q+q^-1)[3|2|2|1]

goodwords:
[ 2, 1, 2, 3 ],
[ 2, 3, 1, 2 ],
[ 2, 3, 2, 1 ],
[ 3, 2, 1, 2 ],
[ 3, 2, 2, 1 ]

polytopes:
conv(1, 1+2, 1+2+2, 1+2+3, 2, hw)
conv(2, 1+2, 1+2+3, 2+3, hw)
conv(2, 1+2, 1+2+2, 2+2, 2+2+3, 2+3, hw)
conv(1, 1+2, 1+2+3, 1+3,3, 2+3, hw)
conv(2, 2+3, 2+2+3, 3, 1+2+3, hw)

mv cycles:
?
?
?
?
?

equivariant multiplicities:
?
?
?
?
?
```

questions

1. Since $\alpha_1 + 2\alpha_2 + \alpha_3$ is the "normalized form" of $2\omega_2$, these (equivalent) lists should fit into $\mathcal{B}(2\omega_2)$ for A_3 but $\mathcal{B}(2\omega_2)$ has cardinality 20.
2. What are goodwords and do they uniquely determine their irreducible characters?
3. These shuffle characters clearly do determine certain MV polytopes. What is the precise question we are posing?

todo

1. implement polytope(goodword)
2. implement lusztig_datum(polytope)

I would like to know if it's possible to recover Pol(Z) from eqm(Z), mdeg(Z) or the torus multigraded Hilbert series of C[Z]. If these shuffle characters biject with dual semicanonical basis, and match Hilb C[Z], then I guess the answer is no. The counterexample from our appendix k

For example

```
IC[1]
=
(q^3+3*q+3*q^-1+q^-3)[1|2|2|3|3|4|4|5]+
          (q^2+2+q^-2)[1|2|2|3|3|4|5|4]+
          (q^2+2+q^-2)[1|2|2|3|4|3|4|5]+
              (q+q^-1)[1|2|2|3|4|3|5|4]+
              (q+q^-1)[1|2|2|3|4|5|3|4]+
          (q^2+2+q^-2)[1|2|3|2|3|4|4|5]+
              (q+q^-1)[1|2|3|2|3|4|5|4]+
              (q+q^-1)[1|2|3|2|4|3|4|5]+
                      [1|2|3|2|4|3|5|4]+
                      [1|2|3|2|4|5|3|4]+
              (q+q^-1)[1|2|3|4|2|3|4|5]+
                      [1|2|3|4|2|3|5|4]+
                      [1|2|3|4|2|5|3|4]+
                      [1|2|3|4|5|2|3|4]+
          (q^2+2+q^-2)[2|1|2|3|3|4|4|5]+
              (q+q^-1)[2|1|2|3|3|4|5|4]+
              (q+q^-1)[2|1|2|3|4|3|4|5]+
                      [2|1|2|3|4|3|5|4]+
              [2|1|2|3|4|5|3|4]+
          (q+q^-1)[2|1|3|2|3|4|4|5]+
              [2|1|3|2|3|4|5|4]+
                      [2|1|3|2|4|3|4|5]+
                      [2|1|3|4|2|3|4|5]+
              (q+q^-1)[2|3|1|2|3|4|4|5]+
                      [2|3|1|2|3|4|5|4]+
                      [2|3|1|2|4|3|4|5]+
                      [2|3|1|4|2|3|4|5]+
                      [2|3|4|1|2|3|4|5]
```