Definition 1. **Lusztig's stratification** of $\mathfrak{g}$

Definition 2. $S \subset \mathfrak{g}$ is a **transverse** or **normal** slice to a nilpotent orbit $\alpha \equiv \mathbb{O}_e$ at the element $e \in \mathfrak{g}$ if

(1) $T_e\alpha \oplus T_eS = \mathfrak{g}$
(2) there is a $\mathbb{G}_m$ action on S
 (a) contracting it to e, and
 (b) preserving **Lusztig's stratification** of $\mathfrak{g}$

Lemma 1. *Let L be a Lusztig stratum.*

- $S \cap \alpha = \{e\}$
- $S \cap L \neq \varnothing \iff \alpha \subset \overline{L}$
- *S intersects L transversely, i.e. for each $x \in S \cap L$, $T_xS \oplus T_xL = \mathfrak{g}$*

Proof.
- The first point is a consecutive of Definition 2 (2)?

□

Lemma 2. *The following data specifies a normal slice S to α at e.*

- *$h \in \mathfrak{g}^{ss}$ such that $ad(h)$ has integer eigenvalues, and $ad(h)(e) = 2e$*
- *$C \overset{\textbf{vect}}{\subset} \mathfrak{g}$ such that $C \cap T_e(\alpha) = C \cap [\mathfrak{g}, e] = 0$, $ad(h)(C) = C$, and if $ad(h)(x) = nx$ for some $x \in C$, then $n \leq 1$*

Proof. Anne: See Lemma 5.2.1, 5.4.2, but most importantly Lemma 4.3.1 of Riche's Kostant section and universal centralizer.

(1) $$e = \sum_{\Delta} e_\alpha; \quad \check{\lambda}_\circ := \sum \tilde{\Phi}^+ \check{\alpha}; \quad t \cdot x := t^{-2} \check{\lambda}_\circ(t) \cdot x$$

Apparently, we can lift the action of h on $\mathfrak{g}$ to a map $\mathbb{G}_m \to G$ and hence an action of $\mathbb{G}_m$ on $\mathfrak{g}$ which fixes e and preserves $e + C$. The element h defines a 1-parameter subgroup e^{sh} in G. □

MVy construct an isomorphism $\psi : T_x \cap \mathcal{N} \to T_b \cap \mathrm{Gr}_N$ where

- $T_x = \{x + f \big| f \in \mathrm{End}(D), [\text{conditions}]\}$
- $\mathcal{N}$ is the nilpotent cone in $\mathrm{End}(D)$
- $T_b := L^{<0}G(\mathcal{K})L_b$ which is the same as to use the notation that we're used to Gr_μ as $L^{<0}G(\mathcal{K}) = \ker(G(\mathbb{C}[z^{-1}]) \xrightarrow{z^{-1} \mapsto 0} G)$
- $\mathrm{Gr}_N = \{L \supset L_0 \big| \dim L/L_0 = N\}$