Weekly

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In these notes we embark on a journey across two major continent: Soergel bimodules and Iwahori–Hecke algebras.

Goals

- work out at least three constructions of bases in the baby sl_2 case
- do the exercises in the SBim textbook

Chapter 1 exercises

Exercise 1.5 Confirm that the set of all reflections agrees with the set of transpositions (i, j). For each transposition (i, j) with $i < j$, find an expression for it of lenght $2(j - i) - 1$.

Proof. First, map simple reflections to simple transpositions $s_i \mapsto (i, i+1)$. Next, extend this map to a homomorphism ϕ of W and S_n. To check that it extends, let $w \in W$ be an element which is conjugate to a simple reflection, i.e. $s_{i_1} \cdots s_{i_a} = w = u s_i u^{-1}$ for some $u = s_{j_1} \cdots s_{j_b} \in W$. Probably it suffices to check the case that W is a product of two simple reflections, so let's start there. If $w = s_p s_q = u \cdot s_i$, then $(p, p+1)(q, q+1) = \phi(u) \cdot (i, i+1)$. □