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Phase space diffusion and anisotropic pick-up ion distributions in the solar wind: an injection study

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Abstract. Pick-up ions are produced all over the interplanetary space by ionization of neutral interstellar atoms. Over the last decade it was generally expected that after pick-up these ions quickly are isotropized in the velocity space comoving with the solar wind by strong pitch-angle scattering, though not assimilating to the thermodynamic state of solar wind ions. Recent studies of pick-up ion data obtained with plasma analyzers on AMPTE and ULYSSES have, however, revealed that during extended time periods substantially anisotropic distributions prevail. In this paper we want to improve the understanding of the evolutionary behaviour of pick-up ions in interplanetary phase space by an pick-up ion injection study taking into account all relevant diffusion terms describing pitch angle scattering, adiabatic cooling, drifts and energy diffusion. For particles injected at 1 AU the resulting distribution function stays substantially anisotropic up to distances of the order of 6 AU, unless increased isotropic turbulence levels and non-dissipative wave spectra are considered. The average bulk velocity of pick-up ions within this distance range is smaller than the solar wind velocity by factors of between 0.6 and 1.0. Pick-ups are shown to substantially become spread out from that solar wind parcel into which they were originally injected. Derivations of interstellar gas parameters using pick-up ion flux data can only be dared with additional care in the interpretation. As a consequence the location of the LISM helium cone axis and the LISM helium temperature are faked in the associated He^+ pick-up ion flux pattern.

Key words: turbulence – acceleration of particles

1. Introduction to pick-up ion behaviour in velocity space

The phenomenon and existence of pick-up ions were already predicted in very early years (Semar (1970), Fahr (1971), Holzer (1972), Fahr (1973)), long before their later detection. It was already clear at these times that interstellar atoms when ionized in the interplanetary medium shall immediately be picked up by local electric induction forces and shall essentially be convected outwards with the solar wind, but it was fairly unclear at these times how these newly generated ions while comoving with the solar wind behave in velocity space.

Meanwhile, since already quite a long time now, such interplanetary pick-up ions also have been detected by space probes. The first detection of such suprathermal ion species succeeded in the years 1984/85 with the plasma analyzer SULEICA onboard the earth-bound satellite AMPTE (see Möbius et al., (1985)). In the years after 1984 many more identifications of cometary, lunar, planetary and interplanetary pick-up ions came up which need not be mentioned here in detail. Perhaps best suited for the study of the velocity-space behaviour of these ions were the recent measurements by the SWICS instrument onboard of the ULYSSES space probe (see Gloeckler et al., (1993), Geiss et al., (1994)). Arising from more recent interpretations of measurements of the SULEICA instrument (Möbius et al., (1996), (1997)) and of the SWICS instrument (Gloeckler et al., (1995), Fisk et al., (1997)) the theoretical discussion on the pick-up ion behaviour in velocity space was again restarted which began with papers by Wu & Davidson (1972), Wu et al. (1973), Hartle & Wu (1973), Wu & Hartle (1974).

These latter papers were studying the instability of the initial pick-up ion distribution against wave excitation and could show that due to the identified instabilities with respect to electrostatic or electromagnetic wave excitations the initial toroidal ring distribution of pick-up ions in the solar wind reference frame may decay into a broadened distribution with reduced values of pick-up ion velocities $v_{||}$ parallel and $v_{\perp}$ perpendicular to the solar wind magnetic field $\mathbf{B}$ within time periods of the order of $3 \cdot 10^2$ sec ($v_{||}$) and $4 \cdot 10^3$ sec ($v_{\perp}$), respectively. How much this decay in fact affects the intensity of pick-up ion resonance emissions in the extreme ultraviolet was analysed earlier by Paresce et al. (1983) and presently in more detail by Fahr et al. (1998) and Gruntman & Fahr (1998).

From the studies by Wu & Davidson (1972), Wu et al. (1973), and Hartle & Wu (1973) it could not be concluded into which specific forms the initial toroidal pick-up ion distribution will actually develop. It was only later tacitly assumed that most probably the toroidal will first develop into a spherical shell distribution under strong energy-conserving pitch-angle scattering processes and then will suffer from an adiabatic cooling effect

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operating in the expanding solar wind. The combined effect of both processes on the pick-up ion distribution function was first described by Vasyliunas & Siscoe (1976). In the period following this publication it had more or less generally been accepted that pick-up ion distributions first undergo a fast pitch angle scattering towards an isotropic function caused by resonant ion interactions with ambient hydromagnetic wave fields (Isenberg, (1987), Lee & Ip, (1987), Fahr & Ziemkiewicz, (1988), Bogdan et al., (1991). On the basis of this expectation also the energy diffusion of pick-up ions by Fermi–2 scattering processes were described in papers by Isenberg (1987), Bogdan et al. (1991), Chalov et al. (1995, 1997), Chalov & Fahr (1996) and it was demonstrated that pick-up ions experience substantial acceleration while being convected outwards with the solar wind.

Nevertheless, it was already recognized by Lee & Ip (1987) that the growth rates γ_t for linear wave excitations by thermally broadened, toroidal pick-up ion functions very much depend on the initial injection conditions (i.e the initial velocities $v_{0\|}$ and $v_{0\perp}$ of newly picked-up ions) since leading to an expression

$$\gamma_t = \sqrt{\frac{\pi}{8e}}\,\frac{v_A|v_{0\|}|\omega_i}{c^2}\,\left(\frac{\omega_i}{\Omega_P}\right)\left(\frac{v_{0\perp}}{c_i}\right)^2, \tag{1}$$

where ω_i and c_i are the plasma frequency and the thermal velocity spread of the pick-up ions, and where v_A and Ω_P are the local Alfvén speed and the proton gyrofrequency. The above expression shows that the growth rates become vanishingly small for vanishing velocities $v_{0\|,\perp}$, meaning that a toroidal function around pitch angle cosines $\mu_0 = \cos\xi_0 = v_{0\|}/\sqrt{v_{0\|}^2 + v_{0\perp}^2} \approx 0$ and $\mu_0 = 1$ are marginally stable. This may be interpreted as saying that whenever pitch angle scattering brings ions from $\mu \le 0$ towards $\mu \approx 0$ then it may become inefficient. It also may point to the fact that pitch angle scattering is not likely to produce complete shell distributions. The evolutionary tendencies of noncomplete shell distributions have been explicitly studied by Freund & Wu (1988) were it has been demonstrated that shell distributions are the less unstable the more complete they are, meaning that completion of the shell and establishment of an isotropic distribution function may be a very time-consuming asymptotic process.

In view of both these theoretical results and most recent pick-up ion flux observations Gloeckler et al. (1995), Isenberg (1997), Möbius et al. (1996, 1997) and Fisk et al. (1997) have restarted the discussion on incomplete and anisotropic pick-up ion distributions. Möbius et al. (1997) have registered pick-up ion flux reductions over periods when the magnetic tilt angle with respect to the solar wind velocity is small (radial field). They ascribe this phenomenon to the fact that at these events when pick-up ions are exclusively injected into the sunward hemisphere of the velocity space substantially anisotropic distribution functions may prevail so that the SULEICA plasma analyzer then may miss essential fractions of the pick-up ions passing over the detector. Fisk et al. (1997) have also analysed these events of reduced fluxes in terms of hemispherical pick-up ion populations underlying differential exchange processes by pitch angle scatterings and can show that the flux reductions seen with SWICS can be explained with effective scattering mean free paths eventually increasing to values of about 1 AU meaning that pitch angle scattering at some periods may be fairly inefficient. Under these auspices we have considered it highly worthwhile to more thoroughly study the evolution of pick-up ion distribution functions taking into account all forms of diffusion processes in configuration and velocity space. In the present work we shall present the first results of these studies demonstrating the effect of general phase space diffusion processes of pick-up protons which are locally injected at one specific event of time.

2. Governing equation and method of solution

The relevant transport equation to describe the evolution of gyrotropic velocity distribution functions $f = f(t, \mathbf{x}, v, \mu)$ of pick-up ions in a background plasma moving at a velocity $\mathbf{U}$ can be written in the following general form (Skilling, (1971), Isenberg, (1997)

$$\begin{aligned}\frac{\partial f}{\partial t} + (U_i + v\mu b_i)\,\frac{\partial f}{\partial x_i} + \Bigg[\frac{1-3\mu^2}{2} b_i b_j\,\frac{\partial u_j}{\partial x_i} - {}\\ \frac{1-\mu^2}{2}\frac{\partial U_i}{\partial x_i} - \frac{\mu b_i}{v}\left(\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j}\right)\Bigg] v\frac{\partial f}{\partial v} + {}\\ \frac{1-\mu^2}{2}\Bigg[v\frac{\partial b_i}{\partial x_i} + \mu\frac{\partial U_i}{\partial x_i} - 3\mu b_i b_j\frac{\partial U_j}{\partial x_i} - {}\\ \frac{2b_i}{v}\left(\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j}\right)\Bigg]\frac{\partial f}{\partial \mu} = \hat{S}\,f + Q(t, \mathbf{x}, v, \mu),\end{aligned} \tag{2}$$

where $\mathbf{x}$ is the vector of spatial variables in an inertial frame (solar rest frame), v and $\mu = \cos(\xi)$ are the velocity and cosine of the particle pitch angle ξ in the solar wind rest frame, $\mathbf{b}$ is the unit vector of the large-scale magnetic field, Q is the local production rate of particles, and $\hat{S}f$ is the scattering operator applied to the function f (see e.g. Schlickeiser, 1989)

$$\begin{aligned}\hat{S}f \;=\; \frac{\partial}{\partial\mu}\left(D_{\mu\mu}\frac{\partial f}{\partial\mu}\right) + \frac{\partial}{\partial\mu}\left(D_{\mu v}\frac{\partial f}{\partial v}\right) + {}\\ \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 D_{\mu v}\frac{\partial f}{\partial \mu}\right) + \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 D_{vv}\frac{\partial f}{\partial v}\right).\end{aligned} \tag{3}$$

In Eq. (3) $D_{\mu\mu}$, $D_{\mu v}$ and D_{vv} are the respective Fokker-Plank diffusion coefficients which will be specified below.

Within the present paper we shall study the evolution of pick-up ion distribution functions in the solar wind environment under the following simplifying assumptions:

1. The solar wind flow is assumed to be a stationary spherically-symmetric radial flow with U=const. Thus the latitudinal dependences of the solar wind velocity and number density are ignored here. Nevertheless the distribution function of pick-up ions is found as a function of latitude because of the latitudinal dependence of the interplanetary magnetic field, i.e its tilt angle ψ with respect to the radial direction as function of $\mathbf{x}$.

2. The source term in Eq. (2) and hence the distribution function do not depend on longitude ϑ.

In a forthcoming study we shall also take into account these two abovementioned complications. The large-scale interplanetary magnetic field is assumed to have the standard Parker spiral configuration

$$B_r = B_{rE}\left(\frac{r_E}{r}\right)^2,\ B_\vartheta = 0,\ B_\varphi = B_{rE}\,\frac{r_E}{r}\omega\,\sin(\vartheta), \tag{4}$$

where B_{rE} = const (see e.g. ULYSSES data in Balogh et al., (1995); Smith & Balogh, (1995), $\omega = r_E\Omega/U(\Omega = 2.7\cdot 10^{-6}c^{-1})$, $r_E = 1$ AU, and ϑ is the polar angle (colatitude) in the heliocentric spherical coordinate system (r,ϑ,φ). With these assumptions the general transport Eq. (3) can be written in the form

$$\begin{aligned}\frac{\partial f}{\partial t} + (U + v\mu\chi)\,\frac{\partial f}{\partial r} + \\ \left[\frac{1-3\mu^2}{2}\frac{1-\chi^2}{r} - \frac{1-\mu^2}{r}\right] U\,v\,\frac{\partial f}{\partial v} + \\ \frac{1-\mu^2}{2}\left[\frac{v}{r^2}\frac{d}{dr}\left(r^2\chi\right) + \frac{2U}{r}\mu - 3\mu U\frac{1-\chi^2}{r}\right]\frac{\partial f}{\partial \mu} \\ = \hat{S}\,f + Q(t,\mathbf{x},v,\mu),\end{aligned} \tag{5}$$

where $\chi = \cos(\psi)$ and ψ = arctg$(\omega\,\sin(\vartheta)\,r/r_E)$. Here and in the following an outward pointing magnetic field is adopted. The second term on the left-hand side of Eq. (5) describes the convective motion of particles, the third term corresponds to the so-called adiabatic cooling, and the fourth one to adiabatic focusing.

Since Eq. (5) does not contain a derivative of f with respect to ϑ it can be solved by specifying the latitude and integrating with respect to r. If in addition we introduce the differential number density of particles in phase space r, v, μ with $N = 2\pi r^2 v^2 f$, Eq. (5) can then be rewritten in the conservation-law form as

$$\begin{aligned}\frac{\partial N}{\partial t} &= -\frac{\partial}{\partial r}(A_1 N) - \frac{\partial}{\partial v}(A_2 N) - \frac{\partial}{\partial \mu}(A_3 N) \\ &+ \frac{\partial^2}{\partial v^2}(D_{vv}N) + 2\frac{\partial^2}{\partial v\partial\mu}(D_{\mu v}N) + \frac{\partial^2}{\partial\mu^2}(D_{\mu\mu}N) \\ &+ 2\pi r^2 v^2\,Q(t,r,\vartheta,v,\mu),\end{aligned} \tag{6}$$

where

$$A_1 = U + v\mu\chi, \tag{7}$$

$$\begin{aligned}A_2 = \left[\frac{1-3\mu^2}{2}\frac{1-\chi^2}{r} - \frac{1-\mu^2}{r}\right] U\,v + \\ \frac{1}{v^2}\frac{\partial}{\partial v}(v^2 D_{vv}) + \frac{\partial D_{\mu v}}{\partial \mu},\end{aligned} \tag{8}$$

$$\begin{aligned}A_3 = \frac{1-\mu^2}{2}\left[\frac{v}{r^2}\frac{d}{dr}\left(r^2\chi\right) + \frac{2U}{r}\mu - 3\mu U\frac{1-\chi^2}{r}\right] + \\ \frac{1}{v^2}\frac{\partial}{\partial v}(v^2 D_{\mu v}) + \frac{\partial D_{\mu\mu}}{\partial \mu}.\end{aligned} \tag{9}$$

Eq. (6) can either be solved directly by use of a finite-difference method or by an alternative approach calculating stochastic trajectories of particles in phase space. In this last case Eq. (6) is replaced by an equivalent system of stochastic differential equations (SDE's) (concerning the mathematical equivalence between Fokker-Plank equations and SDE's one may study e.g. Gardiner, (1990)). The SDE method has been used, for example, to model transport and acceleration of electrons (MacKinnon & Craig, (1991)), cosmic rays (Achterberg & Krülls, (1992); Krülls & Achterberg, (1994)), and pick-up ions (Chalov et al., (1995, 1997); Chalov & Fahr, (1996); Fichtner et al., (1996)). It can be shown that Eq. (6) is equivalent to the following system of SDE's (for more details see Chalov et al., (1997))

$$dr = A_1\,dt, \tag{10}$$

$$dv = A_2\,dt + (\lambda\eta/2 + U\,D_{\mu\nu}/\eta)\;dW_1 + \eta\,dW_2, \tag{11}$$

$$d\mu = A_3\,dt + \frac{1}{U}\left[\left(\frac{\eta}{U}\right)dW_1 + \left(\frac{U\,D_{\mu\nu}}{\eta} - \frac{\lambda\eta}{2}\right)dW_2\right], \tag{12}$$

where

$$\lambda = \frac{D_{vv} - U^2 D_{\mu\mu}}{U\,D_{\mu v}}, \tag{13}$$

$$\eta = \sqrt{\frac{(D_{vv} + U^2 D_{\mu\mu})/2 + \sqrt{U^2 D_{\mu\mu}D_{vv} - (UD_{\mu v})^2}}{\lambda^2/4 + 1}}.$$

In Eqs. (11-12) $\mathbf{W}(t) = W_1(t), W_2(t)$ represents a two-variable Wiener process (see e.g. Gardiner, (1990)) describing uncorrelated variable fluctuations with Gaussian distributions, and $d\mathbf{W}(t) = \mathbf{W}(t+dt) - \mathbf{W}(t)$ is a vector describing two statistically independent differential increments dW_1 and dW_2 satisfying a Gaussian probability distribution with an e-folding-width increasing with time dt according to

$$(d\mathbf{W}) = \frac{1}{2\pi dt}\exp\left(-\frac{d\mathbf{W}^2}{2dt}\right). \tag{14}$$

This distribution itself results as the solution of a Fokker-Planck type diffusion equation.

Each solution of Eqs. (10-12) is a stochastic trajectory in phase space. To obtain the differential number density of particles, N, we must simulate a statistically relevant set of stochastic trajectories and determine the corresponding density of these trajectories in phase space. To integrate Eqs. (10-12) numerically a simple Cauchy-Euler finite-difference method can be used.

3. Diffusion coefficients

In the present paper only ion interactions with nondispersive parallel and antiparallel propagating Alfvén waves are considered. In the case of nonrelativistic particles the quasilinear Fokker-Planck coefficients can then be written in the following form (Schlickeiser, 1989)

$$D_{\mu\mu} = \frac{\pi\Omega_c^2}{2B^2}\left(1-\mu^2\right)\left[\left(1-\frac{\mu v_A}{v}\right)^2 \alpha_+(\mu) + \left(1+\frac{\mu v_A}{v}\right)^2 \alpha_-(\mu)\right], \quad (15)$$

$$D_{\mu v} = \frac{\pi\Omega_c^2}{2B^2}\left(1-\mu^2\right) v_A\left[\left(1-\frac{\mu v_A}{v}\right) \alpha_+(\mu) - \left(1+\frac{\mu v_A}{v}\right) \alpha_-(\mu)\right], \quad (16)$$

$$D_{vv} = \frac{\pi\Omega_c^2}{2B^2}\left(1-\mu^2\right) v_a^2\left[\alpha_+(\mu)+\alpha_-(\mu)\right], \quad (17)$$

where $\Omega_c = eB/mc$ is the ion gyrofrequency, v_A is the Alfvén speed. The quantities $\alpha_+(\mu)$ and $\alpha_-(\mu)$ are connected with particle-resonating waves propagating outwards and inwards, respectively, and are given by the following expressions

$$\alpha_+(\mu) = \frac{1}{|v\mu - v_A|}\left[I^L\left(k=-\frac{\Omega_c}{v\mu - v_a}\right) + I^R\left(k=+\frac{\Omega_c}{v\mu - v_a}\right)\right], \quad (18)$$

$$\alpha_-(\mu) = \frac{1}{|v\mu + v_A|}\left[I^L\left(k=-\frac{\Omega_c}{v\mu + v_a}\right) + I^R\left(k=+\frac{\Omega_c}{v\mu + v_a}\right)\right]. \quad (19)$$

In Eqs. (18) and (19) k is the wavenumber, I^L and I^R are the differential intensities of left-hand and right-hand circularly polarized Alfvén waves, respectively, so that the mean-squared amplitude of the associated fluctuations is

$$< \delta B^2 > = \int\limits_{k_{min}}^{\infty} \left(I_+^L + I_-^L + I_+^R + I_-^R\right)\, dk, \quad (20)$$

where k_{min} corresponds to the largest scale of energy containing fluctuations and the signs $(\pm)$ correspond to parallel and antiparallel moving waves.

A representative power spectrum of hydromagnetic field fluctuations in the solar wind consists of the inertial and dissipative ranges (e.g. see Zhou & Matthaeus, (1990), Miller & Roberts, (1995)). It is assumed here that the dissipative range has an exponential form, so that the differential intensity can be written as (see Bieber et al., (1988); Smith et al., (1995); Schlickeiser et al., (1991))

$$I_\pm^{L,R}(k) = I_{0\pm}^{L,R}(k)\; k^{-q}\, \exp\left(-k l_d\right), \quad (\text{for } k > k_{min}), \quad (21)$$

where l_d is the dissipative scale. In the following we will consider the case when left- and right-hand polarized waves have the same intensities, that is $I_{0\pm}^L = I_{0\pm}^R = I_0^\pm$. If we now substitute Eqs. (18), (19), and (21) into Eqs. (15)-(17) taking into account that $k_{min} \ll \Omega_c/(v+v_A)$, we obtain

$$\begin{aligned} D_{\mu\mu} \;=\; & \frac{U}{r_E} D_0\left(1-\mu^2\right)\Bigg(\left(1-\mu v_A/v\right)^2 \times \\ & |1-\mu v/v_A|^{q-1}\, \exp\left[-\frac{L}{|1-\mu v/v_A|}\right] \\ & + \epsilon\left(1+\mu v_A/v\right)^2 \times \\ & |1+\mu v/v_A|^{q-1}\, \exp\left[-\frac{L}{|1+\mu v/v_A|}\right]\Bigg), \end{aligned} \quad (22)$$

$$\begin{aligned} D_{\mu v} \;=\; & \frac{U}{r_E} v_A D_0\left(1-\mu^2\right)\Bigg(\left(1-\mu v_A/v\right) \times \\ & |1-\mu v/v_A|^{q-1}\, \exp\left[-\frac{L}{|1-\mu v/v_A|}\right] \\ & - \epsilon\left(1+\mu v_A/v\right) \times \\ & |1+\mu v/v_A|^{q-1}\, \exp\left[-\frac{L}{|1+\mu v/v_A|}\right]\Bigg), \end{aligned} \quad (23)$$

$$\begin{aligned} D_{vv} \;=\; & \frac{U}{r_E} v_A^2 D_0\left(1-\mu^2\right) \times \\ & \Bigg(|1-\mu v/v_A|^{q-1}\, \exp\left[-\frac{L}{|1-\mu v/v_A|}\right] \\ & + \epsilon\, |1+\mu v/v_A|^{q-1}\, \exp\left[-\frac{L}{|1+\mu v/v_A|}\right]\Bigg). \end{aligned} \quad (24)$$

In Eqs. (22)-(24) we have used the following denotations

$$L = \frac{\Omega_c l_d}{v_A}, \qquad \epsilon = I_0^-/I_0^+, \quad (25)$$

$$\begin{aligned} D_0 \;=\; & \frac{\pi}{4}\frac{\Omega_c r_E}{U}\left(\frac{v_A k_{min}}{\Omega_c}\right)^{q-1} \times \\ & \frac{1}{(1+\epsilon)\, E_q(l_d k_{min})}\; \frac{< \delta B^2 >}{B^2}, \end{aligned} \quad (26)$$

where $E_q(z)$ denotes the exponential integral

$$E_q(z) = \int\limits_1^\infty x^{-q}\exp(-z\,x)\, dx. \quad (27)$$

To derive Eq. (26) for the diffusion coefficient D_0 Eq. (20) has been used. The parameter ϵ is related to the cross helicity h_c of Alfvénic turbulence as $h_c = (1-\epsilon)/(1+\epsilon)$.

4. Parameters of the problem

Taking into account Eq. (4) we obtain for the cosine of the angle ψ between the magnetic field vector and the radial direction

$$\chi \equiv \cos(\psi(r,\vartheta)) = \frac{1}{\sqrt{1+\omega^2(r/r_E)^2\sin^2(\vartheta)}}. \quad (28)$$

Furthermore we shall assume that

$$< \delta B^2 >=< \delta B_E^2 > \left(\frac{r_E}{r}\right)^{\alpha}, \qquad (29)$$

where $< \delta B_E^2 >$ and α in view of ULYSSES observations may depend on the colatitude ϑ (see Forsyth et al., (1996). Then by use of Eq. (4) we shall have for the last factor on the right-hand side of Eq. (26)

$$\frac{< \delta B^2 >}{B^2} = \frac{< \delta B_E^2 >}{B_E^2} \frac{1}{H^2} \left(\frac{r_E}{r}\right)^{\alpha-2}, \qquad (30)$$

where $B_E = B(r_E, \vartheta)$, and

$$H(r,\vartheta) = \sqrt{\frac{(r_E/r)^2 + \omega^2 \sin^2(\vartheta)}{1+\omega^2 \sin^2(\vartheta)}}. \qquad (31)$$

It is known from ULYSSES observations that both $< \delta B_E^2 >$ and B_E^2 are latitude-dependent values (see e.g. Forsyth et al., (1996)). But we shall assume here that the ratio $\delta =< \delta B_E^2 > /B_E^2$ essentially behaves as a constant. On the other hand we shall consider the magnitude of this value as a free parameter. By use of Eq. (4) and adopting an electron number density according to $n_e = n_{eE}\,(r_E/r)^2$, where n_{eE} = const, we can obtain

$$v_A(r,\vartheta) = h(r,\vartheta)\, v_{A0}. \qquad (32)$$

In Eq. (32) $v_{A0} = v_A(r_E, \pi/2)$, and

$$h(r,\vartheta) = \sqrt{\frac{(r_E/r)^2 + \omega^2 \sin^2(\vartheta)}{1+\omega^2}}. \qquad (33)$$

Furthermore for this paper we shall specify pick-up ions as pick-up protons. The ratio v_A/Ω_c for protons can be written as

$$\frac{v_A}{\Omega_c} = 2.3 \cdot 10^7\, (n_e/1\ \mathrm{cm}^{-3})^{-1/2}\mathrm{cm}. \qquad (34)$$

All calculations presented here were carried out using the following values for the basic parameters: $U = 450$ km/s, $v_{A0} = 50$ km/s, $n_{eE} = 7$ cm^{-3}, $q = 5/3$. Also it was assumed that $k_{min} = 10^{-11}$ cm^{-1} (see Belcher & Davis, (1971); Jokipii & Coleman, (1968)), and $l_d = 3 \cdot 10^7$ cm (Schlickeiser et al., (1991)). Then the dimensionless value L (see Eq. (25)) can be written as $L = 3.45 r_E/r$. Calculations with $L = 0$ (the dissipative range is absent) were also carried out. What concerns the spatial dependence of $< \delta B^2 >$ (see Eq. (29)) we assume that $\alpha = 3$. This dependence follows from the WKB theory for the transport of nondissipative Alfvén waves in the solar wind (Hollweg, (1974)) and is close to observations with the HELIOS spacecraft (Tu & Marsch, (1995)).

It can be shown from the definition (27) that at $z \ll 1$

$$E_q(z) = \frac{1}{q-1} + O(z^{q-1}).$$

Since $z \equiv l_d k_{min} = 3 \cdot 10^{-4}$ we can admit here that $E_{5/3}(z) \approx 1.5$. Thus the diffusion coefficient D_0 (see Eq. (26)) depends only on two free parameters, namely $\epsilon = I_0^-/I_0^+$ and $\delta = < \delta B^2 > /B_E^2$.

5. Numerical results

In this first paper on pitch-angle and velocity diffusions of pick-up ions in the solar wind we consider the time evolution of particles injected at $t = 0$ on a spherical shell with radius $r = r_E$. One should have in mind that this can only somehow approximate the reality for earth-injected H-pick-up ions of geocoronal origin or for the case of He^+-pick-up ions mainly originating at regions close to the sun, while heliospheric pick-up protons in order to be described realistically would need the adequate consideration of a continuous injection all over the heliosphere according to the actual, local injection rates. However, for the above mentioned injection the source term in Eq. (5) attains the following form

$$\begin{aligned} Q(t,r,\vartheta,v,\mu) \ &= \ Q_0(\vartheta)\,\delta(t)\,\delta(r-r_E) \times \\ &\quad \delta(v-U)\,\delta(\mu-\mu_0(\vartheta)), \end{aligned} \qquad (35)$$

where $\mu_0(\vartheta) = -\chi(r_E, \vartheta)$ (see Eq. (28)) and $\delta(x)$ is the Dirac delta function. This means that here we assume an injection of pick-up ions at $t = 0$ with vanishing velocity in the solar frame (i.e the original peculiar velocity of the parent neutral atom is taken to be negligibly small in comparison with the solar wind velocity U). Thus here we intend to treat a clean test case to be able to study a clearcut response to all diffusion effects which are operating. In a forthcoming paper we shall then also consider realistic, spatially distributed sources and continuous production rates.

The system of SDE's given by Eqs. (10-12) has been solved numerically by use of the Cauchy-Euler finite-difference method for a statistically relevant set of particles (in the present calculations about 10^5 particles have been used). A set of calculations has been carried out for various values of the governing parameters to find out their role in determining the phase space evolution of pick-up protons in the solar wind. The essential information on all runs is presented in Table (1). Column 1 gives the run number. Columns 2, 3, and 4 give the level of Alfvénic turbulence at the Earth's orbit $< \delta B_E^2 > /B_E^2$, the ratio of abundances of backward and forward moving waves $\epsilon = I_0^-/I_0^+$, and the dissipative scale l_d, respectively. The last column gives the colatitude (n.b.: $\vartheta = 90°$ corresponds to the ecliptic plane).

Figs. 1–5 show spatial distributions of pick-up protons at subsequent times τ = 1, 2, 3, 4, 5, where the dimensionless time $\tau = U\,t/r_E$ (i.e the solar wind passage time over 1 AU) is introduced. Consequently a solar wind parcel, started at τ = 0 from r = 1 AU, reaches at times τ = 1, 2,... solar distances of r = 2, 3,...AU. The number density F of particles in Figs. 1–5 is given by the formula

$$F(t,r) = C_1 \int_0^{\infty} \int_{-1}^{+1} v^2 f\, dv d\mu, \qquad (36)$$

where the constant C_1 is defined with the normalization

$$\int_0^{\infty} r^2 F\, dr = 1. \qquad (37)$$

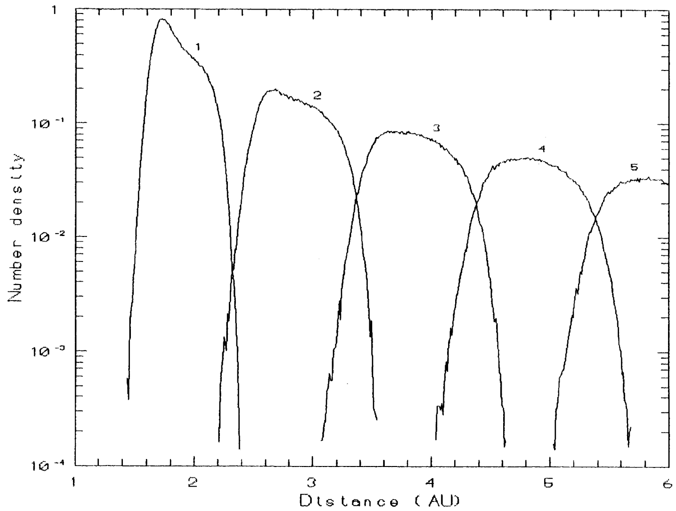

Fig. 1. Spatial distributions of pick-up protons injected at τ = 0 and $r = r_E$. The numbers denote the subsequent moments: τ = 1, 2, 3, 4, 5. a) run 1.

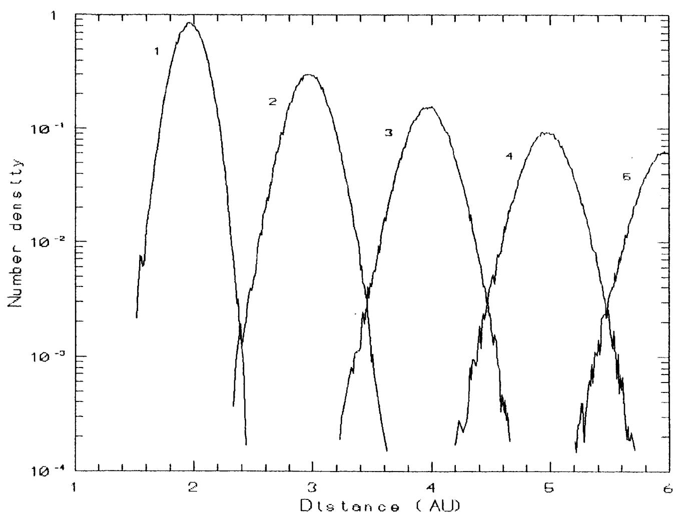

Fig. 2. Spatial distributions of pick-up protons injected at τ = 0 and $r = r_E$. The numbers denote the subsequent moments: τ = 1, 2, 3, 4, 5. b) run 2.

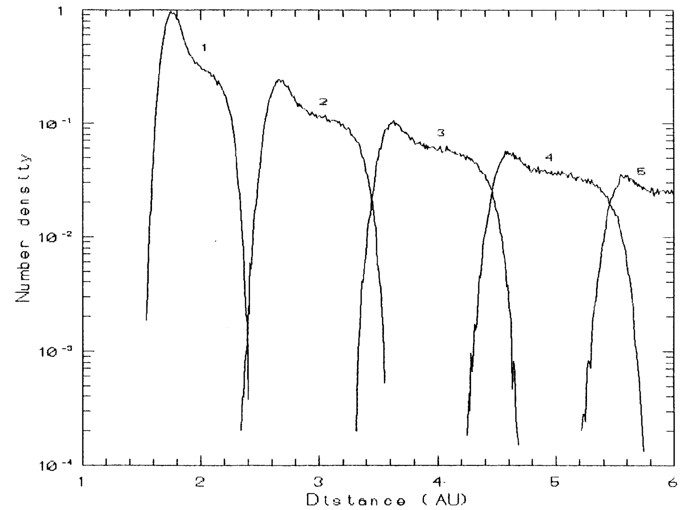

Fig. 3. Spatial distributions of pick-up protons injected at τ = 0 and $r = r_E$. The numbers denote the subsequent moments: τ = 1, 2, 3, 4, 5. c) run 3.

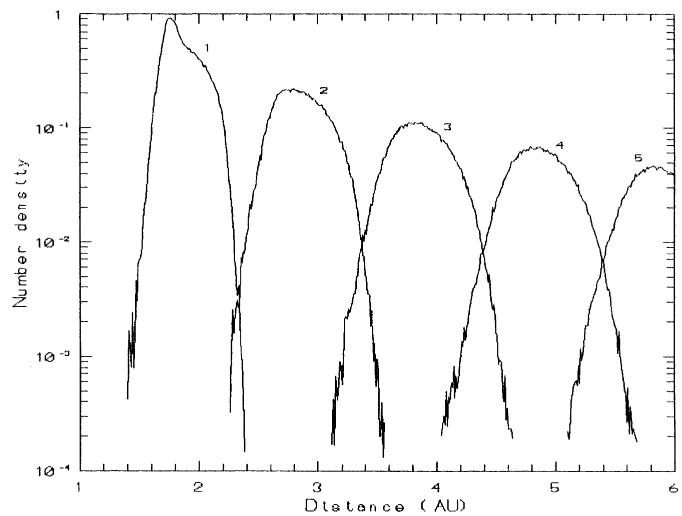

Fig. 4. Spatial distributions of pick-up protons injected at τ = 0 and $r = r_E$. The numbers denote the subsequent moments: τ = 1, 2, 3, 4, 5. d) run 4.

Table 1. Parameters of runs

Run number	δ	ϵ	$l_d(cm)$	ϑ
1	0.004	1	0	90°
2	0.02	1	0	90°
3	0.004	0	0	90°
4	0.02	1	$3 \cdot 10^7$	90°
5	0.02	1	$3 \cdot 10^7$	30°

In Figs. 1–5 one can study the spatial broadening of pick-up protons which were initially injected according to a spatial δ-function representing runs 1 through 5 given in Table (1). This broadening of the spatial distribution can be explained as due to an individual parallel (along the magnetic field lines) motion and thus spatial diffusion of protons with an anisotropic pitch-angle distribution. The pitch-angle anisotropy is also reflected by the fact that the average velocity of the pick-up proton bulk is smaller than the solar wind velocity. This is clearly seen in Figs. 1a,c–e. The maxima of spatial distributions in these figures are systematically shifted towards the Sun with respect to the isochronical solar wind bulk. Only in the case labeled as run 2 (see Table 1 and Fig. 2) the bulk of the pick-up protons comoves with the isochronical solar wind parcel. Run 2 differs from runs 1 and 3 by the adopted much higher level of turbulence, and from runs 4 and 5 by the fact that an identical level of turbulence, however here, together with an absence of a dissipative range in the turbulence spectrum is adopted (see Table 1). As we show in the following figures this results from the fact that the velocity distribution in run 2 is isotropic. As it was expected the maxima in Fig. 5 are closer to the Sun than in Fig. 4 since the magnetic

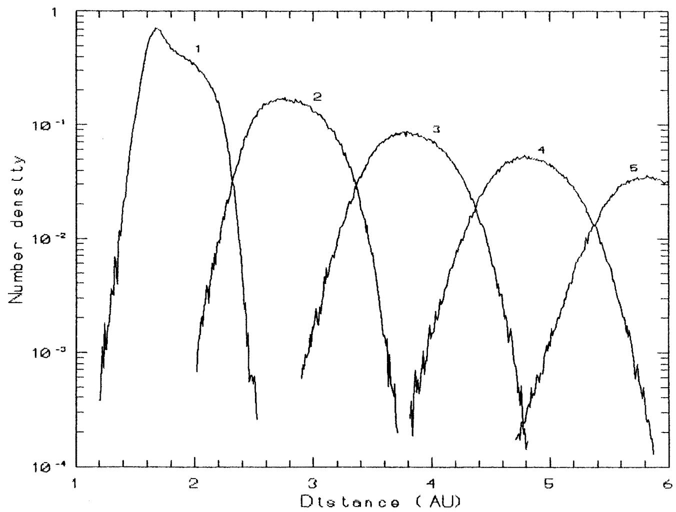

Fig. 5. Spatial distributions of pick-up protons injected at τ = 0 and $r = r_E$. The numbers denote the subsequent moments: τ = 1, 2, 3, 4, 5. e) run 5.

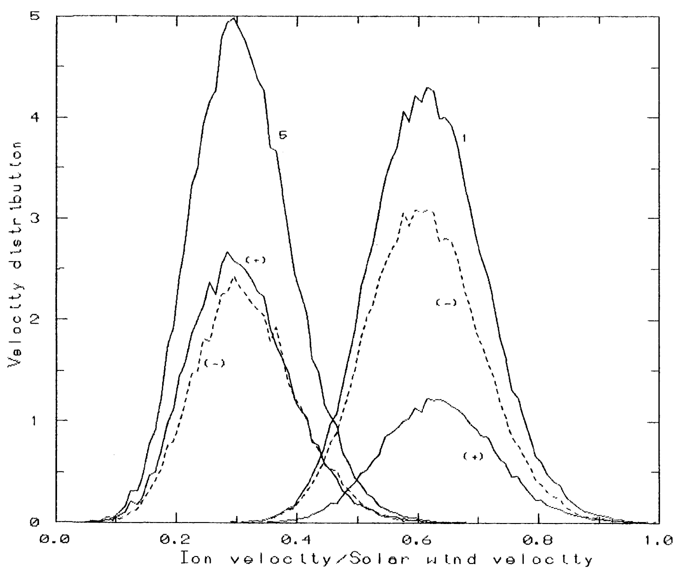

Fig. 6. Velocity distributions of pick-up protons corresponding to run 1 at times τ = 1, 5. The dashed curves labeled by (–) represent particles with $\mu < 0$ and the solid ones labeled by (+) represent particles with $\mu > 0$ (see text). a) velocity distributions at the maxima of the spatial distributions at τ = 1 and 5.

field in the first case ($\vartheta = 30°$) is closer to a radial field than in the second case ($\vartheta = 90°$, ecliptic plane!).

To reveal the degree of the velocity anisotropy we present Figs. 6 and 7. These figures show pitch-angle averaged velocity distributions of pick-up protons corresponding to run 1 at times t = 1 and 5. The velocity distribution functions f_v, $f_v^{(+)}$, and $f_v^{(-)}$ shown in Figs. 6 and 7 are given by the following formulae

$$f_v(t, r, v) = C_2 \int_{-1}^{+1} f \, d\mu, \tag{38}$$

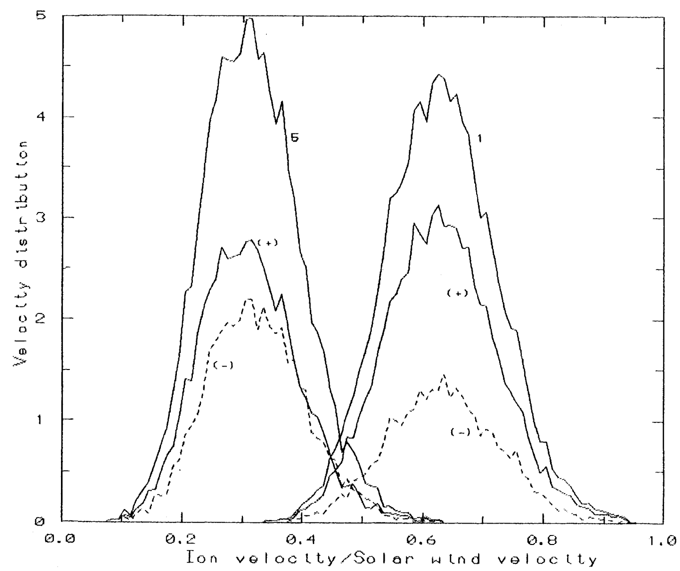

Fig. 7. Velocity distributions of pick-up protons corresponding to run 1 at times τ = 1, 5. The dashed curves labeled by (–) represent particles with $\mu < 0$ and the solid ones labeled by (+) represent particles with $\mu > 0$ (see text). b) velocity distributions at r = 2AU (at τ = 1) and r = 6AU (at τ = 5).

$$f_v^{(+)}(t, r, v) = C_3 \int_{0}^{+1} f \, d\mu, \tag{39}$$

$$f_v^{(-)}(t, r, v) = C_3 \int_{-1}^{0} f \, d\mu, \tag{40}$$

where the following normalization is used:

$$\int_0^\infty f_v \, d(v/U) = 1, \tag{41}$$

$$f_v = f_v^{(+)} + f_v^{(-)}. \tag{42}$$

Fig. 6 shows velocity distributions at the maxima of the spatial distributions at $\tau = 1$ and 5 (see Fig. 1), while Fig. 7 shows velocity distributions at r = 2AU at time τ = 1 and r = 6AU at time τ = 5, that is in a parcel comoving with the solar wind. The dashed curves labeled by (–) represent particles with $\mu < 0$ and the solid ones labeled by (+) represent particles with $\mu > 0$. First of all one can identify considerable adiabatic cooling which is in accordance with most recent findings in observational data obtained with the SWICS instrument (Fisk et al., (1997)). Fig. 6 shows that at τ = 1 the bulk of pick-up protons have negative pitch angles, that is, they move inward as judged by the solar wind reference frame (see Gloeckler et al., (1995); Möbius et al., (1997)). At τ = 5 the distribution is almost isotropic. Another interesting feature in Fig. 6 is a shift between maxima of the curves (+) and (–) at τ = 1 denoting that particles with negative pitch angles suffer more efficient cooling than particles with positive ones. An explanation of this effect has recently been

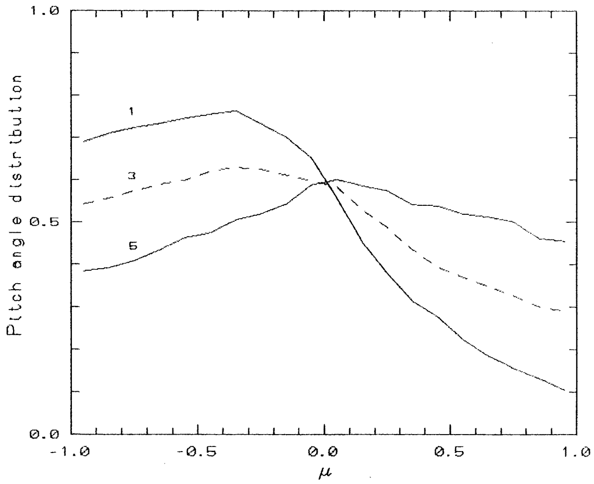

Fig. 8. Pitch-angle distributions of pick-up protons at the maxima of the spatial distributions (see Figs. 1–5) for different run numbers. The numbers in the figures indicate various times. a) run 1.

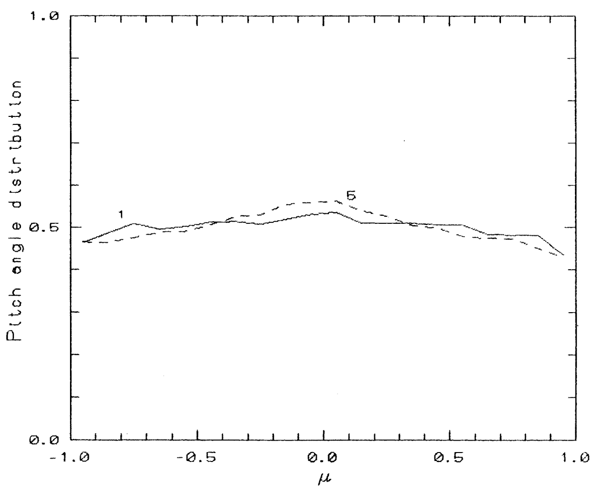

Fig. 9. Pitch-angle distributions of pick-up protons at the maxima of the spatial distributions (see Figs. 1–5) for different run numbers. The numbers in the figures indicate various times. b) run 2.

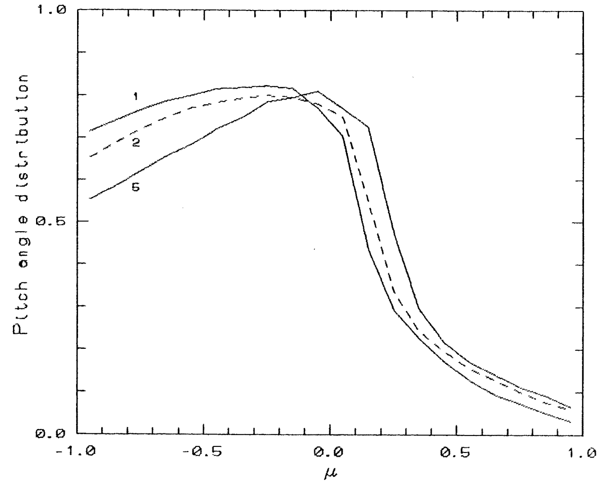

Fig. 10. Pitch-angle distributions of pick-up protons at the maxima of the spatial distributions (see Figs. 1–5) for different run numbers. The numbers in the figures indicate various times. c) run 3.

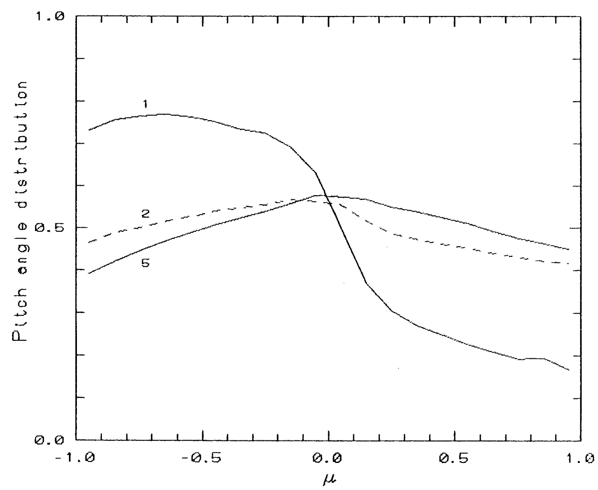

Fig. 11. Pitch-angle distributions of pick-up protons at the maxima of the spatial distributions (see Figs. 1–5) for different run numbers. The numbers in the figures indicate various times. d) run 4.

given by Fisk et al. (1997): particles propagating inward in the frame of the solar wind have a longer than average dwell time in the inner heliosphere and as result of it suffer substantial adiabatic cooling. Fig. 7 as distinct from Fig. 6 shows velocity distributions at a parcel comoving with the solar wind. One can see that in this case particles with positive pitch angles prevail and even at τ = 5 the distribution is still anisotropic.

Figs. 8–12 show pitch-angle distributions of pick-up protons at the maxima of the spatial distributions (see Figs. 1–5 for different run numbers. The numbers in the figures indicate various times. The pitch-angle distribution function f_μ shown in Figs. 8–9 is given by the formula

$$f_\mu(t, r, \mu) = C_4 \int_0^\infty v^2 f \, dv, \tag{43}$$

with the normalization

$$\int_{-1}^{+1} f_\mu \, d\mu = 1. \tag{44}$$

Fig. 9 shows that the velocity distribution corresponding to run 2 is already isotropic at τ = 1 (see the shapes of the spatial distributions in Fig. 1b), while in the case with the lower level of turbulence (Fig. 8) isotropy is reached at τ = 3 only. Fig. 10 shows the case when Alfvénic waves propagate only in one direction (ϵ = 0). In this case the velocity distribution of the bulk of pick-up protons still is anisotropic even at τ = 5 because of the well-known resonance gap in the pitch-angle scattering (Schlickeiser, (1989)). Comparison between runs 2 and 4 (Figs. 9 and 11, respectively) which differ only by the presence or absence of the dissipation range in the last case shows that

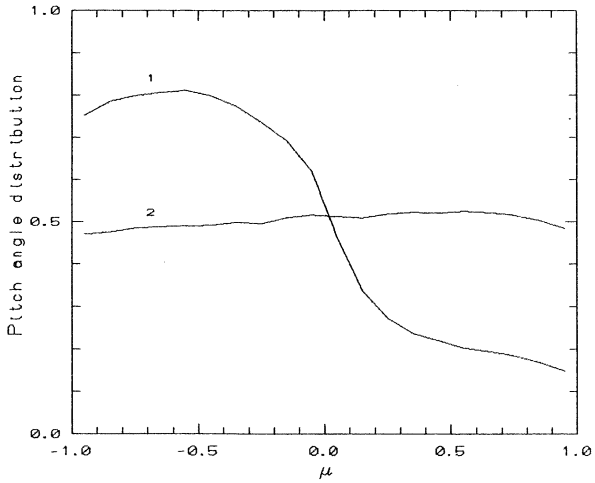

Fig. 12. Pitch-angle distributions of pick-up protons at the maxima of the spatial distributions (see Figs. 1–5) for different run numbers. The numbers in the figures indicate various times. e) run 5.

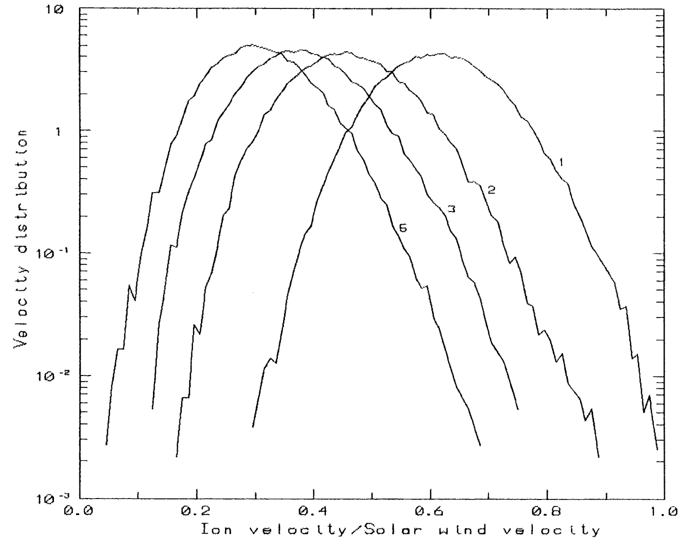

Fig. 13. Velocity distributions of pick-up protons at the maxima of the spatial distributions for different run numbers. The numbers in the figures indicate various times. a) run 1.

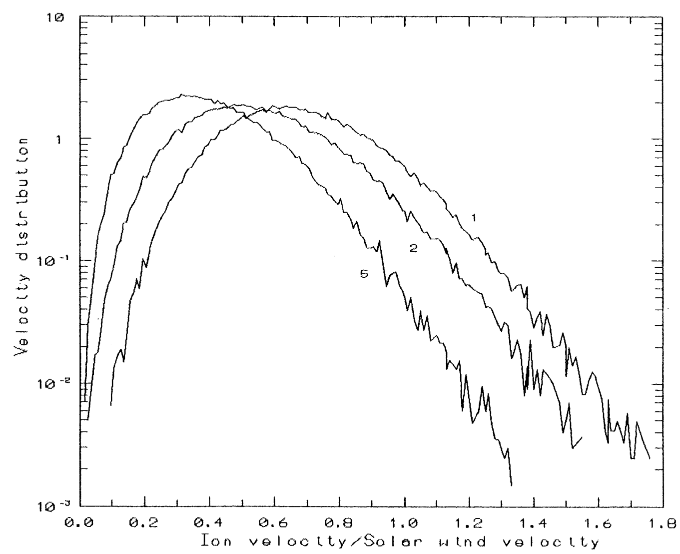

Fig. 14. Velocity distributions of pick-up protons at the maxima of the spatial distributions for different run numbers. The numbers in the figures indicate various times. b) run 2.

a deficit of high frequency waves in the solar wind can essentially modify the process of scattering towards isotropy. More isotropic velocity distribution of pick-up protons at $\vartheta = 30°$ (Fig. 12) than at $\vartheta = 90°$ Fig. 11 at times $\tau > 1$ can be explained by the fact that adiabatic focusing transports particles from negative pitch angles to positive ones more efficiently under radial magnetic field conditions (it can be seen from Eqs. (9), (12)).

Effects of velocity diffusion can be seen in Figs. 13–17. These figures show velocity distributions of pick-up protons at the maxima of the spatial distributions for different run numbers. As usually the numbers in the figures indicate various times. The velocity distribution function f_v is given by Eqs. (38),(41). Considerable stochastic acceleration of pick-up protons takes place only for a sufficiently high level of turbulence (Fig. 14). The presence of the dissipation range in the spectrum of magnetic field fluctuations reduces the rate of acceleration (Fig. 16). As it could be expected, stochastic acceleration does not operate in the case when Alfvénic waves propagate only in one direction (i.e. $\epsilon = 0$).

6. Conclusions and outlook

Perhaps the most interesting outcome of the calculations which we presented in this paper is the fact not anticipated in earlier theoretical representations that pick-up ions substantially spread out from the plasma volume comoving with the solar wind into which they were initially injected. Most of the papers in the past dealing with pick-up ion motions in configuration and velocity space are based on the assumption that these ions are sticking to the solar wind plasma box into which they were injected (see e.g. Fahr & Rucinski, (1989); Lee & Ip, (1987); Isenberg, (1997); Bogdan et al., (1991); Rucinski et al., (1993); Chalov et al., (1995), (1997); Chalov & Fahr, (1996); Rucinski et al., (1998). This means that due to spatial diffusion of pick-up ions any spatial pattern of the pick-up ion injection rapidly and systematically loses its stigmatic imprint on the subsequently evolving pick-up ion distribution in space. In Figs. 1–5 one can see that when the bulk of pick-up ions injected at 1 AU has reached a distance of 3 AU this sample of pick-up ions is already spread out over a distance range of 1.5 AU.

This phenomenon especially touches the problem of He^+-pick up ions emanating from the downwind cone of neutral interplanetary helium. These He^+-pick-up ions had first been detected by Möbius et al. (1985, 1988) with the SULEICA plasma-analyzer onboard of the AMPTE satellite. In later publications these authors have interpreted their He^+-flux measurements in terms of interstellar helium gas parameters, like the LISM He-density and He-temperature (see Möbius, 1990, Möbius et al.,

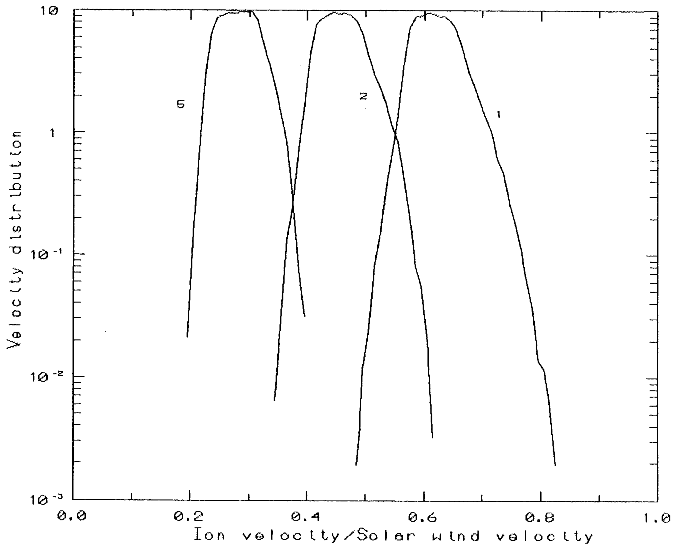

Fig. 15. Velocity distributions of pick-up protons at the maxima of the spatial distributions for different run numbers. The numbers in the figures indicate various times. c) run 3.

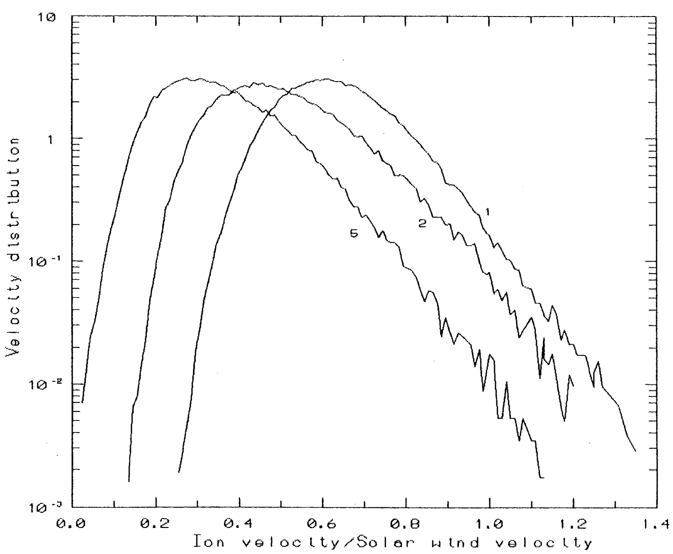

Fig. 16. Velocity distributions of pick-up protons at the maxima of the spatial distributions for different run numbers. The numbers in the figures indicate various times. d) run 4.

1995, Möbius, 1996, Möbius et al., 1996). While in the earlier of the just mentioned publications it was assumed that He^+-pick up ions are strictly convected outwards with the solar wind, in the later publications it was realized that pick-up ions undergo pitch angle scattering processes and thereby are diffusing out of their co-moving injection volumes thus spreading out from the cone (see Möbius et al., 1996). Considering a 3-d random walk process connected with a pitch-angle scattering mean free path λ_{ps} these authors could argue that their earlier LISM helium temperature derivations of between 12000 to 18000 K can then be reduced to temperature ranges of between 6500 to 8500 K derived by Witte et al. (1993) from neutral helium gas detections for scattering mean free paths λ_{ps} of the order of 1 AU.

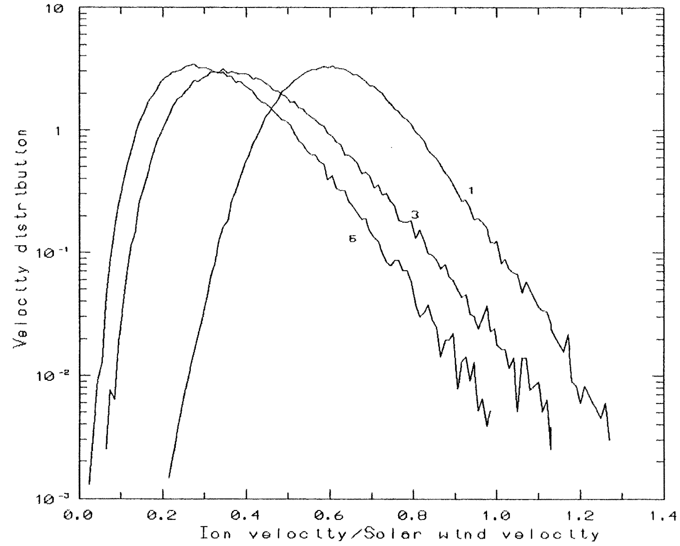

Fig. 17. Velocity distributions of pick-up protons at the maxima of the spatial distributions for different run numbers. The numbers in the figures indicate various times. e) run 5.

With the results of this paper here we shed interesting new light onto these derivations by showing both that spatial diffusion of pick-up ions in fact is important and, even more important, that it takes place in an anisotropic form. In the solar rest frame spatial diffusion predominantly takes place in the plane defined by the solar wind velocity $\mathbf{U}$ and the frozen-in magnetic field $\mathbf{B}$, whereas the drifts perpendicular to this plane are of very minor importance (see Rucinski et al., 1993). As we have shown here seen from the reference frame co-moving with the solar wind more particles are diffusing in the hemisphere of negative pitch-angles ($\mu < 0$) compared to the hemisphere with positive pitch angles ($\mu > 0$). This means that the cone-injected particles may preferentially diffuse to the side left of the cone axis as seen from the sun. Looking to He^+-fluxes reaching the earth orbit this allows to conclude that left of the cone axis one may find a milder flux decrease compared to the decrease right of the cone axis which should be steeper. In fact this seems to be evidently appearing in the data shown by Möbius (1990) where the flux increase from November towards December is steeper than the decrease from December towards January. Thus neither should the flux maximum be identified with the cut of the helium cone axis and the earth orbit, nor should the left or right flux decrease profile be taken as direct indication for the LISM helium temperature. More theoretical work has to be done on He^+ pick-up ion phase space diffusion before safe conclusions with respect to LISM parameters can be drawn.

Pick-up ions may also be observationally accessible by means of their resonance emissions. Especially interesting is this access for the case of He^+-pick-up ions which are resonantly excited by the solar 304 Å line emission. As already discussed in the paper by Paresce et al.(1983) the resulting resonance intensity strongly depends on the prevailing He velocity distribution function and practically is determined by the fraction of pick-up ions within a velocity range of less than 100 km/s relative to the sun. The results in this paper show that pick-up ions can be expected with predominance in the sunward hemisphere of the velocity space and thus with this predominance have effective outward velocities smaller than the solar wind velocity $|\mathbf{U}|$. Therefore they are good candidates to become resonantly excited. A measurement of the existing interplanetary He^+-pick-up ion resonance glow intensity (from the earth depending on the solar offset angle one could expect intensities between 10^1 and 10^{-1} Rayleighs!) thus could be an effective means to observationally study phase space diffusion of He^+ pick-up ions (see Fahr et al. (1998)).

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References

Achterberg A., Krülls W.M., A&A 265, L13
Balogh A., Smith E.J., Tsurutani B.T., et al., Science 268, 1007
Belcher J.W., Davis L., J.Geophys.Res. 76, 3534
Bieber J.W., Smith C.W., Matthaeus W.H., 1988, ApJ 334, 470
Bogdan T.J., Lee M.A., Schneider P., 1991, J.Geophys.Res. 96, 161
Chalov S.V., Fahr H.J., 1996, Solar Phys. 168, 389
Chalov S.V., Fahr H.J., Izmodenov V., 1995, A&A 304, 609
Chalov S.V., Fahr H.J., Izmodenov V., 1997, A&A 320, 659
Fahr H.J., 1971, Planet. Space Sci. 19, 1121
Fahr H.J., 1973, Solar Phys. 30, 193
Fahr H.J., Ziemkiewicz J., 1988, A&A 202, 295
Fahr H.J., Rucinski D., 1989, Planet.Space Sci. 37, 555
Fahr, H.J., Fichtner, H., and Scherer,H., 1998, Astron.Astrophys.,in press
Fichtner H., le Roux J.A., Mall U., Rucinski D., 1996, A&A 314, 650
Fisk L.A., Schwadron N.A., Gloeckler G., 1997, Geophys.Res.Lett. 24, 93
Forsyth,R.J., Balogh,A., Horbury,T.S., Erdös,G., Smith,E.J. and Burton,M.E.,1996, A&A 316, 287
Freund H.P., Wu C.S., 1988, J.Geophys.Res. 93, 14277
Gardiner C.W., 1990, Handbook of Stochastic Methods (2d ed.,New York: Springer)
Geiss J., Gloeckler G., Mall U., et al., 1994, A&A 282, 924
Gloeckler G., Geiss J., Balsiger H., et al., 1993, Science 261, 70
Gloeckler G., Schwadron N.A., Fisk L.A., Geiss J., 1995, Geophys.Res.Lett. 22, 2665
Gruntman,M., and Fahr,H.J., 1998, Geophys.Res.Lett., in press
Hartle R.E., Wu C.S., 1973, J.Geophys.Res. 78, 5802
Hollweg J.V., 1974, J.Geophys.Res. 79, 1539
Holzer T.E., 1972, J.Geophys.Res. 77, 5407
Isenberg P.A., 1987, J.Geophys.Res. 92, 1067
Isenberg P.A., 1997, J.Geophys.Res. 102, 4719
Jokipii J.R., Coleman P.J., 1968, J.Geophys.Res. 73, 5495
Krülls W.M., Achterberg A., 1994, A&A 286, 314
Lee M.A., Ip W.-H., 1987, J.Geophys.Res. 92, 11041
MacKinnon A.L., Craig I.J.D., 1991, A&A 251, 693
Miller J.A., Roberts D.A., 1995, ApJ 452, 912
Möbius E., 1990, in: 1.COSPAR Colloquium (Warsaw, Poland) on “Physics of the Outer Heliosphere”, Page E., Grzedzielski S. (eds). Pergamon Press, Oxford, pp.345-354
Möbius E., 1996, Space Sci.Rev. 78, 375
Möbius E., Hovestadt D., Klecker B., Gloeckler G., Ipavich F.M., 1985, Nature 318, 426
Möbius E., Klecker B., Hovestadt D., Scholer M., 1988, Astrophys.Space Sci. 144, 487
Möbius E., Rucinski D., Hovestadt D., Klecker B., 1995, A&A 304, 505
Möbius E., Rucinski D., Isenberg P.A., Lee M.A., 1996, Ann.Geophys. 14, 492
Möbius E., Rucinski D., Lee M.A., Isenberg P.A., 1997, J.Geophys.Res. (in press)
Paresce F., Fahr H., Lay G., 1983, J.Geophys.Res. 86, 10038
Rucinski D., Fahr H.J., Grzedzielski S., 1993, Planet.Space Sci. 41, 773
Rucinski D., Browski M., Fahr H.J., 1998, A&A, in press
Schlickeiser R., 1989, ApJ 336, 243
Schlickeiser R., Dung R., Jaekel U., 1991, A&A 242, L5
Semar C.L., 1970, J.Geophys.Res. 75, 6892
Skilling J., 1971, ApJ 170, 265
Smith C.W., Bieber J.W., Matthaeus W.H., 1990, ApJ 363, 283
Smith E.J., Balogh A., 1995, Geophys.Res.Lett. 22, 3317
Tu C.-Y., Marsch E., 1995, Space Sci. Rev. 73, 1
Vasyliunas V.M., Siscoe G.L., 1976, J.Geophys.Res. 81, 1247
Witte M., Rosenbauer H., Banaszkiewicz M., Fahr H., 1993, Adv.Space Res. 13(6), 121
Wu C.S., Davidson R.C., 1972, J.Geophys.Res. 77, 5399
Wu C.S., Hartle R.E., 1974, J.Geophys.Res. 79, 283
Wu C.S., Hartle R.E., Ogilvie K.W., 1973, J.Geophys.Res. 78, 306
Zhou Y., Matthaeus W.H., 1990, J.Geophys.Res. 95, 14881