Astron. Astrophys. 101, L17–L18 (1981)

ASTRONOMY AND ASTROPHYSICS

Letter to the Editor

Two Differing Definitions of the Dynamical Equinox and the Mean Obliquity

E. M. Standish, Jr.

California Institute of Technology, Jet Propulsion Laboratory 264–664, Pasadena, California 91109, USA

Received July 3, accepted July 14, 1981

SUMMARY

When one computes (either implicitly or explicitly) the location of the dynamical equinox, E, or the value of the mean obliquity, ε, in a rotating reference frame, one gets different results from those computed in an inertial frame. The differences, E(rotating) -E(inertial) = 0".09363 (for B1950.0), = -0".09366 (for J2000) and ε(rotating) -ε(inertial) = + 0".00364 (for B1950.0), = + 0".00334 (for J2000), must be accounted for when intercomparing results.

KEY WORDS - Astrometry, Catalogues, Ephemerides

INTRODUCTION

It has recently become apparent that there exist, in use, two differing definitions of the dynamical equinox and of the mean obliquity which will produce different resulting values between them. The discrepancy arises from the fact that one definition implicitly considers the location of the mean ecliptic of epoch computed in a rotating reference frame while the other refers to an inertial system. The differences are of a significant size and will affect the intercomparison of the resulting values.

This paper illustrates the differing definitions and derives their corresponding values.

DERIVATION

In order to evaluate the mean dynamical equinox and obliquity of some given epoch, one may, for example, compute the instantaneous angular momentum vector of the Earth-Moon barycenter about the Sun as a function of time, expressed in an inertial equatorial reference frame:

$$\underline{h} = \underline{r} \times \underline{\dot{r}}$$

By eliminating the secular and periodic terms, one produces the time-average, centered on the epoch, of this vector, $\underline{\bar{h}}$. The equinox and obliquity are then computed from

$$-E = \Omega = \tan^{-1} (\bar{h}_x / - \bar{h}_y) \text{ , and}$$

$$\varepsilon = \cos^{-1} (\bar{h}_z / \bar{h}).$$

If, however, one were to compute $\underline{h}' = \underline{r}' \times \underline{\dot{r}}'$ where the coordinates, $\underline{r}'$, are now referred to the rotating ecliptic, one would find an additional term:

$$\underline{h}' = \underline{h} - \underline{r} \times (\underline{b} \times \underline{r}),$$

where $\underline{b}$, expressed in the equatorial system, is

$$\underline{b} = \dot{\pi}_A \begin{pmatrix} \cos \bar{\Pi}_A \\ \sin \bar{\Pi}_A \cos \varepsilon \\ \sin \bar{\Pi}_A \sin \varepsilon \end{pmatrix}.$$

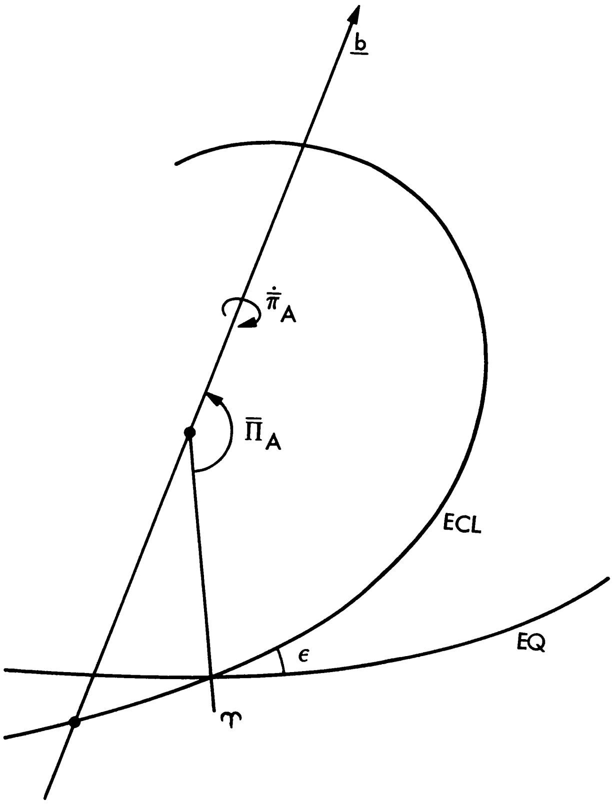

Figure 1. The ecliptic rotates about the vector, $\underline{b}$, located by the angle, $\bar{\Pi}_A$, with a rate, $\dot{\pi}_A$.

The vector, b, (see Fig. 1) is the axis about which the ecliptic rotates, given by the angle

$$\overline{\Pi}_A = 174^\circ\!.876 + 0^\circ\!.914\ T_{2000},$$

and its magnitude, rate of rotation, is

$$\dot{\pi}_A = (47''\!.003 - 0''\!.066\ T_{2000})/\mathrm{cty}.$$

(Lieske et al. 1977)

It may be shown that the time-average of the additional term is not equal to 0, and as such

$$\underline{\overline{h}} \neq \underline{\overline{h}}'$$

If we consider the earth in circular orbit at 1 A.U. and choose an equatorial reference frame such that

$$\underline{\overline{h}} = \begin{pmatrix} 0 \\ -\omega \sin \varepsilon \\ \omega \cos \varepsilon \end{pmatrix} \text{ and } \underline{r} = \begin{pmatrix} \cos \omega t \\ \sin \omega t \cos \varepsilon \\ \sin \omega t \sin \varepsilon \end{pmatrix}$$

where ω is the earth's sidereal mean motion, we find that

from which $$\underline{\overline{h}}' = \underline{\overline{h}} - \frac{1}{2}\,\underline{b} \quad ,$$

and
$$\Omega' = \Omega - \dot{\pi}_A \cos \overline{\Pi}_A/2\omega \sin \varepsilon$$

$$\varepsilon' = \varepsilon + \dot{\pi}_A \sin \overline{\Pi}_A/2\omega \ .$$

From these formulae, we compute for B1950.0,

$$E(\text{rotating}) - E(\text{inertial}) = -(\Omega' - \Omega) = -0''\!.09363,$$

and

$$\varepsilon(\text{rotating}) - \varepsilon(\text{inertial}) = +\ 0''\!.00364,$$

For the J2000, the corresponding values are $-0''\!.09366$ and $+0''\!.00334$ respectively.

DISCUSSION

In former times it has been the practice to operate in the rotating system. Such is the case, for example, with Newcomb's analytical theory of the Sun, where the motion is expressed with respect to the rotating ecliptic. The inertial velocity vector of the particle does not, in general, lie in the rotating plane, and, as such, the instantaneous node of that plane does not coincide with the node of the plane which is normal to the instantaneous angular momentum vector.

On the other hand, numerical integrations of the equations of motion do refer to inertial space. So does the recent analytical theory of Bretagnon (1980) which uses elements referred to an inertial system.

The recent determination of the dynamical equinox with respect to the Fk4 stellar catalogue (Fricke, 1981) seems to be founded upon the rotating definitions of E and ε since much of the data considered there has been reduced using Newcomb's theory. As such, it might seem prudent to adopt this definition as a standard for the sake of continuity.

CONCLUSIONS

The classical definitions of the dynamical equinox and mean obliquity at epoch are those which imply computation in a rotating reference frame (i.e., with respect to the rotating ecliptic). The adoption of these defintiions as standards requires the application of corrective factors when the quantities are computed with respect to an inertial system.

ACKNOWLEDGEMENTS

The above-mentioned discrepancies were noted during a comparison of results between the author and members of the U.S. Naval Observatory. T. C. Van Flandern suggested the possible cause.

The research in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under NASA Contract NAS 7-100.

REFERENCES

Bretagnon, P. 1980, Astron. and Astrophys., 84, 329

Fricke, W. 1981, in "Reference Coordinate Systems for Earth Dynamics" p. 359, Eds. E. M. gaposchkin, B. Kolaczek, Reidel - Dordrecht.

Lieske, J. H., Lederle, T., Fricke, W., and Morando, B. 1977, Astron. and Astrophys., 58, 1.