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A hemispherical model of anisotropic interstellar pickup ions

Philip A. Isenberg

Institute for the Study of Earth, Oceans and Space and Department of Physics, University of New Hampshire, Durham

Abstract. We present an analytical model for the distribution function of interstellar pickup ions in a radially directed heliospheric magnetic field which naturally results in anisotropic particle distributions. The model includes the effects of convection and spatial transport in the solar wind, adiabatic deceleration in the radial flow, adiabatic focusing in the radial field, and pitch angle scattering toward isotropy in the frame of the solar wind. The pitch angle scattering is approximated by the hemispherical assumption: we take the scattering to be very efficient within each pitch angle range $\mu \gtrless 0$ (where μ is the cosine of the pitch angle) but inhibited between the hemispheres separated by $\mu = 0$. The analytical solution is obtained for the case where the scattering rate across $\mu = 0$ scales as the particle speed divided by the radial position of the fluid parcel. The model distribution functions can be used to interpret recent observations of anisotropic pickup ions.

1. Introduction

Interstellar pickup ions are neutral atoms from the interstellar medium which flow into the heliosphere and become ionized by solar EUV and the particles of the solar wind when they move close enough to the Sun. Once ionized, they interact with the electromagnetic fields carried by the solar wind, becoming singly charged components of the solar wind plasma. Their very slow inflow speed with respect to the Sun (20 - 30 km/s) corresponds to an initial speed comparable to the solar wind speed V_{sw} in the reference frame moving with the solar wind plasma. Their initial pitch angle depends on the angle between the interplanetary magnetic field **B** and the radially flowing solar wind, such that the cosine of the initial pitch angle $\mu_0 = - B_r/B$, where B_r is the radial component of the magnetic field. These suddenly superthermal ions form a component of the solar wind which is unstable to the generation of magnetohydrodynamic waves [*Wu and Davidson*, 1972; *Winske and Gary*, 1986; *Sharma and Patel*, 1986; *Lee and Ip*, 1987]. The action of these self-generated waves, along with the ambient resonant waves in the solar wind, is to scatter the pickup ions in pitch angle toward an isotropic state in the frame of the solar wind. Quasi-linear calculations [e.g., *Lee and Ip*, 1987] predict timescales for isotropization of these distributions which are short compared to the timescale for ionization of new ions, so the body of work on interstellar pickup ions over the past 20 years has tended to assume that these particles are essentially isotropic in the solar wind frame and comove with the solar wind [*Vasyliunas and Siscoe*, 1976; *Isenberg*, 1986, 1987, 1991, 1993; *Isenberg and Lee*, 1995; *Burlaga et al.*, 1990, 1994; *Lee*, 1997]. We include in this general category the nondispersive bispherical models of *Galeev and Sagdeev* [1988], *Johnstone et al.* [1991], and *Williams and Zank* [1994] in which the anisotropies are limited to the order of V_A/V_{sw}, where V_A is the Alfvén speed.

When interstellar pickup ions were finally observed, the initial measurements appeared consistent with this assumption of isotropy [*Möbius et al.*, 1985, 1988; *Möbius*, 1986; *Gloeckler et al.*, 1993; *Gloeckler and Geiss*, 1997]. However, these measurements were restricted to the particles moving faster than the solar wind speed in the spacecraft frame, equivalent to the antisunward hemisphere of phase space in the solar wind frame. More recent observations, including a direct measurement of pickup ions in the sunward hemisphere of the distribution, have indicated that substantial anisotropies may often be present.

Gloeckler et al. [1995] reported a 30-day integration of pickup protons by the Solar Wind Ion Composition Spectrometer on Ulysses over the south polar coronal hole. In the steady high-speed solar wind there, pickup protons with radial speeds slower than the solar wind speed are energetic enough to be detected by the instrument. These particles, which populate the sunward hemisphere of phase space in the solar wind frame, were found to be significantly more numerous than the antisunward particles, implying radial anisotropies of greater than 50%.

The average magnetic field during these observations was oriented within ~ 45° of the heliocentric radial direction [*Forsyth et al.*, 1995]. The importance of a quasi-radial field orientation is emphasized by the observations of *Möbius et al.* [1995], who investigated the antisunward intensities of pickup He^+ at the AMPTE/IRM satellite under various magnetic field conditions. The Suprathermal Energy Ionic Charge Analyzer instrument on AMPTE can only detect the energetic, antisunward portion of the pickup ion distribution, but Möbius et al. found that the measurable portion of the pickup He^+ distribution under steady conditions was highly dependent on the orientation of the magnetic field. When the angle between the field and the radial direction was greater than ~ 45°, the pickup ion spectrum exhibited the expected sharp cutoff at spacecraft-frame particle speeds $v_s = 2V_{sw}$ and was fairly flat below that speed. During extended periods when the magnetic field direction was within 45° of the radial, however, the antisunward fluxes were strongly reduced compared to the fluxes during nonradial conditions, and the spectrum approaching $v_s = 2V_{sw}$ fell gradually with increasing energy toward the level of the ambient energetic particles.

These observations imply that pickup ions have greater difficulty reaching the antisunward hemisphere when the average field is quasi-radial. Newly generated ions appear with spacecraft-frame speeds near zero. If the magnetic field is highly

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oblique to the radial direction, the cyclotron motion induced by the solar wind fields will act to bring the ions into the antisunward hemisphere within a gyroperiod. However, if the field is more radially aligned, the ions must be pitch angle scattered into the antisunward hemisphere (with a little help from adiabatic focusing). If this pitch angle transport is not fast compared to the ionization rate of new particles, a sunward anisotropy will result. Furthermore, if the scattering rate is slow enough, the ions will lose substantial energy by adiabatic deceleration before reaching the antisunward regime. In this case, pickup ions would not be seen in the most energetic parts of the expected distribution, such as near the standard cutoff at $v_s = 2V_{sw}$, resulting in spectra similar to those reported by *Möbius et al.* [1995]. Thus these observations indicate that pitch angle scattering of interstellar pickup ions is much less efficient than predicted by the standard quasi-linear calculations.

In providing new insights on the process of pitch angle scattering, these observations may also have important implications for the theory of charged particle transport in magnetized plasmas. The interaction of energetic charged particles with magnetic irregularities is largely determined by the resulting changes in pitch angle, and the mean free path for the spatial propagation of these particles is closely related to the properties of the pitch angle diffusion coefficient [e.g., *Jokipii*, 1968, 1971]. There are well-known difficulties with the predictions of quasi-linear theory for the transport of low-energy cosmic rays [*Palmer*, 1982; *Bieber et al.*, 1994], and the behavior of pickup ions, whose rigidity is comparable to the cosmic rays in question, could yield valuable clues in this puzzle.

Clearly, an important step in analyzing these developments is construction of a model for the anisotropic pickup ions. A model which incorporates the known physics of the system and successfully fits the data implies the validity of the model assumptions and furthers our understanding. The model can then be used to infer additional properties of the system, suggesting future measurements, which are in turn used to test the model. Ideally, a model of these anisotropic pickup ions should include realistic descriptions of the following effects: spatially dependent ionization, pitch angle scattering toward isotropy in the solar wind frame on a timescale perhaps comparable to the ionization time, expansion and deceleration of the ions in the spherically diverging wind, and transport of streaming ions with respect to the solar wind.

A model of these ions was included in the work of *Gloeckler et al.* [1995], derived from the steady state convection-diffusion equation normally applied to nearly isotropic cosmic rays. They presented an expression for the isotropic part of the distribution function which took into account the spatially dependent ionization and the spherical effects, assuming a constant value for λ, the diffusive mean free path of the ions. The diffusive formalism then required the anisotropy to be given by the product of λ and the spatial gradient of the isotropic part, and this identification yielded a value for λ = 1 - 2 AU. However, as noted by Gloeckler et al., the diffusive formalism leading to the equation and results of that paper are not rigorously applicable to these particles. The cosmic ray equation assumes that the ion speeds are much greater than the bulk flow speed of the solar wind, but pickup ion speeds are comparable to, or smaller than, V_{sw}. Furthermore, the diffusive approach is derived as an expansion of the distribution function about the isotropic state and assumes that near isotropy is maintained by a scattering process with mean free path much less than other spatial scales in the system. The fact that the anisotropy is large, resulting in a derived mean free path comparable with the scales of the other processes in the model, calls into question the basic assumption of the diffusive model for these particles.

An additional concern with the model of *Gloeckler et al.* [1995] is that the spatial transport of the ions is not described correctly. Inflowing interstellar atoms are nearly at rest with respect to the Sun when they are ionized, and the boost that the new ions are given by the solar wind fields is always directed outward from the Sun. Thus interstellar pickup ions do not penetrate closer to the Sun than their ionization point, and the distribution at any radial position cannot be affected by the particles ionized at larger radii. However, solutions of a steady state diffusion equation describe particles which have at least some access to the entire system, and the distribution at any point is affected by conditions everywhere else in the system. This diffusive character is evident in the model distribution of *Gloeckler et al.* [1995].

The work of *Möbius et al.* [1995] also contained a model for these ions. The starting point for this model was a Boltzmann-type equation for the evolution of the gyrotropic distribution of pickup ions along a field line under the action of pitch angle scattering. The pitch angle dependence was approximated by a two-stream structure [*Fisk and Axford*, 1969], where the distribution consisted of two shells, streaming along a field line with speeds in the solar wind frame of $\pm V_{sw\parallel}/2$, half the parallel component of the solar wind speed. The effect of pitch angle scattering was modeled by a transfer of particles from one stream to the other at a rate proportional to the pitch angle gradient, defined by the density difference between the two streams. This simple picture resulted in a model of anisotropic pickup ions under conditions of radial magnetic field in which the ions were prevented from traveling inward in the frame of the Sun. However, the effects of spherical expansion were not included, and the reduction of the distribution to two streams removed any spectral information from the model.

In this paper, we will construct a model of anisotropic pickup ions which does not have these deficiencies. We will start by deriving a general equation for the distribution function of gyrotropic particles of any nonrelativistic energy and pitch angle dependence, interacting with a flowing medium by pitch angle scattering toward isotropy in the frame of that medium. We will then concentrate on the steady state case with a constant-speed radial solar wind as the background medium, assuming conditions of radially directed magnetic field and a source of particles appropriate for interstellar pickup ions. This form of the equation includes the spherical effects of adiabatic deceleration and adiabatic focusing in the diverging magnetic field. We obtain solutions of this equation under a "hemispherical" assumption, where we consider that pitch angle scattering is very efficient within each pitch angle range $\mu \gtrless 0$ (where μ is the cosine of the pitch angle), but the scattering is inhibited between the hemispheres separated by $\mu = 0$. We then discuss the behavior of these solutions and further implications of this model.

2. A General Transport Equation

We are seeking an equation describing the behavior of a distribution of particles in a flowing medium such as the solar wind. We want the distribution to be a function of position and time variables $(\mathbf{x}, t)$ measured in an inertial frame and of a velocity variable $\mathbf{v}$ measured in the frame which is moving at a velocity $\mathbf{U}$ with respect to the inertial frame. Loosely following the procedure used by *Skilling* [1971] and *Burgers* [1969], we write the Boltzmann equation for the distribution function $f(\mathbf{x}, \mathbf{v}', t)$ in the inertial frame as

$$\frac{\partial f}{\partial t} + \mathbf{v}' \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}'}{m} \cdot \frac{\partial f}{\partial \mathbf{v}'} = (\text{scattering terms}) + Q \tag{1}$$

where $\mathbf{v}'$ is the velocity variable measured in the inertial frame, m is the particle mass, and Q is a particle source term. Here the external forces in the system have been heuristically split into ordered large-scale forces $\mathbf{F}'$, and small-scale forces designated "scattering terms" which will act to stochastically scatter the particles. In this system, the important large-scale forces are electromagnetic, $\mathbf{F}' = q(\mathbf{E}_0 + \mathbf{v}' \times \mathbf{B}_0)/c$, where $\mathbf{E}_0$ and $\mathbf{B}_0$ are the inertial-frame electric and magnetic fields, respectively, suitably averaged over small spacescales and timescales, q is the particle charge, and c is the speed of light.

Now define $\mathbf{v}' = \mathbf{U} + \mathbf{v}$, where $\mathbf{v}$ is the velocity variable measured in the reference frame moving with velocity $\mathbf{U}(\mathbf{x}, t)$. Transforming (1) into the new mixed variables $(\mathbf{x}, \mathbf{v}, t)$ yields

$$\frac{\partial f}{\partial t} + (U_i + v_i)\frac{\partial f}{\partial x_i} + \left[\frac{F_i}{m} - \frac{\partial U_i}{\partial t} - (U_j + v_j)\frac{\partial U_i}{\partial x_j}\right]\frac{\partial f}{\partial v_i} = (\text{scattering terms}) + Q \tag{2}$$

where the subscripts refer to vector components and repeated indices are summed over. Here $\mathbf{F}$ is the large-scale force measured in the moving frame. If we identify $\mathbf{U}$ as the velocity of a background conducting plasma such as the solar wind, the large-scale electric field is the motional field $\mathbf{E}_0 = -\mathbf{U} \times \mathbf{B}_0/c$. This field is transformed away in the rest frame of the plasma, leaving $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}_0/c$.

We next assume that the Lorentz force dominates the motion of the particles, so their gyroradius is much smaller than other spatial scales and their gyroperiod is much smaller than other timescales in the system. Thus the distribution function is nearly gyrotropic, $f(\mathbf{x}, \mathbf{v}, t) = f(\mathbf{x}, v, \mu, t)$, where $\mu = \mathbf{v} \cdot \mathbf{b}/v$ is the cosine of the particle pitch angle and $\mathbf{b} = \mathbf{B}_0/|B_0|$ is the unit vector along the large-scale magnetic field. We will assume complete gyrotropy here, recognizing that we are neglecting the action of perpendicular drift effects on the distributions. Averaging (2) over gyrophase then yields our form of the Skilling equation

$$\begin{aligned}\frac{\partial f}{\partial t} &+ (U_i + v\mu b_i)\frac{\partial f}{\partial x_i} + \left[\frac{1-3\mu^2}{2} b_i b_j \frac{\partial U_j}{\partial x_i} - \frac{1-\mu^2}{2}\frac{\partial U_i}{\partial x_i} - \frac{\mu b_i}{v}\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right)\right] v \frac{\partial f}{\partial v} \\ &+ \frac{1-\mu^2}{2}\left[v\frac{\partial b_i}{\partial x_i} + \mu \frac{\partial U_i}{\partial x_i} - 3\mu b_i b_j \frac{\partial U_j}{\partial x_i} - \frac{2 b_i}{v}\left(\frac{\partial U_i}{\partial t} + U_j\frac{\partial U_i}{\partial x_j}\right)\right]\frac{\partial f}{\partial \mu} \\ &= \frac{\partial}{\partial \mu}\left[K(1-\mu^2)\frac{\partial f}{\partial \mu}\right] + Q(x_i, v, \mu, t)\end{aligned} \tag{3}$$

for the distribution function of particles whose velocity dependence is measured in the frame of the moving background plasma. Having achieved this frame transformation of the velocity variable, we assume that the scattering by small-scale field variations on the right-hand side of (1) and (2) is given by the usual pitch angle scattering toward isotropy in the frame of the moving plasma, with scattering coefficient $K(\mathbf{x}, v, \mu)$. We note that the parenthetical terms on the left-hand side of (3) containing the convective derivative of $\mathbf{U}$ are not included in the equation of *Skilling* [1971], who assumed that $U \ll v$.

3. Hemispherical Model of Interstellar Pickup Ions

To apply (3) to the case of interstellar pickup ions in the solar wind under radial magnetic field conditions, we assume a steady state $\partial/\partial t = 0$, a constant radial solar wind as the background plasma $\mathbf{U}(\mathbf{x}, t) = U\,\hat{\mathbf{r}}$, and a large-scale magnetic field pointing away from the Sun $\mathbf{b} = \hat{\mathbf{r}}$. (We take an outward pointing magnetic field for the purpose of definiteness, leading to the identification of antisunward particles as those with $\mu > 0$. The analysis is identical for an inward pointing field with the antisunward hemisphere given by $\mu < 0$.) These assumptions reduce (3) to

$$(U + \mu v)\frac{\partial f}{\partial r} - \frac{1-\mu^2}{r} U v \frac{\partial f}{\partial v} + \frac{1-\mu^2}{r}(v + \mu U)\frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu}\left[K(1-\mu^2)\frac{\partial f}{\partial \mu}\right] + Q(r, \theta, v, \mu) \tag{4}$$

The terms on the left-hand side of (4) correspond to convection, adiabatic deceleration in the spherically expanding solar wind, and adiabatic focusing in the spherically expanding magnetic field, respectively. The angular variable θ in the source term Q is a polar angle measured from the direction pointing into the inflowing interstellar wind. It is clear that as long as $v \le U$, this equation does not permit particles to propagate to smaller radial positions.

To obtain a solution of (4) in closed form, we make a further assumption on the character of the pitch angle scattering of pickup ions. We assume that under conditions of nearly radial magnetic field, the scattering is so efficient within each hemisphere of the distribution that the pitch angle gradients away from $\mu = 0$ can be neglected. Further, we assume that for some region around $\mu = 0$ the scattering is inhibited, perhaps as a manifestation of the well-known resonance gap for the cyclotron interaction of ions with parallel-propagating waves [e.g., *Rowlands et al.*, 1966; *Dusenbery and Hollweg*, 1981]. This picture is in basic agreement with the recent analysis of *Fisk et al.* [1996], who concluded that the pickup proton distributions seen on Ulysses were consistent with reduced scattering in a localized region around $\mu = 0$. In reality, the width and character of this gap in the scattering will depend on the power spectra of the various resonant wave modes present [e.g., *Schlickeiser*, 1988, 1989] as well as the relevant wave dissipation scales [*Davila and Scott*, 1984; *Smith et al.*, 1990; *Schlickeiser et al.*, 1991] and the effectiveness of nonlinear processes such as mirroring [*Goldstein*, 1980]. We will not consider these details here but approximate the gap to be infinitesimal in width with a finite particle scattering rate across it. Thus the model pickup ion distribution consists of nested hemispheres in phase space separated by the $\mu = 0$ plane, allowing the density in the $\mu < 0$

hemisphere to be different than that in the $\mu > 0$ hemisphere. This hemispherical distribution is described by

$$f(r, \theta, v, \mu) = f_{-}(r, \theta, v)\, S(-\mu) + f_{+}(r, \theta, v)\, S(\mu) \tag{5}$$

where f_+ (f_-) refers to the antisunward (sunward) hemisphere in a radial field and $S(x)$ is the step function.

Since interstellar pickup ions originate only in the sunward hemisphere of phase space along the magnetic field and reach the antisunward hemisphere by scattering through $\mu = 0$, ions in this model will be inhibited from reaching the antisunward hemisphere. Thus sunward anisotropies under near-radial field conditions are a natural consequence of this model. This hemispherical model is basically an extension of the two-stream model of *Fisk and Axford* [1969], *Möbius et al.* [1995], and *Fisk et al.* [1996], where here we retain the information on the energy spectrum of the particles. A similar construction has been used by *Kóta* [1994] to model cosmic ray diffusion.

Now, integrating (4) and (5) over μ separately from −1 to 0 and then from 0 to 1 yields the coupled equations for $f_\pm$

$$\left(U \pm \frac{v}{2}\right)\frac{\partial f_\pm}{\partial r} - \frac{2Uv}{3r}\frac{\partial f_\pm}{\partial v} + \frac{v}{2r}(f_+ - f_-) = \mp \Gamma (f_+ - f_-) + \left\{ \begin{array}{c} 0 \\ Q(r, \theta, v) \end{array} \right\} \tag{6}$$

where $\Gamma \equiv K(\mu = 0)$ gives the rate of scattering across $\mu = 0$ and we have explicitly indicated that there is no ionization source for ions in f_+.

We find that an analytical solution of (6) is available in the case that the scattering rate scales as $\Gamma \sim v/r$. Such a dependence is not unreasonable. Ions encounter the resonance gap near $\mu = 0$ for a value of their speed parallel to the magnetic field $v_{\|g}$, determined only by the charge and mass of the ion and the properties of the wave field. For a given ion species, the effective width of the gap with respect to μ is given by $v_{\|g}/v$. A scattering rate inversely proportional to the gap width then yields $\Gamma \sim v$. The specific radial dependence is less compelling, but one expects the fluctuations that provide scattering through $\mu = 0$ to fall off with distance from the Sun. We note that although the WKB evolution of the low-frequency transverse wave power might imply $\Gamma \sim r^{-3}$, the scattering processes in the gap involve both resonant interactions with waves in the high-frequency dissipation range and nonlinear interactions with the longitudinal fluctuations, and these processes are not well understood at this time.

The method of solution is as follows: Decouple the first-order equations (6) into separate second-order equations for f_+ and f_- as functions of r and $w = v/U$. Transform the variables in the f_+ [f_-] equation to ρ_+ [ρ_-] and w, where $\rho_\pm = r\, w^{3/2} \exp(\pm 3w/4)$. The derivatives with respect to w and $\ln\rho$ in the resulting equations have constant coefficients and can be easily solved by standard techniques.

The ionization source term for interstellar pickup ions into the sunward hemisphere in steady state is given by the product of the ionization rate and the inflowing neutral particle density. For an ionization rate which is spherically symmetric about the Sun, the source term has the form

$$Q(r, \theta, v) = \frac{\beta_0 r_0^2}{2\pi r^2 v^2} N(r, \theta)\, \delta(v - U) \tag{7}$$

where β_0 is the ionization rate at $r = r_0$ and $N(r,\theta)$ is the inflowing neutral particle density. The delta function represents the fact that the ions first appear with speed equal to U in the solar wind frame, where here we neglect the small speed of the neutral particles at the point of ionization. We note that the angular position variable θ enters the problem only through the neutral particle density, and once $N(r)$ at a given θ is specified, the angle does not affect the solution further. This is due to the fact that the radial solar wind accumulates pickup ions along its path of constant θ. Consequently, we will suppress the θ dependence of the solution in what follows, understanding that $N(r)$ carries with it an implicit value of the constant angle at the observation point.

The solution of the system (6) with the source (7) is given by

$$f_+(r, w) = \frac{3\beta_0 r_0^2}{8\pi U^4}\frac{D+1}{r w^{3/2}} \exp(-aD) \times \int_0^{2a} dz\, N\left[r w^{3/2} \exp(z - a)\right] J_0(\Phi) \tag{8}$$

in the antisunward ($\mu > 0$) hemisphere of the distribution and

$$f_-(r,w) = \frac{3\beta_0 r_0^2}{4\pi U^4}\frac{\exp(-aD)}{r w^{3/2}}\left\{ N\left[r w^{3/2} \exp(a)\right] - \frac{\sqrt{1-D^2}}{2}\int_0^{2a} dz\, N\left[r w^{3/2} \exp(a - z)\right]\sqrt{\frac{2a - z}{z}}\, J_1(\Phi) \right\} \tag{9}$$

in the sunward ($\mu < 0$) hemisphere, where the dimensionless scattering parameter $D = 2r\Gamma/v$, $a = 3(1 - w)/4$, and the argument of the Bessel functions is $\Phi = [z(2a - z)(1 - D^2)]^{1/2}$.

To illustrate these expressions, we choose a simple form of the neutral particle density, $N(r) = N_0 \exp(-A/r)$, appropriate for a cold interstellar neutral gas moving in straight lines through the heliosphere. In Figure 1, we plot the normalized phase-space density

$$F = f(r, v, \mu)\left[\frac{3\beta_0 r_0^2}{8\pi r U^4} N(r)\right]^{-1} \tag{10}$$

as a function of particle speed in the spacecraft frame v_s along a radial cut through the distribution ($\mu = \pm 1$), for $r = 2$ AU, $A = 4$ AU (such as might apply to pickup protons seen at Ulysses), and a range of values for D. The curves on the left-hand side of Figure 1 correspond to f_- using (10), and the curves on the right-hand side correspond to f_+ using (9). The full distribution may be visualized by taking these curves to wrap around into hemispheres centered on the $v_s/U = 1$ point and separated by the plane containing that point.

We see first that even in the case of no scattering ($D = 0$), some pickup ions are transported to the f_+ hemisphere as a consequence of magnetic focusing in the diverging field. As D increases, the ions can leave the sunward hemisphere more easily and the density in the antisunward hemisphere increases.

Another characteristic of this model is that there is no sharp cutoff at $v_s/U = 2$, and, in fact, f_+ falls smoothly to zero as w approaches 1. This is in distinct contrast to isotropic models, which maximize the density at $w = 1$ as a consequence of taking the source of newly generated ions to be at $v = U$ in all directions. However, in reality, newly generated ions only appear in the sunward hemisphere along the magnetic field, and unless the

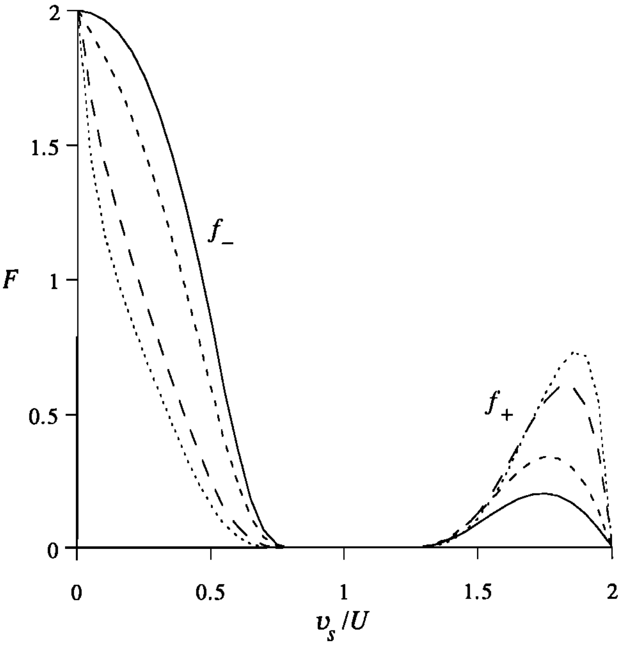

Figure 1. The normalized phase-space density along a radial cut through the distribution in the spacecraft frame, as given by equation (10), for r = 2 AU, A = 4 AU, and a range of values for D. The curves are D = 0 (solid line), D = 1 (short-dashed line), D = 5 (long-dashed line), and D = 10 (dotted line).

field is perpendicular to the radial direction, they must always be transported to $v_s/U = 2$ by a combination of scattering and adiabatic focusing. During this transport, they lose at least some energy to adiabatic deceleration, so there really should be no pickup ions at $v_s/U = 2$ in any nonperpendicular configuration, and the density should vanish there. The sharpness of this fall-off of course depends on how much time the ions need to reach the antisunward direction ($\mu = 1$ in our radial configuration), and larger values of D result in more abrupt decreases near $v_s/U = 2$. Similarly, the source at $w = 1$ in the sunward hemisphere (7) is twice as large as it is for isotropic models, since it is spread over half the area in phase space, and this leads to $F = 2$ at $v_s/U = 0$. As D increases, the fall in density near $v_s/U = 0$ also becomes sharper. In the limit that $D \to \infty$, the density changes at $v_s/U = 0$ and 2 become infinitesimally narrow, and the solution given by (8) and (9) reduces to the isotropic solution of *Vasyliunas and Siscoe* [1976].

4. Discussion

We have presented an analytical model for the distribution of interstellar pickup ions in a radial magnetic field which includes the important physical processes of convection, adiabatic deceleration and focusing, and pitch angle scattering toward isotropy in the frame moving with the solar wind. This model naturally produces radial anisotropies as a consequence of the anisotropic, sunward source for pickup ions, together with a finite time for transport to the antisunward direction. The hemispherical assumption, that the only impediment to instantaneous pitch angle scattering to isotropy occurs at $\mu = 0$, is central to the detailed results of the model, but this assumption is apparently supported by the analysis of the pickup proton observations given by *Fisk et al.* [1996].

The hemispherical assumption can also resolve a paradox pointed out by *Gloeckler et al.* [1995], who noted that the presence of these large, apparently stable anisotropies seemed incompatible with considerable theoretical work on the hydromagnetic instability of streaming ion distributions and the self-consistent reduction of the anisotropy [e.g., *Lee and Ip*, 1987 and references therein]. However, these instabilities are excited by pitch angle gradients in the distribution, and in our model the only gradients are around $\mu = 0$. In principle, these gradients in the model are maintained by the fact that there are very few waves to resonate with these particles. This same absence of resonant wave power for ions with μ near zero can explain the squelching of the streaming instability and the survival of the anisotropy for hemispherical distributions.

A quantity of frequent interest in studies of charged particle transport in the inner heliosphere is the diffusive mean free path along the magnetic field $\lambda_\parallel$. Though any quantity derived from diffusion theory has questionable validity in the presence of significant anisotropies, we can formally obtain an expression for the effective $\lambda_\parallel$ from our model which may be of interest. We take the standard definition of $\lambda_\parallel = 3\,\kappa_\parallel/v$, where the parallel spatial diffusion coefficient is formally given by

$$\kappa_\parallel = \frac{v^2}{8}\int_{-1}^{1} \frac{1-\mu^2}{K(\mu)}\,d\mu \tag{11}$$

in terms of the pitch angle scattering coefficient $K(\mu)$ [*Jokipii*, 1966; *Hasselmann and Wibberenz*, 1968, 1970]. In our model, the pitch angle scattering is finite only at $\mu = 0$, so

$$\frac{1}{K(\mu)} = \Gamma^{-1}\delta(\mu) = \frac{2r}{vD}\,\delta(\mu) \tag{12}$$

which leads to $\lambda_\parallel = 3r/(4D)$.

The next task at hand is naturally to compare the predictions of this model with the measurements of anisotropic pickup ions. Unfortunately, a preliminary attempt to fit the detailed spectrum presented by *Gloeckler et al.* [1995] has not been successful (G. Gloeckler, personal communication, 1996). There are many potential causes for this failure, and it is difficult at this point to identify the relevant ones. The magnetic field may not have been quasi-radial for some significant fraction of the 30-day integration time of the measurement, resulting in a superposition in the data of the hemispherical distribution with a more isotropic state. Alternatively, there may be important deviations from other assumptions of the model, such as the specific v/r dependence assumed for the scattering rate. Finally, there still may be some vital piece of physics that has simply not been thought of here. We will continue to investigate the applicability of this hemispherical model to the available pickup ion observations.

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P. A. Isenberg, Institute for the Study of Earth, Oceans and Space, Morse Hall, University of New Hampshire, Durham, NH 03824-3525. (e-mail: phil.isenberg@unh.edu)

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