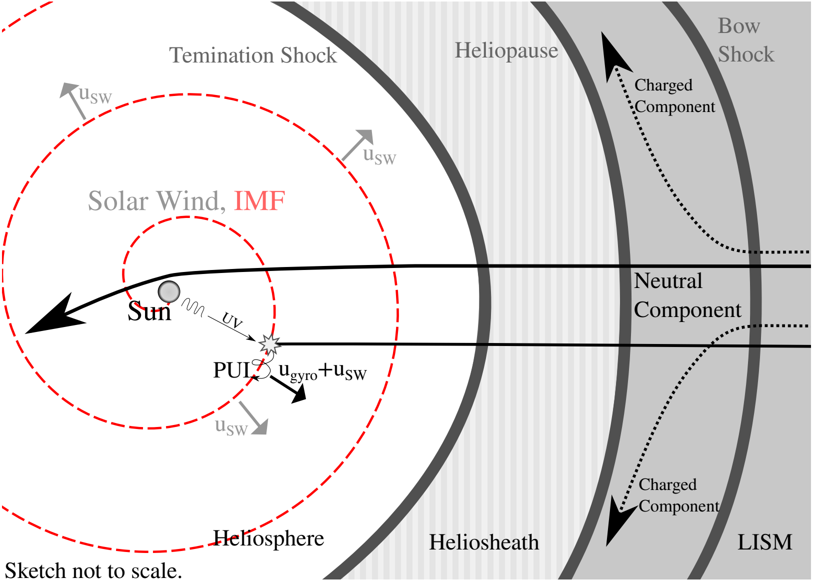
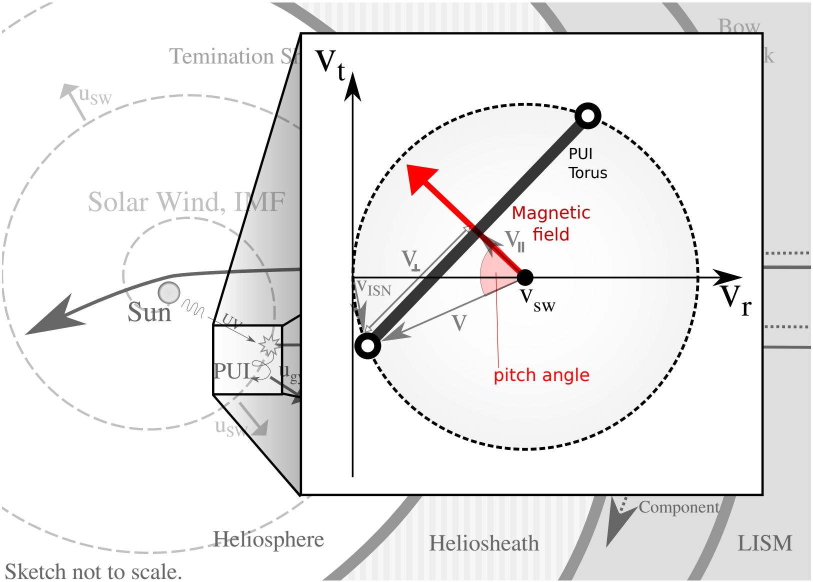
Kinetic simulations of PUI transport and pitchangle-scattering during turbulent conditions of the solar wind

Duncan Keilbach


Temination Shock
Heliopause
Bow Shock
u_{SW}
u_{SW}
Charged Component
Solar Wind, IMF
Neutral Component
Sun
UV
PUI
$u_{gyro}+u_{SW}$
u_{SW}
Charged Component
Heliosphere
Heliosheath
LISM
Sketch not to scale.



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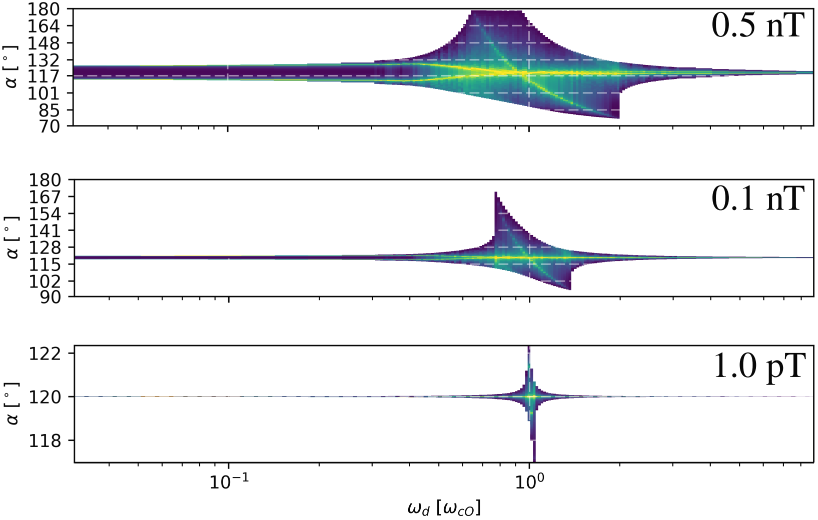

0.5 nT
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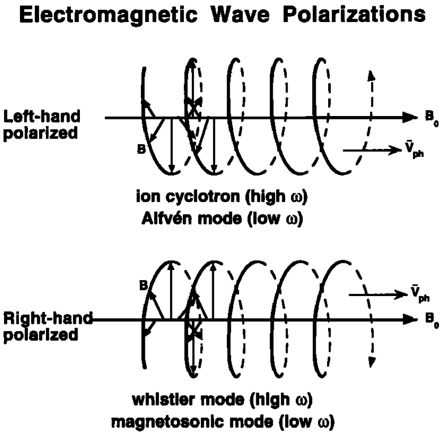
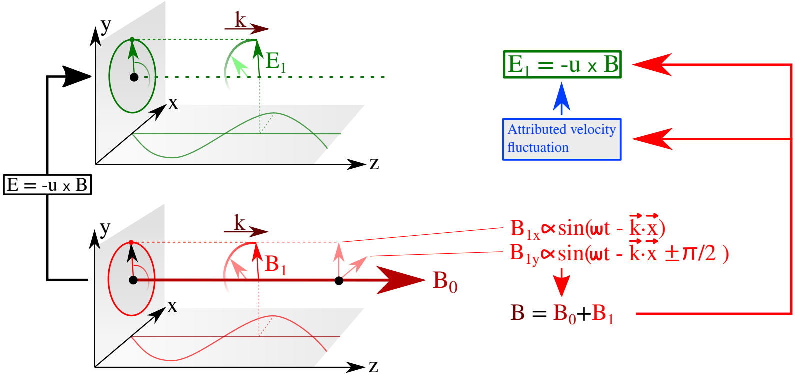


- Circular polarized wave along $\vec{B}$ $\rightarrow v_A = \frac{B}{\sqrt{\mu_0 \rho}}$
- $\delta \vec{u} = \pm \frac{\delta \vec{B}}{\sqrt{\mu_0 \rho}}$
- Induction: $\vec{E} = -\vec{u} \times \vec{B}$

Full MHD-simulation is beyond the scope of this work.

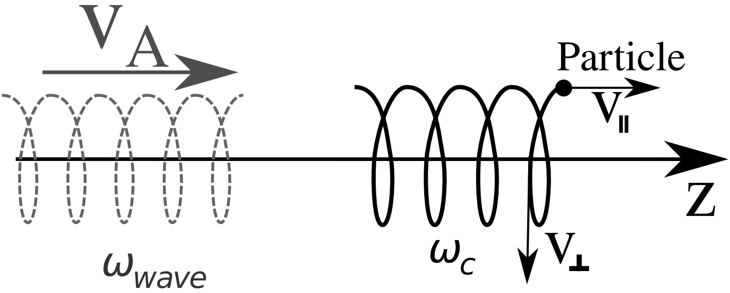
$\Rightarrow$ Analytical expressions for the waves



Dispersion relation:

$$\left(\frac{\omega}{\Omega_p}\right)^2 = (kl_p)^2 + \frac{1}{2}(kl_p)^4 - \frac{1}{2}\sqrt{(kl_p)^2+4}\ , l_p = \frac{c}{\omega_p}$$

General principles



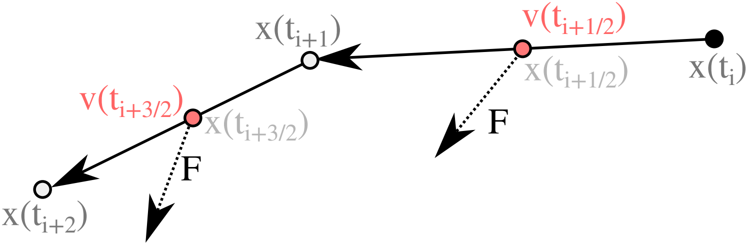
Resonance condition: $n \cdot \omega_c = \omega_d := \omega_{wave} - \vec{k} \cdot \vec{v}_{||}$

$\varepsilon := \left(v_{||} - v_{ph}\right)^2 + v_\perp^2$ **is conserved**

Implementation of the PUI simulation

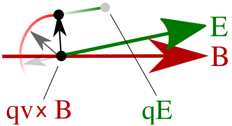
Timestep-Algorithm (Leapfrog)

$$\frac{\Delta\vec{v}}{\Delta t} \approx \vec{a} = \frac{q}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



Optimization (Boris-Scheme)

- $q\vec{E}$: Accelleration along $\vec{E}$
- $q\vec{v} \times \vec{B}$: Rotation around $\vec{B}$

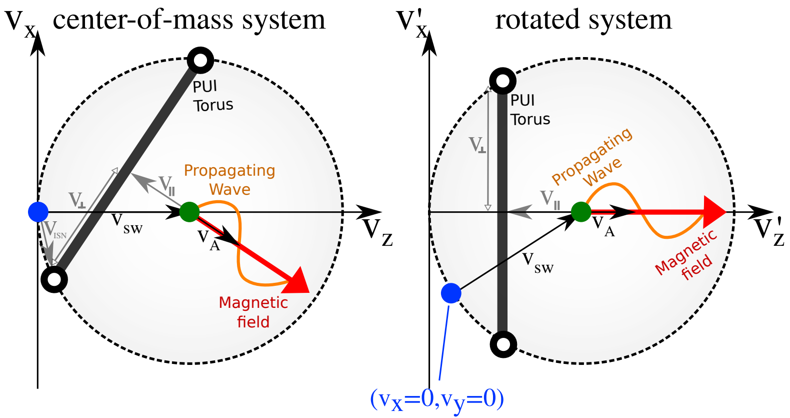


Rotation around $\vec{\mathrm{B}}$ Accelleration along $\vec{\mathrm{E}}$

1. Add half electric impulse
2. Rotate around magnetic field (angle: $\omega_c \cdot \Delta t$)
3. Add another half electric impulse

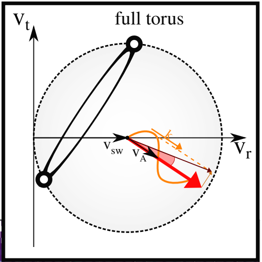
Particle coordinate system

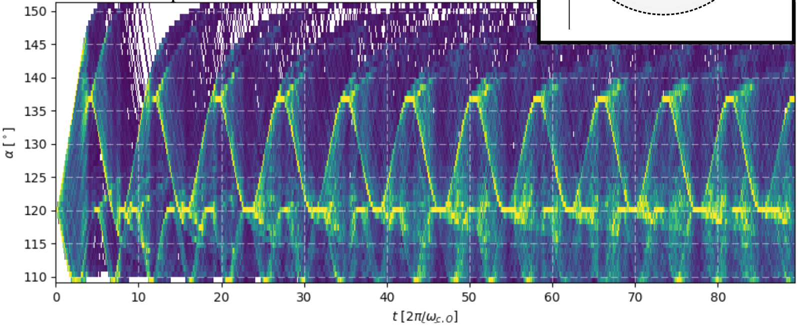
Rotated solar wind bulk system



Boundary Conditions

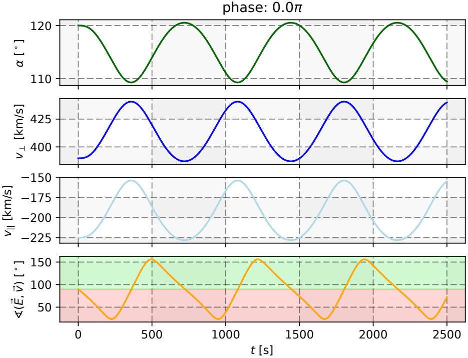
-Background field: 5 nT
-Wave amplitude: 0.1nT
-Wave frequency: 0.28 of oxygen gyro frequency
-Simulated time: 25 wave cycles
-500 test particles, initial pitch angle: 120°
-solar wind speed: 450 km/s

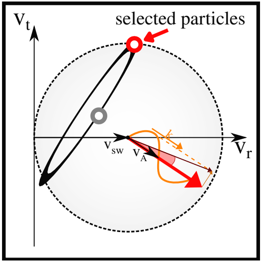


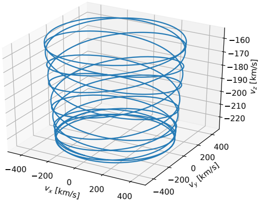


Boundary Conditions

-2 test particles, initial pitch angle: 120°







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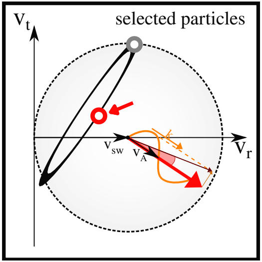

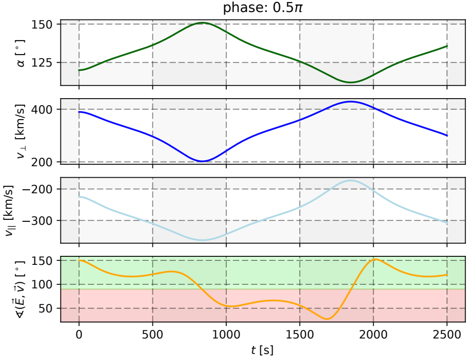

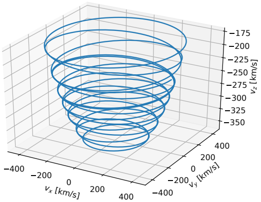

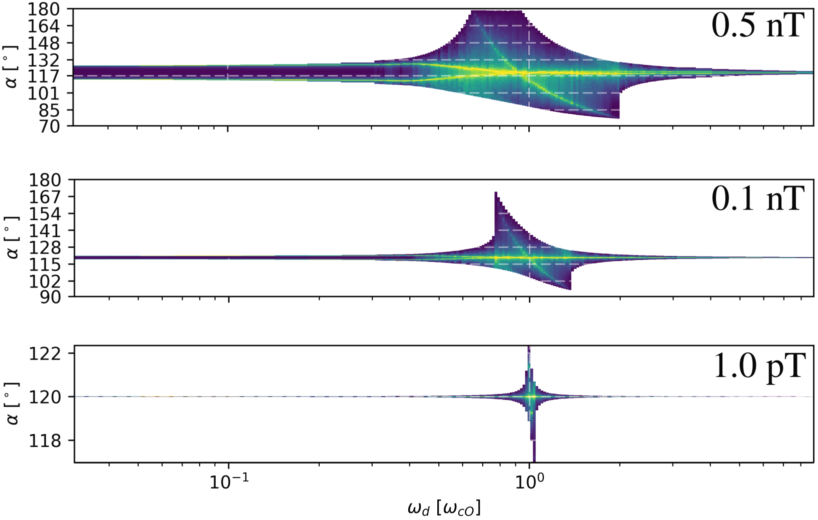
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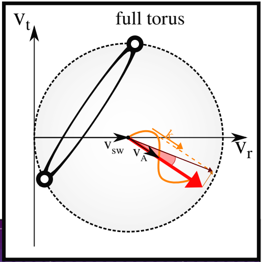
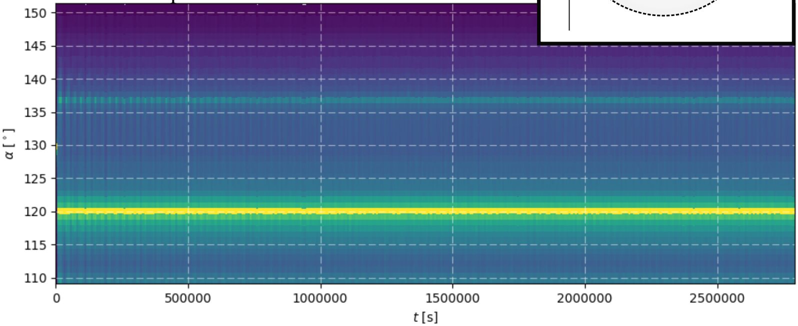
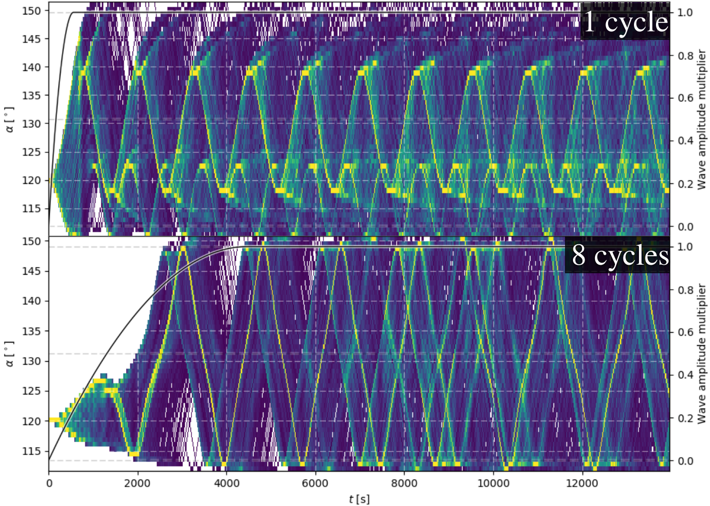
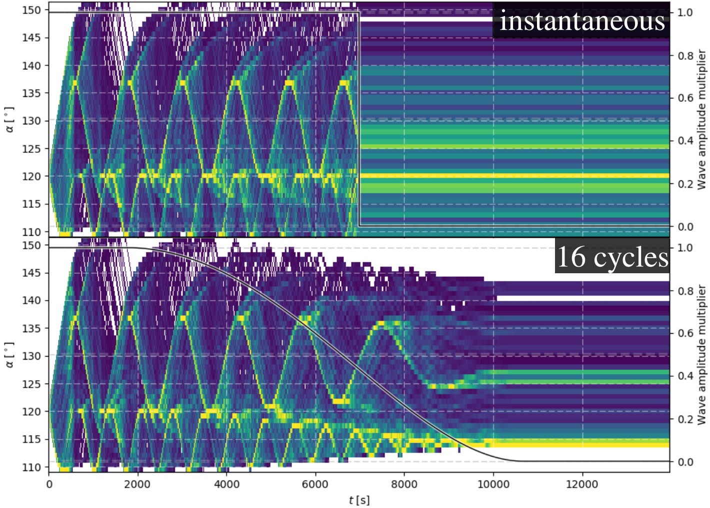




Turning waves on

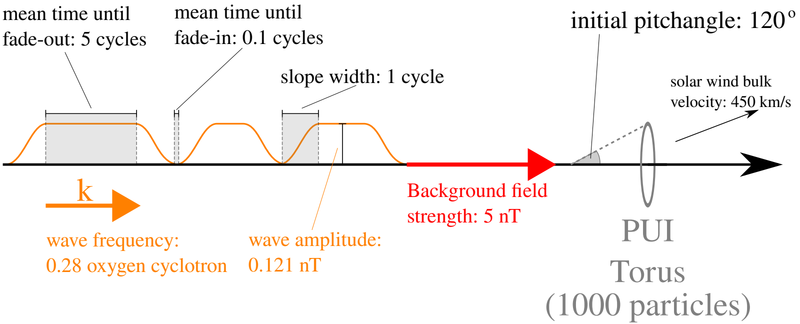


Turning waves off



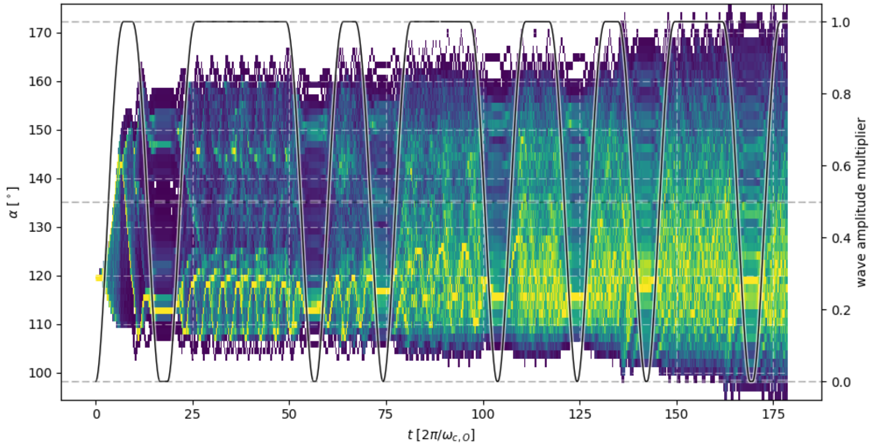
Turning waves on and off randomly

Boundary conditions:

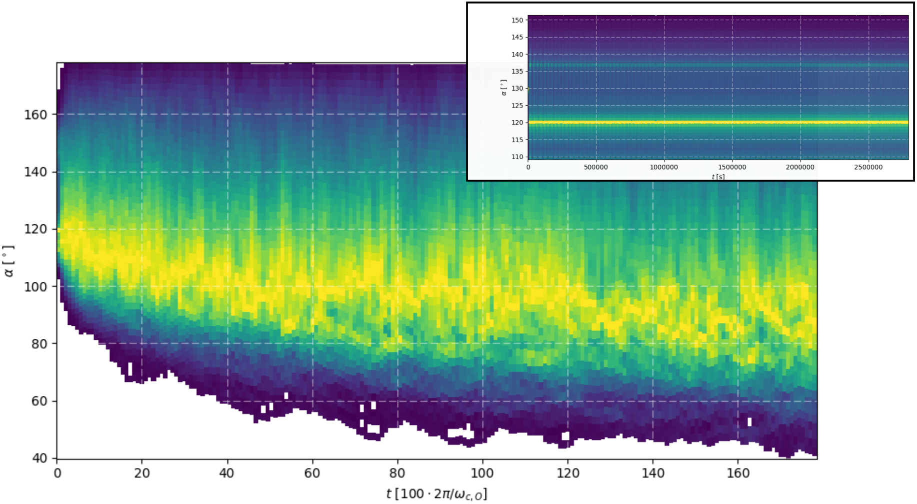


200 consecutive runs of 25 wave cylces each.

Turning waves on and off randomly - first 50 cycles



Turning waves on and off randomly - full simulation

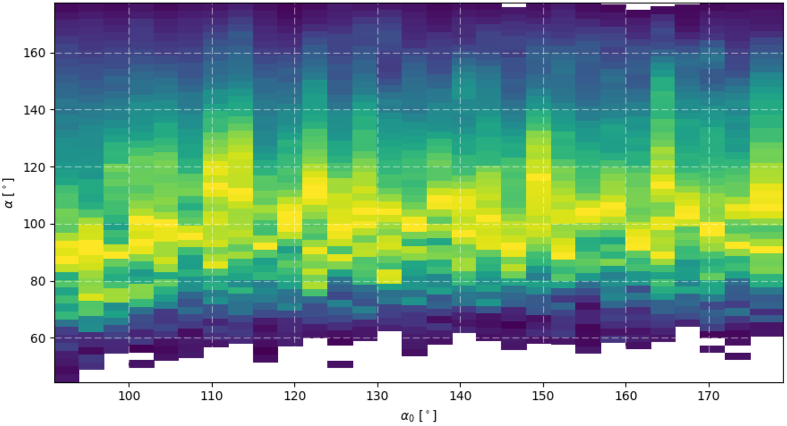


Dependency on the wave amplitude

Standard deviation of the pitch angle distribution after 5000 wave cycles

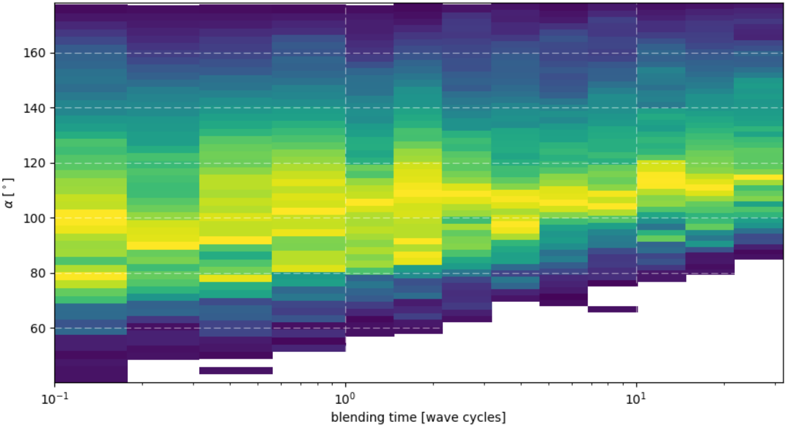
Dependency on the initial pitch angle

Pitch angle distribution after 2500 wave cycles



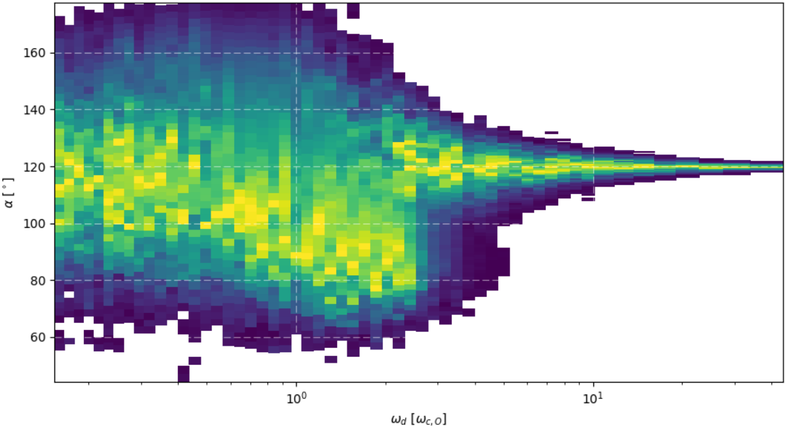
Dependency on the fading - time

Pitch angle distribution after 2500 wave cycles



Dependency on the wave frequency

Pitch angle distribution after 2500 wave cycles



Conclusions and Outlook

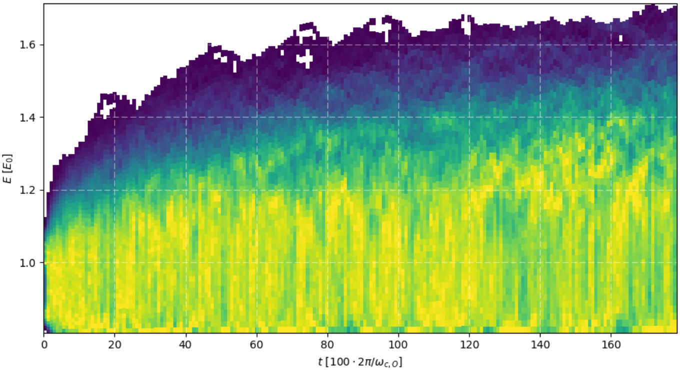
A numerical study to understand wave particle interactions qualitatively on microscopic scales has been performed.

- **Continuous, monofrequent wave fields...**
    - cause stationary oscillations in pitch angle.
    - do not cause the distribution to become broader over time.
- **Intermittent, monofrequent wave fields...**
    - freeze and restore the oscillations at random times
    - cause a systematic broadening of the pitch angle distribution over time

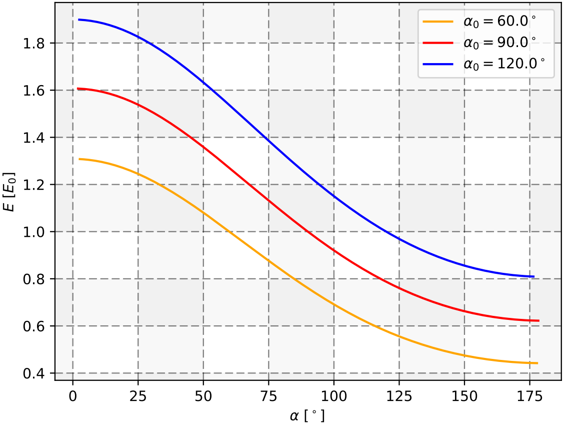
Outlook

- Gradient and curvature in the background field
- Interactions with polyfrequent waves
- Modification by collisions with solar wind particles
- Amplitude modulation

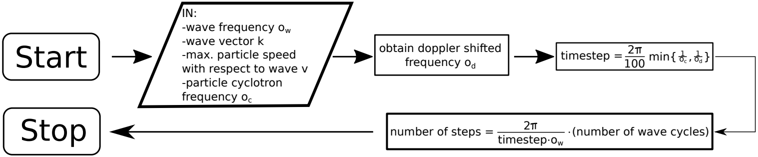
Particle energy distribution



Energy as a function of the pitch angle



Choice of time step



Basic idea:

At least 100 points per gyro period or particle-sampled wave period.

The shortest of these time scales is considered for the time step.

Time derivatives of $v_{||}$ and $v_\perp$

From Lorentz-Force:

$$\dot{\vec{v}} = \frac{q}{m}\left(\delta\vec{E} + \vec{v}\times\left(\vec{B}_0 + \delta\vec{B}\right)\right) \tag{1}$$

$$\dot{\vec{v}} = \frac{q}{m}\delta\vec{E} + \frac{q}{m}\vec{v}\times\vec{B}_0 + \frac{q}{m}\vec{v}\times\delta\vec{B}\ . \tag{2}$$

Obtain the z-Component:

$$\dot{v}_{||} = \frac{q}{m}v_\perp\delta B\sin(\Delta\varphi). \tag{3}$$

Conservation of energy in wave frame:

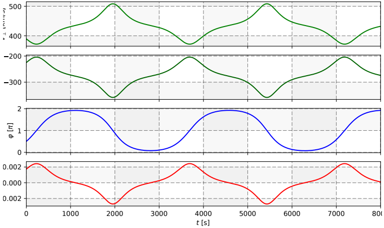
$$\dot{v}_\perp = \dot{v}_{||}\frac{v_{||} - v_\varphi}{v_\perp} \tag{4}$$

Derivative of $v_\perp$:

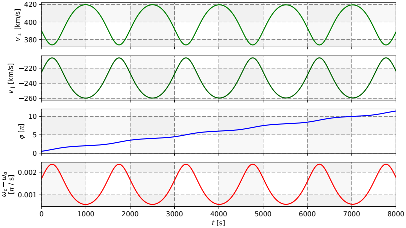
$$\dot{v}_\perp = \frac{q\delta B}{m}\sin(\Delta\varphi)\left(v_{||} - v_\varphi\right) \tag{5}$$

Euler-integration

0.0045 nT



0.0039 nT



Overtaking-criterion

Criterion:
$k|v_{||}| > |\omega_c - \omega|$, before $\Delta\varphi$ reaches 2π.
Linearization: $v_{||} = v_{||,0} + \vartheta \cdot t$ with

$$\vartheta := \dot{v}_{||}(t=0) = \frac{q\delta B}{m}\sqrt{\varepsilon - (v_{||} - v_\varphi)^2} \tag{6}$$

Change in phase difference:

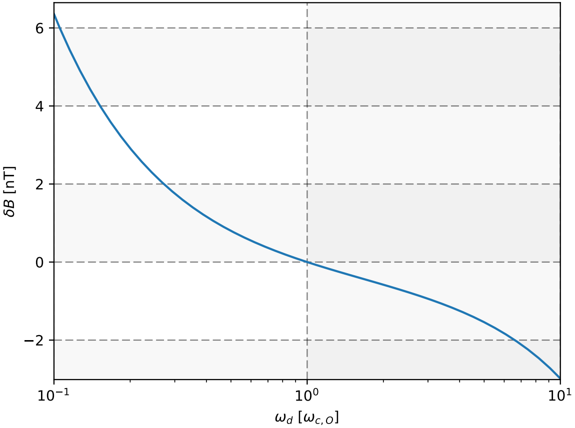
$$\frac{d}{dt}\varphi = \omega_c - \omega = kv_{||,0} + k\vartheta t. \tag{7}$$

Result: Required amplitude for overtaking:

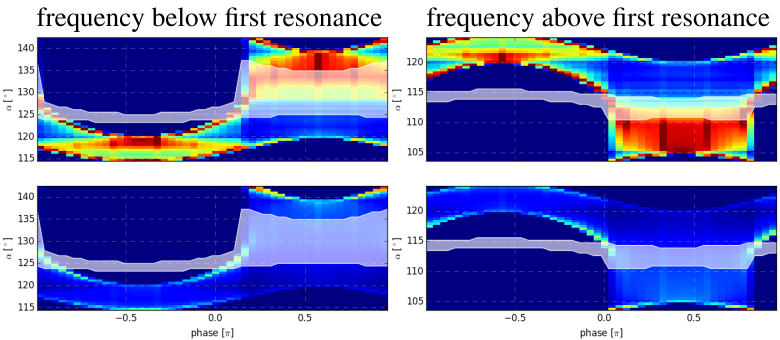
$$\delta B = \frac{m}{q} \cdot \frac{\left(\frac{\omega_c - \omega}{k} - v_{||,0}\right) \cdot \frac{\omega_c - \omega + kv_{||,0}}{2\pi}}{\sqrt{\varepsilon - (v_{||,0} - v_\varphi)^2}} \tag{8}$$

Overtaking-criterion

Required amplitude for overtaking

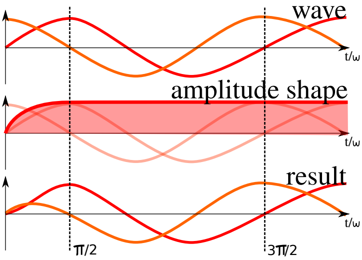


Time-averaged spectra of the pitch angle as a function of $\Delta\varphi$



Implementation of the fields - Remarks

- **Coupling with electrical field:** $\vec{E} = -\vec{v} \times \vec{B}$, $\delta v = \pm \frac{\delta B}{\sqrt{\mu_0 \rho}}$
- **Fade-in:** Wave faded in with a parabolic shape over the first quarter period (for continuous waves).
- Parabolic or fourth-oder shape functions have been used to model intermittent waves.



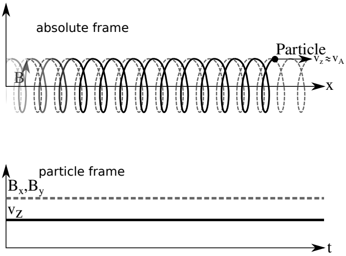
shape functions:

$$f_1(x) = 1 - \left(\left(\frac{x}{a}\right)\right)^2 \qquad (9)$$

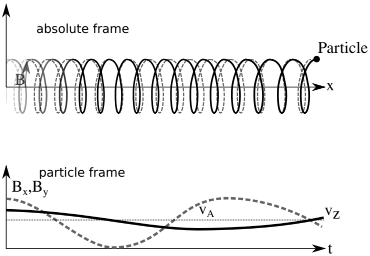
$$f_2(x) = \left(1 - \left(\frac{x}{a}\right)^2\right)^2 \qquad (10)$$

0^{th} order resonance

without interaction



with interaction



Condition: $v_z \approx v_A$
Particle is „surfing“on the wave.
v_z: Synchronization with v_A.

General theoretical considerations

- **initial parallel velocity**
  from resonance condition

$$v_{||} = v_{ph} - \frac{n\omega_c}{k_{||}} \tag{11}$$

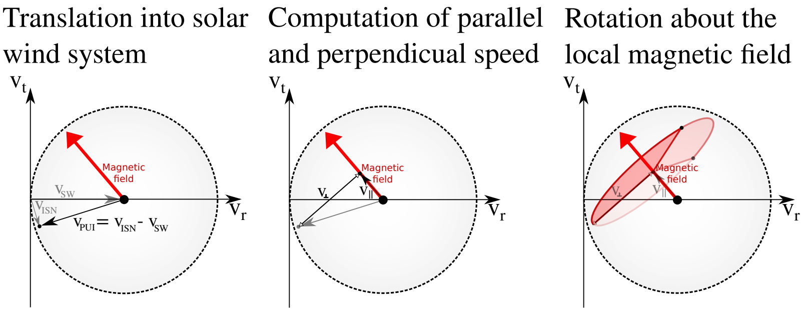
- **The energy in the waveframe is conserved**
  (absolute) energy gain: $\Delta E = h\omega/2\pi$

$$const. = \frac{m}{2}\left(\left(v_{||} - v_{ph}\right)^2 + v_\perp^2\right) \tag{12}$$

Implications:

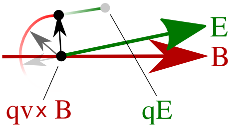
- Parameter for "goodness" of simulation:
  $\varepsilon := \left(v_{||} - v_{ph}\right)^2 + v_\perp^2$
- Change in parallel velocity difference causes perpendicular accelleration/decelleration.

Sampling of torus distributions



Optimization (Boris-Scheme)

- $q\vec{E}$: Accelleration along $\vec{E}$
- $q\vec{v} \times \vec{B}$: Rotation around $\vec{B}$



Rotation around $\vec{\mathrm{B}}$ Accelleration along $\vec{\mathrm{E}}$

1. Add half electric impulse to $\vec{v}_{old}$: $\vec{v}^- := \vec{v}_{old} + q\Delta t\vec{E}/2m$.
2. Rotate $\vec{v}^-$ around $\vec{B}$ for $\vec{v}^+$: $\vec{v}^+ := \vec{v}^- + \vec{v}' \times \vec{s}$,where $\vec{s} = 2\vec{t}/(1 + \vec{t}^2)$ and $\vec{t} = q\vec{B}\Delta t/2m$.
3. Add the other half electric impulse to get $\vec{n}_{new}$: $\vec{v}_{new} := \vec{v}^+ + q\Delta t\vec{E}/2m$

Correllation between $\delta\vec{u}$ and $\delta\vec{B}$

Equation of motion and equation of induction:

$$\rho_m \frac{\partial \vec{v}_m}{\partial t} = \vec{j} \times \vec{B} \tag{13}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\nabla \times \vec{B}). \tag{14}$$

Assumptions:

$$\vec{B} = \vec{B}_0 + \vec{B}_1 \quad , \vec{v}_m = \vec{v}_0 + \vec{v}_1 \tag{15}$$

$$\vec{B}_0 = (0, 0, B_0) \quad , \vec{B}_1 = (B_{1x}, 0, 0) \quad , \vec{v}_0 = 0 \quad , \vec{v}_1 = (v_{1x}, 0, 0) \tag{16}$$

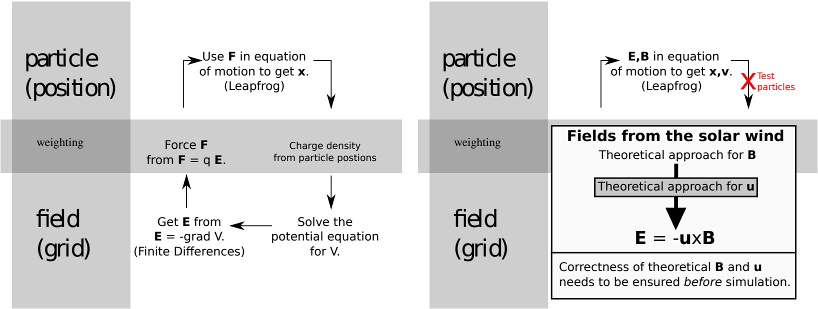
Equations simplify to: $\rho_m \frac{\partial B}{\partial t} = \mu_0 \frac{\partial u}{\partial z}$

Wave equations for $\vec{u}_1$ and $\vec{B}_1$ predict a sinusoidal disturbance, which is correllated.

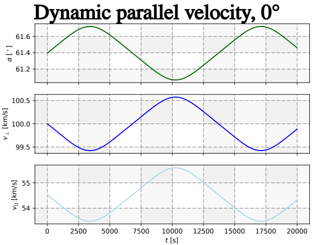
This yields $\vec{u}_1, \vec{B}_1 \propto \sin(\omega t \pm \vec{k} \cdot \vec{x})$. Amplitudes: $\delta\vec{u}, \delta\vec{B}$.

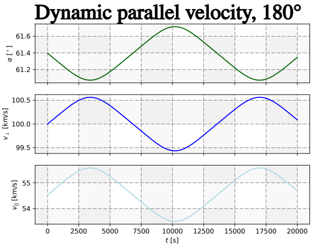
$\Rightarrow \quad \delta\vec{u} = \pm\delta\vec{B}/\sqrt{\mu_0 \rho_m}$

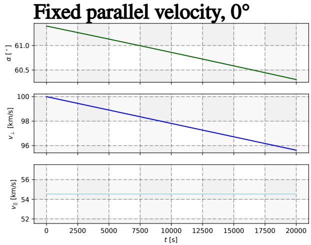
Comparison: PIC vs. test particles

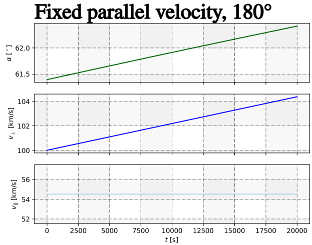


Boundary conditions: $\omega = 0.5\ \omega_{c,O},\ \ \delta B = 5 \cdot 10^{-4}$ nT

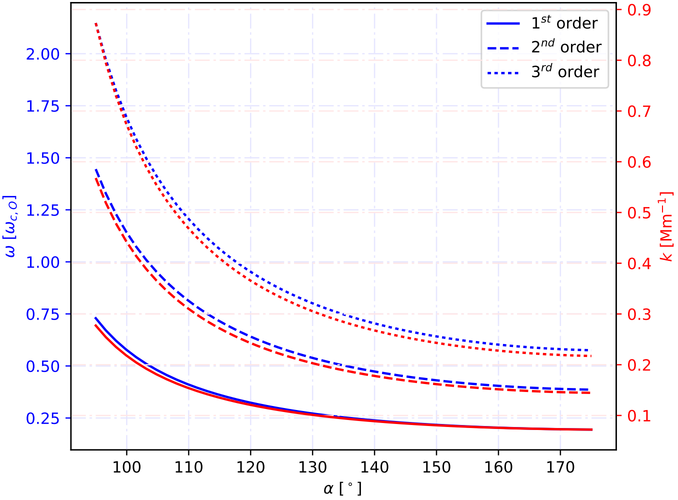


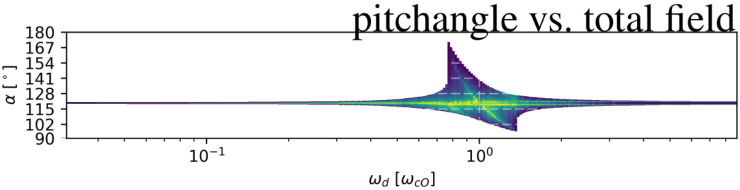






Resonant frequencies and wavevectors

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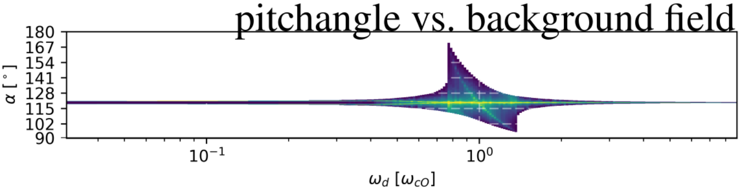
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10^{-1}
10^{0}
ω_d [ω_{cO}]