Problem Set 1

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1

For reference: $1day * (24hrs/1day) * (3600s/1hr) * (10^6/1s) = 8.64x10^4 \mu s$

A

In this case, Acme should pay professor Flitwick.
$n = 41$
$f(n) = 1.99^n$
$g(n) = n^3$
$t = 17days = 1.4688x10^{12} \mu s$

Without Flitwick, it takes $f(41) = 1.99^{41} = 1.79x10^{12} \mu s$
With Flitwick it takes $t + g(n) = 1.47x10^{12} \mu s + 41^3 = 1.46x10^{12} \mu s$

It will take Flitwick less time to spend 17 days working and then running his program than it will take to just run the program by itself:

Flitwick's alg is $17days, 0.069s$
The other lag will take $20days, 17hours, 21min, 47.45seconds$

B

In this case, Acme should not pay professor Flitwick.
$n = 10^6$
$f(n) = n^{2.00}$
$g(n) = n^{1.99}$
$t = 2days = 2 * 8.64x10^{10} \mu s = 1.728x10^{11} \mu s$

Without Flitwick, it takes $f(10^6) = (10^6)^{2.00} = 10^{12} \mu s =$
$11.574days = 11days, 13hours, 46minutes, 40seconds$

With Flitwick it takes $t + g(n) = 2days + (10^6)^{1.99} = 1.73x10^{11}\mu s + 8.71x10^{11}\mu s = 1.044x10^{12}\mu s$
$1.044x10^{12}/8.64x10^{10}\mu s = 12.08days = 12days, 1hr, 56min, 3.6s$

It is a close call, but it will be faster to just run the original algorithm and not pay Flitwick.

2

3

A

Is there a c for which $0 \leq 2^{nk} \leq c2^n$ for $k > 1$?

Dividing both sides by 2^n:
$(2^n)^{k-1} \leq c$

No. There is no constant c that is greater or equal to $(2^n)^{k-1}$ for sufficiently large n.

B

Is there a c for which $0 \leq 2^{n+k} \leq c2^n$ for $0 \leq k \leq c$ (some positive constant)?

$2^n * 2^k \leq c2^n$
Dividing both sides by 2^n:
$2^k \leq c$for Yes. For $n_o = 0, c \geq 2^k$ where $k \geq 0$.

4

A

B

Final order: 1, $n^{1/lgn}$, 2^{lg*n}, $(\sqrt{2})^{lgn}$, n, $nlgn$, $lg(n!)$, n^2, $(3/2)^n$, e^n, $(lgn)!$, $n!$