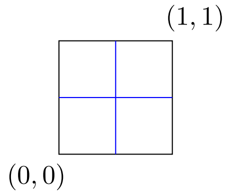
1 2D Cubes

1.1 Division into Cubes

We divided a cube into several sub-cubes. The integral over the large cube should correspond to the sum of the integrals over the smaller cubes.

The division used is shown below.



We calculated the following integral and looked at the convergence of the right hand side against the value calculated for the left hand side. This convergence is shown in figure 1.

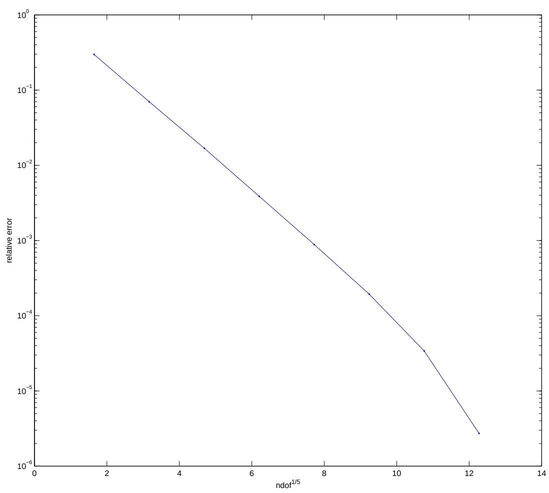


Figure 1: Convergence of the sum of integrals over cubes. $\mathrm{ndof}^{1/5}$ is plotted against the logarithm of the relative error.

This test can be found in the file `testcube.m`.

1.2 Tests with $\|y-x\|^{\alpha}$

We can calculate the exact solution to

$$I = \int_{x\in[0,1]^2}\int_{y\in[0,1]^2} \|y-x\|^{\alpha} dydx,$$

where $\alpha = -2 + \frac{1}{\pi}$.

The solution is approximately $I = 14.5558268793833356$. (The calculation of this result can be found in the file `QuadLib/ndxnd/CubexCube/identicalcubes2d.tex`)

The convergence of the quadrature towards this value is shown in Figure 2.

This test can be reproduced by executing the following lines.

```
alpha = (-2+1/pi);
F = @(x,y) sqrt(sum(z.^2,2)).^alpha;
vertexlist=[0 1 0; 0 0 1];
[t,wt]=squad2dquad_tensor(nr,2*nr, vertexlist);
Q(cnt)=sum(F(t(:,5:6)).*wt);
```

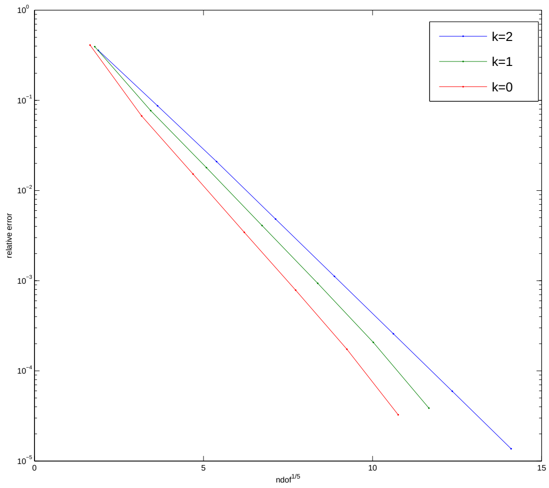


Figure 2: Convergence against the analytically calculated value. $\mathrm{ndof}^{1/5}$ is plotted against the logarithm of the relative error.

We can calculate the exact solution to

$$I = \int_{x\in[0,1]^2} \int_{y\in[-1,0]\times[0,1]} \|y-x\|^\alpha dydx,$$

where $\alpha = -3 + \frac{1}{\pi}$.

The solution is approximately $I = 5.82649697$.(The calculation of this result can be found in the file `QuadLib/ndxnd/CubexCube/comedgecubes2d.tex`)

The convergence of the quadrature towards this value is also shown in Figure 2.

This test can be reproduced by executing the following lines.

```
alpha = (-3+1/pi);
F = @(x,y) sqrt(sum(z.^2,2)).^alpha;
vertexlist=[0 1 0 0; 0 0 1 -1];
[t,wt]=squad2dquad_tensor(nr,2*nr, vertexlist);
Q(cnt)=sum(F([t(:,5) t(:,4)-t(:,2)]).*wt);
```

We can calculate the exact solution to

$$I = \int_{x\in[0,1]^2} \int_{y\in[-1,0]^2} \|y-x\|^\alpha dydx,$$

where $\alpha = -4 + \frac{1}{\pi}$.

The solution is approximately $I = 1.81135918$.(The calculation of this result can be found in the file `QuadLib/ndxnd/CubexCube/comvertexcubes2d.tex`)

The convergence of the quadrature towards this value is also shown in Figure 2.

This test can be reproduced by executing the following lines.

```
alpha = (-4+1/pi);
F = @(x,y) sqrt(sum(z.^2,2)).^alpha;
vertexlist=[0 1 0 -1 0; 0 0 1 0 -1];
[t,wt]=squad2dquad_tensor(nr,2*nr, vertexlist);
Q(cnt)=sum(F(t(:,3:4)-t(:,1:2).*wt);
```

Thus we have tested all the different configurations for cubes in two dimensions against analytically calculated values.

2 Tests with $\log(\|y-x\|)y_1^3x_2^2$

We use Mathematica to calculate the values of the integral

$$I = \int_{x\in C^{(1)}} \int_{y\in C^{(2)}} \log(\|y-x\|)y_1^3x_2^2 dydx$$

for different configurations of the cubes $C^{(1)}$ and $C^{(2)}$.

1. $C^{(1)} = C^{(2)} = [0, 1]^2$:
$$I \approx -0.06105493109$$
This test can be reproduced by executing the following lines.

```
F=@(x,y)(log(sqrt((y(:,1)-x(:,1)).^2+(y(:,2)-x(:,2)).^2)).*y(:,1).^3.*x(:,2).^2);
vertexlist=[0 1 0; 0 0 1];
[t,wt]=squad2dquad_tensor(nr,2*nr, vertexlist);
Q(cnt)=sum(F(t(:,1:2),t(:,3:4)).*wt);
```

2. $C^{(1)} = [0, 1]^2$, $C^{(2)} = [-1, 0] \times [0, 1]$:
$$I \approx 0.02368248462$$
This test can be reproduced by executing the following lines.

```
F=@(x,y)(log(sqrt((y(:,1)-x(:,1)).^2+(y(:,2)-x(:,2)).^2)).*y(:,1).^3.*x(:,2).^2);
vertexlist=[0 1 0 0; 0 0 1 -1];
[t,wt]=squad2dquad_tensor(nr,2*nr, vertexlist);
Q(cnt)=sum(F(t(:,1:2),t(:,5:6)).*wt);
```

3. $C^{(1)} = [0, 1]^2$, $C^{(2)} = [-1, 0]^2$:
$$I \approx -0.04915397545$$
This test can be reproduced by executing the following lines.

```
F=@(x,y)(log(sqrt((y(:,1)-x(:,1)).^2+(y(:,2)-x(:,2)).^2)).*y(:,1).^3.*x(:,2).^2);
vertexlist=[0 1 0 -1 0; 0 0 1 0 -1];
[t,wt]=squad2dquad_tensor(nr,2*nr, vertexlist);
Q(cnt)=sum(F(t(:,1:2),t(:,5:6)).*wt);
```

The convergence against these values is shown in Figure 3.

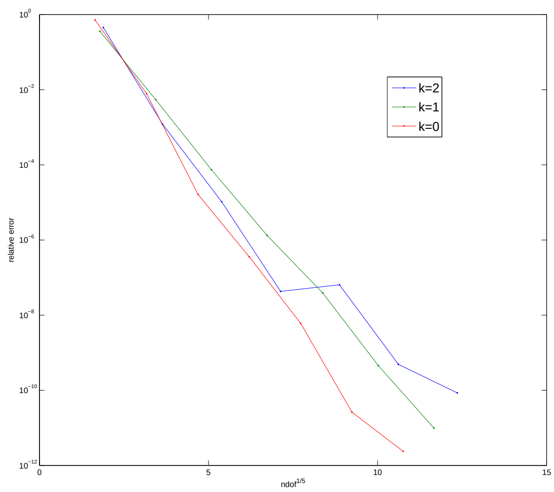
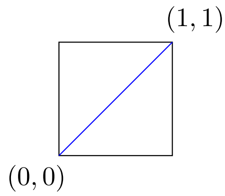


Figure 3: Convergence against the value calculated by Mathematica. ndof$^{1/5}$ is plotted against the logarithm of the relative error.

3 2d Simplices

3.1 Division of a Cube

The unit Cube $[0,1]^2$ can be divided into simplices as below. The value of



$$I = \int_{x\in[0,1]^2} \int_{y\in[0,1]^2} \|y-x\|^{\alpha} dy dx,$$

where $\alpha = -2 + \frac{1}{\pi}$ is approximately $I = 14.5558268793833356$.(The calculation of this result can be found in the file `QuadLib/ndxnd/CubexCube/identicalcubes2d.tex`)

We now look at the convergence of the sum of integrals over simplices against the value I. It is given in Figure 4.

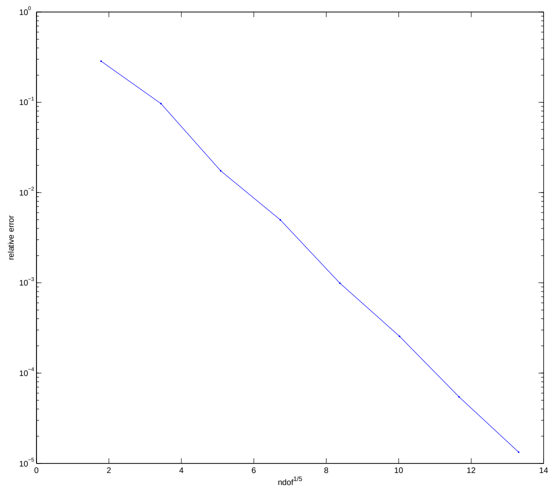
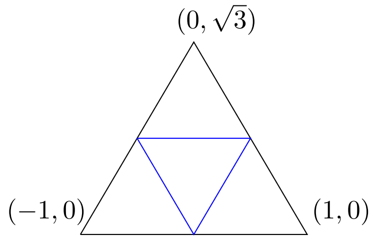


Figure 4: Convergence against the value I. ndof$^{1/5}$ is plotted against the logarithm of the relative error.

This test can be found in the file `testsimp.m`.

3.2 Division of Simplex

We can divide a Simplex into sub-simplices as shown below.



Let the simplex with vertices $\{(-1,0),(1,0),(0,\sqrt(3))\}$ be called $S^{(1)}$.

3.2.1 The integrand $\|y-x\|^\alpha$

We integrate

$$I = \int_{x\in S^{(1)}} \int_{y\in S^{(1)}} \|y-x\|^\alpha dydx,$$

where $\alpha = -2 + \frac{1}{\pi}$.

We now look at the convergence of the sum of integrals over simplices against the value of the integral over $S^{(1)}$. It is given in Figure 5.

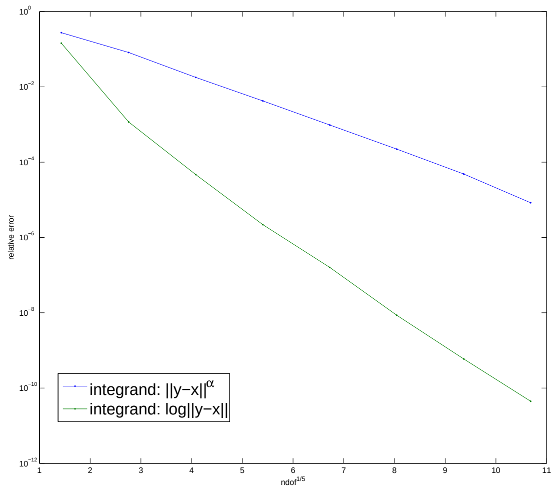


Figure 5: Convergence against the value I. ndof$^{1/5}$ is plotted against the logarithm of the relative error.

This test can be found in the file `testsimp2.m`.

3.2.2 The integrand $\log(\|y-x\|)$

We integrate

$$I = \int_{x\in S^{(1)}} \int_{y\in S^{(1)}} \log(\|y-x\|)dydx.$$

We now look at the convergence of the sum of integrals over simplices against the value of the integral over $S^{(1)}$. It is given in Figure 5.

This test can be found in the file `testsimp2.m`.