

Modeling Household Carbon Footprints: Methods, Metrics, and Estimation Frameworks

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1 Introduction

Climate change poses a pressing global challenge, driven not only by industrial activity and energy production but also by the cumulative impact of household-level demand. Residential consumption, encompassing everything from energy use and food choices to investment behaviors, accounts for nearly 17–40% of global greenhouse gas (GHG) emissions, depending on measurement approach and boundary definitions. Accurately accounting for household carbon footprints (HCFs) is therefore pivotal to designing mitigation strategies that are both equitable and effective.

To date, HCF estimation methods have evolved along multiple trajectories, each addressing different facets of systemic emissions. Emission factor–based accounting, including the GHG Protocol and IPCC Guidelines, offers a clear rule-based framework by tying activity data to standardized emissions coefficients. Life cycle assessment (LCA) enriches this by providing cradle-to-grave tracing of product-related emissions, though it remains vulnerable to data truncation, boundary limitations, and uneven standardization. Environmentally extended input-output analysis (EEIOA) captures the full spectrum of supply-chain emissions by linking household expenditures with sectoral emissions embedded across global trade networks. Moreover, hybrid methods integrate product-level LCA detail into IO frameworks, while recent dynamic EEIO applications have further allowed for the analysis of capital stock evolution and cross-regional interdependencies.

Despite their sophistication, these frameworks often treat households as passive actors, failing to account for price-mediated behavior, investments, or feedback effects across markets. The general equilibrium model developed by Hakenes and Schliephake (2024) addresses this deficiency by explicitly modeling how household consumption and investment decisions trigger economy-wide emissions through price mechanisms and behavioral spillovers. This thesis compares these four methodological paradigms—emission factor accounting, LCA, EEIOA, and general equilibrium modeling—by applying each to a unified empirical dataset and evaluating their implications for carbon responsibility, policy priorities, and supply-side intervention.

The primary objective of this work is twofold: first, to assess the analytical robustness and policy relevance of each HCF estimation method; second, to demonstrate that while household-level behavioral change has symbolic value, lasting climate impact requires structural interventions in supply chains and production systems. As energy and food demand grow with demographic and economic trends, this study posits that upstream policy—rather than targeting aggregate household behavior—is the most potent lever for sustainable decarbonization.

The paper is structured as follows: Section 2 reviews existing literature, Sections 3–7 present each methodology and implementation, Section 8 offers a comparative discussion, and Section 9 concludes with policy and research implications.

2 Literature Review

The household carbon footprint has been studied extensively using varying definitions and scopes. Early contributions by Pachauri and Spreng (2002) and Lenzen et al. (2004) laid the groundwork for estimating household emissions using input-output techniques. Druckman and Jackson (2009) expanded this literature by linking household expenditure patterns with emission intensities, emphasizing the heterogeneity of household behavior. Baiocchi and Minx (2010) provided one of the first large-scale comparative assessments across European countries. Subsequent studies such as Hertwich and Peters (2009), Ivanova et al. (2016), and Moran et al. (2018) further refined household-level carbon accounting, incorporating global supply chains and inequality dimensions. These foundational studies established the analytical relevance of household carbon footprints in understanding consumption-based emissions and set the stage for methodological innovations.

A bibliometric analysis of 1,311 peer-reviewed articles published between 2000 and 2025 reveals a substantial and accelerating growth in research on household carbon footprints. The volume of publications increased significantly after 2010, with a pronounced surge following the adoption of the Paris Agreement in 2015. To account for incomplete data in 2025, the publication count for the year was estimated by doubling the observed mid-year value, assuming a consistent publication rate throughout the year. This adjustment avoids visual misinterpretation of a declining trend and is reflected in Figure 2.1, which shows the annual publication trajectory. The most frequent publication outlets include the *Journal of Cleaner Production*, *Science of the Total Environment*, and *Environmental Science and Technology*. Leading contributors in terms of publication count and influence include Long et al., Heinonen et al., and Li et al., whose work has focused on input-output modeling, behavioral drivers, and carbon inequality. Keyword co-occurrence analysis (Figure 2.2) indicates strong thematic linkages among “input-output analysis,” “life cycle assessment,” “sustainable consumption,” and “GHG emissions.” Co-authorship network patterns also reveal the formation of dense collaboration clusters, particularly around institutions such as the University of Tokyo, Sun Yat-sen University, and the University of Maryland. A full breakdown of annual publication counts, author rankings, and keyword frequencies is provided in Appendix A (Table A.1–A.3).

The estimation of household carbon footprints in the literature has relied on a range of methods, each grounded in distinct assumptions about attribution, causality, and system boundaries. The Greenhouse Gas (GHG) Protocol developed by the World Resources Institute and the World Business Council for Sustainable Development (WRI and WBCSD, 2004) remains a foundational standard for measuring direct (Scope 1), energy-related (Scope 2), and indirect (Scope 3) emissions. Its approach, based on multiplying activity data by standardized emission factors, is also formalized in the IPCC Guidelines (IPCC, 2019), and has been applied in national inventories and corporate disclosures. Weber and Matthews (2008) adapted this framework for household-level analysis in the United States, producing emission estimates based on household expenditure data. Life cycle assessment (LCA) methods represent a more detailed approach, es-

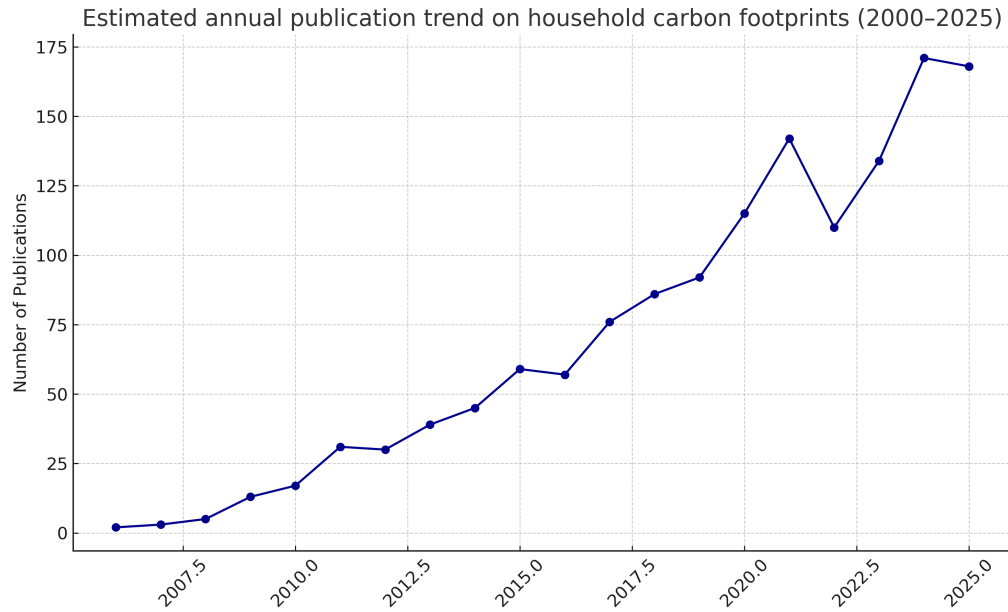


Figure 2.1: Estimated annual publication trend on household carbon footprints (2000–2025). The 2025 value is a linear extrapolation based on mid-year data.

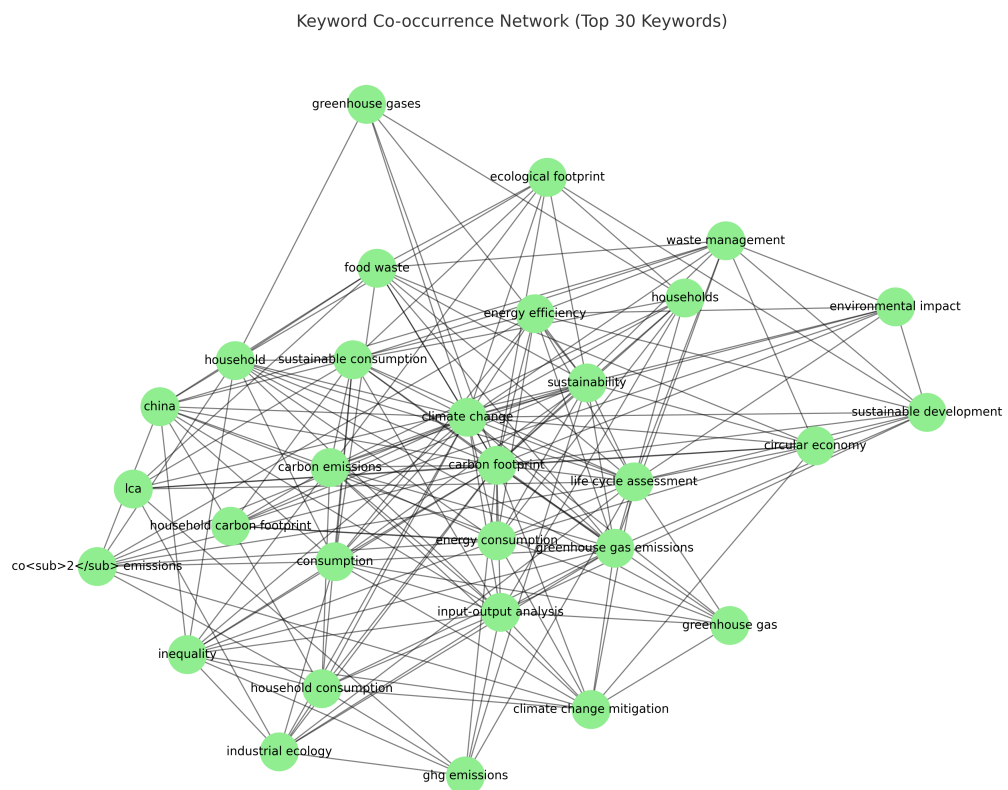


Figure 2.2: Keyword co-occurrence network for the 30 most frequently used terms in household carbon footprint literature (2000–2025).

timating cradle-to-grave emissions across all stages of a product's life. Studies such as Guinée (2011) and Steubing et al. (2022) apply LCA to household consumption, though the method often requires high-resolution data and faces boundary truncation issues. Input-output analysis (IOA), especially in its environmentally extended form (EEIOA), has enabled system-wide footprinting by linking household expenditures to upstream emissions. Baiocchi and Minx (2010) applied EEIOA to compare household footprints across Europe, while Wiedmann et al. (2009) introduced a theoretical foundation for multi-regional models. Ivanova et al. (2016) added a spatial inequality perspective across global households. Hybrid models have sought to combine the detail of LCA with the coverage of IOA, as in Sonesson et al. (2017) for food systems and in Chard (2024) and Hasegawa et al. (2021) for investment-based emissions. Finally, the model of Hakenes and Schliephake (2024) represents a general equilibrium approach in which household consumption and investment affect emissions indirectly through price changes and behavioral responses across sectors. This model introduces inter-household spillovers and endogenized market feedbacks, offering a causally coherent framework for footprint attribution.

Despite substantial methodological innovation, critical gaps remain in the literature on household carbon footprint estimation. Emission factor models and life cycle assessment frameworks often treat household preferences and market interactions as exogenous or fixed, limiting their ability to simulate policy-induced behavioral change (Guinée 2011; IPCC 2019). Environmentally extended input-output models, while capable of tracing upstream emissions across global supply chains, typically rely on static technical coefficients and lack behavioral realism in their representation of household decision-making (Wiedmann et al. 2009; Ivanova et al. 2016). Hybrid approaches that combine life cycle data with IO frameworks improve sectoral resolution but generally do not incorporate the endogeneity of prices, income effects, or inter-household spillovers (Sonesson et al. 2017; Chard 2024). Moreover, recent studies that examine the carbon intensity of investment portfolios have expanded the scope of household responsibility (Hasegawa et al. 2021), yet often do so in partial equilibrium settings that abstract from feedback effects across markets. The model introduced by Hakenes and Schliephake (2024) addresses many of these limitations by embedding household consumption and investment behavior within a general equilibrium framework. This approach allows for the propagation of individual decisions through price mechanisms and sectoral reallocation, thereby capturing indirect responsibility in a structurally consistent way.

Taken together, the reviewed literature offers a broad but fragmented understanding of household carbon responsibility. While methodological advances have enhanced precision and coverage, questions remain about the causal validity and policy relevance of different approaches. This thesis contributes by systematically comparing four major estimation methods—emission factor accounting, life cycle assessment, input-output analysis, and the general equilibrium model proposed by Hakenes and Schliephake—through empirical illustrations and critical evaluation. Beyond methodological comparison, the analysis is motivated by a broader concern: that household-level behavioral change, while symbolically important, offers limited mitigation potential without structural reform on the supply side. As energy and food demands grow

with population pressures, the study advocates for shifting the focus of policy design toward upstream interventions in production systems rather than overemphasizing downstream consumption.

3 Methodology

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4 The GHG Protocol

The Greenhouse Gas (GHG) Protocol is a globally recognized standard for accounting and reporting greenhouse gas emissions. Developed in the late 1990s through a collaboration between the World Resources Institute (WRI) and the World Business Council for Sustainable Development (WBCSD), the GHG Protocol was officially launched in 2001 with the primary aim of providing a consistent and comprehensive framework for emissions accounting across corporate and public sectors. Over the years, its importance has grown significantly, with subsequent expansions such as the development of the GHG Protocol Scope 3 Standard in 2011, which broadened the accounting boundary to include indirect emissions across a company's or household's value chain. The historical motivation behind its creation was rooted in the need for greater transparency and comparability in emissions disclosures, especially as climate policy instruments and stakeholder expectations became increasingly sophisticated.

The principal reason for employing the GHG Protocol in household-level emissions analysis lies in its capacity to provide a standardized and granular approach to calculating emissions across different dimensions of behavior. It allows for a full inventory of climate impacts arising from everyday life—from fueling a car to investing in equity portfolios. Additionally, the protocol facilitates benchmarking across time and geography, making it possible to compare the carbon intensities of different households or regions. This is particularly valuable for policy-

making, where a reliable basis for comparison is needed to design effective incentives, taxes, or subsidy programs aimed at reducing emissions. Moreover, with the rise of ESG (Environmental, Social, and Governance) investing, households are increasingly motivated to assess not only their consumption patterns but also the environmental implications of their financial choices. The GHG Protocol’s inclusion of Scope 3 investment-related emissions is thus particularly timely and relevant.

The mathematical formulation under the GHG Protocol for calculating a household’s carbon footprint begins with the aggregation of emissions across three main scopes of emissions. The total carbon footprint of a household is expressed as:

$$CF_{\text{household}} = E_{\text{Scope 1}} + E_{\text{Scope 2}} + E_{\text{Scope 3}} \quad (1)$$

Each of these components is calculated based on the product of activity data and corresponding emission factors. For Scope 1, this includes the quantity of fuel combusted in household-controlled devices or vehicles, multiplied by the fuel-specific emission factor. Scope 2 emissions are determined by multiplying electricity or district heating usage by grid-specific emission factors. Scope 3 is more complex and can be further disaggregated into emissions from the consumption of goods and services, and emissions from household investments. For the consumption subcategory, expenditures are multiplied by lifecycle emission factors derived from environmentally extended input-output models or product-level lifecycle assessments. For investment-based emissions, the monetary value of investments is multiplied by portfolio-weighted emission intensities of the respective industries.

Despite its widespread adoption, the traditional GHG Protocol approach has recognized limitations, particularly regarding its static nature and reliance on average emission factors. Recent literature emphasizes that more accurate household carbon footprint estimates require better representation of behavioral responses to price and policy signals. For example, Hertwich and Peters (2009) highlight that static inventories fail to capture the rebound effects and indirect market shifts that arise when households change consumption patterns in response to taxes or subsidies. Integrating behavioral elasticities into Scope 3 calculations would help address this gap by accounting for how consumers substitute goods or adjust spending in response to changing costs (Munksgaard et al., 2000). Additionally, several studies argue that using national average emission factors can mask significant regional differences. Lenzen et al. (2004) and Wiedmann (2009) both recommend refining emission estimates through regionally disaggregated electricity grid mixes, heating fuel profiles, and transport modes, as local supply chains often diverge substantially from national averages. By combining region-specific emission data with behavioral modelling, household carbon footprint accounting can better align with real-world dynamics and provide a stronger basis for targeted mitigation measures.

Moreover, the GHG Protocol framework can be further refined by incorporating time-series data, which would enable longitudinal comparisons and provide clearer insights into progress toward emissions reduction targets and the effectiveness of policy measures over time (Hertwich & Peters, 2009). In addition, expanding the focus to include complementary sustainability met-

rics, such as water use, material intensity, and land footprint, would allow for a more comprehensive assessment of household environmental impacts beyond carbon alone (Wiedmann & Minx, 2008). Taken together, these enhancements illustrate the evolving potential of the GHG Protocol to deliver a robust and nuanced picture of household sustainability.

To demonstrate how the GHG Protocol method can be applied in practice, the following section uses household consumption data from Spain for the year 2022 as an illustrative case. This example shows how direct, indirect energy, and value chain emissions are calculated within the three-scope framework and highlights the method's practical use for identifying major sources of emissions. Based on this calculation, the total household carbon footprint is estimated at approximately 11,828 kg CO₂e per year, with indirect emissions accounting for the largest share, which reflects a pattern widely documented in studies of household carbon footprints in high-income countries (Hertwich and Peters, 2009; Ivanova et al., 2016).

Such insights can be pivotal for policy recommendations, such as encouraging low-carbon food choices, promoting public transport, or offering green investment options to households.

4.1 Illustration: Application of the GHG Protocol to Spanish Household Consumption (2022)

As an empirical demonstration, the GHG Protocol framework is applied to household expenditure data for Spain in 2022, illustrating how emissions are quantified and distributed across Scopes 1, 2, and 3 in practice. The data is sourced from the Spanish National Statistics Institute (INE), which reports average annual consumption expenditures per household disaggregated by COICOP classification. This dataset provides category-specific expenditure values and percentage structures, enabling assignment of appropriate emission factors to each type of consumption.

The total mean annual household expenditure is reported at €31,568. This amount is allocated across major consumption areas, including food and non-alcoholic beverages, housing and energy, transport, communication, and services such as restaurants and recreation. The corresponding structure percentages indicate the relative contribution of each category to total spending. For example, housing and energy account for approximately 32.4 percent of total expenditure, while food and beverages represent about 16.0 percent. Transport expenditures stand at 12.0 percent, with notable growth observed in service-related categories such as restaurants and hotels. A detailed breakdown of these categories is provided in Table 1 of the appendix.

Next, we calculate Scope 1 emissions. These emissions arise from the direct combustion of fossil fuels by households, primarily through private vehicle use and home heating. We use energy consumption data expressed in gigajoules (GJ) per capita and apply appropriate emission factors. According to Spain's INE and international emission factor databases such as DEFRA and IPCC, petrol used for transport has an emission factor of 73.3 kg CO₂e/GJ, while natural gas used in heating has a slightly lower factor of 56.1 kg CO₂e/GJ.

We then proceed to Scope 2 emissions, which pertain to purchased energy—namely electricity and district heating—used within the household but generated off-site. The average house-

Table 4.1: Direct Emissions from Household Energy and Transport (Scope 1)

Energy Source	Consumption (GJ/hab)	Emission Factor (kg CO ₂ e/GJ)	Emissions (kg CO ₂ e)
Natural Gas (Transport)	0.04	56.1	2.24
Petrol (Transport)	14.44	73.3	1058.45
Natural Gas (Heating)	0.73	56.1	40.95
Petrol (Other)	0.18	73.3	13.19
Total			1114.83

hold in Spain consumed approximately 8.96 GJ of heating and cooling energy. The national grid's emission factor for such energy consumption, based on 2022 data, is estimated at 92.6 kg CO₂e per GJ.

Table 4.2: Indirect Emissions from Heating and Cooling (Scope 2)

Energy Source	Consumption (GJ/hab)	Emission Factor (kg CO ₂ e/GJ)	Emissions (kg CO ₂ e)
Heating/Cooling Energy	8.96	92.6	829.70
Total			829.70

The most complex and voluminous part of the analysis involves Scope 3 emissions. These emissions arise from the indirect impacts of household consumption decisions, including the carbon embedded in food, manufactured goods, services, and transportation infrastructure. Each expenditure category is multiplied by a category-specific emission factor derived from lifecycle assessment databases. For example, the food category carries an emission factor of 0.50 kg CO₂e per euro spent, reflecting emissions from agriculture, processing, and distribution. Clothing, by contrast, has a lower factor of 0.25 kg CO₂e/€, while restaurant services, due to their energy intensity, have a higher factor of 0.40 kg CO₂e/€.

Table 4.3: Consumption-Based Emissions (Scope 3)

Category	Expenditure (€)	Emission Factor (kg CO ₂ e/€)	Emissions (kg CO ₂ e)
Food and non-alcoholic beverages	5,050	0.50	2,525.00
Alcoholic beverages and tobacco	481	0.30	144.30
Clothing and footwear	1,232	0.25	308.00
Housing and utilities	10,243	0.25	2,560.75
Furnishings	1,296	0.30	388.80
Health	1,228	0.20	245.60
Transport services	3,794	0.30	1,138.20
Communications	925	0.15	138.75
Recreation	1,534	0.35	536.90
Education	468	0.10	46.80
Restaurants and hotels	2,953	0.40	1,181.20
Miscellaneous goods and services	2,364	0.30	709.20
Total			9,883.55

Finally, by summing the results from all three scopes, we obtain the total household carbon footprint for a typical Spanish household in 2022. The emissions distribution clearly reveals that Scope 3 emissions dominate, comprising nearly 84% of the total. This insight aligns with

broader research indicating that in high-income settings, the indirect emissions associated with consumption patterns far exceed direct household emissions.

Table 4.4: Total Household Carbon Footprint by Scope

Scope	Emissions (kg CO ₂ e)
Scope 1	1,114.83
Scope 2	829.70
Scope 3	9,883.55
Total	11,828.08

This empirical illustration not only validates the functionality of the GHG Protocol when applied to real-world household data but also emphasizes the critical role of consumption behavior in shaping emissions outcomes. The findings suggest that while improvements in home energy efficiency and cleaner fuels are valuable, the most substantial reductions may be achieved through systemic shifts in consumption patterns, such as transitioning to plant-based diets, reducing air travel, or shifting investments away from carbon-intensive industries.

5 Life Cycle Assessment (LCA) Method

The LCA method calculates emissions throughout the entire life cycle of a product or service, from production to disposal. This model captures emissions from every stage of the supply chain and provides a comprehensive assessment of indirect emissions.

The carbon footprint for a single industry using the LCA approach is:

$$fp_h = q_h \cdot LCA_j \quad (2)$$

where q_h is the quantity consumed by household h , and LCA_j represents the life cycle emissions per unit in industry j .

5.1 Methodology for Household Carbon Footprint Calculation based on LCA approach

This section adapts the framework developed by Peng et al. (2021) to estimate household carbon footprints by combining complementary life cycle assessment (LCA) techniques. Aligned with principles outlined in Guinée (2011) and comparative reviews such as Matthews et al. (2008) and Steubing et al. (2022), the approach quantifies both direct and embodied greenhouse gas (GHG) emissions, together with net carbon sequestration linked to household-level activities.

The methodology integrates three LCA variants to address the system boundary limitations inherent in partial assessments: (1) Process-based LCA is applied to quantify emissions from agricultural operations and livestock production, capturing emissions embedded in material in-

puts and field-level activities; (2) Input–Output LCA extends the boundary to indirect emissions embedded in household consumption of energy, food, housing, and transport, using environmentally extended input–output (EEIO) tables (Weber and Matthews, 2008); (3) Hybrid LCA bridges both levels by combining process inventory detail with macroeconomic input–output linkages, reducing truncation errors and capturing carbon flows related to afforestation, durable goods, and other land-use changes (Crawford et al., 2018; Shen et al., 2023).

Following Peng et al. (2021), the model distinguishes five core domains: direct energy use, short-lived and durable consumption, household agriculture, afforestation, and livestock management. Activity-specific emissions are parameterized using survey-derived data and regionally adjusted emission factors in accordance with IPCC (2019) guidelines. This combined accounting framework quantifies net household carbon flows as the balance of annual GHG emissions and sequestration within a unified system boundary:

$$CF_i = \sum_n E_{in} + \sum_m S_{im} \quad (3)$$

where CF_i represents the Carbon footprint of household i , E_{in} is the annual carbon emissions of household i in category n and S_{im} is the annual carbon sequestration of household i in category m . This disaggregated yet integrated structure provides the basis for the following activity-specific equations, which formally express how annual household emissions and sequestration are calculated within each functional domain.

5.2 Carbon Emissions from Direct Energy Consumption

The emissions from household direct energy use are estimated by multiplying the quantity of fuel consumed by its corresponding emission factor. For each household i , the total emissions attributable to direct fuel combustion are calculated as:

$$E_{id} = \sum_d (F_{id} \cdot EF_d) \quad (4)$$

where F_{id} denotes the annual consumption of household i for fuel type d . The fuel-specific emission factor EF_d combines oxidation efficiency, fuel composition, and calorific value, consistent with IPCC (2019) guidelines:

$$EF_d = OX_d \cdot \left(C_{o,d} \cdot \frac{12}{44} + C_{h,d} \cdot \frac{12}{16} \right) \cdot H_d \cdot 10^{-9} \quad (5)$$

Here, OX_d represents the oxidation efficiency, typically assumed to be 100% for complete combustion; $C_{o,d}$ and $C_{h,d}$ are the fuel-specific emission coefficients for CO_2 and CH_4 , respectively; and H_d denotes the net calorific value of the fuel. The conversion factors ensure that carbon content is expressed in consistent units of tonnes CO_2 equivalent per unit of fuel input.

5.3 Carbon Emissions from Living Consumption

Emissions attributable to household consumption of goods and services are divided into two categories: short-lived products and durable consumer goods. The total annual emissions from short-lived consumption are given by:

$$E_{if} = \sum_f (EF_f \cdot C_{if}) \quad (6)$$

where E_{if} denotes the annual carbon emissions from short-lived good f , C_{if} is the quantity of product f consumed by household i , and EF_f is its corresponding life cycle emission factor. For durable consumer products, the embodied emissions are amortized over the product's expected service life to obtain an annualized footprint:

$$E_{ij} = \sum_j \frac{(EF_j \cdot C_{ij})}{L_j} \quad (7)$$

Here, E_{ij} represents the annual emissions associated with durable product j , C_{ij} is the total quantity purchased, EF_j is the relevant life cycle emission factor, and L_j is the product's average lifetime (in years). This treatment ensures that emissions embedded in capital household goods are allocated proportionally across their period of use, consistent with standard LCA accounting conventions.

5.4 Carbon Footprint in Agricultural Activities

Emissions and sequestration associated with household-level agricultural production are accounted for through inputs, on-site operations, and net biomass growth. The total annual carbon footprint from agricultural activities for household i is calculated as:

$$CF_{ia} = \sum_a (EF_a \cdot M_{ia}) + \sum_t (EF_t \cdot FS_{ia}) + \sum_v (B_v \cdot 0.475) \quad (8)$$

Here, CF_{ia} denotes the net carbon impact of agricultural activities. M_{ia} represents the quantity of material input a (such as fertilizers or pesticides) applied by household i , with an associated emission factor EF_a . The second term accounts for emissions from field operations, where FS_{ia} is the cultivated field size and EF_t is the emission factor per unit area for operation t (e.g., tillage, irrigation). The final term reflects the carbon sequestered in above-ground biomass, with B_v indicating the dry biomass yield for crop or vegetation type v , multiplied by a standard carbon content coefficient of 47.5% (IPCC default for plant biomass).

5.5 Carbon Sequestration from Afforestation

Carbon sequestration through household-level afforestation is estimated based on the area of land dedicated to tree planting and the average carbon stock of the tree species established. The annual sequestration from afforestation for household i is given by:

$$S_{iaf} = FS_{iaf} \cdot CS_{\text{citrus}} \quad (9)$$

In this expression, S_{iaf} represents the amount of carbon sequestered through afforestation activities, FS_{iaf} denotes the field size (in hectares) allocated for tree planting by household i , and CS_{citrus} is the mean carbon stock per unit area for citrus trees or other comparable species. The carbon stock factor reflects the average annual carbon uptake, accounting for biomass accumulation under local growth conditions.

5.6 Carbon Emissions from Livestock Raising

Emissions from household livestock activities include fodder production, enteric fermentation, and manure management. The total annual emissions from livestock raising for household i are calculated as:

$$E_{il} = \sum_f (EF_{if} \cdot F_{if}) + \sum_l (EF_{il} \cdot N_{il}) \quad (10)$$

In this formulation, E_{il} represents the total carbon emissions attributable to livestock-related activities. F_{if} denotes the annual quantity of fodder consumed by livestock, multiplied by the corresponding emission factor EF_{if} to account for upstream impacts of fodder cultivation and transport. The second term captures direct emissions from livestock, where N_{il} is the number of animals of type l and EF_{il} is the per-animal emission factor, which includes methane emissions from enteric fermentation and nitrous oxide from manure management.

5.7 Aggregate Formula for Household Carbon Footprint

The total household carbon footprint (CF_{total}) aggregates emissions and sequestration from direct energy use, living consumption, agricultural production, afforestation, and livestock raising, as specified in Equations (4) to (10).

$$\begin{aligned} CF_{\text{total}} = & \underbrace{\sum_d (F_{id} \cdot EF_d)}_{\text{Direct energy}} + \underbrace{\sum_f (EF_f \cdot C_{if}) + \sum_j \frac{(EF_j \cdot C_{ij})}{L_j}}_{\text{Living consumption}} \\ & + \underbrace{\sum_a (EF_a \cdot M_{ia}) + \sum_t (EF_t \cdot FS_{ia}) + \sum_v (B_v \cdot 0.475)}_{\text{Agriculture}} \\ & - \underbrace{\sum_{iaf} (FS_{iaf} \cdot CS_{\text{citrus}})}_{\text{Afforestation}} + \underbrace{\sum_f (EF_{if} \cdot F_{if}) + \sum_l (EF_{il} \cdot N_{il})}_{\text{Livestock}}. \end{aligned} \quad (11)$$

By integrating disaggregated activity data with region-specific emission factors, this aggregate formulation is a consistent representation of the net annual GHG balance at the house-

hold level, consistent with best practices in household carbon accounting (Ivanova et al., 2016; Dubois et al., 2019).

Beyond its measurement function, this structure highlights how household decisions on energy use, consumption patterns, and land management jointly influence emissions outcomes (Hertwich and Peters, 2009). Such clarity enables scenario analysis of behavioral shifts and technological choices, informing targeted interventions like carbon price adjustments, renewable energy adoption incentives, or compensation for sequestration through land-based measures (Rogelj et al., 2018; Steubing et al., 2022). As shown in recent household-level studies, aligning micro-level incentives with broader climate objectives is essential to internalize externalities and realize cost-effective emission reductions (Wiedmann et al., 2020).

5.8 Household Emissions Breakdown under LCA

An integrated life cycle assessment (LCA) illustration clarifies how the total household carbon footprint, as defined by Equations (4) to (10), is distributed across major activity domains. Drawing on estimates from Peng et al. (2021), Sala et al. (2014), and Matthews et al. (2008), direct household energy use, including heating fuels and private vehicle fuels, accounts for approximately 20% to 30% of total greenhouse gas emissions. Living consumption, which covers short-lived goods, food purchases, and services, represents the largest share at about 50% to 60%. This highlights the significant upstream emissions embodied in supply chains. Within this category, food-related impacts illustrate the importance of production and distribution processes rather than any single consumption choice. For instance, staples such as cereals, fruits, and vegetables generally have lower average emission intensities than more resource-intensive foods, but all contribute to the overall footprint. Durable goods, including household appliances and furniture, add an additional 5% to 10% of total emissions when annualized, showing the relevance of product lifespan and embedded material flows.

Together, this evidence shows that focusing only on direct household fuel use underestimates the total climate impact of residential lifestyles because indirect emissions embedded in goods and services account for most impacts, as discussed by Hertwich and Peters (2009). A comprehensive, activity-based LCA offers a more realistic basis for identifying mitigation options such as improving building energy performance, extending product lifespans, sourcing materials more sustainably, and investing in land-based carbon sequestration. This broader perspective supports the development of household-level strategies that address systemic drivers of emissions rather than isolating individual consumption categories.

6 Input–Output Model and Carbon Footprint Estimation

The environmentally extended Input-Output (EEIO) framework provides a macroeconomic approach for quantifying household carbon footprints by tracing both direct and upstream greenhouse gas (GHG) emissions embedded in goods and services.

Illustrative Breakdown of Household Carbon Footprint
(adapted from Peng et al. 2021 & Notarnicola et al. 2017)

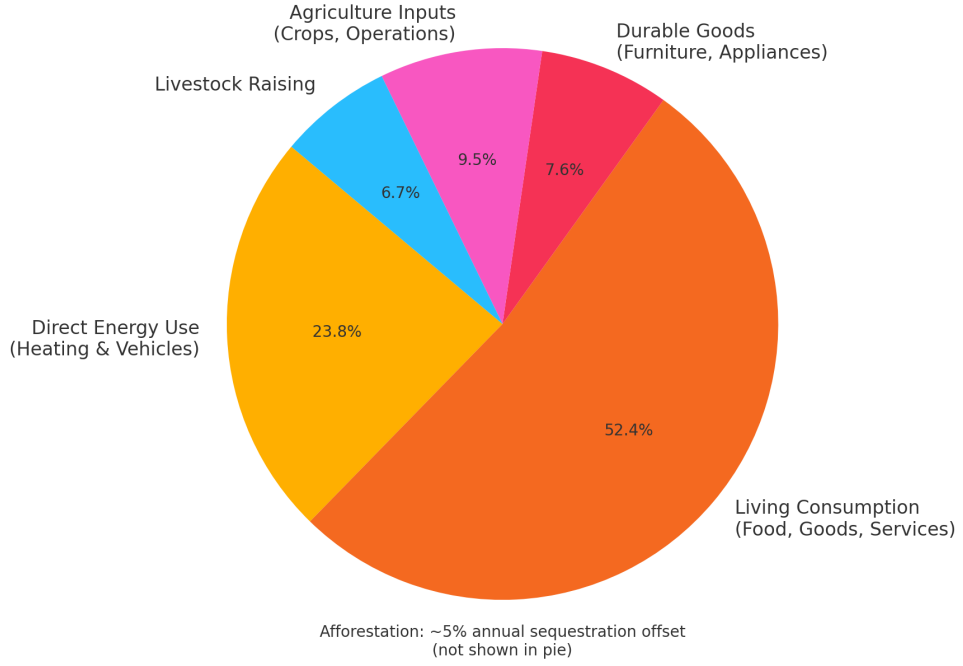


Figure 1: Relative contributions of household activities to carbon footprints

6.1 Leontief Input–Output Framework

We begin with the standard Leontief system, where total output \mathbf{X} is the sum of intermediate input requirements and final demand:

$$\mathbf{X} = \mathbf{A}\mathbf{X} + \mathbf{F}. \quad (12)$$

Solving for total output yields the fundamental input–output identity:

$$\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{F}. \quad (13)$$

Here, \mathbf{A} is the technical coefficient matrix, \mathbf{F} is the vector of household final demand, and $(\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse, which accounts for the total production required (both direct and indirect) to satisfy final demand.

6.2 Technical Coefficient Matrix

Each element A_{ij} of the matrix \mathbf{A} is defined as:

$$A_{ij} = \frac{z_{ij}}{x_j}, \quad (14)$$

where z_{ij} denotes the monetary value of inputs from sector i to sector j , and x_j is the total output of sector j . The matrix \mathbf{A} reflects the technological input structure of the economy.

6.3 Stability of the Leontief Inverse

The Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$ exists and is finite if the matrix \mathbf{A} satisfies the stability condition $\rho(\mathbf{A}) < 1$, where $\rho(\cdot)$ denotes the spectral radius (i.e., the largest absolute eigenvalue). This condition ensures that the production system is productive and does not require infinite inputs. In applied input–output tables, a sufficient (but not necessary) condition is that the column sums of \mathbf{A} are each less than one:

$$\sum_i A_{ij} < 1 \quad \text{for all } j.$$

This implies that each sector uses less than one unit of intermediate input to produce one unit of output, a condition that is generally satisfied in empirical datasets (Miller and Blair, 2009). A numerical illustration of the stability condition, along with matrix-based examples and emission multiplier computation, is provided in Appendix B.

6.4 Carbon Footprint Estimation via Input–Output Modelling

The environmentally extended input–output (EEIO) model estimates household carbon footprints by applying sectoral emission intensities to the total output vector required to satisfy final demand:

$$\mathbf{E} = \mathbf{C}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{F}, \quad (15)$$

where \mathbf{C} is the vector of direct emission intensities (e.g., kg CO₂e per euro of output), and \mathbf{E} is the resulting emissions attributable to household consumption. This formulation captures both direct and upstream (supply chain) emissions associated with consumption.

6.5 Tiered Decomposition of Household Emissions

Following Matthews et al. (2008) and Long et al. (2019), the total household footprint can be analytically decomposed into three tiers:

Tier 1: Direct Emissions

$$\mathbf{E}_1 = \mathbf{C}_d \cdot \mathbf{F}_d, \quad (16)$$

where \mathbf{F}_d is household consumption of directly combusted fuels and \mathbf{C}_d is the corresponding emission intensity vector.

Tier 2: Indirect Energy Emissions

$$\mathbf{E}_2 = \mathbf{C}_e \cdot (\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{F}_e, \quad (17)$$

with \mathbf{F}_e representing consumption of electricity and district heating, and \mathbf{C}_e their emission

intensities.

Tier 3: Indirect Supply Chain Emissions

$$\mathbf{E}_3 = \mathbf{C} \cdot [(\mathbf{I} - \mathbf{M})(\mathbf{I} - \mathbf{A})]^{-1} \cdot [(\mathbf{I} - \mathbf{M}) \cdot \mathbf{F} + \mathbf{EX}], \quad (18)$$

where \mathbf{M} is a diagonal matrix of sectoral import shares, and \mathbf{EX} accounts for exports. This import-adjusted EEIO formulation ensures emissions are assigned to domestic demand (Long et al., 2019; Sheng et al., 2024).

Total Household Footprint. Combining all tiers, the household footprint is:

$$\mathbf{E}_{\text{total}} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3. \quad (19)$$

This integrated EEIO framework mitigates truncation and boundary errors typical of process-based LCA, capturing the full feedback loops of modern economies (Matthews et al., 2008; Steubing et al., 2022).

6.6 Illustrative Application of the Input-Output Model

In this illustration, pre-calculated environmentally extended emission intensities (kg CO₂e per euro spent) are applied to household consumption data for France, Spain, and Germany for 2021. These intensities represent the aggregated effect of $\mathbf{C}(\mathbf{I} - \mathbf{A})^{-1}$ and are derived from the EXIOBASE multi-regional input-output (MRIO) model, as accessed via Climatiq.io.¹ The method aligns with the tier-3 comprehensive accounting approach discussed in Matthews et al. (2008), Long et al. (2019), and Sheng et al. (2024).

6.7 Data and Methodology

The Leontief inverse $(\mathbf{I} - \mathbf{A})^{-1}$ then expands final demand to include both direct and indirect production requirements.

$$EF_i = \sum_j C_j L_{ji}, \quad \text{where } L = (\mathbf{I} - \mathbf{A})^{-1}.$$

Here, C_j denotes the direct emissions intensity for producing sector j , and L_{ji} represents the total requirements linking producing sectors j and i . The resulting spend-based factors are published through Climatiq.io, which provides up-to-date coefficients aggregated from EXIOBASE under a permissive data license (Creative Commons Attribution-ShareAlike 4.0 International License).

To ensure numerical stability, EXIOBASE balances its input-output tables using harmonised supply-use statistics, preventing divergence in the Leontief inverse. An additional feasibility

¹These coefficients implicitly incorporate trade-adjusted upstream emissions, though no explicit import share matrix was applied in this illustration.

check confirms that the sum of each column in **A** remains below unity, preserving the productive structure required for invertibility.

Annual household final consumption expenditure for France, Spain, and Germany was sourced from Eurostat for the year 2021 and harmonised to euros at the average annual exchange rate. For each expenditure category i and country c , the household carbon footprint is estimated by multiplying the national expenditure by the corresponding spend-based factor:

$$E_{i,c} = F_{i,c} \times EF_i.$$

For instance, for France, the estimated annual household spending on food and non-alcoholic beverages is approximately

$$F_{\text{food,FR}} = 1.322 \times 10^9 \times 0.139 = 183.8 \times 10^9 \text{ EUR.}$$

Multiplying this by the category-specific factor of 0.48 kg CO₂e per euro yields an estimated

$$E_{\text{food,FR}} = 183.8 \times 10^9 \times 0.48 = 88.2 \times 10^6 \text{ tonnes CO}_2\text{e.}$$

The same procedure is applied across all final demand categories and for Spain and Germany. This method follows the tier-3 EEIO approach and systematically allocates upstream supply chain emissions, providing a comprehensive perspective on the consumption-driven climate impact of households. All emission factors are listed in Appendix A, Table 2.

6.8 Results

Table 6.1 summarizes the estimated household carbon footprints for France, Spain, and Germany in 2021, derived using the environmentally extended input–output model. The full breakdown by expenditure category is reported in Appendix A (Tables 3–5).

Table 6.1: Total estimated household carbon footprints (2021)

Country	Expenditure (bn €)	Emissions (Mt CO ₂ e)
France	1322.0	420.0
Spain	747.9	227.0
Germany	1794.8	545.9

In all three countries, the dominant drivers of household emissions are housing, food, and transport. These sectors together account for over 60% of the footprint, reflecting the carbon intensity of residential energy use, agri-food supply chains, and private mobility. The relative magnitude and structure of results are consistent with earlier multi-regional IO studies (Matthews et al. 2008; Long et al. 2019; Sheng et al. 2024), confirming the utility of EEIO methods for policy-relevant footprint accounting.

Thus, the footprint estimates capture the sectoral and cross-country variation in household carbon intensity, illustrating the effectiveness of the EEIO approach in quantifying consumption-

driven emissions at national scale.

6.9 Comparative Assessment of LCA and EEIOA for Household Carbon Footprint Estimation

Life cycle assessment (LCA) and environmentally extended input–output analysis (EEIOA) are both established approaches for estimating household carbon footprints but differ in system boundaries and detail. LCA quantifies emissions by summing process-based impacts across all life cycle stages, often including capital goods and infrastructure explicitly.

In this illustration, the household carbon footprint is quantified using an LCA framework adapted from Steubing et al. (2022). The structure of the total footprint, previously defined in Equation (11), can be re-stated as:

$$CF_{\text{total}} = \sum_d (F_{id} \cdot EF_d) + \sum_f (C_{if} \cdot EF_f) + \sum_j \left(\frac{C_{ij} \cdot EF_j}{L_j} \right) + \sum_a (M_{ia} \cdot EF_a) - \sum_t (S_{it} \cdot CS_t). \quad (20)$$

where fuel use, short-lived and durable goods, capital goods lifespan, material inputs, and carbon stock changes are all explicitly accounted for.

In comparison, the EEIOA model, as defined in Equation (14), estimates the footprint as:

$$CF_{\text{EEIOA}} = R(I - A)^{-1}y. \quad (21)$$

In standard practice, the final demand vector y typically excludes gross fixed capital formation, meaning that capital goods for future production are not fully reflected in the EEIOA results. This distinction explains why LCA-based footprints can exceed EEIOA estimates in capital-intensive sectors. Steubing et al. (2022) demonstrate that for electricity, fossil-based power systems show close agreement between LCA and EEIOA estimates, whereas renewable electricity systems diverge more significantly due to the inclusion of construction and infrastructure impacts in the LCA boundary.

This comparative perspective highlights that LCA captures emissions from long-lived assets more comprehensively, while EEIOA better reflects systemic supply chain emissions embedded in everyday household expenditure. Using both approaches together clarifies how immediate consumption interacts with infrastructure and capital investment to shape total household carbon footprints.

7 The Hakenes & Schliephake Model

Traditional methods for estimating household carbon footprints attribute emissions based on direct consumption or financial ownership in emitting industries. However, they often ignore the market feedback loops triggered by individual decisions — such as how a household reducing demand might simply shift that demand to other consumers or investors.

The model developed by Hakenes and Schliephake (2024) addresses this issue through a general equilibrium framework. By embedding both product and financial markets, the model assigns carbon footprints based not only on what households consume or invest in, but also on the spillover effects of those choices across the economy. This consequentialist approach attempts to capture the true marginal impact of household behavior on aggregate emissions.

7.1 Deriving the Household Footprint in a One-Industry Economy

We consider a simplified version of the model developed by Hakenes and Schliephake (2024), focusing on an economy with a single industry. A representative good is produced using capital as the only input. Firms operate under constant returns to scale, with a marginal cost of production c . Let Q denote the aggregate quantity produced and consumed, and I the total capital invested. Given the linear technology, we have:

$$I = cQ.$$

Each unit of the good generates emissions x , which aggregates both production-related and consumption-related emissions. Thus, total emissions in the economy are given by:

$$X = xQ.$$

7.1.1 Firms and Capital Market

Firms raise capital I from households and produce output Q . After selling the output at price P , they repay investors using the liquidation value λ and a noise term ε , which follows a normal distribution with zero mean and variance σ^2 . The return on investment is:

$$r = \frac{P}{c} + \lambda + \varepsilon.$$

Profits are distributed to investors in proportion to their capital contributions. Firms operate competitively, so expected profits are zero in equilibrium.

7.1.2 Household Optimization Problem

Household h is endowed with wealth w and allocates it between investment i_h and consumption q_h . The portion not invested yields a risk-free return r_f . The budget constraint is:

$$m_h = ri_h + r_f(w - i_h) - Pq_h,$$

where m_h is the leftover wealth after investment and consumption. The household derives utility from consumption and terminal wealth. The expected utility function is given by:

$$U_h = \mathbb{E} \left[-e^{-\alpha \left(aq_h - \frac{b}{2} q_h^2 + m_h - xQ \right)} \right],$$

where a represents the marginal utility of the first unit of the good, $b > 0$ captures diminishing marginal utility, α is the coefficient of absolute risk aversion, and xQ reflects the disutility from global emissions.

Substituting m_h into the utility function and linearizing expectations due to the exponential-normal structure, we obtain:

$$\mathbb{E}[U_h] = -\exp \left\{ -\alpha \left[(a - P)q_h - \frac{b}{2}q_h^2 + r_f w + \left(\frac{P}{c} + \lambda - r_f \right) i_h - \frac{\alpha}{2}\sigma^2 i_h^2 - xQ \right] \right\}.$$

7.1.3 Market Equilibrium and Footprint Derivation

To calculate the household's consequentialist footprint, we compare the equilibrium outcome with and without household h . In equilibrium, the market clears:

$$Q = q_h + (n - 1)q_{-h}, \quad I = i_h + (n - 1)i_{-h}, \quad I = cQ.$$

Other households maximize the same utility, taking P as given. Their optimal demand and investment are derived from the first-order conditions:

$$q_{-h} = \frac{a - x - P}{b}, \quad i_{-h} = \frac{1}{\alpha\sigma^2} \left(\frac{P}{c} + \lambda - r_f \right).$$

Substituting these into the equilibrium conditions and solving, we obtain the aggregate quantity:

$$Q = \phi q_h + (1 - \phi) \frac{i_h}{c} + (\text{terms independent of } h),$$

where the weighting parameter ϕ is defined as:

$$\phi = \frac{b}{b + c^2\alpha\sigma^2}.$$

This weight determines how the household's choices affect equilibrium quantities and, consequently, emissions. The consequentialist footprint of household h is defined as the marginal impact of their participation on total emissions:

$$fp_h = x(Q(q_h, i_h) - Q(0, 0)) = x \left(\phi q_h + (1 - \phi) \frac{i_h}{c} \right).$$

The parameter ϕ captures the relative influence of consumption and investment. When the financial asset is risk-free ($\sigma^2 = 0$), we obtain $\phi = 1$, and the entire footprint is attributed to consumption. Conversely, if consumption utility is linear ($b = 0$), then $\phi = 0$, and the footprint depends entirely on investment. This formulation ensures full accounting of emissions across households:

$$\sum_h fp_h = xQ = X.$$

7.1.4 Derivation of the Weighting Parameter ϕ

To derive the footprint weighting parameter ϕ , we begin with the assumption that aggregate output Q is produced by a linear technology using capital I with constant marginal cost c . Hence,

$$Q = \frac{I}{c}.$$

The total capital in the market is supplied by n households. We distinguish a representative household h from the remaining $n-1$ households, and denote their investment and consumption decisions by (i_h, q_h) and (i_{-h}, q_{-h}) , respectively.

In equilibrium, market clearing implies:

$$Q = q_h + (n-1)q_{-h}, \quad I = i_h + (n-1)i_{-h}, \quad I = cQ.$$

Substituting into the identity $I = cQ$, we obtain:

$$i_h + (n-1)i_{-h} = c(q_h + (n-1)q_{-h}).$$

Now consider how the quantity Q changes when household h changes its behavior. Holding the other households' behavior fixed, the marginal effect of h 's consumption and investment on output is given by the total differential:

$$\frac{\partial Q}{\partial q_h} = 1, \quad \frac{\partial Q}{\partial i_h} = \frac{1}{c}.$$

However, these effects are attenuated by the endogenous reactions of other households. If household h increases consumption q_h , market price P rises. Other households respond by lowering their own consumption q_{-h} and adjusting their investment i_{-h} to the new return. Conversely, if h increases investment i_h , the capital supply rises, which reduces price and affects others' choices.

We now derive the explicit behavioral responses.

The other households' optimal consumption satisfies:

$$\frac{\partial \mathbb{E}[U_{-h}]}{\partial q_{-h}} = 0 \quad \Rightarrow \quad a - x - bq_{-h} - P = 0,$$

which yields:

$$q_{-h} = \frac{a - x - P}{b}.$$

Their optimal investment satisfies:

$$\frac{\partial \mathbb{E}[U_{-h}]}{\partial i_{-h}} = 0 \quad \Rightarrow \quad \frac{P}{c} + \lambda - r_f - \alpha\sigma^2 i_{-h} = 0,$$

so that:

$$i_{-h} = \frac{1}{\alpha\sigma^2} \left(\frac{P}{c} + \lambda - r_f \right).$$

Now insert these behavioral responses into the aggregate equilibrium conditions:

$$Q = q_h + (n - 1) \left(\frac{a - x - P}{b} \right), \quad I = i_h + (n - 1) \left(\frac{1}{\alpha \sigma^2} \left(\frac{P}{c} + \lambda - r_f \right) \right).$$

Combining these with $Q = \frac{I}{c}$, we solve for the dependence of Q on q_h and i_h . Define the partial footprint of household h as the difference in total output caused by its activity:

$$fp_h = x (Q(q_h, i_h) - Q(0, 0)).$$

Linearizing Q in q_h and i_h , and denoting the resulting coefficients as footprint weights, we obtain:

$$fp_h = x \left(\phi q_h + (1 - \phi) \frac{i_h}{c} \right),$$

where

$$\phi = \frac{b}{b + c^2 \alpha \sigma^2}.$$

This expression reflects how much of the household's carbon footprint is attributed to consumption versus investment. It arises from the equilibrium interactions between price responses and household behavioral elasticities in both the product and capital markets.

7.1.5 Comparative Statics of the Weighting Parameter ϕ

We now investigate how the footprint weighting parameter ϕ , defined as

$$\phi = \frac{b}{b + c^2 \alpha \sigma^2},$$

responds to changes in the underlying structural parameters of the model.

Differentiating ϕ with respect to the coefficient of absolute risk aversion α , we obtain

$$\frac{\partial \phi}{\partial \alpha} = -\frac{bc^2 \sigma^2}{(b + c^2 \alpha \sigma^2)^2} < 0.$$

This implies that as households become more risk-averse, the footprint share attributed to consumption declines, while the relative importance of investment decisions increases.

With respect to the volatility of financial returns, captured by σ^2 , we find

$$\frac{\partial \phi}{\partial \sigma^2} = -\frac{bc^2 \alpha}{(b + c^2 \alpha \sigma^2)^2} < 0.$$

An increase in financial risk similarly reduces ϕ , shifting the footprint burden from consumption to investment channels.

Finally, consider the effect of changing the curvature of the utility function through the pa-

parameter b . Differentiation yields

$$\frac{\partial \phi}{\partial b} = \frac{c^2 \alpha \sigma^2}{(b + c^2 \alpha \sigma^2)^2} > 0.$$

A higher value of b , indicating stronger diminishing marginal utility from consumption, increases the share of the footprint attributed to consumption activities.

In sum, the weighting parameter ϕ is decreasing in both risk aversion and return volatility, and increasing in the concavity of consumption preferences. These results highlight how the relative responsibility of consumption and investment for carbon emissions is endogenous to household behavior and financial risk, making the model responsive to empirical variation across households or economies.

7.2 Empirical Illustration: Application of the Single-Industry Model

Here, the simplified version of the Hakenes and Schliephake (2024) model is applied to the U.S. wheat market, using USDA data from 2010 to 2016. Production volumes serve as a proxy for quantity supplied, while total domestic use approximates quantity demanded. Farm prices are taken as observed average annual prices.

To estimate supply behavior, an ordinary least squares (OLS) regression of price is fitted on observed production, yielding the empirical supply curve. In the empirical illustration, the demand curve is specified as linear and downward sloping. Its slope is calibrated using average values from the dataset, consistent with observed market behavior in the U.S. wheat sector. While the curve is not estimated directly via regression (due to data limitations on price responsiveness), it reflects a stylized elasticity based on domain knowledge. This contrasts with the supply curve, which is estimated using OLS on observed price and production data. We simulate the 2016–2017 wheat supply shock, during which production declined by 15.6%. The intersection of the two curves provide the empirical equilibrium quantities and prices before and after the 2016–2017 supply shock. This is modeled by proportionally shifting the supply curve upward. Equilibrium price and quantity before and after the shock are obtained by solving the intersection between the demand curve and the respective supply curves.

7.2.1 Carbon Footprint Estimation under Empirical Supply Curve

We compute the carbon footprint associated with each equilibrium using an emission factor of 10.88 kg CO₂e per bushel (based on FAO and USDA estimates).

7.2.2 Carbon Footprint Estimation under Theoretical Supply Curve

To simulate the same supply shock within the Hakenes and Schliephake (2024) framework, the demand curve from the empirical estimation was retained. However, instead of using a supply curve estimated via ordinary least squares, a theoretically derived supply curve was constructed based on model assumptions. In this approach, firms were assumed to raise capital

Scenario USDA data	Quantity (million bushels)	Price (USD)	Carbon Footprint (million kg CO₂e)
Before Shock	2100.71	5.58	22859.68
After Shock	2068.38	5.82	22500.32
Change	—	—	-359.36

Table 1: Carbon footprint before and after the supply shock using real market data.

from households, who in turn optimally allocate their investments under risk.

The equilibrium supply curve in this setup is derived from the market-clearing condition and the household's optimal investment response under uncertainty, and takes the form:

$$P(Q) = c(r_f - \lambda) + \frac{c^2 \alpha \sigma^2}{n - 1} Q,$$

where c denotes the marginal cost of production, r_f the risk-free rate, λ the liquidation value of capital, α the coefficient of absolute risk aversion, σ^2 the variance of investment returns, and n the total number of households.

This expression yields a linear and upward-sloping supply curve. The theoretical supply curve applied in this illustration was constructed using parameter values selected to reflect realistic conditions in the U.S. wheat and financial markets during the study period. The marginal cost of production was assumed to be $c = 4$, which is consistent with per-bushel production costs observed in U.S. wheat farming and allows the resulting equilibrium prices to align with historical market levels. The risk-free rate was set to $r_f = 0.05$, corresponding to the average yield on 10-year U.S. Treasury bonds between 2010 and 2016. The liquidation value of capital was taken as $\lambda = 0.01$, reflecting the reduced resale value of farm-specific capital such as machinery or equipment. The coefficient of absolute risk aversion was assumed to be $\alpha = 0.5$, a value that captures moderate household risk sensitivity consistent with empirical estimates from investment literature. The volatility of investment returns was specified as $\sigma = 0.4$, implying a variance of $\sigma^2 = 0.16$, which falls within the range typically observed for U.S. agricultural investments and related financial instruments. Finally, the number of households was assumed to be $n = 100,000$, representing an approximation of the number of wheat-producing farms in the United States during the relevant years. These parameter values were used to generate a supply curve that reflects theoretical investment behavior under risk, providing a basis for comparison with the empirically estimated curve, and the intercept reflects the opportunity cost of capital. The new equilibrium values were obtained by solving the intersection of this supply curve with the demand curve used previously.

By solving the intersection of this supply curve with the same demand curve used in the empirical case, equilibrium values for price and quantity were obtained both before and after the simulated shock. The corresponding carbon footprints were then computed using the same emissions factor of 10.88 kg CO₂e per bushel.

Scenario Theory	Quantity (million bushels)	Price (USD)	Carbon Footprint (million kg CO ₂ e)
Before Shock	2112.36	5.69	22983.46
After Shock	2095.13	5.85	22808.35
Change	—	—	-175.11

Table 2: Model-based carbon footprint before and after the supply shock.

7.2.3 Structural Sources of Difference in Emissions Outcomes

Although the same demand curve was used in both the empirical and theoretical approaches, the estimated reduction in carbon footprint differed considerably. The empirical estimation yielded a reduction of 359.36 million kg CO₂e, while the theoretical model predicted a more modest reduction of 175.11 million kg CO₂e.

This difference can be attributed entirely to the way supply was modeled. In the empirical estimation, the supply curve was estimated via OLS using observed data on price and quantity. This approach captured market behavior as it appeared in the historical record but did not account for underlying decision-making under uncertainty or equilibrium responses. In contrast, the theoretical supply curve was derived from the model’s structural assumptions, incorporating risk preferences, investment volatility, and optimal capital allocation. It reflected how households would respond to market changes under forward-looking behavior, leading to a more muted response in output and, correspondingly, in emissions.

Additionally, the theoretical model introduced a consequentialist perspective by assigning carbon responsibility based on the marginal impact of a household’s consumption or investment. In doing so, it internalized substitution effects and capital reallocation, which were not accounted for in the empirical estimation. As a result, while the same emissions formula was applied in both cases, the theoretical model predicted a smaller footprint change due to the buffering effects of equilibrium adjustments. This difference underscores the importance of integrating behavioral dynamics into footprint assessment, particularly when evaluating the impact of shocks or policy interventions.

8 Results

9 Discussion

10 Conclusion

References

Statement of authorship

I hereby confirm that the work presented has been performed and interpreted solely by myself except for where I explicitly identified the contrary.

Date: _____

Signature: _____