The Hakenes & Schliephake Model

The consequentialist carbon footprint model developed by Hakenes and Schliephake (2024) provides a structured approach to attributing carbon responsibility between consumption and investment under conditions of market risk and uncertainty.

$$f_{ph} = x\left(\varphi q_h + (1-\varphi)\frac{i_h}{c}\right) \tag{1}$$

Derivation of the Weighting Parameter φ

$$U = bq_h - \frac{1}{2}\alpha\sigma^2\left(\frac{i_h}{c}\right)^2 \tag{2}$$

$$\frac{\partial U}{\partial q_h} = b - \lambda = 0 \tag{3}$$

$$\frac{\partial U}{\partial i_h} = -\alpha\sigma^2\left(\frac{i_h}{c^2}\right) - \lambda\frac{1}{c} = 0 \tag{4}$$

$$\lambda = b \tag{5}$$

$$\alpha\sigma^2\left(\frac{i_h}{c^2}\right) - \frac{b}{c} = 0 \tag{6}$$

$$i_h = -\frac{bc^2}{\alpha\sigma^2} \tag{7}$$

$$\varphi = \frac{b}{b + c^2\alpha\sigma^2} \tag{8}$$

Sensitivity of φ

$$\frac{\partial\varphi}{\partial b} = \frac{\alpha\sigma^2}{(b+\alpha\sigma^2)^2} \tag{9}$$

$$\frac{\partial\varphi}{\partial\alpha} = -\frac{b\sigma^2}{(b+\alpha\sigma^2)^2} \tag{10}$$

$$\frac{\partial\varphi}{\partial\sigma^2} = -\frac{b\alpha}{(b+\alpha\sigma^2)^2} \tag{11}$$

Market Equilibrium Analysis

$$P = a - bQ \tag{12}$$

$$P = c(r_f - \lambda) + \frac{c^2\alpha\sigma^2}{n-1}Q \tag{13}$$

$$a - bQ = c(r_f - \lambda) + \frac{c^2 \alpha \sigma^2}{n-1} Q \tag{14}$$

$$Q^* = \frac{(a-x) - c(r_f - \lambda)}{b + \frac{c^2 \alpha \sigma^2}{n-1}} \tag{15}$$

Carbon Footprint Function and Sensitivity

$$f_{ph} = x \left(\frac{b q_h + \alpha \sigma^2 \frac{i_h}{c}}{b + \alpha \sigma^2} \right) \tag{16}$$

$$\frac{\partial f_{ph}}{\partial b} = x \frac{\alpha \sigma^2 \left(q_h - \frac{i_h}{c} \right)}{(b + \alpha \sigma^2)^2} \tag{17}$$

$$\frac{\partial f_{ph}}{\partial \alpha} = -x \frac{b \sigma^2 \left(q_h - \frac{i_h}{c} \right)}{(b + \alpha \sigma^2)^2} \tag{18}$$

$$\frac{\partial f_{ph}}{\partial \sigma^2} = -x \frac{b \alpha \left(q_h - \frac{i_h}{c} \right)}{(b + \alpha \sigma^2)^2} \tag{19}$$

Discussion and Implications

The analytical framework captures the consequentialist perspective of carbon attribution, emphasizing how risk, marginal utility, and volatility shift the responsibility for emissions between consumption and investment.