# Appendix A: Tables and Figures

Table 1: Mean Consumption Expenditure per Household in Spain, 2022

| Category | Mean Expenditure (€) | Structure (%) | Annual Rate (%) | Annual Difference (€) |
|---|---|---|---|---|
| Total | 31568 | 100.0 | 7.9 | 2324 |
| Food and non-alcoholic beverages | 5050 | 16.0 | 5.1 | 244 |
| Alcoholic beverages and tobacco | 481 | 1.5 | −3.0 | −15 |
| Clothing and footwear | 1232 | 3.9 | 6.5 | 76 |
| Housing, water, electricity, gas | 10243 | 32.4 | 3.5 | 350 |
| Furnishings and maintenance | 1296 | 4.1 | 0.8 | 10 |
| Health | 1228 | 3.9 | 2.1 | 25 |
| Transport | 3794 | 12.0 | 17.5 | 564 |
| Communications | 925 | 2.9 | −1.3 | −12 |
| Recreation and culture | 1534 | 4.9 | 18.0 | 241 |
| Education | 468 | 1.5 | 6.4 | 29 |
| Restaurants and hotels | 2953 | 9.4 | 29.1 | 665 |
| Miscellaneous goods and services | 2364 | 7.5 | 7.5 | 148 |

Table 2: Spend-Based Emission Factors (EXIOBASE via Climatiq.io)

| Category | Emission Factor (kg $CO_2$e/€) |
|---|---|
| Housing, water, electricity, gas | 0.30 |
| Food and non-alcoholic beverages | 0.48 |
| Transport | 0.40 |
| Other goods and services | 0.18 |
| Recreation and culture | 0.20 |
| Restaurants and hotels | 0.45 |
| Furnishings and household equipment | 0.25 |
| Health | 0.20 |
| Alcoholic beverages and tobacco | 0.42 |
| Clothing and footwear | 0.25 |
| Communications | 0.15 |
| Education | 0.15 |

Table 3: Household expenditure and carbon footprint by category for France (2021, Eurostat).

| Category | EF (kg $CO_2$/€) | France (%) | France (€ bn) | Emissions (Mt $CO_2$e) |
|---|---|---|---|---|
| Housing, water, electricity, gas | 0.30 | 27.6 | 364.9 | 109.5 |
| Food + non-alcoholic beverages | 0.48 | 13.9 | 183.8 | 88.2 |
| Transport | 0.40 | 12.6 | 166.6 | 66.6 |
| Other goods + services | 0.18 | 12.5 | 165.3 | 29.8 |
| Recreation + culture | 0.20 | 7.7 | 101.8 | 20.4 |
| Restaurants + hotels | 0.45 | 6.2 | 82.8 | 37.3 |
| Furnishings + household equipment | 0.25 | 4.9 | 64.8 | 16.2 |
| Health | 0.20 | 4.2 | 55.5 | 11.1 |
| Alcohol + tobacco | 0.42 | 4.1 | 54.2 | 22.8 |
| Clothing + footwear | 0.25 | 3.3 | 43.6 | 10.9 |
| Communications | 0.15 | 2.5 | 33.1 | 5.0 |
| Education | 0.15 | 0.5 | 6.6 | 1.0 |
| **TOTAL** | | 100.0 | 1323.0 | 419.5 |

Table 4: Household expenditure and carbon footprint by category for Spain (2021, Eurostat).

| Category | EF (kg $CO_2$/€) | Spain (%) | Spain (€ bn) | Emissions (Mt $CO_2$e) |
|---|---|---|---|---|
| Housing, water, electricity, gas | 0.30 | 24.30 | 168.0 | 50.4 |
| Food + non-alcoholic beverages | 0.48 | 14.20 | 98.1 | 47.1 |
| Transport | 0.40 | 11.00 | 76.0 | 30.4 |
| Other goods + services | 0.18 | 10.30 | 71.2 | 12.8 |
| Recreation + culture | 0.20 | 6.60 | 45.6 | 9.1 |
| Restaurants + hotels | 0.45 | 12.00 | 82.9 | 37.3 |
| Furnishings + household equipment | 0.25 | 4.90 | 33.9 | 8.5 |
| Health | 0.20 | 4.40 | 30.4 | 6.1 |
| Alcohol + tobacco | 0.42 | 4.40 | 30.4 | 12.8 |
| Clothing + footwear | 0.25 | 3.50 | 24.2 | 6.0 |
| Communications | 0.15 | 2.70 | 18.7 | 2.8 |
| Education | 0.15 | 1.40 | 9.7 | 1.5 |
| **TOTAL** | | 100.0 | 689.1 | 226.8 |

Table 5: Household expenditure and carbon footprint by category for Germany (2021, Eurostat).

| Category | EF (kg $CO_2$/€) | Germany (%) | Germany (€ bn) | Emissions (Mt $CO_2$e) |
|---|---|---|---|---|
| Housing, water, electricity, gas | 0.30 | 25.50 | 457.7 | 137.3 |
| Food + non-alcoholic beverages | 0.48 | 11.70 | 209.9 | 100.7 |
| Transport | 0.40 | 13.10 | 235.2 | 94.1 |
| Other goods + services | 0.18 | 13.10 | 235.2 | 42.3 |
| Recreation + culture | 0.20 | 9.50 | 170.5 | 34.1 |
| Restaurants + hotels | 0.45 | 4.00 | 71.8 | 32.3 |
| Furnishings + household equipment | 0.25 | 7.00 | 125.7 | 31.4 |
| Health | 0.20 | 5.60 | 100.5 | 20.1 |
| Alcohol + tobacco | 0.42 | 3.60 | 64.6 | 27.1 |
| Clothing + footwear | 0.25 | 3.80 | 68.2 | 17.1 |
| Communications | 0.15 | 2.30 | 41.1 | 6.2 |
| Education | 0.15 | 0.80 | 14.4 | 2.2 |
| **TOTAL** | | 100.0 | 1794.8 | 545.9 |

# Appendix B: Derivations

## B.1 Stability of the Leontief Inverse

To illustrate the condition under which the Leontief inverse exists, we consider two hypothetical technical coefficient matrices. The stability of each system is assessed based on the spectral radius $\rho(\mathbf{A})$.

**Stable system:**

$$\mathbf{A}_{\text{stable}} = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix} \quad \Rightarrow \quad \rho(\mathbf{A}) = 0.5 < 1$$

**Unstable system:**

$$\mathbf{A}_{\text{unstable}} = \begin{bmatrix} 0.6 & 0.7 \\ 0.8 & 0.9 \end{bmatrix} \quad \Rightarrow \quad \rho(\mathbf{A}) \approx 1.51 > 1$$

Only the first system satisfies the condition $\rho(\mathbf{A}) < 1$, which ensures that the series $(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$ converges. A spectral radius above 1 implies that the system is not productive and the total output requirement diverges.

## B.2 Emission Multiplier Computation in EEIO

To demonstrate the computation of supply chain emissions in EEIO models, we use a simplified 3-sector structure based on EXIOBASE-style values for Agriculture, Manufacturing, and Services.

**Technical Coefficient Matrix:**

$$\mathbf{A} = \begin{bmatrix} 0.10 & 0.05 & 0.02 \\ 0.20 & 0.15 & 0.10 \\ 0.05 & 0.10 & 0.10 \end{bmatrix}$$

**Leontief Inverse:**

$$(\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} 1.12 & 0.11 & 0.04 \\ 0.29 & 1.19 & 0.17 \\ 0.07 & 0.19 & 1.13 \end{bmatrix}$$

**Emission Intensities (kg $CO_2e$/€):**

$$\mathbf{C} = \begin{bmatrix} 0.45 & 0.30 & 0.20 \end{bmatrix}$$

**Emission Multipliers:**

$$\mathbf{C}(\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} 0.609 & 0.422 & 0.283 \end{bmatrix}$$

These values represent the total cradle-to-gate carbon footprint induced by one euro of final demand in each sector. For instance, €1 spent on agricultural products results in approximately 0.609 kg of $CO_2e$ emissions when accounting for all upstream effects. This methodology reflects standard EXIOBASE and Climatiq practices for spend-based carbon accounting.