## Statistical Learning (5454) - Assignment 1

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Due: 2024-03-25

## Exercise 1

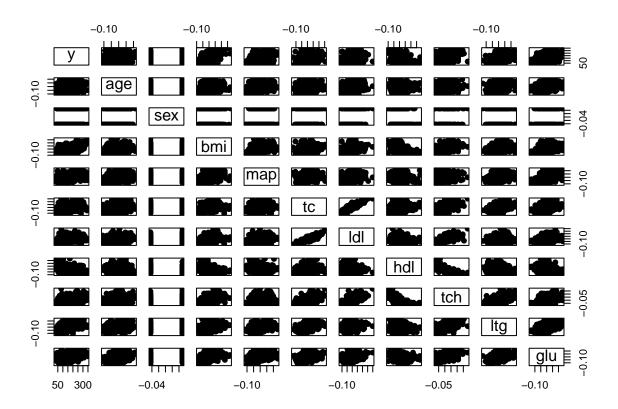
```
##
                                                                     bmi
                           age
                                                sex
##
    Min.
           : 25.0
                     Min.
                             :-0.107226
                                           Min.
                                                  :-0.04464
                                                               Min.
                                                                       :-0.090275
##
    1st Qu.: 87.0
                     1st Qu.:-0.037299
                                           1st Qu.:-0.04464
                                                               1st Qu.:-0.034229
    Median :140.5
                     Median: 0.005383
                                           Median :-0.04464
                                                               Median :-0.007284
                                                  : 0.00000
##
    Mean
            :152.1
                             : 0.000000
                                                                       : 0.000000
                     Mean
                                           Mean
                                                               Mean
##
    3rd Qu.:211.5
                     3rd Qu.: 0.038076
                                           3rd Qu.: 0.05068
                                                               3rd Qu.: 0.031248
##
    Max.
            :346.0
                     Max.
                             : 0.110727
                                           Max.
                                                  : 0.05068
                                                               Max.
                                                                       : 0.170555
##
                                                     ldl
         map
                                tc
##
    Min.
            :-0.112400
                         Min.
                                 :-0.126781
                                               Min.
                                                       :-0.115613
##
    1st Qu.:-0.036656
                         1st Qu.:-0.034248
                                               1st Qu.:-0.030358
    Median :-0.005671
                         Median :-0.004321
                                               Median :-0.003819
##
           : 0.000000
                                 : 0.000000
                                               Mean
                                                       : 0.000000
    Mean
                         Mean
    3rd Qu.: 0.035644
                                               3rd Qu.: 0.029844
                          3rd Qu.: 0.028358
##
##
    Max.
           : 0.132044
                         Max.
                                 : 0.153914
                                               Max.
                                                       : 0.198788
##
         hdl
                               tch
                                                    ltg
##
    Min.
            :-0.102307
                         Min.
                                 :-0.076395
                                               Min.
                                                       :-0.126097
    1st Qu.:-0.035117
                          1st Qu.:-0.039493
                                               1st Qu.:-0.033249
##
##
    Median :-0.006584
                         Median :-0.002592
                                               Median :-0.001948
                                 : 0.000000
##
            : 0.000000
                                               Mean
                                                       : 0.000000
    3rd Qu.: 0.029312
                          3rd Qu.: 0.034309
                                               3rd Qu.: 0.032433
##
##
    Max.
            : 0.181179
                                 : 0.185234
                                               Max.
                                                       : 0.133599
##
         glu
##
            :-0.137767
##
    1st Qu.:-0.033179
    Median :-0.001078
    Mean
           : 0.000000
    3rd Qu.: 0.027917
##
    {\tt Max.}
            : 0.135612
```

• Explanation: Random selection

Table 1: Correlation Matrix

	У	age	sex	bmi	map	tc	ldl	hdl	tch	ltg	glu
у	1.00	0.19	0.04	0.59	0.44	0.21	0.17	-0.39	0.43	0.57	0.38
age	0.19	1.00	0.17	0.19	0.34	0.26	0.22	-0.08	0.20	0.27	0.30
sex	0.04	0.17	1.00	0.09	0.24	0.04	0.14	-0.38	0.33	0.15	0.21
bmi	0.59	0.19	0.09	1.00	0.40	0.25	0.26	-0.37	0.41	0.45	0.39
map	0.44	0.34	0.24	0.40	1.00	0.24	0.19	-0.18	0.26	0.39	0.39
tc	0.21	0.26	0.04	0.25	0.24	1.00	0.90	0.05	0.54	0.52	0.33

	У	age	sex	bmi	map	tc	ldl	hdl	tch	ltg	glu
ldl	0.17	0.22	0.14	0.26	0.19	0.90	1.00	-0.20	0.66	0.32	0.29
hdl	-0.39	-0.08	-0.38	-0.37	-0.18	0.05	-0.20	1.00	-0.74	-0.40	-0.27
$\operatorname{tch}$	0.43	0.20	0.33	0.41	0.26	0.54	0.66	-0.74	1.00	0.62	0.42
ltg	0.57	0.27	0.15	0.45	0.39	0.52	0.32	-0.40	0.62	1.00	0.46
glu	0.38	0.30	0.21	0.39	0.39	0.33	0.29	-0.27	0.42	0.46	1.00



- Explanation standardized - Interpret correlation

```
##
## Call:
## lm(formula = y ~ ., data = train)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
                                            Max
## -154.436 -37.748
                       -1.375
                                37.421 153.466
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 152.706
                             2.711 56.319 < 2e-16 ***
                  9.856
                            62.721
                                     0.157 0.875213
## age
## sex
               -240.347
                            64.936
                                    -3.701 0.000245 ***
                499.266
                            70.415
                                     7.090 6.35e-12 ***
## bmi
## map
                354.976
                            70.187
                                     5.058 6.55e-07 ***
                           436.264
               -861.163
                                    -1.974 0.049095 *
## tc
## ldl
               541.190
                           354.923
                                    1.525 0.128119
```

```
## hdl
               116.045
                           221.425
                                     0.524 0.600518
                          166.601
## tch
                                     0.999 0.318178
               166.516
               773.896
                           179.728
## ltg
                                     4.306 2.11e-05 ***
                                     0.925 0.355729
                63.631
                            68.817
## glu
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 54.18 on 389 degrees of freedom
## Multiple R-squared: 0.5258, Adjusted R-squared: 0.5136
## F-statistic: 43.13 on 10 and 389 DF, p-value: < 2.2e-16
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 152.71
                             2.71
                                     56.32
                                             <2e-16 ***
                   9.86
                            62.72
                                     0.16
                                              0.88
## age
## sex
               -240.35
                            64.94
                                     -3.70
                                             <2e-16 ***
                499.27
                                     7.09
                                             <2e-16 ***
## bmi
                            70.41
## map
                354.98
                            70.19
                                     5.06
                                            <2e-16 ***
## tc
               -861.16
                            436.26
                                    -1.97
                                              0.05 *
## ldl
                541.19
                            354.92
                                     1.52
                                              0.13
                                     0.52
## hdl
                116.05
                           221.42
                                              0.60
## tch
                166.52
                           166.60
                                     1.00
                                              0.32
                                     4.31
## ltg
                773.90
                           179.73
                                            <2e-16 ***
## glu
                 63.63
                            68.82
                                     0.92
                                              0.36
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## [1] 2854.869
## [1] 2945.384
##
## Call:
## lm(formula = y ~ sex + bmi + map + tc + ltg, data = train)
##
## Residuals:
##
       Min
                  1Q
                      Median
                                    3Q
                                            Max
## -154.487 -39.583
                       -2.167
                               36.677 143.460
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 152.676
                            2.744 55.634 < 2e-16 ***
                            60.008 -2.393 0.01716 *
## sex
              -143.624
## bmi
                            67.332
                                    8.621 < 2e-16 ***
               580.467
                                     5.067 6.23e-07 ***
               344.751
                            68.041
## map
                            67.313 -3.243 0.00128 **
## tc
               -218.311
## ltg
               657.293
                           75.344
                                    8.724 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 54.85 on 394 degrees of freedom
## Multiple R-squared: 0.5077, Adjusted R-squared: 0.5014
## F-statistic: 81.26 on 5 and 394 DF, p-value: < 2.2e-16
              Estimate Std. Error t value Pr(>|t|)
                                     55.63
## (Intercept)
                152.68
                              2.74
                                             <2e-16 ***
                                     -2.39
                                              0.02 *
## sex
               -143.62
                             60.01
```

```
580.47
## bmi
                          67.33
                                   8.62
                                         <2e-16 ***
## map
                                  5.07
               344.75
                          68.04
                                         <2e-16 ***
## tc
              -218.31
                          67.31
                                  -3.24
                                         <2e-16 ***
               657.29
## ltg
                          75.34
                                  8.72
                                         <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## [1] 2963.644
## [1] 3022.301
## Analysis of Variance Table
## Model 1: y ~ age + sex + bmi + map + tc + ldl + hdl + tch + ltg + glu
## Model 2: y ~ sex + bmi + map + tc + ltg
## Res.Df
             RSS Df Sum of Sq F Pr(>F)
## 1
       389 1141947
## 2
       394 1185458 -5
                      -43510 2.9643 0.01221 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Start: AIC=3204.71
## y ~ age + sex + bmi + map + tc + ldl + hdl + tch + ltg + glu
##
         Df Sum of Sq
                       RSS
                               AIC
                72 1142020 3202.7
## - age
         1
                806 1142754 3203.0
## - hdl 1
## - glu 1
               2510 1144457 3203.6
## - tch 1
                2933 1144880 3203.7
                     1141947 3204.7
## <none>
## - ldl 1
              6825 1148773 3205.1
            11438 1153386 3206.7
## - tc 1
             40216 1182164 3216.6
## - sex 1
## - ltg 1
            54429 1196377 3221.3
## - map
        1
              75090 1217038 3228.2
## - bmi
            147581 1289529 3251.3
        1
## Step: AIC=3202.74
## y ~ sex + bmi + map + tc + ldl + hdl + tch + ltg + glu
##
##
         Df Sum of Sq
                       RSS
## - hdl 1
                824 1142844 3201.0
## - glu 1
                2656 1144676 3201.7
## - tch 1
                2916 1144936 3201.8
                     1142020 3202.7
## <none>
## - ldl 1
              6890 1148910 3203.1
            11478 1153497 3204.7
## - tc 1
             40274 1182294 3214.6
## - sex 1
## - ltg 1
               54900 1196920 3219.5
## - map
        1
              79224 1221244 3227.6
## - bmi
        1
            147570 1289590 3249.3
##
## Step: AIC=3201.03
## y ~ sex + bmi + map + tc + ldl + tch + ltg + glu
##
##
        Df Sum of Sq RSS
                               AIC
```

```
2185 1145029 3199.8
## - tch
          1
## - glu 1
                2705 1145549 3200.0
## <none>
                    1142844 3201.0
## - ldl
                8808 1151653 3202.1
          1
## - tc
          1
               27555 1170400 3208.6
## - sex 1
              40811 1183656 3213.1
## - map
              78720 1221564 3225.7
          1
## - ltg
              92523 1235368 3230.2
          1
## - bmi
          1
              147071 1289915 3247.4
##
## Step: AIC=3199.79
## y ~ sex + bmi + map + tc + ldl + ltg + glu
##
         Df Sum of Sq
                         RSS
                                AIC
## - glu
                3071 1148100 3198.9
## <none>
                1145029 3199.8
## - ldl 1
               36551 1181580 3210.4
## - sex 1
              39159 1184188 3211.2
## - tc
              61374 1206403 3218.7
          1
## - map
         1
               76944 1221973 3223.8
## - bmi
        1
             146794 1291823 3246.0
## - ltg
          1
              239636 1384665 3273.8
##
## Step: AIC=3198.86
## y ~ sex + bmi + map + tc + ldl + ltg
##
         Df Sum of Sq
                       RSS
                                AIC
             1148100 3198.9
## <none>
## - sex 1
               37042 1185142 3209.6
## - ldl 1
              37358 1185458 3209.7
## - tc
          1
              61253 1209352 3217.7
## - map
          1
              84790 1232890 3225.4
## - bmi
        1 158343 1306443 3248.5
## - ltg
          1 262231 1410331 3279.1
##
## Call:
## lm(formula = y ~ sex + bmi + map + tc + ldl + ltg, data = train)
##
## Residuals:
## Min
                1Q Median
                                 3Q
## -157.214 -38.027 -2.143 36.163 149.530
##
## Coefficients:
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 152.723
                          2.704 56.477 < 2e-16 ***
## sex
             -225.947
                          63.453 -3.561 0.000415 ***
                         69.234
                                 7.362 1.07e-12 ***
## bmi
             509.713
## map
              362.152
                        67.222
                                  5.387 1.23e-07 ***
             -775.933
                         169.455 -4.579 6.28e-06 ***
## tc
## ldl
              554.531
                         155.071
                                   3.576 0.000392 ***
                        84.993
## ltg
             805.250
                                 9.474 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 54.05 on 393 degrees of freedom
## Multiple R-squared: 0.5232, Adjusted R-squared: 0.5159
## F-statistic: 71.88 on 6 and 393 DF, p-value: < 2.2e-16
##
            Estimate Std. Error t value Pr(>|t|)
                              56.48 < 2.2e-16 ***
## (Intercept)
             152.72
                        2.70
## sex
             -225.95
                        63.45
                              -3.56 < 2.2e-16 ***
## bmi
              509.71
                        69.23
                               7.36 < 2.2e-16 ***
                       67.22
                               5.39 < 2.2e-16 ***
## map
              362.15
## tc
             -775.93
                       169.46
                              -4.58 < 2.2e-16 ***
              554.53
                       155.07
                              3.58 < 2.2e-16 ***
## ldl
## ltg
              805.25
                       84.99
                               9.47 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## [1] 2870.25
## [1] 2966.798
## Analysis of Variance Table
## Model 1: y ~ age + sex + bmi + map + tc + ldl + hdl + tch + ltg + glu
## Model 2: y ~ sex + bmi + map + tc + ldl + ltg
   Res.Df
             RSS Df Sum of Sq
                               F Pr(>F)
## 1
      389 1141947
      393 1148100 -4 -6152.4 0.5239 0.7182
## 2
## Subset selection object
## Call: regsubsets.formula(y ~ ., data = train, nvmax = 9, really.big = TRUE)
## 10 Variables (and intercept)
     Forced in Forced out
         FALSE
                  FALSE
## age
## sex
         FALSE
                  FALSE
         FALSE
                  FALSE
## bmi
         FALSE
                  FALSE
## map
## tc
         FALSE
                  FALSE
## ldl
         FALSE
                  FALSE
## hdl
        FALSE
                  FALSE
                  FALSE
## tch
         FALSE
         FALSE
                  FALSE
## ltg
## glu
         FALSE
                  FALSE
## 1 subsets of each size up to 9
## Selection Algorithm: exhaustive
##
         age sex bmi map to ldl hdl tch ltg glu
    ## 1
## 3 (1) " " " " * " * " " " " " " " " " "
    ## 7 (1) " "*" "*" "*" "*" "*" "*" "*"
    ## 9 (1) " "*" "*" "*" "*" "*" "*" "*" "*"
```

```
SSB 2 4 6 8 Subset size
```

```
Adj.R2 BIC AIC
## 1
          7
              6
##
## Call:
## lm(formula = select_model(5, lm_subset, "Y"), data = train)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     ЗQ
                                             Max
## -148.699 -38.009
                       -0.413
                                36.673 148.969
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
                 152.69
                              2.72 56.131 < 2e-16 ***
## (Intercept)
                -233.35
                             63.90
                                    -3.652 0.000295 ***
## sex
                                     7.344 1.20e-12 ***
## bmi
                 506.03
                             68.90
                             67.63
                                     5.308 1.86e-07 ***
## map
                 358.97
## hdl
                -289.95
                             68.92
                                    -4.207 3.21e-05 ***
## ltg
                 467.58
                             68.89
                                     6.787 4.22e-11 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 54.37 on 394 degrees of freedom
## Multiple R-squared: 0.5163, Adjusted R-squared: 0.5101
## F-statistic: 84.1 on 5 and 394 DF, p-value: < 2.2e-16
## [1] 2911.967
```

```
## [1] 2956.157

## Analysis of Variance Table
##

## Model 1: y ~ age + sex + bmi + map + tc + ldl + hdl + tch + ltg + glu
## Model 2: y ~ sex + bmi + map + hdl + ltg
## Res.Df RSS Df Sum of Sq F Pr(>F)
## 1 389 1141947
## 2 394 1164787 -5 -22839 1.556 0.1716

## No id variables; using all as measure variables
## No id variables; using all as measure variables
## No id variables; using all as measure variables
## No id variables; using all as measure variables
## No id variables; using all as measure variables
```

Table 2: Results all models

	full	$\operatorname{small}$	stepwise	subset
X.Intercept.	152.71	152.68	152.72	152.69
age	9.86	NA	NA	NA
sex	-240.35	-143.62	-225.95	-233.36
bmi	499.27	580.47	509.71	506.03
map	354.98	344.75	362.15	358.97
tc	-861.16	-218.31	-775.93	NA
ldl	541.19	NA	554.53	NA
hdl	116.05	NA	NA	-289.96
tch	166.52	NA	NA	NA
ltg	773.90	657.29	805.25	467.58
glu	63.63	NA	NA	NA
MSE in sample	2854.87	2963.64	2870.25	2911.97
MSE out of sample	2945.38	3022.30	2966.80	2956.16

- Exercise 2
- Exercise 3
- Exercise 4
- Exercise 5