Project 1: Martingale

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Experiment 1

1. Estimate the probability of winning $80 within 1000 sequential bets

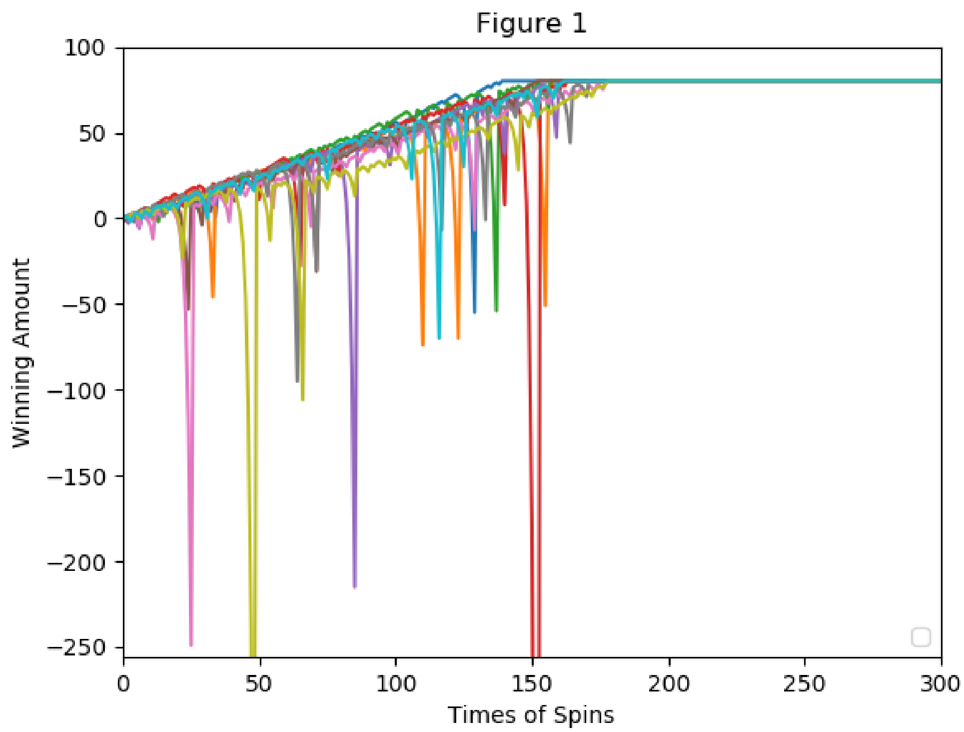

Figure 1. The winning amount of each spin when running simple simulator for 10 times.

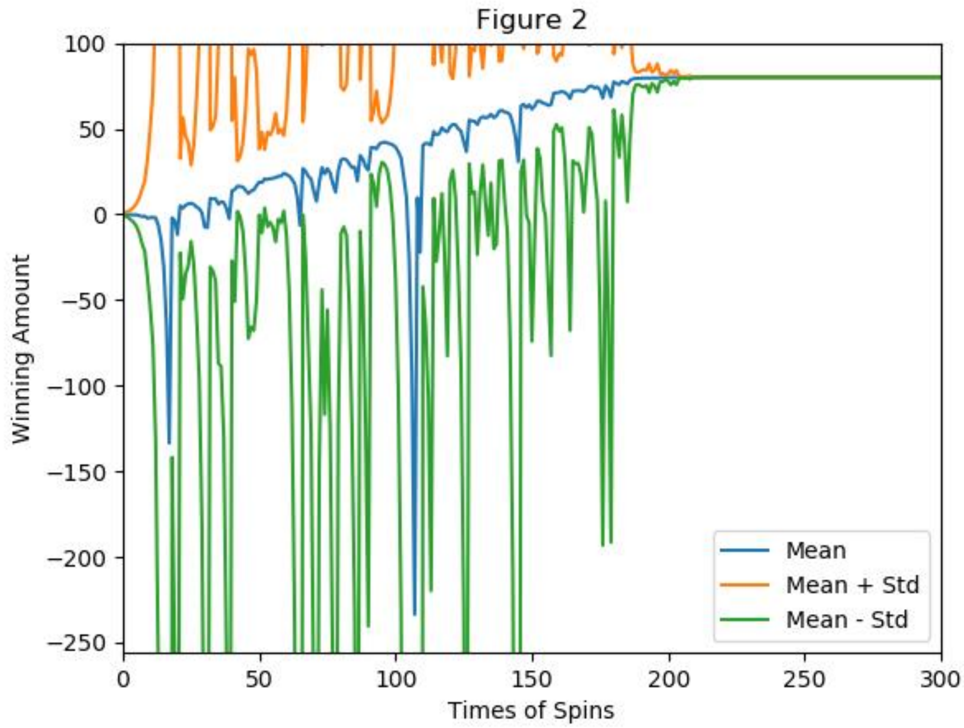

Figure 2. The mean value of winning amount with standard deviation of each spin when running simple simulator for 1000 times.

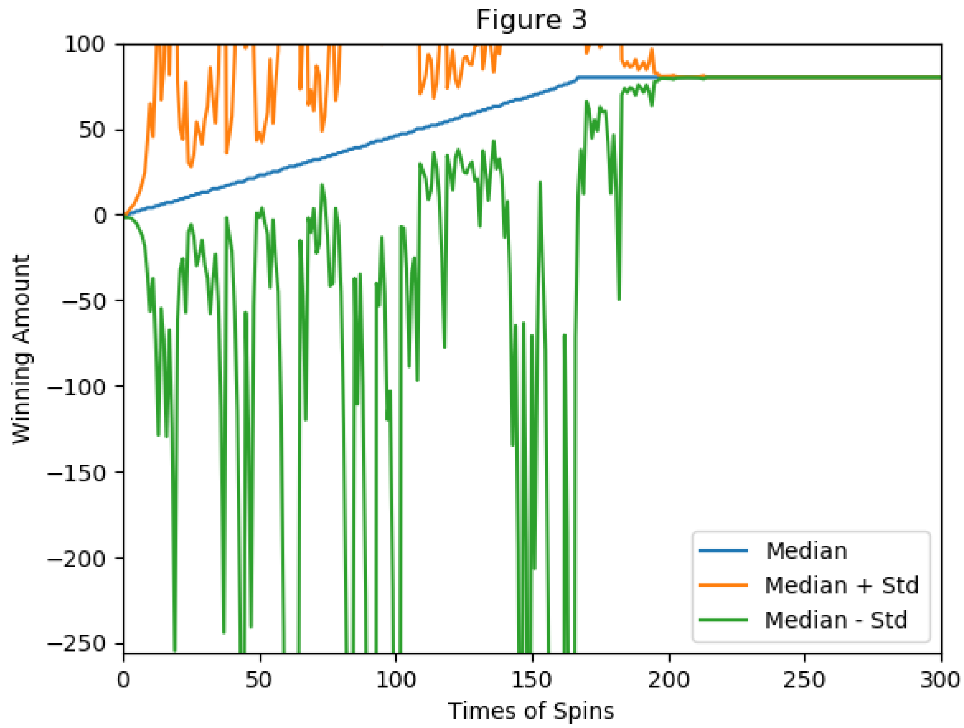

Figure 3. The median value of winning amount with standard deviation of each spin when running simple simulator for 1000 times.

As shown in Figure 1, in the 10 runs, the winning amount reaches $80 every time at around 175 spins. According to the information provided by Figure 2 and 3, after 1000 runs, both mean and median value of winning arrive at $80 in less than 250 spins, with the standard deviation approaching 0.

To sum up, within 1000 sequential bets, we could estimate that the probability of winning $80 is 100% if we have unlimited money.

2. What is the estimated expected value of our winnings after 1000 sequential bets?

The expected value of a random variable is the long-run average value of repetitions of the same experiment it represents ("Expected value", 2019). In this case, the average value is shown as mean in Figure 2. We could see that after 250 spins, it stabilizes at $80.

Hence, it should be $80.

3. Does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases?

As shown in Figure 2 and figure 3, the standard deviation vibrates initially with different vale. But as the number of sequential bets increases to more than 250 times, mean+standard deviation and mean-standard deviation both come to $80, which means that standard deviation converges to $0. With unlimited money, the balance of win and loss could be reached after 1000 times, despite the final winning.

Experiment 2

1. Estimate the probability of winning $80 within 1000 sequential bets

By calculating the total of sessions when the final winning reaches $80 in 1000 situation, I estimated that this probability should be around 64.1%.

2. What is the estimated expected value of our winnings after 1000 sequential bets?

By calculating the mean of second simulator, I found that when the final winning reaches $80 in 1000 situation, the estimated expected value should be around

-$44.0. This number also corresponds to the figure 4.

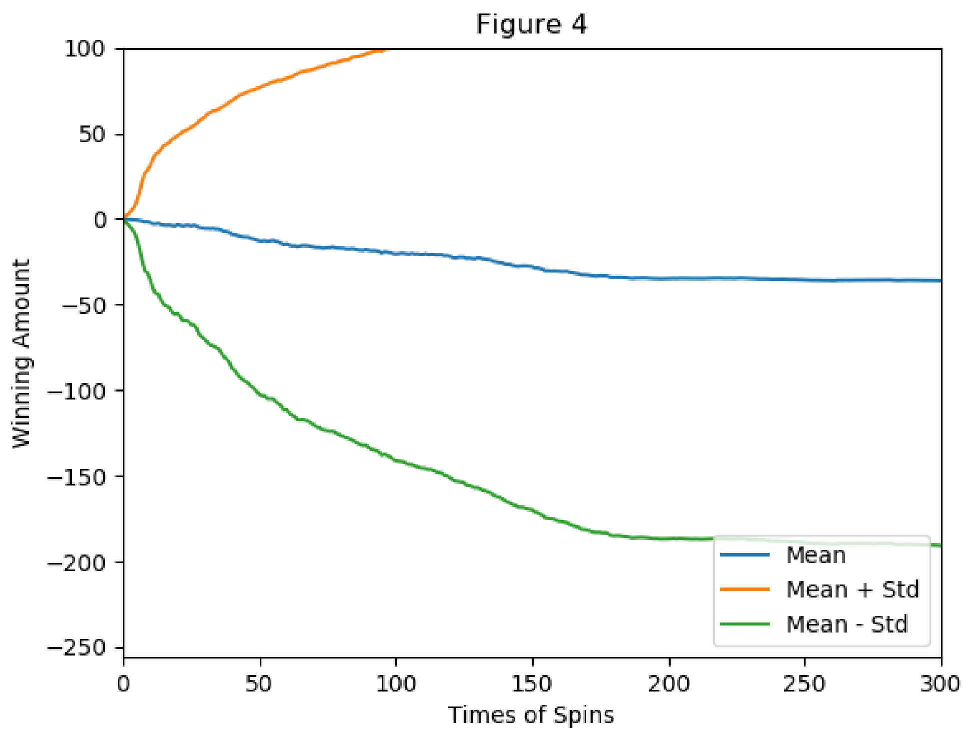

Figure 4. The mean value of winning amount with standard deviation of each spin when running a more realistic simulator for 1000 times.

3. Does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases?

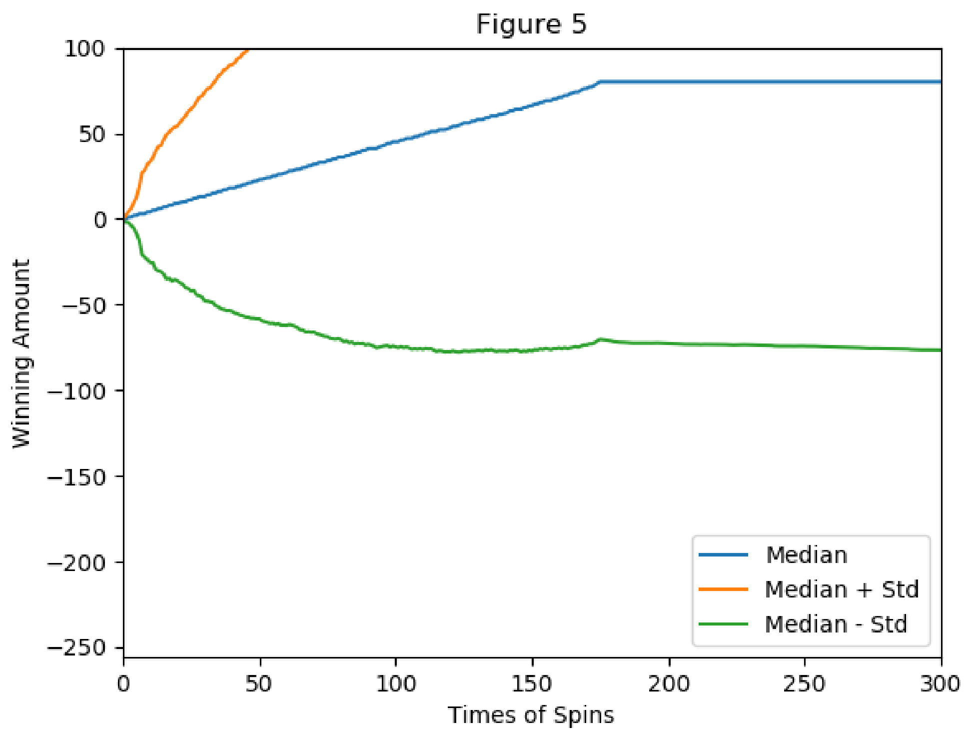

Figure 5. The median value of winning amount with standard deviation of each spin when running a more realistic simulator for 1000 times.

After calculating the standard deviation that are close to 1000 times, it could be found that the standard deviation stabilizes at $161.2. With limited money, the gamblers are very likely to lose all their money in earlier runs. Hence, they would have to give up the game due to bankrupt and fail to see the final winning.

Reference

Expected value. (2019). Retrieved from https://en.wikipedia.org/wiki/Expected_value